IV. Reply to Comment 4

In the literature, there exist many quadratic TFDs whose kernels do not have a two–dimensional (2-D) lowpass shape but are well known for their cross-terms suppression property. One particular example is the cone-shape distribution whose Doppler-lag kernel [1, p. 146] is displayed in Fig. 1 for two values of its tuning parameter α, specifically, 0.01 and 100. By continuously varying the value of α from small to large, the kernel evolves from an allpass filter shape (i.e., the distribution is not appropriate for cross-terms suppression) to the shape displayed in the figure (i.e., the kernel removes all cross-terms, except those on the axis $\tau = 0$). *There exists no value of* α *for which this kernel has a 2-D lowpass shape*. For an arbitrary signal, the user has to select an appropriate value of α to decide the amount of cross-terms suppression.

Similarly, by continuously varying its parameter α, the BD Doppler-lag kernel shape evolves as shown in the figure of the comments. In particular, for small values of α, the kernel removes all cross-terms, except those on the axis $\nu = 0$, and for large values, the BD becomes inappropriate for cross-terms suppression, as explained in the original paper. Note that because of its zero value at the origin, the BD may cause some extra energy distortion for the auto-terms; however, this does not seem to adversly affect its time-frequency representation, as shown by the various examples provided in the paper. This is why the major interest of the paper is not only to define a new quadratic TFD with useful resolution properties but to also open a new direction of research in the design of quadratic TFDs with new criteria that are not limited by old thinking.

In short, one can say that *each kernel has its own characteristics that are, in general, different from those of the others*. This situation is very normal and expected because each kernel defines a *different* member of the quadratic class with different properties. Further details can be found in a recent tutorial on this question [3, ch. 3].

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A Fine Blind Frequency Offset Estimator for OFDM/OQAM Systems

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***Abstract*—Like other orthogonal frequency division multiplexing (OFDM) systems, OFDM systems based on offset quadrature amplitude modulation (OFDM/OQAM) are very sensitive to carrier frequency offset. In this paper, a new blind carrier frequency offset estimator is developed for OFDM/OQAM systems by exploiting the noncircularity of the received OFDM/OQAM signal. Since the received signal exhibits conjugate cyclic frequencies at twice the carrier frequency offset, the frequency estimator is designed by maximizing a cost function expressed in terms of the sample conjugate cyclocorrelations. The theoretical asymptotic (large sample) performance analysis of the proposed estimator is established. Computer simulations are presented to illustrate the performance of the estimator. It is shown that the proposed estimator is very accurate whenever it is well initialized. Therefore, the proposed estimator appears to be very well adapted for a tracking mode rather than for an acquisition mode.**

***Index Terms*—Carrier frequency offset, conjugate cyclocorrelation, estimation, OFDM system.**

I. Introduction

The orthogonal frequency division multiplexing (OFDM) system, which belongs to the family of multicarrier transmission schemes, has been developed to combat efficiently the intersymbol interference (ISI) effects on frequency-selective channels. Its main advantages are the very low computational cost [the receiver consists only of a fast Fourier transform (FFT) and, if necessary, of a more general filterbank] and the simplified equalization step [1]. During the last few years, OFDM-like techniques have received increasing attention and are currently employed in the European digital radio broadcasting (DAB), digital terrestrial TV broadcasting (DVBT), indoor wireless systems (HIPERLAN), and broadband access on twisted pair (ADSL).

However, it is well-known that OFDM-like techniques are more sensitive to carrier frequency offset than single carrier techniques [2]. The frequency offset (due to Doppler shifts and local oscillator drifts) gives rise to intercarrier interference (ICI), which dramatically degrades the performance. Therefore, removing the frequency offset at the front end of the receiver is a crucial task.

A lot of techniques based on the OFDM principle have been proposed in the literature. The structure of a standard OFDM transmitter consists of the concatenation of an IFFT transform and a guard interval and a linear modulation shaped by means of a rectangular window. As the time-frequency localization of such shaping windows is not compact, considerable research attention has recently been allocated to developing alternative modulations such as OFDM/offset quadrature amplitude modulation (OQAM), which constitutes a combination between an offset quadrature-amplitude modulation and a square-root Nyquist pulse-shaping filter (see, e.g., [3]–[9]). For OFDM/OQAM schemes,

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the guard interval is omitted, which guarantees a better spectral efficiency relative to regular OFDM schemes. However, OFDM/OQAM systems require an equalization step, which can be efficiently implemented [9].

