Blind Carrier Frequency Offset Estimation for Noncircular Constellation-Based Transmissions

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***Abstract*—We address the problem of nondata aided frequency offset estimation for noncircular transmissions over frequency-selective channels in a linear precoder-based communications context. Linear precoding is representative of a downlink direct-sequence code division multiple access (DS-CDMA) system or of an orthogonal frequency division multiplexing (OFDM) system. We observe that twice the frequency offset is a cyclic frequency of the received signal. We thus introduce an estimator relying on the maximization of the empirical cyclocorrelations. We analyze its asymptotic behavior and obtain a closed-form expression for the asymptotic covariance. This enables us to design relevant system parameters. Numerical illustrations are provided and confirm our theoretical assertions. Comparisons with existing methods are also reported.**

***Index Terms*—Blind estimation, carrier frequency offset, cyclic frequency, DS-CDMA, fading channel, harmonic retrieval, noncircular constellation, OFDM, synchronization.**

I. Introduction

FOR several years, the orthogonal frequency division multiplexing (OFDM) based multicarrier communication systems and the direct-sequence code division multiple access (DS-CDMA) based multiuser communication systems have received increasing attention. For instance, the OFDM technique is employed in the European digital broadcast radio system and will be used in the future indoor and local area networks. The third-generation mobile communication network is based on the DS-CDMA technique. Although these two systems have been developed separately to solve different problems, it is possible to treat them in a similar way. Indeed, these two systems rely on a transformation of the transmitted symbols by means of a linear mapping. Therefore, these systems can be described with a general formalism based on linear precoders (see [1] and [2]). In this paper, the theoretical derivations are developed for any linear precoder. Regarding the numerical simulations, we only concentrate on either a DS-CDMA or an OFDM system. We are aware that other linear precoder-based techniques (like transmitter induced cyclostationarity [2] or fractional sampling [3]) are also of interest. Since these techniques are not widely used and only concern a single-user and single-carrier communication scheme, we do not deeply focus on them. Nevertheless, the numerical study of these techniques can be laid in further papers [4].

The transmitted signal that has propagated over the channel is mainly disturbed by two kinds of impairments: on the one hand, the frequency selectivity and, on the other hand, the carrier frequency offset. This offset corresponds to a mismatch between the receive and transmit carrier frequencies and may be caused by a Doppler effect or a local oscillator drift.

For a DS-CDMA system, a carrier frequency offset leads to an additional[1] loss of orthogonality between the different users and affects the performance. In an OFDM system, the presence of a carrier frequency offset is more critical [5]. Therefore, extensive literature exists on this topic. More precisely, intercarrier interference (ICI) appears after the discrete Fourier transform. This ICI prevents the equalizer from distinguishing between the effects caused by the channel and the frequency offset [6]. In order to overcome this drawback, we have to counteract the presence of the carrier frequency offset before any other processing (namely, before the compensation of the channel time dispersion). Thus, it is relevant to estimate the carrier frequency offset directly from the received signal that is still corrupted by the channel.

Sending a known training sequence leads traditionally to accurate estimates of the carrier frequency offset and of the filter corresponding to the channel. Nevertheless, this method reduces the effective transmission rate. It also cannot be achieved in some applications (e.g., passive listening in a military context). Therefore, we only focus on nondata-aided (NDA) estimation methods.

Several works have been reported about this problem. For a DS-CDMA system, two methods have recently been developed. A subspace-based estimator has been introduced in [7]. The estimator only makes sense if the spreading factor is larger than the channel length while the number of users is bounded and cannot reach the spreading factor. An ESPRIT-like estimator is described in [8]. Unfortunately, this estimator can only be applied if the filter is not time-selective relative to the chip duration. For an OFDM system, several methods have been developed. Under the strong assumption of the absence of dispersive channel, at least two powerful estimators have been proposed: i) a maximum likelihood-based estimator described in [9] as well as ii) an estimator relying on an angle of a correlation product introduced in [10]. In the presence of a frequency-selective fading channel, we can mention the following estima-

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[1]if the channel is dispersive

tors: Subspace based estimators have been studied in [11] and [12]. An ESPRIT-like estimator has been introduced in [13], and its associated adaptive version is carried out in [14]. A deterministic maximum-likelihood based estimator has been proposed in [15]. Nevertheless, all estimators require the presence of virtual sub-carriers. Although such an assumption prevents the full load of the systems, virtual sub-carriers are encountered in practice. Indeed, in IEEE 802.11 or the HIPERLAN2 standard, the conditions on out-of-band interference lead to unused the band borders. Furthermore, the optimal localization of the null subcarriers is discussed in [14]–[16]. In [17], the deterministic maximum-likelihood approach is developed in the case where the constellation is constant modulus, e.g., phase-shift keying (PSK). It is proved that null subcarriers are not required and that the system can be then fully loaded.

So far, besides these restrictive assumptions, the above-mentioned estimators only consider the specific cyclic-prefix OFDM scheme. For instance, no estimator has been developed for a zero-padded OFDM scheme described in [18], except in the following current work [19].

A nondata aided estimator has been recently introduced in [20] for a single-carrier and single-user transmission context. It is designed for noncircular transmissions, i.e., the symbol constellation is assumed to be noncircular, which means that the mathematical expectation of the square symbol is not equal to zero. Notice that new noncircular constellations have been also recently built in [21] for improving phase estimation and for avoiding differentiel encoding. A noncircular process $\{p_n\}_{n\in\mathbb{Z}}$ admits a so-called conjugate correlation function $(n,\tau)\mapsto r_c(n,\tau)$, which is the correlation between the process and its shifted versions, namely, $r_c(n,\tau) = \mathbb{E}[p(n+\tau)p(n)]$. In [20], the authors have noticed that if the symbol constellation is noncircular, then twice the carrier frequency offset is the sole cyclic frequency of the baud-rate sampled received signal associated with its conjugate correlation function. Thus, the estimator is provided by maximizing a weighted sum of conjugate cyclocorrelations in the cyclic frequency domain. The asymptotic behavior of its estimate has also been analyzed in [20]. The analysis has shown that the performance is almost independent of the frequency-selective channel if the number of cyclocorrelation coefficients taken into account is large enough.

Our purpose is to extend such an approach to any linearly precoded noncircular transmission scheme, especially to multi carrier (OFDM) and multiuser (downlink DS-CDMA) noncircular communications.

The paper is organized as follows: In Section II, we briefly review the transmission using linear precoders. More precisely, we mention the general formalism putting together the cyclic-prefix OFDM, zero-padded OFDM, and downlink DS-CDMA systems in a unique framework. In Section III, we express the new estimator in the unifying framework. In Section IV, we analyze the asymptotic behavior of the estimator. We obtain a closed-form expression for the asymptotic covariance. In Section V, we analyze the influence of design parameters. For a DS-CDMA system, we conclude that the performance of the well-designed proposed estimator is almost insensitive to the dispersive channel. This property holds if each user code and the symbol constellation are real valued. For an OFDM system, we show that the system can be fully loaded and that the null subcarrier is not required. Section VI is devoted to numerical illustrations and comparisons.

II. Problem Statement

At first, we consider the single-carrier and single-user communications framework. A linearly modulated symbol sequence, which is denoted by $\{s_n\}_{n\in\mathbb{Z}}$, is transmitted. This sequence is assumed to be zero-mean, independent and identically distributed (i.i.d.), noncircular, and unit-variance. The transmitted signal passes through a multipath propagation channel. Hence, the continuous-time baseband received signal $y_a(t)$ can be written as

$$y_a(t) = \left(\sum_{n\in\mathbb{Z}} s_n h_a(t-nT_s)\right) e^{2i\pi\delta f_0 t} + w_a(t) \qquad (1)$$

where T_s and δf_0 are the symbol period and the carrier frequency offset, respectively. The unknown filter $h_a(t)$ results from the convolution of the shaping filter and the propagation multipath channel. Without loss of generality, one can assume that the mapping $t\mapsto h_a(t)$ is time limited and causal with time support $[0,T_h]$. Finally, $w_a(t)$ denotes an additive zero-mean circular Gaussian noise.

As the transmitter is equipped with a linear precoder, it sends the so-called "pseudo-symbol" sequence $\{v_n\}_{n\in\mathbb{Z}}$ at the baud rate $1/T_v$ instead of the usual symbol sequence $\{s_n\}_{n\in\mathbb{Z}}$ at baud rate $1/T_s$ [1], [2]. We consider the integers P and Q such that $P\leq Q$. The "pseudo-symbol" sequence is defined as follows:

$$V_Q(n) = \mathbf{K}S_P(n) \qquad (2)$$

with $V_Q(n) = [v_{nQ},\ldots,v_{nQ+Q-1}]^T$. $S_P(n)$ can be expressed in a similar way [2]. Moreover, the "pseudo-symbol" baud rate $1/T_v$ is equal to Q/PT_s. Matrix $\mathbf{K}$ is tall and has full column rank denotes and is called a "precoding matrix." The previous general formalism encompasses the cyclic-prefix OFDM, zero-padded OFDM, and downlink DS-CDMA systems [1], [2]. Indeed, matrix $\mathbf{K}$ is equal to a particular Vandermonde matrix for a cyclic-prefix OFDM system. A zero-padded OFDM system is obtained by concatenating a null matrix and an IFFT matrix. For a DS-CDMA system, each input of the vector $S_P(n)$ corresponds to one user. Then, each column of $\mathbf{K}$ corresponds to a user-specific spreading code. Furthermore, the "pseudo-symbol" baud rate $1/T_v$ corresponds to the chip rate. The single-carrier and single-user system is obtained by setting $P=Q=1$ and $\mathbf{K} = 1$. In fact, the model (2) is more general since it also appears to any transmitted induced cyclostationarity (TIC) scheme (see [22] for more details).

