

Channel estimation and Superresolution in UWB system

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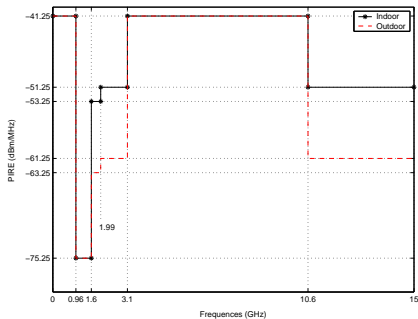
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Outline

- 1 UWB system
 - Impulse Radio
 - Multi-band
 - Channel Model
- 2 Channel estimation
 - Cramer-Rao Bound
 - Existing estimates
 - Comparison
- 3 Superresolution

Introduction

Digital communications system satisfies the following spectral mask :



Interest

- Spread spectrum technique
- Localization

Techniques

Approaches

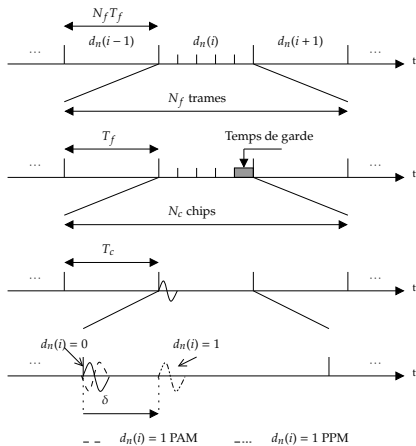
- Impulse Radio (IR)
- Multi-band (MB)

We hereafter focus on Impulse-Radio technique

- Pierce and Hopper 1952
- Winthington and Fullerton 1992
- Win and Scholtz 1993

IR-UWB transmit signal

- Time-Hopping (TH) IR-UWB signal associated with user n



Data stream

$$s(t) = \sum_{i=0}^{M-1} d_i b(t - iN_f T_f)$$

where

- M is the number of transmit symbols
- $\mathbf{d} = [d_0, \dots, d_{M-1}]$ belongs to PAM
- N_f is the number of frame per symbol
- T_f is the duration of each frame

Superframe structure

The super frame composed by N_f frames is structured as follows

$$b(t) = \sum_{j=0}^{N_f-1} g(t - jT_f - \tilde{c}_j T_c)$$

where

- T_c is the chip duration
- N_c is the number of chips in one frame
- Time-hopping code in the j^{th} frame is given by $\tilde{c}_j \in \{0, \dots, N_c - 1\}$
- $g(t)$ is the mono-cycle with the temporal support $[0, T_g]$

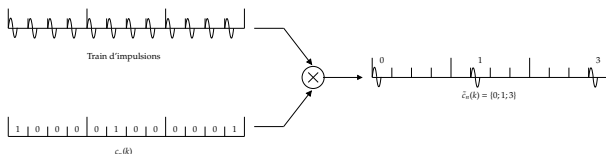
Developed code

For each frame j , let $\mathbf{c}_j = [c_j(0), \dots, c_j(N_C - 1)]$ defined as follows

$$c_j(i) = \begin{cases} 1 & \text{if } i = \tilde{c}_j \\ 0 & \text{otherwise} \end{cases}$$

Then $\mathbf{c} = [\mathbf{c}_0, \dots, \mathbf{c}_{N_f-1}] = [c(0), \dots, c(N_f N_C - 1)]$

$$s(t) = \sum_{i=0}^{M-1} d_i \sum_{j=0}^{N_f N_C - 1} c(j) g(t - jT_C - iN_f T_f)$$



- Status of the chip (occupied/free) outside $g(t)$
- Le Martret & Giannakis 2002

Channel model

- Multi-path random channel
- Molish 2003

Impulse response

$$h(t) = \sum_{k=1}^{N_p} A_k \delta(t - \tau_k)$$

where

- A_k is the attenuation associated with the k^{th} -path
- τ_k is the delay associated with the k^{th} -path

