TELECOM204

"Deterministic Multiple Access Techniques"

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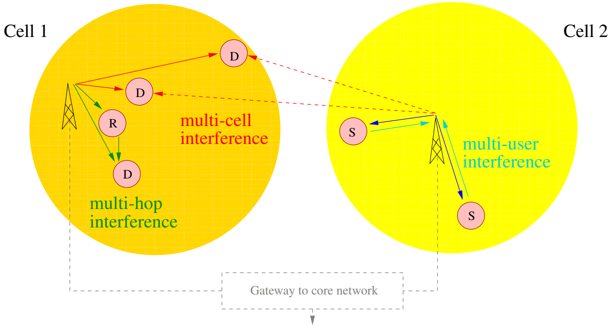
Interference issues

Cellular network (3G/4G/5G)



- Several information flows to manage
- Different kinds of interference: multi-hop (green), multi-cell (red), multi-user “uplink/downlink” (cyan/blue)

$\Rightarrow$ **Multi-User Interference (MUI)**

Toy example 1

- One link of interest (1TX $\rightarrow$ 1RX) but $N-1$ interferers (with the same receive power):

$$y = x + \sum_{k=1}^{N-1} x_k + w$$

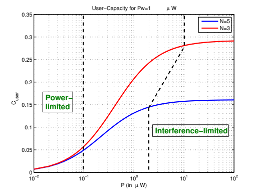
- **Assumption :** interference seen as an extra (Gaussian) noise:

$$C_{\text{user}} = \log_2\left(1 + \frac{P}{(N-1)P + P_w}\right)$$

with user power P and noise power P_w

Result

- $C_{\text{user}} \rightarrow \log_2\left(1 + \frac{1}{(N-1)}\right)$ when $P \rightarrow \infty$
- C_{target} achievable iff $N \leq 1 + \frac{1}{2^{C_{\text{target}}}-1}$



Toy example 2

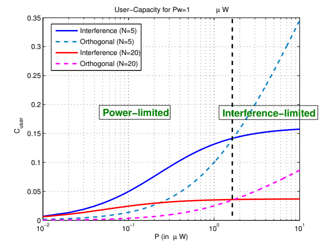
Simple idea: separate the flows ⇔ **orthogonality principle**

- in time (deterministic TDMA: 2G ; random CSMA/CA: Wifi)
- in frequency (FDMA: 2G ; **OFDMA**: 4G/5G)
- more generally through a particular signature (**CDMA**: 3G)

Result

$$C_{\text{user}}^{\perp} = \frac{1}{N}\log_2\left(1 + \frac{P}{P_w}\right)$$

- $C_{\text{user}}^{\perp} \to \infty$ when $P \to \infty$, no upper bound
- For low and medium P (depending on N) $C_{\text{user}} > C_{\text{user}}^{\perp}$.



⇒ **Orthogonality can not be used for any flow (N too large)**
- in practice in downlink and uplink only, ...

⇒ **Even if orthogonality used, partially broken at the receiver**
- in practice multi-path, Doppler effect, ...

Outline

- Section 1: Multi-user Information Theory
  - Capacity region
  - Special case: orthogonal access
- Section 2: Code-Division Multiple Access (CDMA)
  - Transmitter
  - Some receivers
  - Performances
- Section 3: Orthogonal Frequency Division Multiple Access (OFDMA)
  - Subcarrier assignement
  - Diversity
- Section 4: Resource allocation
  - Single-user : waterfilling
  - Problem 1: SNR-target based problem (Perron-Frobenius theorem)
  - Problem 2: Iterative power allocation
  - Problem 3: Sum-rate maximization

Section 1 : Multi-user Information Theory

General scheme

Rates depend on power, receiver algorithm, multiple access, ...

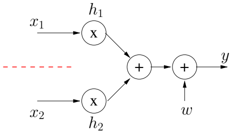
Question

achievable rates regardless of the technique ?
$\Rightarrow$ **Multi-user Information Theory**

- Multi Access Channel (MAC/uplink)

$$y = h_1 x_1 + h_2 x_2 + w$$

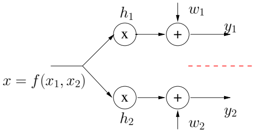
$\rightsquigarrow$ Decode x_1 and x_2 from y