Thus far, numerous works regarding the blind (nondata aided) estimation of the frequency offset in standard OFDM systems have been reported. Among these, the subspace-based methods have been recently proposed in [10] and [11] and shown to be equivalent to the maximum likelihood (ML) estimator in [12]. Nevertheless, the validity domain of such methods is quite restrictive since the presence of virtual subcarriers has to be assumed. In the absence of virtual subcarriers, a maximum likelihood based estimator has also been introduced under the strong assumption of a flat-fading channel [13]. However, simulation experiments show that the ML-based estimator is not quite robust to ISI-channel effects. In [14], a new blind frequency estimator is proposed by exploiting the second-order cyclostationary statistics of the oversampled received signal. This estimator represents an extension of the approach proposed in [15] to the OFDM framework and is specifically designed only for flat-fading channels [14]. However, it performs well for flat-fading channels and is just robust over multipath propagation channels. Recently, a new blind carrier frequency offset estimator for OFDM systems was introduced in [16]. This estimator can only be applied under the restrictive assumption of a noncircular input symbol constellation[1] and exploits the conjugate second-order cyclostationary statistics of the received signal [16].

A natural extension of the estimator [14], which has been designed in the context of a standard OFDM system, to an OFDM/OQAM type system is proposed in [17]. One can observe that the performance and the properties of these two estimators are close. Similar to the analysis performed in [17], we propose herein to extend and analyze the estimator [16] to OFDM/OQAM systems.

As the real and the imaginary parts of the transmitted signal in an OFDM/OQAM system are not statistically identical, such a modulating signal is intrinsically noncircular. Therefore, the estimator introduced in [16] can be extended without restriction to the considered OFDM/OQAM system. The purpose of the paper is to introduce and analyze this new estimator in the context of OFDM/OQAM systems.

As it will be seen later, the proposed estimator is equivalent to estimating the parameters of a harmonic embedded in additive noise by means of the maximization of a periodogram in the frequency domain. It is well known that the periodogram has several local maxima. Therefore, the search of the maximum of the periodogram has to be computed in two steps—i) the *coarse* acquisition step and ii) the *fine* acquisition step—which is in general iterative and initialized by the coarse step. As pointed out in [18], the first step may be affected by outliers.[2] One of the main objectives of this paper is to evaluate the asymptotic (large sample) variance of the proposed frequency offset estimator. The proposed asymptotic performance analysis only studies the behavior of the cost function around the true frequency, and it is not relevant for evaluating the performance of the first step. The performance of the coarse acquisition step will be studied experimentally by means of the probability of detecting a false peak.

Computer simulations illustrate that the proposed estimator is quite sensitive to outliers and but that it yields very good performance for the fine acquisition step whenever it is well initialized. Therefore, the proposed estimator is particularly well suited for the fine tracking mode rather than to the coarse acquisition mode.

The rest of this correspondence is organized as follows. In Section II, for the sake of simplicity, we introduce the estimator for flat-fading channels as in [17]. In Section III, the asymptotic analysis is reported in order to evaluate the performance of the fine acquisition step. In Section IV, computer simulations are presented to corroborate the theoretical performance analysis and to illustrate that the proposed estimator is quite robust to ISI-channel effects and symbol timing errors.

[1]Noncircularity translates into the condition that the expected value of the squared input symbol is nonzero.

[2]The argument corresponding to the maximum of the periodogram may be far away from the expected true value.

II. New Estimator

For an OFDM/OQAM system, the transmitted continuous-time baseband signal $x_a(t)$ can be expressed as follows:

$$x_a(t) = \sum_{q=0}^{Q-1} x_q(t)$$

where the qth signal component

$$x_q(t) := \sum_{l\in\mathbb{Z}} (a_{q,l} g_a(t - lT) + i b_{q,l} g_a(t - lT + T/2)) \times e^{2i\pi(q/Q)(t-lT)}$$

corresponds to a linear offset modulation translated to the sub-carrier q/Q. The sequences $a_{q,l}$ and $b_{q,l}$ are real-valued and belong to PAM-type modulations. Thus, the complex valued sequence $s_{q,l} = a_{q,l} + i b_{q,l}$ can be interpreted as a QAM-type modulation. Moreover, we assume that $a_{q,l}$ and $b_{q,l}$ are independently and identically distributed (i.i.d.) with unit-variance. The duration of a complete OFDM symbol that corresponds to the set $\{s_{0,l}, \ldots, s_{Q-1,l}\}$ is equal to T. Consequently, the period of each information symbol $s_{q,l}$ is $T_s = T/Q$. Furthermore, the pulse $g_a(t)$ is usually a square-root raised cosine (with roll-off ρ and built assuming the OFDM-rate $1/T$) instead of a rectangular window. Without any restriction, one can assume that the mapping $t \mapsto g_a(t)$ is time limited with the time support $[-LT_s, LT_s]$. Last, no guard interval is inserted even in the presence of a frequency-selective channel.