In order to treat OFDM and DS-CDMA systems simultaneously, we now consider the unifying framework [1], [2]. We assume that the receiver knows only the value of Q, i.e., the length of the codes (resp. the number of the subcarriers) if a DS-CDMA (resp. OFDM) system is considered. Then, the continuous-time received signal can be written as follows:

$$y_a(t) = \left(\sum_{n\in\mathbb{Z}} v_n h_a(t-nT_v)\right) e^{2i\pi\delta f_0 t} + w_a(t). \qquad (3)$$

At the front–end of the receiver, we wish to estimate the carrier frequency offset from the sole knowledge of the "pseudo-symbol" baud-rate sampled received signal $y(n) = y_a(nT_v)$. According to (3), the discrete-time signal $y(n)$ can be written in the following form:

$$y(n) = a(n)e^{2i\pi\Delta f_0 n} + w(n) \tag{4}$$

with $a(n) = [h(z)]v_n$ and $w(n) = w_a(nT_v)$. The notation $[]$ stands for the convolution product. The "pseudo-symbol" baud-rate sampled version of the continuous-time filter $h_a(t)$ is denoted by $h(z)$. In an OFDM communication system, the filter $h_a(t)$ depends on a shaping filter built at a $1/T_s$ rate. In a DS-CDMA communication system, the shaping filter is constructed at chip rate, namely, $1/T_v$. The z-transform $h(z)$ represents a causal FIR filter of degree $M = \lceil T_h/T_v \rceil$, where $\lceil x \rceil$ is the upper integer closest to x. We also denote $\Delta f_0 = (\delta f_0 T_v \bmod 1)$. The notation $(a \bmod b)$ stands for a modulo b. By convention, $(a \bmod b)$ belongs to $[0, b)$.

Equation (4) shows that estimating the carrier frequency offset is equivalent to harmonic retrieval in multiplicative and additive noise. In a single-carrier and single-user system, the multiplicative noise $a(n)$ is stationary, whereas it becomes cyclostationary in a multicarrier or multiuser system. Indeed, the "pseudo-symbol" sequence is cyclostationary because it obeys the structure defined by (2). More precisely, as the symbol sequence is noncircular, the "pseudo-symbol" sequence and the process $a(n)$ are cyclostationary with respect to their autocorrelation and conjugate autocorrelation functions. The set of their cyclic frequencies is as follows: $\{k/Q \mid 0 \le k \le Q-1\}$. We recall that a zero-mean discrete-time stochastic process $p(n)$ is said to be cyclostationary if the correlation coefficients-based sequence[2] $\{\mathbb{E}[p(m+n)\overline{p(n)}]\}_{m\in\mathbb{Z}}$ (or the conjugate correlation coefficients-based sequence $\{\mathbb{E}[p(m+n)p(n)]\}_{m\in\mathbb{Z}}$) is "almost-periodic," which means that

$$\mathbb{E}[p(m+n)\overline{p(n)}] = \sum_{k=0}^{\infty} r^{(\alpha_k)}(m)e^{2i\pi\alpha_k n}$$

where $\{\alpha_k\}_{k\ge 0}$ are the so-called cyclic frequencies of $p(n)$. The sequence $\{r^{(\alpha_k)}(m)\}_{m\in\mathbb{Z}}$ denotes the cyclocorrelation sequence at cyclic frequency α_k of $p(n)$. The Fourier expansion

$$S_p^{(\alpha_k)}(e^{2i\pi f}) = \sum_{m\in\mathbb{Z}} r^{(\alpha_k)}(m)e^{-2i\pi m f}$$

represents the cyclospectrum at cyclic frequency α_k of $p(n)$.

On account of the multiplicative noise cyclostationarity, we can not directly apply the estimator introduced by [20] in the context of a single-carrier and single-user transmission to our more general context. Section III, therefore, addresses suitable modifications and extensions.

III. New Estimator

Let $r_c(n,\tau) = \mathbb{E}[y(n+\tau)y(n)]$ be the conjugate correlation at lag τ of $y(n)$. We put $\alpha_0 = (2\Delta f_0 \bmod 1)$. As the noise is circular, we get $r_c(n,\tau) = \mathbb{E}[a(n+\tau)a(n)]e^{i\pi\alpha_0\tau}e^{2i\pi\alpha_0 n}$. As $a(n)$ is cyclostationary with cyclic frequencies $\{k/Q \mid 0 \le k \le Q-1\}$, we obtain that

$$r_c(n,\tau) = \sum_{l=0}^{Q-1} r_c^{(\alpha_0+l/Q)}(\tau)e^{2i\pi(\alpha_0+l/Q)n}. \tag{5}$$

Consequently, $y(n)$ is cyclostationary with respect to its conjugate correlation coefficients. Since Q is known, if $|\Delta f_0| < \min(1/4, 1/2Q)$, then one can check that the knowledge of the cyclic frequencies of $y(n)$ provides the value of α_0. Let $\mathcal{A}_0$ be a compact set included in $(0, \min(1/2, 1/Q))$. Then

$$\forall\, \alpha \in \mathcal{A}_0, \quad \alpha \ne \alpha_0,\ \forall\, \tau,\ \forall\, l, \quad r_c^{(\alpha+l/Q)}(\tau) = 0.$$

Thus, α_0 satisfies the following equality:

$$\alpha_0 = \arg\max_{\alpha\in\mathcal{A}_0} J(\alpha), \quad J(\alpha) = \sum_{l=0}^{Q-1} \left\| \mathbf{r}_c^{(\alpha+l/Q)} \right\|_{W_l}^2$$

with $\mathbf{r}_c^{(\alpha)} = \left[r_c^{(\alpha)}(-T), \ldots, r_c^{(\alpha)}(T) \right]^T$, where T is an integer, and $\{W_l\}_{l=0,Q-1}$ is a set of positive Hermitian weighting matrices.[3] In practice, the vector $\mathbf{r}_c^{(\alpha)}$ of cyclocorrelations has to be estimated because only N observations are available. Let $\hat{r}_{c,N}^{(\alpha)}(\tau)$ be the empirical estimate of $r_c^{(\alpha)}(\tau)$. It is defined by

$$\hat{r}_{c,N}^{(\alpha)}(\tau) = \frac{1}{N}\sum_{n=0}^{N-1} y(n+\tau)y(n)e^{-2i\pi\alpha n}.$$

We now consider the stacking vector

$$\hat{\mathbf{r}}_{c,N}^{(\alpha)} = \left[\hat{r}_{c,N}^{(\alpha)}(-T), \ldots, \hat{r}_{c,N}^{(\alpha)}(T) \right]^T.$$

It thus can be written as

$$\hat{\mathbf{r}}_{c,N}^{(\alpha)} = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{y}_2(n)e^{-2i\pi\alpha n}$$

where $\mathbf{y}_2(n) = [y(n-T)y(n), \ldots, y(n+T)y(n)]^T$. Then, the α_0 estimate, which is denoted by $\hat{\alpha}_N$, is defined by

$$\hat{\alpha}_N = \arg\max_{\alpha\in\mathcal{A}_0} J_N(\alpha), \quad J_N(\alpha) = \sum_{l=0}^{Q-1} \left\| \hat{\mathbf{r}}_{c,N}^{(\alpha+l/Q)} \right\|_{W_l}^2.$$

This estimator extends the one introduced in [20] to a more general scheme including cyclic-prefix OFDM, zero-padded OFDM, and downlink DS-CDMA communication systems. Considering $Q = 1$ leads to the single-carrier and single-user case. This estimator has been partially introduced for a system using nonredundant precoding dedicated to a particular transmitter induced cyclostationary scheme ([23]). Nevertheless, these authors only consider the cyclocorrelation coefficient at lag 0 (i.e., $T = 0$) and the identity weighting matrices (i.e., $W_l = 1$ for each l). In the sequel, we rigorously analyze the asymptotic behavior of this estimator. This enables us to

[2]Let $\mathbf{x}$ be a scalar or a vector. The overline $\overline{x}$ stands for the complex-conjugate of $\mathbf{x}$. The superscripts $\mathbf{x}^T$ and $\mathbf{x}^H$ denote its transpose and transpose-conjugate, respectively.

[3]Let $\mathbf{x}$ and W be a vector and a positive Hermitian matrix, respectively. Then, $\|\mathbf{x}\|_W^2 = \mathbf{x}^H W \mathbf{x}$.

forecast the influence of the weighting matrices and the number of considered cyclocorrelation coefficients on the performance.

IV. Asymptotic Analysis

In this section, we prove that the proposed estimator is consistent and asymptotically normal. We also determine the convergence rate. Finally, we obtain a closed-form expression for the asymptotic covariance.

In the context of single-carrier and single-user communications, such an asymptotic analysis has already reported in [20]. The authors exploited successfully the fact that their proposed estimator can be described by means of harmonic retrieval tools. Therefore, we consider the same approach as that of [20]. In the present paper, we only sketch the proof dealing with consistency and asymptotic normality. The derivations leading to the closed-form expression for the asymptotic covariance are described more precisely. Nevertheless, we focus on the main steps, and for sake of clarity, some straightforward details are omitted. For more details, see [20] and [24] and references therein.

We consider the following zero-mean process: $\mathbf{e}(n) = \mathbf{y}_2(n) - \mathbb{E}[\mathbf{y}_2(n)]$. Equation (5) thus leads to

$$\mathbf{y}_2(n) = \sum_{l=0}^{Q-1} \mathbf{r}_c^{(\alpha_0+l/Q)} e^{2i\pi(\alpha_0+l/Q)n} + \mathbf{e}(n). \tag{6}$$

Therefore, $\mathbf{y}_2(n)$ corresponds to a sum of sinusoids with multivariate amplitudes disturbed by the additive noise $\mathbf{e}(n)$. The cost function $J_N(\alpha)$ can be rewritten as follows:

$$J_N(\alpha) = \sum_{l=0}^{Q-1} \left\| \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{y}_2(n) e^{-2i\pi(\alpha+l/Q)n} \right\|_{W_l}^2. \tag{7}$$

It thus represents a sum of weighted periodograms. We observe that estimating α_0 is connected with multivariate amplitude-based sinusoid frequency retrieval corrupted by an additive noise.

We assume that the noise $w(n)$ satisfies the traditional mixing condition [25]. As $h(z)$ represents an FIR filter and thus has finite memory, one can show that $\mathbf{e}(n)$ satisfies the following condition. Indeed, in such a case, sums reduce to finite ones, and the maximization only works on a finite set of integers.