- Broadcast Channel (BC/downlink)

$$\begin{cases} y_1 = h_1 x + w_1 \\ y_2 = h_2 x + w_2 \end{cases}$$

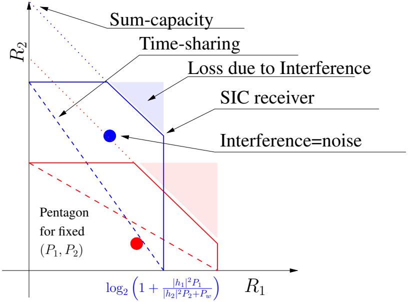
$\rightsquigarrow$ Decode x_1 from y_1 (resp. x_2, y_2)



Answer for MAC

Capacity region [1974, 2004]

$$R_1 \leq \log_2\left(1 + \frac{|h_1|^2 P_1}{P_w}\right), R_2 \leq \log_2\left(1 + \frac{|h_2|^2 P_2}{P_w}\right),$$
$$R_1 + R_2 \leq \log_2\left(1 + \frac{|h_1|^2 P_1 + |h_2|^2 P_2}{P_w}\right)$$



⇒ **Loss due to interference is the triangle (weak or strong)**

⇒ **Large loss if nothing done (the points)**

⇒ **Sum-capacity $R = R_1 + R_2$**

Practical orthogonal multiple access schemes

- TDMA : time separation
- FDMA : frequency separation
- CDMA : code separation
  - Time Hopping (TH) $\rightarrow$ Ultra-Wide Band (UWB)
  - Frequency Hopping (FH) $\rightarrow$ 4G (diversity in Rayleigh channel)
  - Direct Sequence (DS) $\rightarrow$ IS95, 3G

Let us consider 2 users

- U1: α% of time with average power P_1. (power $\frac{P_1}{\alpha}$ when active)
- U2: $(1-\alpha)$% of time with average power P_2. (power $\frac{P_2}{1-\alpha}$ when active)

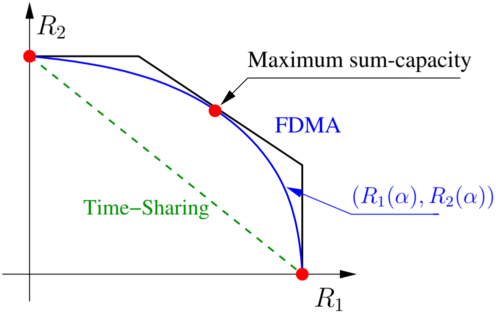
Warning: $\neq$ Time-Sharing (U1 with power P_1 when active, U2 with power P_2 when active)

Result for TDMA/FDMA/CDMA

$$R_1 = \alpha \log_2\left(1 + \frac{|h_1|^2 P_1}{\alpha P_w}\right) \text{ and } R_2 = (1-\alpha)\log_2\left(1 + \frac{|h_2|^2 P_2}{(1-\alpha) P_w}\right)$$

Any orthogonal scheme (in AWGN) offers same capacity region

Sum-capacity



$$C^{\perp} = \alpha \log\left(1 + \frac{|h_1|^2 P_1}{\alpha P_w}\right) + (1-\alpha)\log\left(1 + \frac{|h_2|^2 P_2}{(1-\alpha)P_w}\right)$$

$$C^{\text{Time-Sharing}} = \alpha \log\left(1 + \frac{|h_1|^2 P_1}{P_w}\right) + (1-\alpha)\log\left(1 + \frac{|h_2|^2 P_2}{P_w}\right)$$

$C^{\perp}$ reaches the sum-capacity for $\alpha^{\star} = \frac{|h_1|^2 P_1}{|h_1|^2 P_1 + |h_2|^2 P_2}$

Section 2 : CDMA

Example

- User 1 : $s^{(1)} \rightarrow \mathbf{x}^{(1)} = [c_1^{(1)} s^{(1)}, c_2^{(1)} s^{(1)}] = \underbrace{[c_1^{(1)}, c_2^{(1)}]}_{\mathbf{c}^{(1)}} s^{(1)}$
- User 2 : $s^{(2)} \rightarrow \mathbf{x}^{(2)} = [c_1^{(2)} s^{(2)}, c_2^{(2)} s^{(2)}] = \underbrace{[c_1^{(2)}, c_2^{(2)}]}_{\mathbf{c}^{(2)}} s^{(2)}$

$$\mathbf{y} = \mathbf{x}^{(1)} + \mathbf{x}^{(2)} + \mathbf{w}$$

- Spread spectrum of factor 2
- User separation (through projection) iff $< \mathbf{c}^{(1)} | \mathbf{c}^{(2)} >= 0$
- In noiseless case,

$$\left\{ \begin{array}{ccc} s^{(1)} & = & < \mathbf{c}^{(1)} | \mathbf{y} > \\ s^{(2)} & = & < \mathbf{c}^{(2)} | \mathbf{y} > \end{array} \right.$$