For the sake of simplicity, we assume first that the transmitted signal passes through a flat-fading channel as in [17]. Hence, the continuous-time baseband received signal $y_a(t)$ takes the following form:

$$y_a(t) = x_a(t) e^{2i\pi\delta f_0 t} + w_a(t) \tag{1}$$

where δf_0 is the carrier frequency offset, and $w_a(t)$ stands for the additive zero-mean Gaussian noise. Our aim is to estimate the carrier frequency offset from the sole knowledge of the symbol-rate sampled received signal.

According to (1), the discrete-time signal $y(n) := y_a(nT_s)$ can be expressed as follows:

$$y(n) = \left(\sum_{q=0}^{Q-1} x_q(n)\right) e^{2i\pi\Delta f_0 n} + w(n) \tag{2}$$

where

$$x_q(n) = \sum_{l\in\mathbb{Z}} (a_{q,l} g(n - lQ) + i b_{q,l} \tilde{g}(n - lQ)) \times e^{2i\pi(q/Q)(n-lQ)T_s}$$

$g(m) := g_a(mT_s), \tilde{g}(m) := g_a(mT_s + QT_s/2)$, and $w(n) := w_a(nT_s)$. Finally, $\Delta f_0 := (\delta f_0 T_s \bmod 1)$, where $(a \bmod b) \in [0, b)$ is the remainder after dividing a by b.

Before proceeding further, we recall that a zero-mean discrete-time stochastic process $p(n)$ is said to be unconjugate (conjugate) cyclostationary if the unconjugate (conjugate) correlation coefficients[3]

[3]The overline stands for complex conjugation.

$\mathbb{E}[p(m+n)\overline{p(n)}]$ ($\mathbb{E}[p(m+n)p(n)]$, respectively) can be expressed in terms of a Fourier series expansion, i.e.,

$$\mathbb{E}[p(m+n)\overline{p(n)}] = \sum_{k=0}^{\infty} r^{(a_k)}(m) e^{2i\pi a_k n}$$

where $F := \{a_k\}_{k\geq 0}$ is the countable set of the so-called cyclic frequencies of $p(n)$. The sequence $\{r^{(a_k)}(m)\}_{m\in\mathbb{Z}}$ denotes the cyclocorrelation sequence at cyclic frequency a_k of $p(n)$.

In [17], it has been proved that the *unconjugate* correlations of the OFDM/OQAM signal $y(n)$ are cyclostationary with cycles $F = \{q/Q\}_{q=0,\ldots,Q-1}$. In addition, it turns out that the phases of the associated unconjugate cyclocorrelations directly depend on the unknown carrier frequency offset. Based on this observation, [17] develops a frequency offset estimator whose performance is improved whenever the subcarriers are weighted with different factors. The weighting of the carriers obviously restricts the application field of this method.

We have observed that the *conjugate* correlations of the received signal are nonzero and are cyclostationary with the set of cycles $F = \{\alpha_0 + q/Q\}_{q=0,\ldots,Q-1}$, where $\alpha_0 = (2\Delta f_0 \bmod 1)$. Since Q is assumed to be known, estimating Δf_0 boils down to estimating α_0. Therefore, a frequency offset estimator can be developed by maximizing a certain cost function expressed in terms of the conjugate cyclocorrelations.

Let $r_c(n,\tau) = \mathbb{E}[y(n+\tau)y(n)]$ denote the conjugate correlation at time index n and lag τ of $y(n)$. After straightforward derivations, it turns out that

$$r_c(n,\tau) = \left(\sum_{q=0}^{Q-1} e^{2i\pi(2q/Q)(n+\tau/2)}\right) G_m(n,\tau) e^{2i\pi\alpha_0(n+\tau/2)}$$

where

$$G_m(n,\tau) := \sum_{l\in\mathbb{Z}} (g(n+\tau-lQ)g(n-lQ) - \tilde{g}(n+\tau-lQ)\tilde{g}(n-lQ)).$$

Obvious manipulations lead to (3), shown at the bottom of the page. As $g(n)$ and $\tilde{g}(n)$ are different, one can verify that $n \mapsto G_m(n,\tau)$ is not reduced to zero and is periodic with period Q. This implies that the conjugate correlation of the received signal is nonzero, cyclostationary, and takes the following generic form:

$$r_c(n,\tau) = \sum_{q=0}^{Q-1} r_c^{(\alpha_0+q/Q)}(\tau) e^{2i\pi(\alpha_0+q/Q)n} \tag{4}$$

where $r_c^{(\alpha_0+q/Q)}(\tau)$ is the conjugate cyclocorrelation at lag τ and can be expressed as

$$r_c^{(\alpha)}(\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} r_c(n,\tau) e^{-2i\pi\alpha n}. \tag{5}$$