Condition 1: Let $\mathbf{e}^{(0)}(n) = \mathbf{e}(n)$ and $\mathbf{e}^{(1)}(n) = \overline{\mathbf{e}(n)}$.

$$\forall\, L, \exists B_L < \infty,\ \forall\,(\nu_1,\ldots,\nu_L) \in \{0,1\}^L,\ \forall\,(N,N')$$
$$\max_{n_1\in\mathbb{Z}} \sum_{n_2,\ldots,n_L=N'}^{N} \|\mathrm{cum}_L(\mathbf{e}^{(\nu_1)}(n_1),\ldots,\mathbf{e}^{(\nu_L)}(n_L))\| \le B_L$$

where $\mathrm{cum}_L(\mathbf{x}_1,\ldots,\mathbf{x}_L)$ stands for the Lth-order cumulant tensor of random vectors $\mathbf{x}_1,\ldots,\mathbf{x}_L$.

In [24], the following lemma has been rigorously proved.

Lemma 1: Let K be a positive integer. We put

$$\mathbf{s}_N^{(K)}(\alpha) = \frac{1}{N^{K+1}} \sum_{n=0}^{N-1} n^K \mathbf{e}(n) e^{-2i\pi\alpha n}.$$

Under condition 1 , for each $K \in \mathbb{N}$, we get

$$\lim_{N\to\infty} \max_{\alpha\in[0,1]} \left\| \mathbf{s}_N^{(K)}(\alpha) \right\| \overset{\text{a.s.}}{=} 0$$

where a.s. stands for "almost surely."

This lemma is the starting point of the asymptotic analysis. Using (6) and (7), one obtains that

$$J_N(\alpha) = T_N(\alpha) + \varepsilon_N(\alpha)$$

with

$$T_N(\alpha) = \sum_{l=0}^{Q-1} \left\| \sum_{m=0}^{Q-1} \mathbf{r}_c^{(\alpha_0+m/Q)} \times \frac{1}{N} \sum_{n=0}^{N-1} e^{2i\pi(\alpha_0-\alpha)n} e^{2i\pi((m-l)/Q)n} \right\|_{W_l}^2$$

and

$$\varepsilon_N(\alpha) = \sum_{l=0}^{Q-1} \left\| \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{e}(n) e^{-2i\pi(\alpha+l/Q)n} \right\|_{W_l}^2 + \sum_{l=0}^{Q-1} 2\, \Re e \left\{ \sum_{m=0}^{Q-1} \mathbf{r}_c^{(\alpha_0+m/Q)^T} W_l \times \left(\frac{1}{N} \sum_{n=0}^{N-1} \mathbf{e}(n) e^{2i\pi(\alpha_0-\alpha+(m-l)/Q)n} \right) \right\}$$

where $\Re e$ stands for the real part of a complex-valued number. According to Lemma 1 , we show that as $N \to \infty$, $\varepsilon_N(\hat{\alpha}_N) \to 0$, i.e., $J_N(\hat{\alpha}_N) - T_N(\hat{\alpha}_N) \to 0$. As $\hat{\alpha}_N$ maximizes $J_N(\alpha)$ on the set $\mathcal{A}_0$, $J_N(\hat{\alpha}_N)$ converges to the maximum value of the theoretical cost function $J(\alpha)$, i.e., $J(\alpha_0)$. Consequently, $T_N(\hat{\alpha}_N)$ also converges to the nonzero valued term $J(\alpha_0)$, i.e.,

$$T_N(\hat{\alpha}_N) \to J(\alpha_0) \text{ as } N \to \infty. \tag{8}$$

In [20], Lemma 3 shows that the term

$$\frac{1}{N} \sum_{n=0}^{N-1} e^{2i\pi(\alpha_0-\hat{\alpha}_N)n}$$

converges i) to a nonzero value if $(\hat{\alpha}_N - \alpha_0) \overset{\text{a.s.}}{\to} 0$ and $N(\hat{\alpha}_N - \alpha_0) \overset{\text{a.s.}}{\to} 0$ and ii) zero otherwise. Therefore, if either $(\hat{\alpha}_N - \alpha_0) \overset{\text{a.s.}}{\to} 0$ or $N(\hat{\alpha}_N - \alpha_0) \overset{\text{a.s.}}{\to} 0$ does not hold, the term $T_N(\hat{\alpha}_N)$ converges to zero, which contradicts the equality (8). Finally, this leads to establish the following theorem

Theorem 1: Under the mixing Condition 1, as $N \to \infty$, we get

$$\hat{\alpha}_N - \alpha_0 \overset{\text{a.s.}}{\to} 0 \text{ and } N(\hat{\alpha}_N - \alpha_0) \overset{\text{a.s.}}{\to} 0.$$

To establish the asymptotic normality of $\hat{\alpha}_N$, we note that $\partial J_N(\alpha)/\partial\alpha|_{\alpha=\hat{\alpha}_N} = 0$. Using a first-order Taylor series expansion of the derivative of J_N around α_0, we obtain that

$$\delta\hat{\alpha}_N = -\left[\left. \frac{\partial^2 J_N(\alpha)}{\partial\alpha^2} \right|_{\alpha=\tilde{\alpha}_N} \right]^{-1} \left. \frac{\partial J_N(\alpha)}{\partial\alpha} \right|_{\alpha=\alpha_0} \tag{9}$$

where $\delta\hat{\alpha}_N = (\hat{\alpha}_N - \alpha_0)$ denotes the estimation error. $\tilde{\alpha}_N$ is a scalar between α_0 and $\hat{\alpha}_N$. We define

$$a_N = \frac{1}{N^2}\left.\frac{\partial^2 J_N(\alpha)}{\partial \alpha^2}\right|_{\alpha=\tilde{\alpha}_N} \tag{10}$$

$$b_N = \frac{1}{\sqrt{N}}\left.\frac{\partial J_N(\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0}. \tag{11}$$

Plugging (10) and (11) back into (9), we obtain

$$N^{3/2}(\hat{\alpha}_N - \alpha_0) = -a_N^{-1} b_N. \tag{12}$$

Due to Lemma 1 , one can easily check that a_N tends almost surely to a positive real-valued number, which is denoted by γ_a, as N tends toward infinity. Applying the mixing Condition 1 and Lemma 1 leads to the fact that b_N tends in distribution to a Gaussian distribution with zero-mean and covariance γ_b as N tends toward infinity. Using both previous results, we deduce the following theorem [26].

Theorem 2: Under the mixing Condition 1, as $N \to \infty$, we get

$$N^{3/2}(\hat{\alpha}_N - \alpha_0) \xrightarrow{\mathcal{L}} \mathcal{N}(0,\gamma) \quad \text{with} \quad \gamma = \frac{\gamma_b}{\gamma_a^2}$$

where $\mathcal{L}$ stands for a distribution convergence, and $\mathcal{N}(m,c)$ is a Gaussian distribution with mean m and variance c.

We notice that the convergence rate is proportional to $1/N^3$, which is a standard result about the harmonic retrieval problem.

As detailed in the following theorem, we have also obtained a closed-form expression for the asymptotic covariance γ.

Theorem 3: Under Condition 1, we get

$$\gamma = \frac{3}{\pi^2}\frac{\sum_{l,l'=0}^{Q-1} \mathbf{R}_l^H \mathbf{W}_l \mathbf{G}_{l,l'} \mathbf{W}_{l'} \mathbf{R}_{l'}}{\left(\sum_{l=0}^{Q-1} \mathbf{R}_l^H \mathbf{W}_l \mathbf{R}_l\right)^2}$$

with

$$\mathbf{R}_l = \begin{bmatrix} \mathbf{r}_c^{(\alpha_0+l/Q)} \\ \overline{\mathbf{r}_c^{(\alpha_0+l/Q)}} \end{bmatrix}, \quad \mathbf{W}_l = \begin{bmatrix} W_l & 0 \\ 0 & \overline{W_l} \end{bmatrix}, \text{ and}$$

$$\mathbf{G}_{l,l'} = \begin{bmatrix} \Gamma^{(l,l')} & -\Gamma_c^{(l,l')} \\ -\overline{\Gamma_c^{(l,l')}} & \overline{\Gamma^{(l,l')}} \end{bmatrix}$$

where

$$\Gamma^{(l,l')} = \lim_{N\to\infty} N\mathbb{E}\left[\delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l/Q)} \cdot \delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l'/Q)^H}\right]$$

$$\Gamma_c^{(l,l')} = \lim_{N\to\infty} N\mathbb{E}\left[\delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l/Q)} \cdot \delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l'/Q)^T}\right]$$

and

$$\delta\hat{\mathbf{r}}_{c,N}^{(\alpha)} = \left(\hat{\mathbf{r}}_{c,N}^{(\alpha)} - \mathbf{r}_c^{(\alpha)}\right).$$