Statistical channel model

- We focus on one cluster model

Statistical model

$$p(\tau_k | \tau_{k-1}) = \lambda e^{-\lambda(\tau_k - \tau_{k-1})}$$

$$A_k = \underbrace{(p_k \cdot b_k)}_{a_k} e^{-\tau_k / \gamma}$$

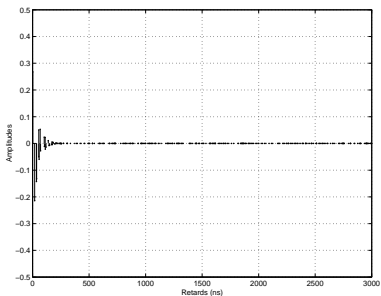
where

- a_k independent of τ_n^k
- p_k binary variable
- b_k log-normal variable

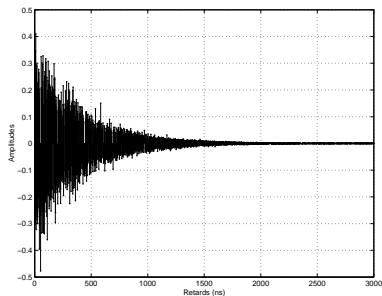
λ and γ are both deterministic parameters

Deterministic parameters

- λ is the path density
- γ is the RMS delay spread (i.e., length of impulse response)



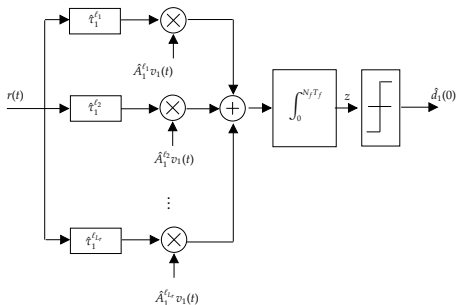
$$\lambda = 0.1 \text{ ns}^{-1} \text{ and } \gamma = 20 \text{ ns}$$



$$\lambda = 1 \text{ ns}^{-1} \text{ and } \gamma = 200 \text{ ns}$$

Receiver

- Rake receiver (for sake of simplicity)
- Correlation with the template $b(t) = \sum_{j=0}^{N_f N_c - 1} c_j g(t - jT_c)$ synchronized at each path



Path estimation is necessary

Fisher Information Matrix

$$J_{A_l, A_k} = \frac{2}{N_0} f_1^{(k,l)}, J_{A_l, \tau_k} = -\frac{2A_k}{N_0} f_2^{(l,k)}, J_{\tau_l, \tau_k} = \frac{2A_k A_l}{N_0} f_3^{(k,l)}$$

where

$$\begin{aligned} f_1^{(k,l)} &= \mathbb{E}_{\mathbf{d}} \left[\int s(t - \tau_k) s(t - \tau_l) dt \right] \\ f_2^{(k,l)} &= \mathbb{E}_{\mathbf{d}} \left[\int s(t - \tau_k) s'(t - \tau_l) dt \right] \\ f_3^{(k,l)} &= \mathbb{E}_{\mathbf{d}} \left[\int s'(t - \tau_k) s'(t - \tau_l) dt \right] \end{aligned}$$

with

- $s'(t) = ds(t)/dt$ and $\mathbb{E}_{\mathbf{d}}[\phi(\mathbf{d})] = \phi(\mathbf{d})$ if \mathbf{d} is a known sequence

↪ CRB for DA scheme and MCRB for NDA scheme

State-of-the-Art

- 1 Laurenti (September 2004) : one path
- 2 Huang (June 2004) : non-overlapping context (i.e., signal echoes are orthogonal)

$$f_m^{(k,l)} = 0 \quad \text{if} \quad k \neq l$$

- 3 Zhang (June 2004) : overlapping taken into account (but no closed-form expression for FIM)

Questions

- Non-overlapping assumption does not hold in realistic situation ?
- Closed-form expressions for $f_m^{(k,l)}$ even when $k \neq l$

Non-overlapping case

Straightforward derivations yield

$$\text{CRB}_{\text{DA}}(A_l) = \text{MCRB}_{\text{NDA}}(A_l) = \frac{N_0}{MN_f} \frac{E_3}{2(E_1 E_3 - E_2^2)}$$

$$\text{CRB}_{\text{DA}}(\tau_l) = \text{MCRB}_{\text{NDA}}(\tau_l) = \frac{N_0}{MN_f} \frac{E_1}{2A_l^2(E_1 E_3 - E_2^2)}$$

with $E_1 = \int g(t)^2 dt$, $E_2 = \int g(t)g'(t)dt$, and $E_3 = \int g'(t)^2 dt$

Remarks

- ↪ In DA scheme, performance does not depend on the training sequence
- ↪ Same expression in the context of single-path (when $N_p = 1$)