According to (3), we obtain the expression at the bottom of the page. As soon as $|\Delta f_0| < 1/Q$, knowledge of the set F provides exactly the value of α_0. Indeed, α_0 is the sole value in the interval $[-1/Q, 1/Q)$ for which $r_c^{(\alpha+q/Q)}(\tau)$ is nonzero, whenever q is an odd integer. Thus, α_0 can be obtained based on the following estimator:

$$\alpha_0 := \arg\max_{\alpha\in[-1/Q,1/Q)} J(\alpha), \quad J(\alpha) := \sum_{q \text{ odd}} \left\| \mathbf{r}_c^{(\alpha+q/Q)} \right\|^2 \tag{6}$$

with $\mathbf{r}_c^{(\alpha)} := [r_c^{(\alpha)}(-L), \ldots, r_c^{(\alpha)}(L)]^{\mathrm{T}}$. The cost function (6) exploits directly all the second-order cyclostationary information (present at any cycle and any lag) in the received signal. By means of numerical computations, we have observed that other cost functions that exploit either only all the cycles or only all the lags are outperformed by the selected cost function. To keep the length of this paper to a minimum, we next concentrate only on the estimator associated with the cost function (6).

In practice, the conjugate cyclocorrelation vector $\mathbf{r}_c^{(\alpha)}$ has to be estimated because only N observations are available. The sample estimate of $\mathbf{r}_c^{(\alpha)}$ is obtained by dropping the limit and the mathematical expectation in (5). This leads to the sample estimate

$$\hat{\mathbf{r}}_{c,N}^{(\alpha)} := \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{z}(n) e^{-2i\pi\alpha n}$$

with $\mathbf{z}(n) := [y(n-L)y(n), \ldots, y(n+L)y(n)]^{\mathrm{T}}$. The corresponding estimate of α_0, which is denoted by $\hat{\alpha}_N$, is defined by

$$\hat{\alpha}_N := \arg\max_{\alpha\in[-1/Q,1/Q)} J_N(\alpha)$$
$$J_N(\alpha) := \sum_{q \text{ odd}} \left\| \hat{\mathbf{r}}_{c,N}^{(\alpha+q/Q)} \right\|^2.$$

The proposed estimator allows an interesting interpretation as a spectral estimation problem. Consider the following zero-mean process: $\mathbf{e}(n) = \mathbf{z}(n) - \mathbb{E}[\mathbf{z}(n)]$. Equation (4) leads to

$$\mathbf{z}(n) = \sum_{q \text{ odd}} \mathbf{r}_c^{(\alpha_0+q/Q)} e^{2i\pi(\alpha_0+q/Q)n} + \mathbf{e}(n). \tag{7}$$

Thus, $\mathbf{z}(n)$ corresponds to a sum of constant amplitude multivariate harmonics embedded in the additive noise $\mathbf{e}(n)$. Estimating α_0 reduces to the problem of estimating the parameters of a number of multivariate harmonics. Note also that the cost function $J_N(\alpha)$ can be rewritten as follows:

$$J_N(\alpha) = \sum_{q \text{ odd}} \left\| \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{z}(n) e^{-2i\pi(\alpha+q/Q)n} \right\|^2$$

which represents a sum of weighted periodograms. This implies that the cost function $J_N(\alpha)$ consists of a number of spectral lines at cycles $\alpha_0 + q/Q$ (where q is an odd) and a ground-like level induced by noise.

$$\begin{aligned} r_c(n,\tau) &= QG_m(n,\tau)e^{2i\pi\alpha_0(n+\tau/2)}, && \text{if } n = (-\tau/2 \bmod Q/2) \\ r_c(n,\tau) &= 0, && \text{otherwise.} \end{aligned} \tag{3}$$

$$\begin{aligned} r_c^{(\alpha_0+q/Q)}(\tau) &= -2G_m((Q-\tau)/2,\tau)e^{2i\pi(q/(2Q)+\alpha_0/2)\tau}, && \text{for } q \text{ odd} \\ r_c^{(\alpha_0+q/Q)}(\tau) &= 0, && \text{for } q \text{ even.} \end{aligned}$$

It is well known that the periodogram $J_N(\alpha)$ has several local maxima. Therefore, the computation of the estimate $\hat{\alpha}_N$ has to be performed in two steps. In the first coarse acquisition step, the objective is to detect the peaks, and the function $J_N(\alpha)$ is evaluated on an FFT grid. In the second, a fine acquisition step carried out by a gradient minimization algorithm of $J_N(\alpha)$, initialized at the estimate provided by the first step, is performed in order to obtain the estimate $\hat{\alpha}_N$.