Proof: See derivations in Appendix A. ■

The previous expressions are not computable as they stand. We need more explicit expressions for Γ and Γ_c, which is the asymptotic covariance of the cyclocorrelation vector empirical estimate. First of all, we derive these matrices in terms of the statistics of the disturbance occurring in the equivalent harmonic retrieval problem, i.e., $\mathbf{e}(n)$. As seen in Appendix A [namely, (20) and (21)], we get

$$\Gamma^{(l,l')} = S_{\mathbf{e}}^{((l-l')/Q)}\left(e^{2i\pi(\alpha_0+l/Q)}\right)$$

$$\Gamma_c^{(l,l')} = S_{\mathbf{e}^{(c)}}^{(2\alpha_0+(l+l')/Q)}\left(e^{2i\pi(\alpha_0+l/Q)}\right)$$

where $f \mapsto S_{\mathbf{e}}^{(\alpha)}(e^{2i\pi f})$ and $f \mapsto S_{\mathbf{e}^{(c)}}^{(\alpha)}(e^{2i\pi f})$ denote the cyclospectra of $\mathbf{e}(n)$ at cyclic frequency α with respect to its autocorrelation function and its conjugate autocorrelation function, respectively. The notation $\mapsto$ stands for "maps to." We now determine the second-order cyclic statistics of the disturbance $\mathbf{e}(n)$. We put $\mathbf{d}_T(e^{2i\pi f}) = [e^{-2i\pi Tf},\ldots,e^{2i\pi Tf}]^T$. After straightforward but tedious derivations, we obtain

$$\begin{aligned}
&S_{\mathbf{e}}^{((l-l')/Q)}(e^{2i\pi(\alpha_0+l/Q)}) \\
&= \sum_{m=0}^{Q-1}\int_0^1 S_y^{(m/Q)}(e^{2i\pi f}) S_y^{(((l-l'-m)/Q)}(e^{2i\pi(\alpha_0+l/Q)-f}) \\
&\quad \cdot \mathbf{d}_T(e^{2i\pi f})\left[\mathbf{d}_T^H(e^{2i\pi(f-m/Q)})\right. \\
&\qquad\qquad \left.+\ \mathbf{d}_T^H(e^{2i\pi(\alpha_0+(l'+m)/Q-f)})\right] df \\
&\quad + \int_0^1 S_{y,4}^{(\alpha)}(e^{2i\pi f_1}; e^{2i\pi f_2}; e^{2i\pi(\alpha_0+l'/Q-f_2)}). \\
&\overline{\mathbf{d}_T(e^{2i\pi(f_1-\alpha_0-l/Q)})}\mathbf{d}_T^H(e^{2i\pi f_2}) df_1 df_2
\end{aligned}$$

$$\begin{aligned}
&S_{\mathbf{e^{(c)}}}^{(2\alpha_0+(l+l')/Q)}(e^{2i\pi(\alpha_0+l/Q)}) \\
&= \sum_{m=0}^{Q-1}\int_0^1 S_{y^{(c)}}^{(\alpha_0+m/Q)}(e^{2i\pi f}) S_{y^{(c)}}^{(\alpha_0+(l-l'-m)/Q)} \\
&\quad \times (e^{2i\pi(\alpha_0+l/Q-f)}). \\
&\mathbf{d}_T(e^{2i\pi f})[\mathbf{d}_T^T(e^{2i\pi(\alpha_0+m/Q-f)}) + \mathbf{d}_T^T(e^{2i\pi(f+(l'-m)/Q)})] df \\
&\quad + \int_0^1 S_{y^{(c)},4}^{(2\alpha_0+(l+l')/Q)}(e^{2i\pi f_1}; e^{2i\pi f_2}; e^{2i\pi(\alpha_0+l'/Q-f_2)}) \\
&\overline{\mathbf{d}_T(e^{2i\pi(f_1-\alpha_0-l/Q)})}\mathbf{d}_T^T(e^{2i\pi f_2}) df_1 df_2
\end{aligned}$$

where $S_y^{(\alpha)}(e^{2i\pi f})$ and $S_{y^{(c)}}^{(\alpha)}(e^{2i\pi f})$ denote the cyclic spectra associated with the autocorrelation function and the conjugate autocorrelation function of $y(n)$. Furthermore, $S_{y,4}^{(\alpha)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})$ and $S_{y^{(c)},4}^{(\alpha)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})$ denote the cyclic trispectra associated with the fourth-order cumulant function and the fourth-order conjugate cumulant function of $y(n)$. In Appendix B, we give a more precise definition of such trispectra.

The third step consists of deriving the expressions of these received signal cyclospectra with respect to the filter $h(z)$ and the cyclic statistics of v_n. After rather simple calculations, we get

$$\begin{aligned}
S_y^{(l/Q)}(e^{2i\pi f}) &= h(e^{2i\pi(f-\alpha_0/2)})\overline{h(e^{2i\pi(f-\alpha_0/2-l/Q)})} \\
&\quad \times S_v^{(l/Q)}(e^{2i\pi(f-\alpha_0/2)}) + \delta_{0,l} S_w(e^{2i\pi f}) \\
S_{y^{(c)}}^{(\alpha_0+l/Q)}(e^{2i\pi f}) &= h(e^{2i\pi(f-\alpha_0/2)}) h(e^{2i\pi(\alpha_0/2+l/Q-f)}) \\
&\quad \times S_{v^{(c)}}^{(l/Q)}(e^{2i\pi(f-\alpha_0/2)})
\end{aligned}$$

and

$$
\begin{aligned}
&S_{y,4}^{(l/Q)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})\\
&= h(e^{2i\pi(-\nu_1+\nu_2+\nu_3-l/Q-\alpha_0/2)})\\
&\quad \times h(e^{2i\pi(\nu_1-\alpha_0/2)})\overline{h(e^{2i\pi(\nu_2-\alpha_0/2)})}\\
&\overline{h(e^{2i\pi(\nu_3-\alpha_0/2)})}\\
&\quad S_{v,4}^{(l/Q)}(e^{2i\pi(\nu_1-\alpha_0/2)}; e^{2i\pi(\nu_2-\alpha_0/2)}; e^{2i\pi(\nu_3-\alpha_0/2)})\\
&S_{y^{(c)},4}^{(2\alpha_0+l/Q)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})\\
&= h(e^{2i\pi(-\nu_1-\nu_2-\nu_3-l/Q+3\alpha_0/2)})\\
&\quad \times h(e^{2i\pi(\nu_1-\alpha_0/2)})h(e^{2i\pi(\nu_2-\alpha_0/2)}).\\
&h(e^{2i\pi(\nu_3-\alpha_0/2)})\\
&\quad S_{v^{(c)},4}^{(l/Q)}\left(e^{2i\pi(\nu_1-\alpha_0/2)}; e^{2i\pi(\nu_2-\alpha_0/2)}; e^{2i\pi(\nu_3-\alpha_0/2)}\right)
\end{aligned}
$$

where $S_v^{(\alpha)}(e^{2i\pi f})$ and $S_{v^{(c)}}^{(\alpha)}(e^{2i\pi f})$ denote the cyclic spectra associated with the autocorrelation function and the conjugate autocorrelation function of $v(n)$. Moreover, the terms $S_{v,4}^{(\alpha)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})$ and $S_{v^{(c)},4}^{(\alpha)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})$ denote the cyclic trispectra associated with the fourth-order cumulant function and the fourth-order conjugate cumulant function of v_n. Finally, $S_w^{(\alpha)}(e^{2i\pi f})$ denotes the power spectral density of the noise $w(n)$, and δ stands for the Kronecker index.

We now derive the cyclic second- and fourth-order cyclospectra of v_n in terms of the elements of matrix $\mathbf{K} = [k_{n,m}]_{\substack{0\le n\le Q-1\\ 0\le m\le P-1}}$. We put $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]$. Then, we obtain that

$$
\begin{aligned}
S_v^{(l/Q)}(e^{2i\pi f}) =& \frac{1}{Q}\sum_{m=0}^{P-1}\sum_{\tau=1-Q}^{Q-1}\sum_{n=A_\tau}^{B_\tau} k_{n+\tau,m}\\
&\times \overline{k_{n,m}}e^{-2i\pi(l/Q)n}e^{-2i\pi f\tau}\\
S_{v^{(c)}}^{(l/Q)}(e^{2i\pi f}) =& \frac{1}{Q}\sum_{m=0}^{P-1}\sum_{\tau=1-Q}^{Q-1}\sum_{n=A_\tau}^{B_\tau}\\
&\times k_{n+\tau,m}k_{n,m}e^{-2i\pi(l/Q)n}e^{-2i\pi f\tau}
\end{aligned}
$$

and

$$
\begin{aligned}
&S_{v,4}^{(l/Q)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})\\
&= \frac{\kappa}{Q}\sum_{m=0}^{P-1}\sum_{\substack{\boldsymbol{\tau}=1-Q\\ C_{\boldsymbol{\tau}}\le D_{\boldsymbol{\tau}}}}^{Q-1}\sum_{n=C_{\boldsymbol{\tau}}}^{D_{\boldsymbol{\tau}}}\\
&\quad \times k_{n,m}k_{n+\tau_1,m}\overline{k_{n-\tau_2,m}k_{n-\tau_3,m}}\\
&\quad \times e^{-2i\pi((l/Q)n+\nu_1\tau_1+\nu_2\tau_2+\nu_3\tau_3)}\\
&S_{v^{(c)},4}^{(l/Q)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})\\
&= \frac{\kappa'}{Q}\sum_{m=0}^{P-1}\sum_{\substack{\boldsymbol{\tau}=1-Q\\ C_{\boldsymbol{\tau}}\le D_{\boldsymbol{\tau}}}}^{Q-1}\sum_{n=C_{\boldsymbol{\tau}}}^{D_{\boldsymbol{\tau}}}\\
&\quad \times k_{n,m}k_{n+\tau_1,m}k_{n-\tau_2,m}k_{n-\tau_3,m}\\
&\quad \times e^{-2i\pi(l/Qn+\nu_1\tau_1-\nu_2\tau_2-\nu_3\tau_3)}\\
&S_{v^{(c)},4}^{(l/Q)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3})\\
&= \frac{\kappa'}{Q}\sum_{m=0}^{P-1}\sum_{\substack{\boldsymbol{\tau}=1-Q\\ C_{\boldsymbol{\tau}}\le D_{\boldsymbol{\tau}}}}^{Q-1}\sum_{n=C_{\boldsymbol{\tau}}}^{D_{\boldsymbol{\tau}}} k_{n,m}k_{n+\tau_1,m}k_{n-\tau_2,m}k_{n-\tau_3,m}\\
&\times e^{-2i\pi((l/Q)n+\nu_1\tau_1-\nu_2\tau_2-\nu_3\tau_3)}
\end{aligned}
$$

with $A_\tau = \max(0, -\tau)$, $B_\tau = Q - 1 + \min(0, -\tau)$, $C_{\boldsymbol{\tau}} = \max(0; -\tau_1; \tau_2; \tau_3)$, and $D_{\boldsymbol{\tau}} = Q - 1 + \min(0; -\tau_1; \tau_2; \tau_3)$. Terms $\kappa = \mathrm{cum}_4\{s_n, s_n, \overline{s_n}, \overline{s_n}\}$ and $\kappa' = \mathrm{cum}_4\{s_n, s_n, s_n, s_n\}$ denote the kurtosis and conjugate kurtosis of s_n, respectively.

Henceforth, it is interesting to point out the influence of the design parameters, namely, the weighting matrices and the number T of considered lags.