Overlapping case

Let

- $\Delta_{\tau_{k,l}} = \tau_k - \tau_l = Q_{k,l}N_f T_f + q_{k,l}T_c + \varepsilon_{k,l}$ with the integer parts $Q_{k,l}$ and $q_{k,l}$, and the remainder $\varepsilon_{k,l}$

Main result

$$\begin{aligned} f_m^{(k,l)} &= M(C(q)\mathcal{A}_m(\varepsilon) + C(q+1)\mathcal{A}_m(\varepsilon - T_c)) \\ &+ D(q)\mathcal{B}_m(\varepsilon) + D(q+1)\mathcal{B}_m(\varepsilon - T_c) \end{aligned}$$

with

- $$C(q) = \sum_{j=0}^{N_f N_c - q - 1} c(j)c(j+q), \quad D(q) = \sum_{j=0}^{q-1} c(j)c(j-q)$$

- $$\mathcal{A}_m(\varepsilon) = \frac{1}{M} \sum_{i=0}^{M-1} \mathbb{E}_{\mathbf{d}}[d_{-Q-1+i}d_i]r_m(\varepsilon), \quad \mathcal{B}_m(\varepsilon) = \frac{1}{M} \sum_{i=0}^{M-1} \mathbb{E}_{\mathbf{d}}[d_{-Q+i}d_i]r_m(\varepsilon)$$

- $r_1(t) = g(t) \star g(-t), r_2(t) = g'(t) \star g(-t), r_3(t) = g'(t) \star g'(-t)$

Comments

- Code collisions plays an important role.
- The more $f_m^{k,l}$ (for $k \neq l$) is high, the more the CRB is high
- If $\varepsilon \in [T_g, T_c - T_g]$, there is no overlapping
- The more the path is dense, the more the CRB taking into account the overlapping is larger than the (simplified) CRB
- Deleuze & Ciblat & Le Martret (July 2004)

Average CRB (I)

$$\mathbb{E}_{\mathbf{x}}[\text{CRB}(\mathbf{x})] = \mathbb{E}_{\mathbf{x}}[\mathbf{J}(\mathbf{x})^{-1}] \geq (\mathbb{E}_{\mathbf{x}}[\mathbf{J}(\mathbf{x})])^{-1}$$

Simplified expressions for \mathcal{A} , \mathcal{B} , \mathcal{C} , \mathcal{D} by averaging over

- *symbol sequence*
- *time-hopping code*

↪ *In DA scheme, average CRB over all possible training sequences*

↪ *In NDA scheme, MCRB is considered*

Average CRB (II)

- $\{d(i)\}_i$ i.i.d. symbols belonging to 2-PAM

Result

$$\mathbb{E}_{\mathbf{d}}[\mathcal{A}_m(\varepsilon)] = \delta_{Q,-1} r_m(\varepsilon), \quad \mathbb{E}_{\mathbf{d}}[\mathcal{B}_m(\varepsilon)] = \delta_{Q,0} r_m(\varepsilon)$$

- \mathbf{c}_j is the realization of i.i.d. random vector whose each component admits the following distribution
 $p(\mathbf{c}) = ((N_c - 1)\delta(\mathbf{c}) + \delta(\mathbf{c} - 1))/N_c$.

Result

$$\left\{ \begin{array}{ll} \mathbb{E}_{\mathbf{c}}[\mathcal{C}(q)] = \frac{N_f N_c - q}{N_c^2} & \text{if } q \neq 0 \\ \mathbb{E}_{\mathbf{c}}[\mathcal{C}(0)] = N_f & \text{if } q = 0 \end{array} \right\}, \quad \left\{ \begin{array}{ll} \mathbb{E}_{\mathbf{c}}[\mathcal{D}(q)] = \frac{q}{N_c^2} & \text{if } q \neq N_f N_c \\ \mathbb{E}_{\mathbf{c}}[\mathcal{D}(N_f N_c)] = N_f & \text{if } q = N_f N_c \end{array} \right.$$

Maximum Likelihood

- Lottici & Andrea & Mengali 2002
- No overlapping context
- Simulations done in a non-overlapping context
- ML carried out in DA and NDA schemes
 - DA scheme : derivations based on likelihood (for PAM or PPM)
 - NDA scheme : derivations based on true likelihood at low SNR (for PPM)