Advantages and Applications

- Hidden information
- Robust to multi-user interference as soon as $| < \mathbf{c}^{(1)} | \mathbf{c}^{(2)} > | \ll 1$
- Simple MAC (Multiple Access Layer) management (see HSPA)
- Inherent –frequency– Diversity (as large bandwidth)

- Military communications
- Multiple access
- Localization (GPS) due to high resolution

Mathematical framework: transmit signal

User k

$$x^{(k)}(t) = \sum_{m=0}^{M-1} s_m^{(k)} h^{(k)}(t - mT_s) \text{ with } h^{(k)}(t) = \sum_{n=0}^{N-1} c_n^{(k)} g(t - nT_c)$$

where

- $g(t)$ shaping filter of bandwidth $\propto 1/T_c$
- T_c chip period, T_s symbol period
- M number of transmit information symbols
- $N = T_s/T_c$ spreading factor
- $\{c_n^{(k)}\}_n$ chip sequence of period N for user k
- $\{s_m^{(k)}\}_{m\in\mathbb{Z}}$ information symbols sequence for user k

Receive signal

- Multipath channel for each user :
  attenuation $\lambda_\ell^{(k)}$ and delay $\tau_\ell^{(k)}$ for the ℓ-th path of user k
- Additive white Gaussian noise $w(t)$
- K users. Usually $K \leq N$

$$
\begin{aligned}
y(t) &= \sum_{k=0}^{K-1} \sum_{\ell=0}^{L_k} \lambda_\ell^{(k)} x^{(k)}(t - \tau_\ell^{(k)}) + w(t) \\
&= \sum_{k=0}^{K-1} \sum_{\ell=0}^{L_k} \sum_{m=0}^{M-1} s_m^{(k)} \lambda_\ell^{(k)} h^{(k)}(t - mT_s - \tau_\ell^{(k)}) + w(t) \\
&= \sum_{k=0}^{K-1} \sum_{\ell=0}^{L_k} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} s_m^{(k)} \lambda_\ell^{(k)} c_n^{(k)} g(t - nT_c - mT_s - \tau_\ell^{(k)}) + w(t)
\end{aligned}
$$

Remark: if downlink, multipath channel independent of k

Chip sequence for synchronous mode

AWGN channel and identical arrival time: $L_k = 0$, $\tau_\ell^{(k)} = 0$

$$y(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} s_m^{(k)} \underbrace{\sum_{n=0}^{N-1} \lambda^{(k)} c_n^{(k)} g(t - nT_c - mT_s)}_{\Phi_{m,k}(t)} + w(t)$$

$\Phi_{m,k}(t)$ **orthogonal basis** $\Leftrightarrow < \mathbf{c}^{(k)} | \mathbf{c}^{(k')} >= \delta_{k,k'}$

Let $N = 2^P$ and $H_0 = [1]$

$$H_p = \begin{bmatrix} H_{p-1} & H_{p-1} \\ H_{p-1} & -H_{p-1} \end{bmatrix}$$

$\Rightarrow$ H_P $N \times N$ orthogonal matrix (divided by $\sqrt{N}$ for energy purpose)

- Walsh-Hadamard sequence
- Impossible to exhibit $(N + 1)$ orthogonal users

Chip sequence for asynchronous mode

neither AWGN channel nor identical arrival time

$$y(t) = \sum_{k=0}^{K-1}\sum_{m=0}^{M-1} s_m^{(k)} \underbrace{\sum_{\ell=0}^{L_k}\sum_{n=0}^{N-1} \lambda_\ell^{(k)} c_n^{(k)} g(t - nT_c - mT_s - \tau_\ell^{(k)})}_{\Psi_{m,k}(t)\ \textbf{not orthogonal basis anymore but ...}} + w(t)$$

- Solution: Pseudo-Noise (PN) sequence of Gold and Kasami
- Let $\mathbf{c}^{(k)}(\tau) = [c_\tau^{(k)}, \cdots, c_{(N-1+\tau) \bmod N}^{(k)}]$ be the shifted sequence by τ. For $k \neq k'$ or $\tau \neq \tau'$, we have

$$< \mathbf{c}^{(k)}(\tau) | \mathbf{c}^{(k')}(\tau') > \approx \frac{1}{\sqrt{N}}$$

Receivers design

Synchronous mode: it is straightforward

Asynchronous mode:

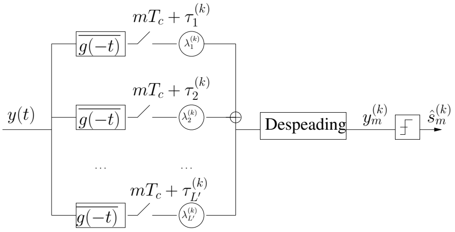
- Single-user detection
  $\Rightarrow$ multi-user interference seen as a noise
- Multi-user detection (MUD)
  $\Rightarrow$ multi-user interference structure used (codes required)

Rake receiver [1958]

$$y(t) = \sum_{m=0}^{M-1} s_m^{(k)} \underbrace{\sum_{\ell=0}^{L-1} \lambda_\ell^{(k)} h^{(k)}(t - mT_s - \tau_\ell^{(k)})}_{\Psi_{m,k}(t)} + \text{other users} + \text{noise}$$

Projection = (truncated) matched filter + sampling at baud rate

$$y_m^{(k)} = < y(t) | \sum_{\ell \in \mathcal{L}} \lambda_\ell^{(k)} h^{(k)}(t - mT_s - \tau_\ell^{(k)}) >$$



Discussion

- Floor effect

$$y_m^{(k)} = s_m^{(k)} \sum_{\ell,\ell'} \lambda_\ell^{(k)} \lambda_{\ell'}^{(k)} < h^{(k)}(t - mT_s - \tau_{\ell'}^{(k)}) | h^{(k)}(t - mT_s - \tau_\ell^{(k)}) >$$

$$+ \underbrace{\sum_{\substack{m' \neq m \\ \ell,\ell'}} s_{m'}^{(k)} \lambda_\ell^{(k)} \lambda_{\ell'}^{(k)} < h^{(k)}(t - m'T_s - \tau_{\ell'}^{(k)}) | h^{(k)}(t - mT_s - \tau_\ell^{(k)}) >}_{\text{ISI}}$$

$$+ \underbrace{\sum_{\substack{k' \neq k \\ m',\ell,\ell'}} s_{m'}^{(k')} \lambda_\ell^{(k)} \lambda_{\ell'}^{(k')} < h^{(k')}(t - m'T_s - \tau_{\ell'}^{(k')}) | h^{(k)}(t - mT_s - \tau_\ell^{(k)}) >}_{\text{MUI}}$$

$$+ \quad \text{noise}$$

- Near-far effect: power control required (IS95, 3G)
- *but* $y_m^{(k)}$ non-exhaustive statistics for symbol $s_m^{(k)}$ [1983]

$$y(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} s_m^{(k)} \Psi_{m,k}(t) + w(t)$$

Optimal detector : Maximum Likelihood

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}$$

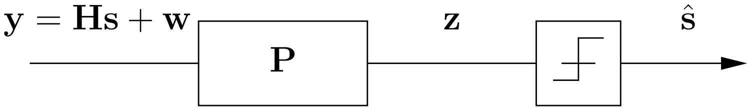
with

- $\mathbf{y} = [\mathbf{y}_0, \cdots, \mathbf{y}_{M-1}]$ with
  - $\mathbf{y}_m = [y_m^{(0)}, \cdots, y_m^{(K-1)}]$
  - $y_m^{(k)} = < y(t) | \Psi_{m,k}(t) >$
- $\mathbf{s} = [s_0^{(0)}, \cdots, s_0^{(K-1)}, \cdots, s_{M-1}^{(0)}, \cdots, s_{M-1}^{(K-1)}]$
- $\mathbf{H}$ is non-diagonal and deals with inter-symbol interference and multi-user interference (actually $\mathbf{H}$ corresponds to the entries $< \Psi_{m,k}(t) | \Psi_{m',k'}(t) >)_{m,m',k,k'}$, so $\mathbf{H}$ is square Hermitian definite-positive matrix)
- $\mathbf{w}$ is Gaussian noise with correlation matrix $2N_0\mathbf{H}$

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg\min_{\mathbf{s}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2_{\mathbf{H}^{-1}}$$

- Exhaustive search: $\mathcal{O}(\mathrm{card}(\mathbb{S})^{KM})$ with the constellation $\mathbb{S}$
- Viterbi algorithm (finite memory): $\mathcal{O}(KM(\mathrm{card}(\mathbb{S}))^{KL}) \Rightarrow$ still huge