V. Parameters Choice

After straightforward derivations, one can check that γ splits into two terms

$$\gamma = \gamma_0 + \mathcal{O}(\sigma^2)$$

where γ_0 is a term independent of the noise $w(n)$, and σ^2 is the variance of the noise $w(n)$ ($w(n)$ does not need to be white). After some obvious but tedious derivations, we obtain the following result.

Theorem 4: We assume that the symbols $\{s_n\}_{n\in\mathbb{Z}}$ are drawn from a real-valued constellation. We also consider that the elements of $\mathbf{K}$ are real-valued.

Let $\mathbf{I}$ be the identity matrix. Recall that M is the degree of the filter $h(z)$.

If $T \geq M + Q$ and $W_l = \delta_{0,l}\mathbf{I}$, then

$$\gamma_0 = 0$$

else

$$\gamma_0 \neq 0.$$

As the most popular noncircular constellation is the binary PSK (BPSK) real-valued constellation, the assumption on the constellation characteristic is not restrictive. Unfortunately, the other assumption on the precoding matrix is more restrictive. Indeed, the OFDM systems do not satisfy such a condition since the associated precoding matrix is complex-valued. Nevertheless, one has to notice that the proposed estimator is consistent for OFDM systems anyway, although the asymptotic covariance is not null in noiseless case for OFDM systems. The previous theorem just asserts that at high SNRs, the floor effect occurs in OFDM system. On the contrary, the condition holds for the DS-CDMA systems since the associated precoding matrices (e.g., Walsh-Hadamard or Gold sequences) are real-valued generally.

In the sequel, the criterion associated with the following weighting matrices set $\{W_l = \delta_{0,l}\mathbf{I} \mid 0 \leq l \leq Q-1\}$ will be called *reduced*. In fact, it only takes advantage of the cyclocorrelations vector around the cyclic frequency α_0. The *complete* criterion is obtained with the following weighting matrices set $\{W_l = \mathbf{I} \mid 0 \leq l \leq Q-1\}$. It is based on the sum of cyclocorrelations vectors around all the cyclic frequencies.

Theorem 4 shows that the asymptotic covariance of the estimator associated with the reduced criterion is proportional to the noise variance if we take into account enough cyclocorrelation coefficients. This covariance then becomes null in the noiseless case. Moreover, it implies that this estimator is almost insensitive to an intersymbol interference effect. Theorem 4 also shows that we should consider the reduced criterion rather than the complete criterion. An explanation is that the cyclocorrelation coefficients at the cyclic frequencies different from α_0 are numerically weak.

Although the asymptotic covariance is null in the noiseless case, the estimate $\hat{\alpha}_N$ is not deterministic unless the filter $h(z)$ is flat fading. Indeed, one can be shown that the first-order derivative of $J_N(\alpha)$ at $\alpha = \alpha_0$, which is denoted by $(dJ_N(\alpha)/d\alpha|_{\alpha=\alpha_0})$, is not null except in the specific flat fading filter context. This implies that in presence of a frequency-selective channel, the theoretical and empirical performances could differ at high SNRs.

VI. Numerical Results

In this section, our aim is twofold. On the one hand, we wish to confirm the accuracy of the theoretical analysis and the parameters choice. For the sake of clarity, these simulations are only done in the CDMA context. On the other hand, we compare our method with other existing methods. We focus on the CDMA context as well as on the OFDM context.

A. Comparison Between Theoretical and Empirical Performances

The symbol stream $\{s_n\}_{n\in\mathbb{Z}}$ is drawn from a BPSK constellation. We denote by N_s the number of transmitted symbols s_n. N_s is equal to N/Q, where N is the number of available received observations. We put $N_s = 200$. The shaping filter is a square-root raised cosine of roll-off $\rho = 0.2$. The symbol-period is fixed to be $T_s = 3\ \mu$s. In order to drop the negligible tails, the complete impulse response of the filter is truncated such that only 99% of the total energy is kept. The propagation multipath channel is modeled by five Rayleigh fading paths with maximal delay $3T_s$. The noise $w(n)$ is assumed to be white.

In this subsection, we only compute a downlink DS-CDMA communication system. The considered downlink DS-CDMA system is as follows: The precoding matrix $\mathbf{K}$ is obtained from a Walsh–Hadamard sequence. The spreading factor and the number of users are $Q = 4$ and $P = 3$, respectively. In order to satisfy the constraint $\alpha_0 < \min(1/2, 1/Q)$, the sought cyclic frequency α_0 is equal to 0.15. Except otherwise stated, each criterion takes into account all the available cyclocorrelation components, i.e., $T = M + Q$.

In Fig. 1, we have plotted one realization of the reduced cost function versus the cyclic frequency α at SNR $= 0$ dB. We observe that the peak around α_0 is much larger than those around the other cyclic frequencies (i.e., $\alpha_0 + l/Q$ with $l \neq 0$). The complete cost function takes *a priori* advantage of the peaks at all the cyclic frequencies for estimating α_0. As the power of these peaks is numerically weak and sometimes below the ground noise level, the resulting estimation is unreliable. Therefore, we must consider that all peaks lead to a degradation of the estimation instead of an improvement of it.

From now on, in each simulation, we average the empirical and theoretical result over 200 Monte Carlo runs. At each run, the channel realization (delay and magnitude of the five paths), the symbol sequence, and the noise are modified.

Fig. 2 depicts the theoretical and empirical mean square error (MSE) versus the SNR of the proposed estimator for different values of the design parameters. According to all the design parameters, we have observed that the filter degree M is about 10. We recall that $N_s = 200$. The theoretical MSE is equal to γ/N^3, and γ is computed via Theorem 3. The empirical MSE is obtained by means of a gradient algorithm initialized at the true point α_0 in order to avoid local minima. This assumption is not so restrictive in practice because in any current (resp. future) system, a training sequence is (resp. remains) available and enables us to estimate roughly the sought parameter. Then, our blind estimate just leads to improve deeply the accuracy of the estimate. As expected, the theoretical and empirical performances are better for the reduced cost function-based estimator than for the complete cost function-based estimator. For the reduced cost function-based estimator, the theoretical performance fits in with that one obtained for a transmission over a flat fading channel (i.e., in the ISI-free case). Nevertheless, there is a gap between the theoretical and empirical performances for high SNRs. It is due to the bad estimation of the cyclocorrelation coefficients for high lags; indeed, an accurate estimation of these cyclocorrelation coefficients requires many samples and, in particular, more than $N_s = 200$ [20]. In fact, this phenomenon can be theoretically interpreted by means of the above-mentioned nondeterminism of the proposed estimator.

In order to inspect the mismatch between the theoretical and empirical performances at high SNRs, we plot in Fig. 3 the theo-

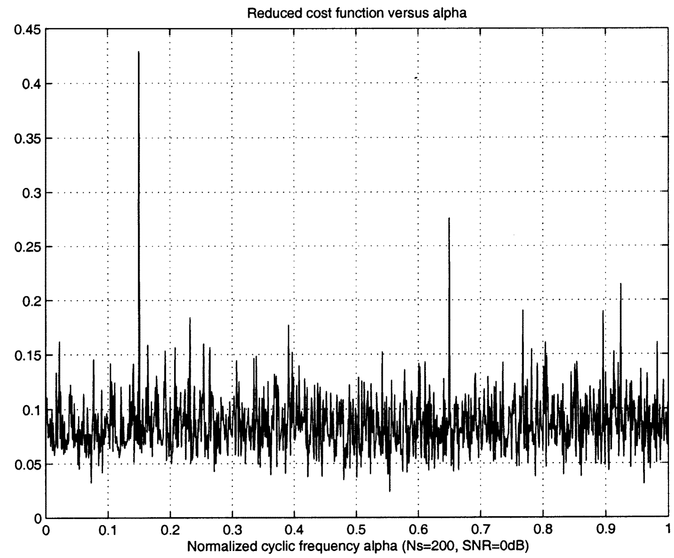

Fig. 1. Cyclocorrelation vector square norm $\|\hat{\mathbf{r}}_{c,N}^{(\alpha)}\|_{\mathbf{I}}^2$ versus α.

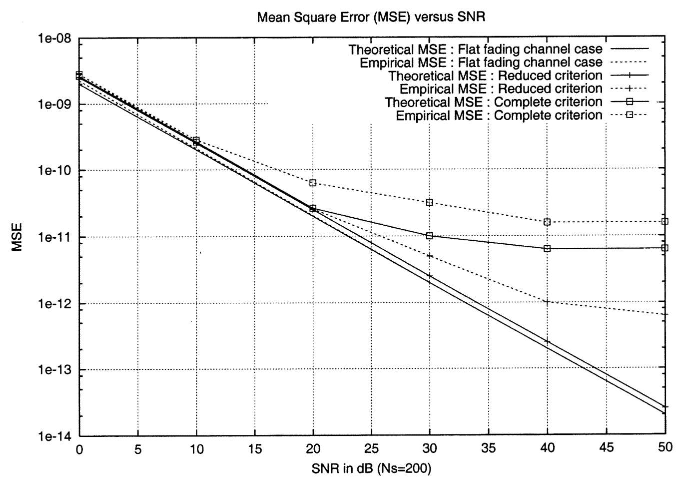

Fig. 2. Theoretical and empirical MSE versus SNR.

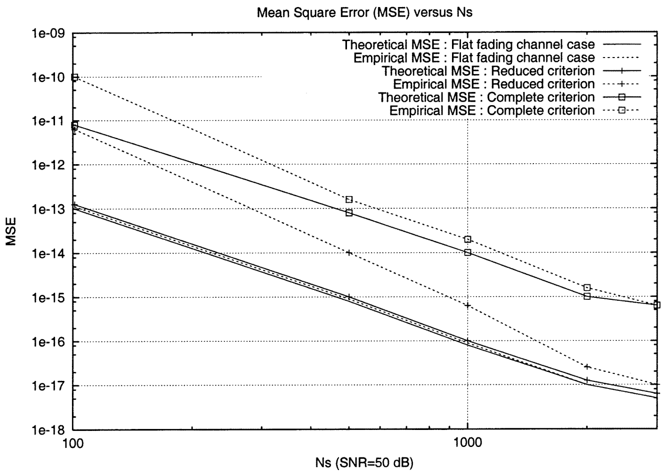

Fig. 3. Theoretical and empirical MSE versus N_s.