Algorithm

$$J_{\text{DA}}(\tau) = \frac{1}{ME_b} \sum_{i=0}^{M-1} z_i(\tau)$$

with $z_i(\tau, d_i) = d_i(r(t) \star b(-t))|_{t=iN_f T_f + \tau}$

- Localizations of peaks provide $\hat{\tau}$
- Magnitudes of peaks provide \hat{A}

Undersampling based method (I)

- Maravic & Vetterli 2003
- DA scheme
- Undersampling at period $T_s \gg T_p$ preceded by Anti-Aliasing Filter

Let $\tilde{r}(t)$ the noiseless receiver signal at the output of AAF

$$\tilde{R}(m) = \text{F.T.}(t \mapsto \tilde{r}(t))|_{f=mf_0} = \sum_{k=1}^{N_p} A_k \tilde{S}(m) e^{-2i\pi\tau_k mf_0}$$

then

$$\tilde{R}_s(m) = \tilde{R}(m) / \tilde{S}(m) = \sum_{k=1}^{N_p} A_k z_k^m$$

with $z_k = e^{-2i\pi\tau_k f_0}$

Undersampling based method (II)

$$\mathbf{R} = \begin{bmatrix} \tilde{R}_s(0) & \tilde{R}_s(1) & \cdots & \tilde{R}_s(N_p - 1) \\ \tilde{R}_s(1) & \tilde{R}_s(2) & \cdots & \tilde{R}_s(N_p) \\ \vdots & \vdots & & \vdots \\ \tilde{R}_s(N_p - 1) & \tilde{R}_s(N_p) & \cdots & \tilde{R}_s(2N_p - 2) \end{bmatrix} \Leftrightarrow [\mathbf{R}]_{\ell, \ell'} = \sum_{k=1}^{N_p} A_k z_k^{\ell + \ell'}$$

Then

$$\mathbf{R} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^H \quad \text{with} \quad \mathbf{V} = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ z_1^{N_p-1} & \cdots & z_{N_p}^{N_p-1} \end{bmatrix}$$

Undersampling based method (III)

Shift invariance

$$\bar{V} = \underline{V} \text{diag}([z_1, \dots, z_{N_p}])$$

where \bar{V} and \underline{V} denote the omission of the first and last row of V respectively

Then it exists a vector \mathbf{x}_k such that

$$\bar{V}\mathbf{x}_k = z_k \underline{V}\mathbf{x}_k$$

$\rightsquigarrow z_k$ is a generalized eigenvalue of (\bar{V}, \underline{V})

Algorithm

For any k , z_k is the root of the polynomial

$$P(s) = \det(\bar{V} - s\underline{V})$$

This obviously provides $\hat{\tau}$ and \hat{A}

First-order cyclostationarity based method (I)

- Luo & Giannakis 2004
- Asymmetric PAM ($d_i \in \{-1, \theta\}$)
- ISI-less context (delay spread $<$ guard-time)

$$r(t) = \sum_{i=0}^{M-1} d_i b_r(t - \tau_1 - iN_f T_f) \quad \text{with} \quad b_r(t) = \sum_{k=1}^{N_p} A_k b(t - \Delta\tau_{k,1})$$

If ISI-less, $\{b_r(t - \tau_1 - iN_f T_f)\}_i$ is a orthogonal set and thus $b_r(t)$ is a square-root Nyquist filter.

Problem

- *Optimal receiver is the matched filter $b_r(-t)$ shifted by τ_1*
- *Knowledge of $b_r(t)$ and τ_1 is needed*

First-order cyclostationarity based method (II)

$$\mathbb{E}[r(t)] = \frac{\theta - 1}{2} \sum_{i=0}^{M-1} b_r(t - \tau_1 - iN_f T_f)$$

The cyclostationary mean contains information about $b_r(t)$ and τ_1

Algorithm

If τ_1 is associated with the strongest path, then

$$\hat{\tau}_1 = \arg \max_{\tau \in [0, N_f T_f]} \left| \int_0^{2N_f T_f} \widehat{\mathbb{E}[r(t)]} b(t - \tau) dt \right|$$

and

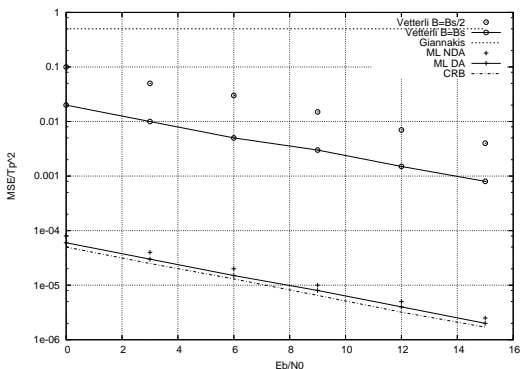
$$\hat{b}_r(t) = \frac{2}{\theta - 1} \widehat{\mathbb{E}[r(t + \hat{\tau}_1)]}, \quad \text{for } t \in [0, N_f T_f)$$