Both steps have to be analyzed separately. The first step, which is connected to the so-called *outliers* effect [18], is theoretically difficult to analyze. A standard and largely adopted criterion to measure the performance of the first step is the probability of right detection of the peak around a given harmonic. This probability is evaluated in Section IV. The analysis of the second step boils down to an asymptotic analysis, which is performed in Section III.

III. Asymptotic Analysis

We herein determine the convergence rate of the estimates and report closed-form expressions for the corresponding asymptotic covariances.

Without any loss of generality, the noise $\mathbf{e}(n)$ is assumed to satisfy the standard mixing condition [19], which resumes to absolute summability of its cumulants: a condition that is satisfied by all finite memory processes. Exploiting the results of previous works about harmonic retrieval in additive noise (see [16] and the references therein), we obtain the following results:

Theorem 1: Asymptotically as $N \to \infty$, the frequency estimate $\hat{\alpha}_N$ converges in distribution to a normal distribution

$$N^{3/2}(\hat{\alpha}_N - \alpha_0) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \gamma).$$

Note that the convergence rate is proportional to $1/N^3$, which is a result that corroborates the standard results encountered in most harmonic retrieval problems. The closed-form expression for the asymptotic covariance γ is detailed in the following theorem (see [16]).

Theorem 2: The asymptotic covariance γ is given by

$$\gamma = \frac{3}{\pi^2} \frac{\sum_{q,q' \text{ odd}} \mathbf{R}_q^{\mathrm{H}} \mathbf{\Psi}_{q,q'} \mathbf{R}_{q'}}{\left(\sum_{q \text{ odd}} \mathbf{R}_q^{\mathrm{H}} \mathbf{R}_q\right)^2}$$

with

$$\mathbf{R}_q := \begin{bmatrix} \mathbf{r}_c^{(\alpha_0+q/Q)} \\ \overline{\mathbf{r}_c^{(\alpha_0+q/Q)}} \end{bmatrix} \quad \text{and}$$

$$\mathbf{\Psi}_{q,q'} := \begin{bmatrix} \Gamma^{(q,q')} & -\Gamma_c^{(q,q')} \\ -\overline{\Gamma_c^{(q,q')}} & \overline{\Gamma^{(q,q')}} \end{bmatrix}$$

where

$$\Gamma^{(q,q')} := \lim_{N\to\infty} N\mathbb{E}\left[\delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+q/Q)} \cdot \delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+q'/Q)\mathrm{H}}\right]$$
$$\Gamma_c^{(q,q')} := \lim_{N\to\infty} N\mathbb{E}\left[\delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+q/Q)} \cdot \delta\hat{\mathbf{r}}_{c,N}^{(\alpha_0+q'/Q)\mathrm{T}}\right]$$

and

$$\delta\hat{\mathbf{r}}_{c,N}^{(\alpha)} := \left(\hat{\mathbf{r}}_{c,N}^{(\alpha)} - \mathbf{r}_c^{(\alpha)}\right).$$

To take advantage of this result, we need more explicit expressions for Γ and Γ_c, which are the asymptotic covariance of the sample estimates corresponding to the vector of cyclocorrelations. First of all, we derive these matrices in terms of the statistics of the disturbance occurring in the equivalent harmonic retrieval problem, i.e., $\mathbf{e}(n)$. According to [16], we have

$$\Gamma^{(q,q')} = S_{\mathbf{e}}^{((q-q')/Q)}\left(e^{2i\pi(\alpha_0+q/Q)}\right)$$
$$\Gamma_c^{(q,q')} = S_{\mathbf{e(c)}}^{(2\alpha_0+(q+q')/Q)}\left(e^{2i\pi(\alpha_0+q/Q)}\right)$$

where $f \mapsto S_{\mathbf{e}}^{(\alpha)}(e^{2i\pi f})$, and $f \mapsto S_{\mathbf{e}(c)}^{(\alpha)}(e^{2i\pi f})$ denote the cyclospectra of $\mathbf{e}(n)$ at cyclic frequency α with respect to its unconjugate and conjugate autocorrelation functions, respectively.