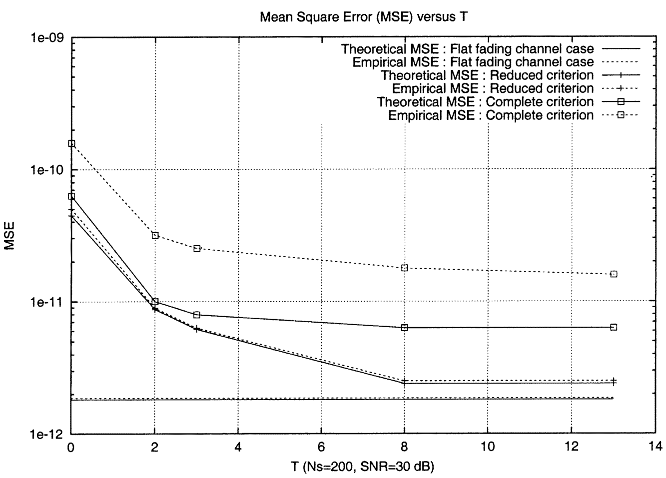

Fig. 4. Theoretical and empirical MSE versus T.

retical and empirical MSE versus N_s at SNR $= 50$ dB. We show that the gap between theoretical and empirical performances decreases as N_s increases.

Fig. 4 represents the theoretical and empirical MSE versus T. We put $N_s = 200$ and SNR $= 30$ dB. The number T of considered conjugate cyclocorrelation lags varies from 0 to $(M+Q)$. We observe that the theoretical and empirical MSE rapidly decrease. For the reduced criterion-based estimator, the optimal performance is reached before taking into account all the cyclocorrelation coefficients. It is due to the filter impulse response weak tails. The weak tails can be neglected, and then, the effective channel length is smaller than M. For the complete criterion-based estimator, the corresponding γ_0 converges to a nonzero term. Therefore, as observed, the influence of T on the performance is less strong.

As a conclusion, even if the previous empirical curves are not always in perfect agreement with the theoretical asymptotic analysis, they nevertheless corroborate the parameter choices.

B. Comparison With Existing Methods

At first, we concentrate on the comparison between estimators introduced in the downlink DS-CDMA communication context.

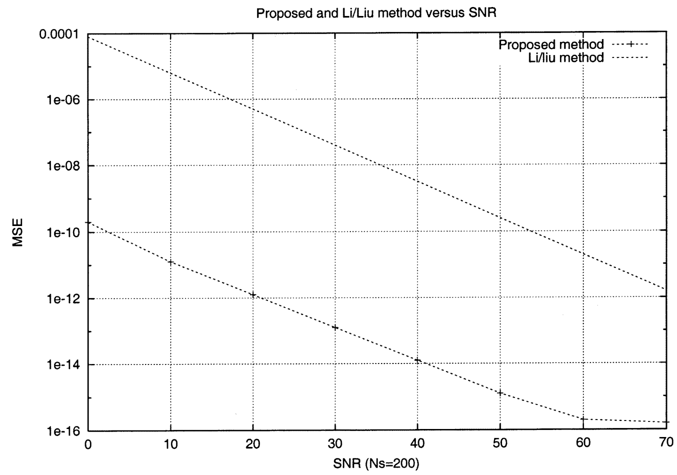

Fig. 5. Empirical MSE for proposed and existing methods in CDMA context versus SNR.

Two methods dealing with the nondata-aided frequency offset estimation in a downlink DS-CDMA communication scheme have been recently introduced: the Fu/Abed method [8] and the Li/Liu method [7], [27]. In order to satisfy the restrictive constraints stated in these two articles, we need to modify two design parameters in the above-mentioned DS-CDMA scheme. Indeed, we now put $Q = 16$ and $P = 3$. Such a choice of parameters was impossible in the previous subsection because it leads to a very high computational duration for the theoretical mean square error given by the asymptotic covariance. On the contrary, the empirical MSE is obtained in a reasonable duration even with this choice of parameters. In order to satisfy the constraint $\alpha_0 < \min(1/2, 1/Q)$, the sought cyclic frequency α_0 is now equal to 0.01.

In [8], it is asserted that the method is better than that described in [7]. In our case, we have observed that the Fu/Abed method performance is very poor and is outperformed by the Li/Liu method. For the Fu/Abed method to work well, the filter impulse response has to be almost flat over a chip duration. It is a very strong assumption that does not hold here.

Therefore, we only compare our method relying on the reduced estimate with the Li/Liu method. In Fig. 5, we plot the empirical MSE versus SNR for these two methods. We notice that our method outperforms the Li/Liu method at each SNR, especially at low SNRs. Nevertheless, a floor effect occurs only for our method, i.e., a stagnation of the MSE appears at very high SNRs. As already mentioned, it is due to the nondeterminism of the estimator. As the Li/Liu method is deterministic, such a phenomenon does not appear. In Fig. 6, we analyze the influence of the number of available observations on the two methods. We recall that our method admits a convergence rate that is proportional to $1/N_s^3$, whereas the Li/Liu estimator has a slower convergence rate proportional to $1/N_s$. An explanation is that their work has be done for frequency estimation issue in complex-valued circular constellation transmission, whereas our work relies on complex-valued noncircular constellation transmission and takes benefit of such constellation property [28], [29]. So, as expected, we notice that the MSE for our method decreases faster than that for the Li/Liu method. We also observe that our method remains powerful even with ten transmitted symbols.

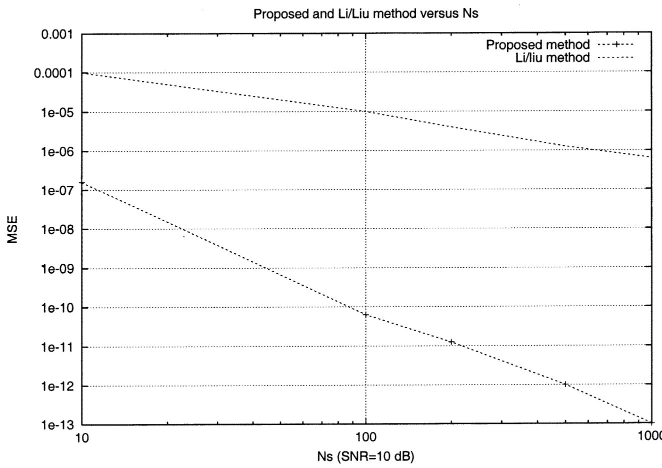

Fig. 6. Empirical MSE for proposed and existing methods in CDMA context versus N_s.

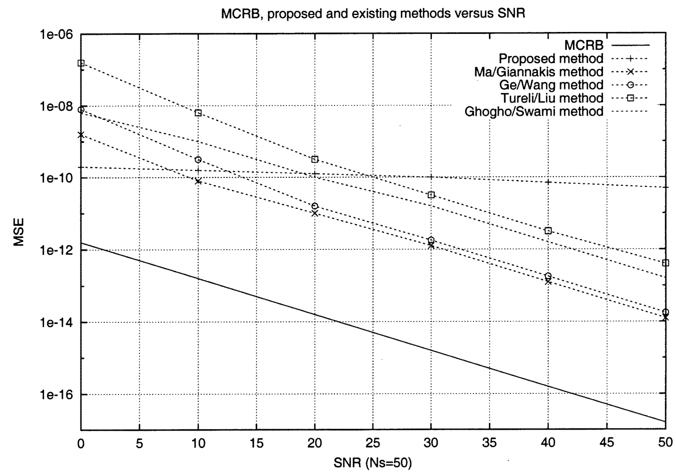

Fig. 7. MCRB and empirical MSE for proposed and existing methods in OFDM context versus SNR.

The better performance of our method is not surprising because we take benefit of the noncircularity of the source, whereas the Li/Liu method does not exploit such an additional knowledge. However, unlike our estimator, the Li/Liu estimator also succeeds with circular constellations.

We now focus on an OFDM system that is designed as the IEEE 802.11 or HIPERLAN2 standards; the number of sub-carriers is $P = 64$. We consider that 16 sub-carriers are virtual, i.e., no information is transmitted through them. The guard interval length is fixed to be 16. This implies that $Q = 80$. As N_s represents the number of transmitted OFDM blocks, it is more reasonable to consider $N_s = 50$ instead of $N_s = 200$. To satisfy the condition $\alpha_0 < \min(1/2, 1/Q)$, we choose $\alpha_0 = 0.01$. As the reduced estimator is better, our method now refers to the reduced estimator based method.

Several methods of NDA frequency offset estimation in an OFDM system scheme are robust against a dispersive channel. For instance, one can cite the Tureli/Liu method [11], [30], the Ge/Wang method [12], the Ma/Giannakis method [19], and the Ghogho/Swami/Giannakis method [15]. Furthermore, a modified Cramér-Rao Bound (MCRB) has been derived in [16]. We thus evaluate the loss in performance of our estimate and the others with respect to this MCRB. We notice that our estimate is not asymptotically efficient.

Fig. 7 depicts the MCRB and the empirical MSE versus SNR for the four above-mentioned methods and ours. For our method, we still notice a floor effect. As γ_0 is not null, this effect is more critical than in the CDMA communication context. We observe that our method is quite insensitive to the SNR. This means that γ_0 is the main term in the expression of γ at each SNR. Therefore, our method has bad performance at high SNRs. On the contrary, our method outperforms the others at low SNRs. This statement was expected since the other methods do not properly treat the noisy case.

In Fig. 8, we have plotted the MCRB and the empirical MSE versus N_s for the five methods. The SNR is fixed to be 0 dB. The convergence rate of the four other simulated methods is proportional to $1/N_s$. Therefore, our method outperforms them as N_s increases, whereas the performance becomes closer at low N_s. At high SNRs, we have observed that the existing methods outperform ours if N_s gets a reasonable value, whereas our method is better only if N_s is chosen large enough.