Non-overlapping case

Set-up

- $T_p = 1\text{ns}$, $T_c = 2T_p$, $N_c = 10$, and $N_f = 10$, $T_s = 200\text{ns}$, $M = 100$
- $\tau = [5T_p, 10T_p, 15T_p]$ and $\mathbf{A} = [0.73, 0.67, 0.35]$

Such assumptions ensure the absence of overlapping

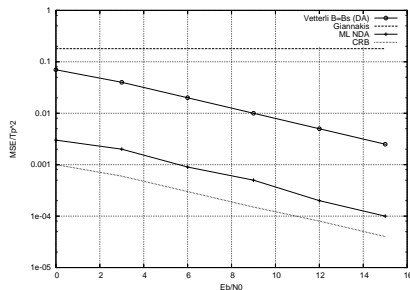


Overlapping case

Set-up

- $\tau = \{kT_p/2\}_{k=1, \dots, 20}$
- \mathbf{A} obeys a normalized exponential decreasing profile

Such assumptions ensure the presence of overlapping



↪ ML non optimal in overlapping case

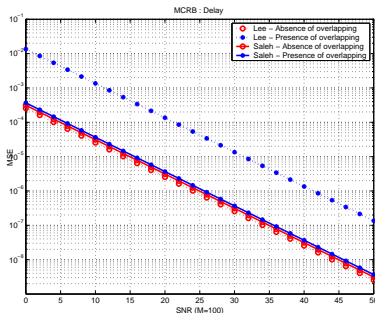
Comparison

Question

Is there overlapping or not in realistic channel ?

Two statistical models :

Molish ($\lambda = 0.2\text{ns}^{-1}$, $\gamma = 20\text{ns}$) and Lee ($\lambda = 2\text{ns}^{-1}$, $\gamma = 5\text{ns}$)



↪ If path density is high, the non-overlapping model does not hold

Definition

- *The superresolution is the smallest gap between two delays that we are able to distinguish from*
- The Cramer-Rao Bound $\text{CRB}(\tau)$ is the smallest mean square error that we may reach when the value of the sought delay is τ

Superresolution definition

The superresolution $\tau_{\text{res.}}$ satisfies the following equation

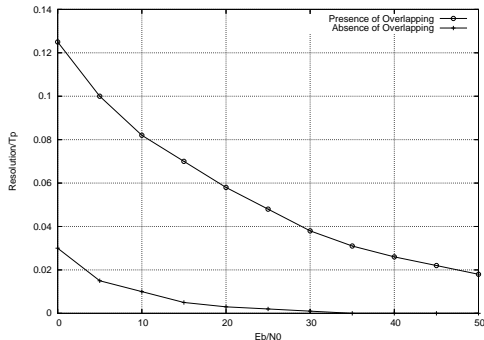
$$\tau_{\text{res.}} = \sqrt{\text{CRB}(\tau_{\text{res.}})}$$

- When τ decreases, the overlapping increases
- To evaluate accurately the superresolution, we need the closed-form expression of $\text{CRB}(\tau)$ in overlapping case

Superresolution versus SNR

Set-up

- $\tau = [0, \tau]$, $\mathbf{A} = [1, 0.5]$, and $M = 100$

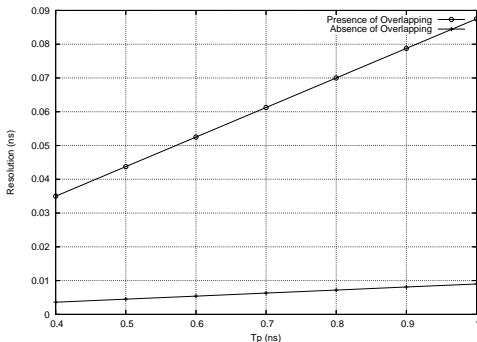


↔ Non-overlapping is too optimistic and does not make sense

Superresolution versus T_p

Set-up

- $E_b/N_0 = 10\text{dB}$ and $M = 100$



\rightsquigarrow Resolution proportional to T_p

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