We now determine the second-order cyclic statistics of the disturbance $\mathbf{e}(n)$. Define $\mathbf{d}_T(e^{2i\pi f}) := [e^{-2i\pi Tf}, \ldots, e^{2i\pi Tf}]^{\mathrm{T}}$. We assume that the source $\{s_{q,l}\}$ is Gaussian. To be perfectly accurate, a term depending on the kurtosis of the source will have to be added in the sought expressions. However, since the cumulants of order higher than two are numerically very weak, the terms depending on the kurtosis of the source may be neglected, and consequently, the Gaussian source approximation holds. Based on the results reported in [16], it turns out that

$$\begin{aligned} &S_{\mathbf{e}}^{((q-q')/Q)}\left(e^{2i\pi(\alpha_0+q/Q)}\right) \\ &= \sum_{m=0}^{Q-1}\int_0^1 S_y^{(m/Q)}(e^{2i\pi f}) S_y^{((q-q'-m)/Q)} \\ &\cdot \left(e^{2i\pi(\alpha_0+q/Q-f)}\right) \\ &\cdot \mathbf{d}_T(e^{2i\pi f})\left[\mathbf{d}_T^{\mathrm{H}}\left(e^{2i\pi(f-m/Q)}\right)\right. \\ &\left.+ \mathbf{d}_T^{\mathrm{H}}\left(e^{2i\pi(\alpha_0+(q'+m)/Q-f)}\right)\right] df \end{aligned}$$

and

$$\begin{aligned} &S_{\mathbf{e(c)}}^{(2\alpha_0+(q+q')/Q)}\left(e^{2i\pi(\alpha_0+q/Q)}\right) \\ &= \sum_{m=0}^{Q-1}\int_0^1 S_{y^{(c)}}^{(\alpha_0+m/Q)}(e^{2i\pi f}) S_{y^{(c)}}^{(\alpha_0+(q-q'-m)/Q)} \\ &\cdot \left(e^{2i\pi(\alpha_0+q/Q-f)}\right) \\ &\cdot \mathbf{d}_T(e^{2i\pi f})\left[\mathbf{d}_T^{\mathrm{T}}\left(e^{2i\pi(\alpha_0+m/Q-f)}\right)\right. \\ &\left.+ \mathbf{d}_T^{\mathrm{T}}\left(e^{2i\pi(f+(q'-m)/Q)}\right)\right] df \end{aligned}$$

where $S_y^{(\alpha)}(e^{2i\pi f})$ and $S_{y^{(c)}}^{(\alpha)}(e^{2i\pi f})$ denote the cyclic spectra associated with the unconjugate and conjugate autocorrelations of $y(n)$, respectively.

The next step resumes finding more suitable expressions for these received signal cyclospectra. The derivations of S_y and $S_{y^{(c)}}$ no longer exploit previous works. Indeed, these calculations are specific for each problem. After rather simple but tedious calculations, we obtain

$$\begin{aligned} S_y^{(q/Q)}(e^{2i\pi f}) &= \sum_{n=0}^{Q-1}\sum_{k\in\mathbb{Z}} G_p(n, kQ) \\ &\quad \times e^{-2i\pi[(f-\alpha_0/2)kQ+(q/Q)n]} + \sigma^2 \\ S_{y^{(c)}}^{(\alpha_0+q/Q)}(e^{2i\pi f}) &= -2\sum_{\tau\in\mathbb{Z}} G_m((Q-\tau)/2, \tau) \\ &\quad \times e^{-2i\pi(f-\alpha_0/2-q/(2Q))\tau} \end{aligned}$$

with

$$\begin{aligned} G_p(n,\tau) = \sum_{l\in\mathbb{Z}} &(g(n+\tau-lQ)g(n-lQ) \\ &+ \tilde{g}(n+\tau-lQ)\tilde{g}(n-lQ)) \end{aligned}$$

and

$$\sigma^2 := \mathbb{E}[|w(n)|^2].$$

As the expressions of the asymptotic covariances can no longer be simplified, we resort to numerical computations of the asymptotic covariances to analyze the theoretical results.

TABLE I
FALSE DETECTION OCCURRENCE (IN PERCENT) FOR $Q = 4$

SNR	0	5	10	15	20
$M = 256$	99	78	38	15	10
$M = 512$	97	30	0.4	0	0
$M = 1024$	95	8	0	0	0

TABLE II
FALSE DETECTION OCCURRENCE (IN PERCENT) FOR $Q = 16$

SNR	0	5	10	15	20
$M = 128$	99	92	71	53	47
$M = 256$	98	70	15	2	1
$M = 512$	95	8	0	0	0

IV. NUMERICAL EVALUATIONS

The following simulation parameters are used throughout this section. The roll-off factor of the shaping filter $g_a(t)$ is equal to $\rho = 0.2$, and the noise $w(n)$ is white with the variance $\sigma^2 = \mathbb{E}[|w(n)|^2]$. The number of OFDM symbols is denoted by $M = N/Q$, and the signal-to-noise ratio (SNR) is defined as SNR $:= 2/\sigma^2$. We average the experimental results over $MC = 500$ independent Monte Carlo runs.