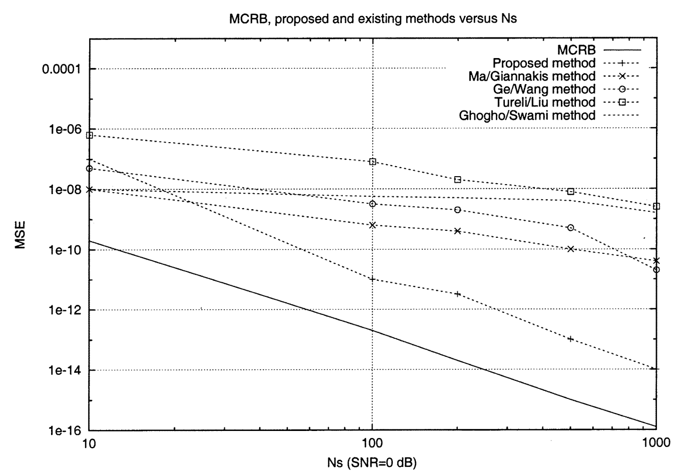

Fig. 8. MCRB and empirical MSE for proposed and existing methods in OFDM context versus N_s.

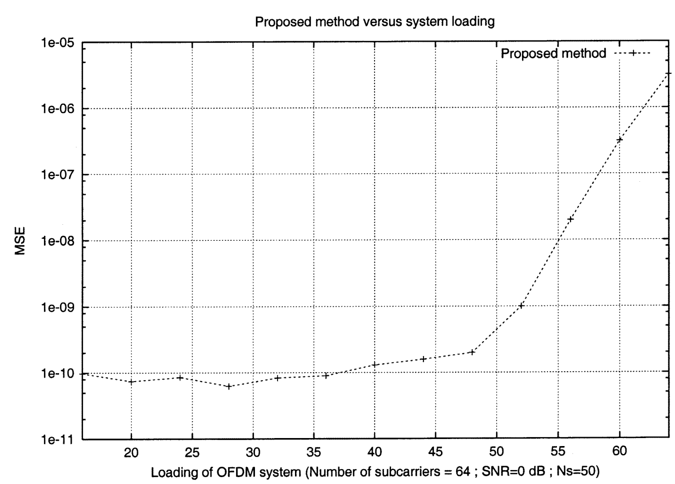

Fig. 9. Proposed method versus the loading of OFDM system.

We now analyze the influence of the number of virtual subcarriers on our method. In Fig. 9, we observe that the performance degrades whenever the number of null subcarriers becomes small. Nevertheless, even in a fully loaded system, the proposed estimate works, whereas the other computed methods fail.

VII. Conclusion

We have investigated a new estimator of the carrier frequency offset for linear precoder-based communication system assuming a frequency-selective channel and a noncircularly distributed symbol stream. We have especially focused on an OFDM system or downlink DS-CDMA technique. We have rigorously analyzed the asymptotic behavior of the proposed estimator. According to this analysis, we have identified relevant design parameters. In a DS-CDMA context, if the user codes and the constellation are real-valued, then we have proved that the well-designed estimator is powerful, even in the presence of inter-symbol interference.

Extending such an approach could be the purpose of further works in two different ways: on the one hand, inspecting the influence of a symbol timing delay and, on the other hand, adapting this method to several nonlinear or offset modulations.

Appendix A
Proof of Theorem 3

For expressing γ, we have to derive the closed-form expression for γ_a and γ_b separately. At first, we focus on γ_a. We put

$$\mathbf{C}_{N,l}^{(K)}(\alpha) = \frac{1}{N^{(K+1)}} \sum_{n=0}^{N-1} n^K \mathbf{y}_2(n) e^{-2i\pi(\alpha+l/Q)n}.$$

After straightforward manipulations, we obtain

$$a_N = 4\pi^2 \sum_{l=0}^{Q-1} \Big(2\mathbf{C}_{N,l}^{(1)}(\tilde{\alpha}_N)^H W_l \mathbf{C}_{N,l}^{(1)}(\tilde{\alpha}_N) - \mathbf{C}_{N,l}^{(0)}(\tilde{\alpha}_N)^H W_l \mathbf{C}_{N,l}^{(2)}(\tilde{\alpha}_N) - \mathbf{C}_{N,l}^{(2)}(\tilde{\alpha}_N)^H W_l \mathbf{C}_{N,l}^{(0)}(\tilde{\alpha}_N) \Big).$$

We put

$$\mathbf{R}_{N,l}^{(K)}(\alpha) = \frac{1}{N^{(K+1)}} \sum_{n=0}^{N-1} n^K \mathbf{r}_c(n) e^{-2i\pi(\alpha+l/Q)n}.$$

According to Lemma 1, $\mathbf{C}_{N,l}^{(K)}(\alpha)$ and $\mathbf{R}_{N,l}^{(K)}(\alpha)$ get asymptotically the same behavior. As $\tilde{\alpha}_N$ belongs to $[\alpha_0, \alpha_0 + \hat{\alpha}_N]$, $N(\tilde{\alpha}_N - \alpha_0)$ converges almost surely to zero. Due to (5), we thus can see that

$$\mathbf{R}_{N,l}^{(K)}(\tilde{\alpha}_N) \rightarrow \frac{\mathbf{r}_c^{(\alpha_0+l/Q)}}{K+1}, \text{ as } N \rightarrow \infty. \quad (13)$$

One can immediately check that

$$\gamma_a = -\frac{2\pi^2}{3} \sum_{l=0}^{Q-1} \left\| \mathbf{r}_c^{(\alpha_0+l/Q)} \right\|_{W_l}^2. \quad (14)$$

We now wish to obtain the term γ_b.

For the sake of clarity, $\mathbf{C}_{N,l}^{(K)}(\alpha_0)$, $\mathbf{R}_{N,l}^{(K)}(\alpha_0)$, and $\mathbf{s}_N^{(K)}(\alpha_0 + l/Q)$ (defined in Lemma 1) are replaced by $\mathbf{C}_{N,l}^{(K)}$, $\mathbf{R}_{N,l}^{(K)}$, and $\mathbf{S}_{N,l}^{(K)}$, respectively. One sees that b_N can be split into three terms

$$b_N = b_N^{(1)} + b_N^{(2)} + b_N^{(3)}$$

where

$$b_N^{(1)} = 2i\pi\sqrt{N} \sum_{l=0}^{Q-1} \left(\mathbf{R}_{N,l}^{(1)^H} W_l \mathbf{R}_{N,l}^{(0)} - R_{N,l}^{(0)^H} W_l \mathbf{R}_{N,l}^{(1)} \right)$$

$$b_N^{(2)} = 2i\pi\sqrt{N} \sum_{l=0}^{Q-1} \left(\mathbf{S}_{N,l}^{(1)^H} W_l \mathbf{S}_{N,l}^{(0)} - \mathbf{S}_{N,l}^{(0)^H} W_l \mathbf{S}_{N,l}^{(1)} \right)$$

$$b_N^{(3)} = 2i\pi\sqrt{N} \sum_{l=0}^{Q-1} \Big(\mathbf{S}_{N,l}^{(1)^H} W_l \mathbf{R}_{N,l}^{(0)} + \mathbf{R}_{N,l}^{(1)^H} W_l \mathbf{S}_{N,l}^{(0)} - \mathbf{S}_{N,l}^{(0)^H} W_l \mathbf{R}_{N,l}^{(1)} - \mathbf{R}_{N,l}^{(0)H} W_l \mathbf{S}_{N,l}^{(1)} \Big).$$

According to (13), one can prove that the deterministic term $b_N^{(1)}$ converges toward zero. As the noise $\mathbf{e}(n)$ satisfies Condition 1, we obtain that for each K, the high order cumulant of $\sqrt{N}\mathbf{S}_{N,l}^{(K)}$ converges to zero. Therefore, $\sqrt{N}\mathbf{S}_{N,l}^{(K)}$ is asymptotically Gaussian. Since $\mathbf{S}_{N,l}^{(K)}$ converges almost surely to zero, $b_N^{(2)}$ converges almost surely to zero [26]. This implies that

$$\gamma_b = \lim_{N\rightarrow\infty} \mathbb{E}\left[b_N^{(3)} b_N^{(3)^H} \right]. \quad (15)$$

It now remains to inspect the term $b_N^{(3)}$. In fact, $b_N^{(3)}$ can be rewritten as follows:

$$b_N^{(3)} = 2i\pi \sum_{l=0}^{Q-1} \mathcal{R}_{N,l} \mathcal{W}_l \mathcal{S}_{N,l}$$

with

$$\mathcal{R}_{N,l} = \left[\mathbf{R}_{N,l}^{(1)^H}, \mathbf{R}_{N,l}^{(0)^H}, \mathbf{R}_{N,l}^{(1)^T}, \mathbf{R}_{N,l}^{(0)^T} \right]$$

$$\mathcal{W}_l = \text{diag}(W_l, -W_l, -W_l^T, W_l^T)$$

$$\mathcal{S}_{N,l} = \sqrt{N}[\mathbf{S}_{N,l}^{(0)^T}, \mathbf{S}_{N,l}^{(1)^T}, \mathbf{S}_{N,l}^{(0)^H}, \mathbf{S}_{N,l}^{(1)^H}]^T.$$

diag(.) stands for the block-diagonal matrix constructed from the mentioned matrices.