In Tables I and II, we display the probability of detecting a false peak versus M and SNR, assuming two scenarios $Q = 4$ and $Q = 16$, respectively. We assume that the coarse search leads to a right detection if it selects the closest frequency of the grid to the sought cyclic frequency. The estimator succeeds in detecting the right peak with almost no error, assuming much lower M or SNR. Moreover, for greater values of Q, a better detection performance can be observed. Nevertheless, our estimation procedure requires large observation windows in order to provide reliable coarse estimates.

In the next experiments, we compare the theoretical and experimental performance of the proposed estimator. We also compare our estimate with the only prior estimate developed in an OFDM/OQAM [17]. As the estimate introduced in [17] is not robust to the frequency-selective propagation channel, we focus on the AWGN channel. Moreover, we have noticed that the performance of his estimate is disastrous without carrier weighting and cannot then be utilized for any purpose. Therefore, Bölcskei's estimate has been carried out by using the carrier weighting of [17], whereas our estimate works whatever the weighting and especially without weighting. The analytical expression of the normalized mean-square error (MSE) is defined as $\gamma/(\alpha_0^2 N^3)$. The dashed curve (with star point) is obtained for the entire algorithm (the first step followed by the second one). The dashed-curve corresponds to the estimate obtained when the second step is initialized with a good estimate. The solid line refers to the MSE obtained by means of asymptotic analysis. Fig. 1 depicts the MSE versus SNR for $Q = 4$. As expected, the effect due to outliers appears only during the coarse search and appears at SNR levels below 10 dB. Furthermore, as soon as the estimate is well initialized, the experimental and theoretical performance are in quite good agreement, except at very low SNR. This mismatch at low SNR can be reduced by considering more data. A floor effect at large SNR values occurs. This is due to the fact that we have observed that γ is non-null, even in the noiseless case. We remark that our estimate outperforms Bölcskei's estimate as soon as the outliers effect vanishes.

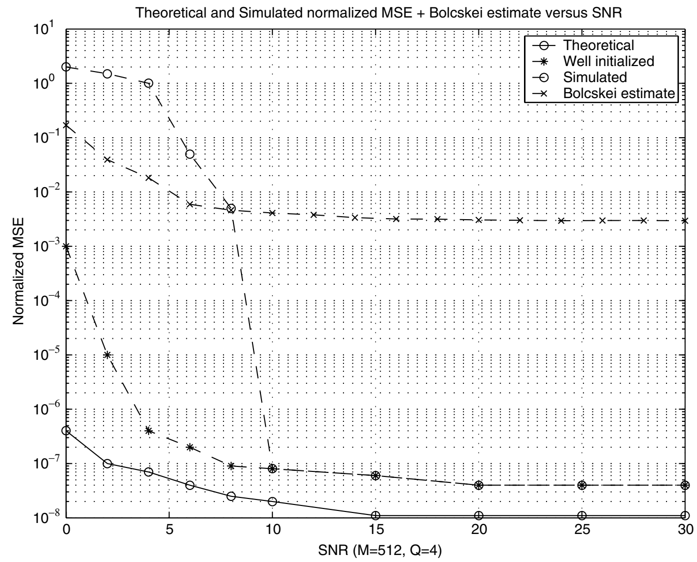

Fig. 1. Practical MSE versus SNR $(\mathrm{M} = 512)$.

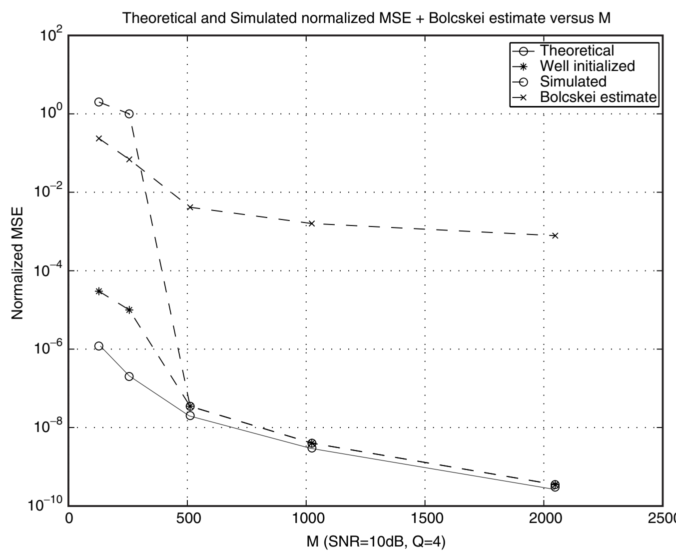

Fig. 2. Practical MSE versus M (SNR $= 10$ dB).

Fig. 2 depicts the MSE versus M for $Q = 4$. The difference between the experimental and theoretical performance vanishes whenever the number of symbols is large enough. Furthermore, the outliers vanish whenever M takes on sufficiently large values. Once again, we observe that our estimate is much better than that of Bölcskei for M sufficiently large.

Now, we consider the scenario when the transmitted signal passes through an unknown frequency-selective channel. The received signal is expressed as follows:

$$y_a(t) = \sum_{k=1}^{K} \lambda_k x_a(t - \tau_k).$$

Two cases are studied: In the first one, only a timing error occurs, i.e., $K = 1$ and $\tau_1 = \tau$. The timing error τ is assumed to be smaller than the sampling period $T_s = T/Q$. In the second case, we consider an actual non-flat-fading channel with $K = 5$. The complex amplitudes $\{\lambda_k\}_{k=1,\ldots,5}$ are Gaussian distributed, and the time delays

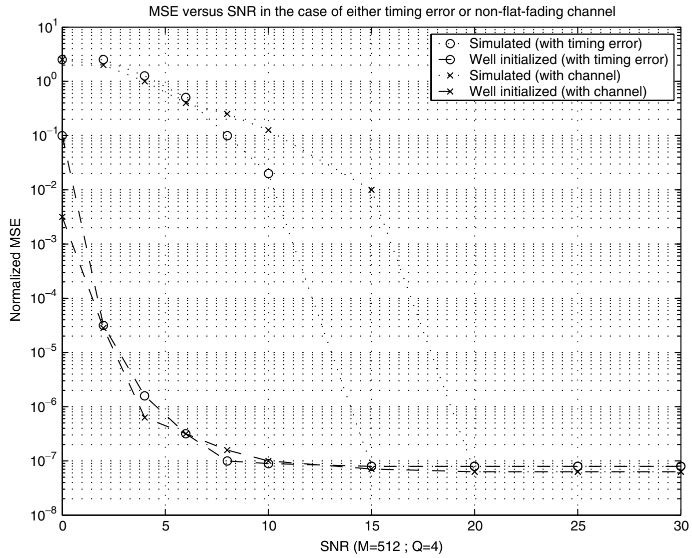

Fig. 3. MSE versus SNR: Timing error case or frequency-selective channel case.

$\{\tau_k\}_{k=1,\ldots,5}$ are uniformly distributed in $[0, 3T_s]$. In a standard OFDM scheme with guard interval, the delay spread is assumed to be smaller than the OFDM symbol duration in order to ensure a reasonable length for the guard interval. Therefore, our assumption concerning the delay spread is not restrictive.

In Fig. 3, we plot the MSE versus SNR for the flat-fading channel (circle point) and for the frequency-selective channel (cross point). The timing error corresponding to the flat-fading channel and the taps of the frequency-selective channel are modified at each Monte Carlo trial. We observe that the estimator is quite robust to timing errors and time-dispersive propagation effects. In the frequency-selective channel case, the performance is worse than in the flat-fading channel framework, and the SNR threshold beyond which no outliers occur increases in the case of the frequency-selective channel. Thus, the coarse acquisition step is more sensitive to the presence of time-dispersive channel. In contrast, the fine acquisition step is robust.

According to the previous numerical results, one may envisage at least two strategies for designing (semi-)blind synchronizers for realistic OFDM/OQAM systems. As the proposed estimator exhibits poor performance in the coarse acquisition step, the first acquisition step may be achieved by means of a different estimator. First, a good candidate for coarse acquisition is the blind estimator introduced in [17], which appears to be less sensitive to outliers effects (cf.[17, Figs. 2 and 3] and Fig. 1 and 2 herein). In contrast, the estimator [17] is much less powerful for the second step (fine acquisition) since its convergence rate is equal to $1/N$ (see [20][4] and Fig. 2). Therefore, the fine acquisition step can be performed via the proposed estimator in order to improve strongly the accuracy of the estimate provided by the coarse acquisition step. Second, a semi-blind approach can be also envisaged: a data-aided estimator relying on a small training sequence inserted in each frame for obtaining a good coarse estimate, which may be subsequently refined by the proposed blind estimator. Finally, one can observe that since a good initialization is necessary, the proposed blind estimator is also particularly adapted to the tracking mode.

[4]In [20], the convergence rate for the estimator proposed in [15] has been evaluated. As the estimator introduced in [17] is a natural extension of that one of [15], similar derivations can be done and lead to the same convergence rate.

V. Conclusion

We have investigated a new blind frequency offset estimator for OFDM/OQAM modulation. The proposed estimator performs well and is quite robust over frequency-selective channels. Nevertheless, this estimator requires quite a large observation window in order to be provided with proper initialization. Therefore, this estimator is rather well suited for a fine search (tracking mode) but not for a coarse search (acquisition mode). The asymptotic performance of this estimator is established, and computer simulations confirm all the theoretical assertions.

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