According to (13) and (15), we get

$$\gamma_b = 4\pi^2 \sum_{l,l'=0}^{Q-1} \mathcal{R}_l \mathcal{W}_l \Gamma_{\mathcal{S}}^{(l,l')} \mathcal{W}_{l'}^H \mathcal{R}_{l'}^H \quad (16)$$

where

$$\mathcal{R}_l = \left[\frac{1}{2}\mathbf{r}_c^{(\alpha_0+l/Q)^H}, \mathbf{r}_c^{(\alpha_0+l/Q)^H}, \frac{1}{2}\mathbf{r}_c^{(\alpha_0+l/Q)^T}, \mathbf{r}_c^{(\alpha_0+l/Q)^T} \right]$$

and

$$\Gamma_{\mathcal{S}}^{(l,l')} = \lim_{N\rightarrow\infty} \mathbb{E}\left[\mathcal{S}_{N,l} \mathcal{S}_{N,l'}^H \right].$$

We now need to obtain a closed-form expression for $\Gamma_{\mathcal{S}}^{(l,l')}$. One can check that we have (17), shown at the bottom of the page, where

$$\mathbf{P}_N^{(l,l')}(K,K') = N\mathbb{E}\left[\mathbf{S}_{N,l}^{(K)}\mathbf{S}_{N,l'}^{(K')^T}\right] = \frac{1}{N^{K+K'+1}}\sum_{\substack{n=0\\n'=0}}^{N-1}\mathbb{E}\left[\mathbf{e}(n)\mathbf{e}(n')^T\right]n^K n'^{K'} \times e^{-2i\pi(\alpha_0+l/Q)n}e^{2i\pi(\alpha_0+l'/Q)n'}$$

and

$$\mathbf{P}_{c,N}^{(l,l')}(K,K') = N\mathbb{E}\left[\mathbf{S}_{N,l}^{(K)}\mathbf{S}_{N,l'}^{(K')^T}\right] = \frac{1}{N^{K+K'+1}}\sum_{\substack{n=0\\n'=0}}^{N-1}\mathbb{E}[\mathbf{e}(n)\mathbf{e}(n')^T]n^K n'^{K'} \times e^{-2i\pi(\alpha_0+l/Q)n}e^{-2i\pi(\alpha_0+l'/Q)n'}.$$

We now analyze the asymptotic behavior of these terms. One can easily prove that

$$\mathbf{R}_{\mathbf{e}}(n,\tau) = \mathbb{E}\left[\mathbf{e}(n+\tau)\mathbf{e}(n)^H\right] = \sum_{l=0}^{Q-1}\mathbf{R}_{\mathbf{e}}^{(l/Q)}e^{2i\pi(l/Q)n}$$

and

$$\mathbf{R}_{\mathbf{e}^{(c)}}(n,\tau) = \mathbb{E}\left[\mathbf{e}(n+\tau)\mathbf{e}(n)^T\right] = \sum_{l=0}^{Q-1}\mathbf{R}_{\mathbf{e}^{(c)}}^{(2\alpha_0+l/Q)}e^{2i\pi(2\alpha_0+l/Q)n}.$$

After some simple manipulations, using Condition 1 and results on Césaro sums leads to

$$\lim_{N\to\infty}\mathbf{P}_N^{(l,l')}(K,K') = \frac{1}{K+K'+1} \times S_{\mathbf{e}}^{((l-l')/Q)}(e^{2i\pi(\alpha_0+l/Q)}) \quad (18)$$

$$\lim_{N\to\infty}\mathbf{P}_{c,N}^{(l,l')}(K,K') = \frac{1}{K+K'+1} \times S_{\mathbf{e}^{(c)}}^{(2\alpha_0+(l+l')/Q)}(e^{2i\pi(\alpha_0+l/Q)}) \quad (19)$$

where

$$S_{\mathbf{e}}^{(\alpha)}(e^{2i\pi f}) = \sum_{\tau\in\mathbb{Z}}\mathbf{R}_{\mathbf{e}}^{(\alpha)}e^{-2i\pi f\tau}$$

and

$$S_{\mathbf{e}^{(c)}}^{(\alpha)}(e^{2i\pi f}) = \sum_{\tau\in\mathbb{Z}}\mathbf{R}_{\mathbf{e}^{(c)}}^{(\alpha)}e^{-2i\pi f\tau}$$

represent the unconjugate/conjugate cyclic spectrum at cyclic frequency α of $\mathbf{e}(n)$, respectively. These spectra can be written more explicitly; since $\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l/Q)} = (1/N)\sum_{n=0}^{N-1}\mathbf{y}_2(n)e^{-2i\pi(\alpha_0+l/Q)n}$ and $\mathbf{y}_2(n) = \sum_{m=0}^{Q-1}\mathbf{r}_c^{(\alpha_0+m/Q)}e^{2i\pi(\alpha_0+m/Q)n} + \mathbf{e}(n)$, it follows that

$$\sqrt{N}\delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+l/Q)} = \sqrt{N}\mathbf{S}_{N,l}^{(0)} + \varepsilon_{N,l}$$

with

$$\varepsilon_{N,l} = \frac{1}{\sqrt{N}}\sum_{\substack{m=0\\m\neq l}}^{Q-1}\mathbf{r}_c^{(\alpha_0+m/Q)}\sum_{n=0}^{N-1}e^{2i\pi((m-l)/Q)n}.$$

We easily obtain that $\varepsilon_{N,l}$ converges to zero as N converges to infinity. We deduce that [26]

$$\Gamma^{(l,l')} = \lim_{N\to\infty}N\mathbb{E}\left[\mathbf{S}_{N,l}^{(0)}\mathbf{S}_{N,l'}^{(0)^H}\right] = \lim_{N\to\infty}\mathbf{P}_N^{(l,l')}(0,0) = S_{\mathbf{e}}^{((l-l')/Q)}(e^{2i\pi(\alpha_0+l/Q)}) \quad (20)$$

$$\Gamma_c^{(l,l')} = \lim_{N\to\infty}N\mathbb{E}\left[\mathbf{S}_{N,l}^{(0)}\mathbf{S}_{N,l'}^{(0)^T}\right] = \lim_{N\to\infty}\mathbf{P}_{c,N}^{(l,l')}(0,0) = S_{\mathbf{e}^{(c)}}^{(2\alpha_0+(l+l')/Q)}(e^{2i\pi(\alpha_0+l/Q)}). \quad (21)$$

Plugging (18)–(21) back into (17) yields

$$\Gamma_{\mathcal{S}}^{(l,l')} = \begin{bmatrix}\Gamma^{(l,l')} & \frac{1}{2}\Gamma^{(l,l')} & \Gamma_c^{(l,l')} & \frac{1}{2}\Gamma_c^{(l,l')}\\ \frac{1}{2}\Gamma^{(l,l')} & \frac{1}{3}\Gamma^{(l,l')} & \frac{1}{2}\Gamma_c^{(l,l')} & \frac{1}{3}\Gamma_c^{(l,l')}\\ \overline{\Gamma_c^{(l,l')}} & \frac{1}{2}\overline{\Gamma_c^{(l,l')}} & \overline{\Gamma^{(l,l')}} & \frac{1}{2}\overline{\Gamma^{(l,l')}}\\ \frac{1}{2}\overline{\Gamma_c^{(l,l')}} & \frac{1}{3}\overline{\Gamma_c^{(l,l')}} & \frac{1}{2}\overline{\Gamma^{(l,l')}} & \frac{1}{3}\overline{\Gamma^{(l,l')}}\end{bmatrix}. \quad (22)$$

Combining (22) and (16) leads to the closed-form expression for γ_b.

APPENDIX B
DEFINITION OF CYCLIC TRISPECTRA

The cyclic trispectra associated with the fourth-order cumulant function and the fourth-order conjugate cumulant function of the process $y(n)$ are defined as follows (for more information, see also [25], [31], and [32], and the references therein). Let $c_{y,4}(n,\tau_1,\tau_2,\tau_3)$ and $c_{y^{(c)},4}(n,\tau_1,\tau_2,\tau_3)$ denote the fourth-order cumulant and fourth-order conjugate cumulant of $y(n)$ at time index n and lags $[\tau_1,\tau_2,\tau_3]$, respectively. We get

$$c_{y,4}(t,\tau_1,\tau_2,\tau_3) := \text{cum}_4\left\{y(n), y(n+\tau_1), \overline{y(n-\tau_2)}, \overline{y(n-\tau_3)}\right\}$$

and

$$c_{y^{(c)},4}(t,\tau_1,\tau_2,\tau_3) := \text{cum}_4\left\{y(n), y(n+\tau_1), y(n+\tau_2), y(n+\tau_3)\right\}.$$

The fourth-order cyclic cumulant at cyclic frequency α is defined by

$$c_{y,4}^{(\alpha)}(\tau_1,\tau_2,\tau_3) = \int_{\mathbb{R}}c_{y,4}(t,\tau_1,\tau_2,\tau_3)e^{-2i\pi\alpha t}dt.$$

In the previous equation, considering the fourth-order conjugate cumulant instead of the fourth-order cumulant leads to the fourth-order conjugate cyclic cumulant at cyclic frequency α.

$$\Gamma_{\mathcal{S}}^{(l,l')} = \lim_{N\to\infty}\begin{bmatrix}\mathbf{P}_N^{(l,l')}(0,0) & \mathbf{P}_N^{(l,l')}(0,1) & \mathbf{P}_{c,N}^{(l,l')}(0,0) & \mathbf{P}_{c,N}^{(l,l')}(0,1)\\ \mathbf{P}_N^{(l,l')}(1,0) & \mathbf{P}_N^{(l,l')}(1,1) & \mathbf{P}_{c,N}^{(l,l')}(1,0) & \mathbf{P}_{c,N}^{(l,l')}(1,1)\\ \overline{\mathbf{P}_{c,N}^{(l,l')}(0,0)} & \overline{\mathbf{P}_{c,N}^{(l,l')}(0,1)} & \overline{\mathbf{P}_N^{(l,l')}(0,0)} & \overline{\mathbf{P}_N^{(l,l')}(0,1)}\\ \overline{\mathbf{P}_{c,N}^{(l,l')}(1,0)} & \overline{\mathbf{P}_{c,N}^{(l,l')}(1,1)} & \overline{\mathbf{P}_N^{(l,l')}(1,0)} & \overline{\mathbf{P}_N^{(l,l')}(1,1)}\end{bmatrix} \quad (17)$$

The corresponding cyclic trispectrum at cyclic frequency α is given by

$$S_{y,4}^{(\alpha)}(e^{2i\pi\nu_1}; e^{2i\pi\nu_2}; e^{2i\pi\nu_3}) = \int_{\mathbb{R}^3} c_{y,4}^{(\alpha)}(\tau_1, \tau_2, \tau_3) \times e^{-2i\pi(\nu_1\tau_1+\nu_2\tau_2+\nu_3\tau_3)} d\tau_1 d\tau_2 d\tau_3.$$

In order to obtain the conjugate cyclic trispectrum, we have to replace the fourth-order cyclic cumulant with the fourth-order conjugate cyclic cumulant in the previous equation.

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