Zero-Forcing (ZF)



$$\mathbf{P}_{\mathrm{ZF}} = \mathbf{H}^{-1}$$

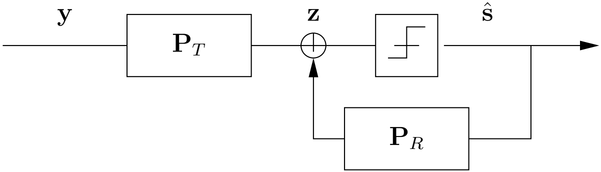
- MUI completely vanishes
- No near-far effect
- Noise enhancement issue

Minimum Mean Square Error (MMSE)

$$\mathbf{P}_{\mathrm{MMSE}} = (\mathbf{H} + 2N_0)^{-1}$$

- If low SNR, then MUI not treated
- If high SNR, close to ZF
- Expensive large matrix inversion (size $KM \times KM$)

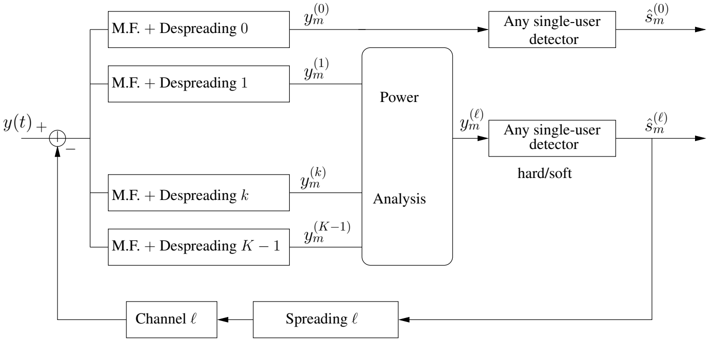
Decision feedback equalizer principle (DFE)



How $\hat{\mathbf{s}}$ available for feeding back the information

- Time causality: straightforward
- Multi-user causality:
  - the first one
  - the strongest one

Example: Successive Interference Canceller (SIC)



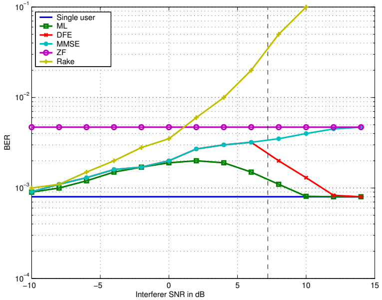
- A lot of combinations via any single-user detector

Performances

- $N = 7$, $K = 4$, and AWGN channels with non-orthogonal codes

$$\mathbf{H} = \frac{1}{7}\begin{bmatrix} 7 & 3 & -1 & -1 \\ 3 & 7 & -1 & 3 \\ -1 & -1 & 7 & -1 \\ -1 & 3 & -1 & 7 \end{bmatrix}$$

- User of interest with fixed SNR of 7dB
- Interferers with variable SNRs



Section 3 : OFDMA

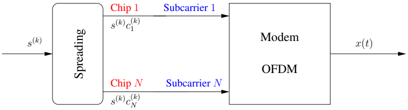
Principle

- In one subcarrier, only one user is assigned
- Each user may have several subcarriers

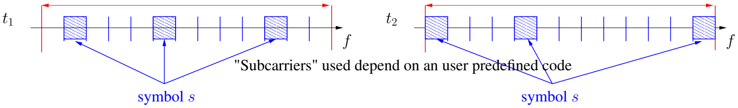
This is an orthogonal technique (inspired by FDMA)

Question: How assigning the subcarriers ?

1. when CSIT: find out relevant subcarrier assignement ?
2. when no CSIT: diversity management ?
    - 2.1 Multi-Carrier-CDMA (MC-CDMA) `[1993]`: not used anymore



    - 2.2 Frequency-Hopping-OFDMA (FH-OFDMA): used in practice



When CSIT, fair subcarrier assignement 1

Let

- $H_k(n)$ be the channel frequency response for user k on subcarrier n
- $P_k(n)$ be the power for user k on subcarrier n
- $\gamma_k(n)$ the assignement policy:
  - $\gamma_k(n) = 1$ iff subcarrier n assigned to user k,
  - $\gamma_k(n) = 0$ otherwise

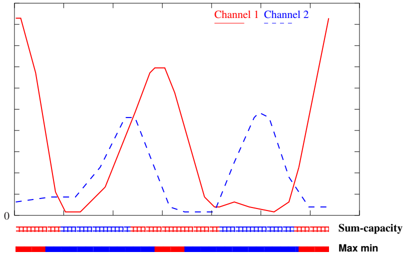
$$R_k = \sum_{n=1}^{N} \gamma_k(n) \log_2 \left(1 + \frac{|H_k(n)|^2 P_k(n)}{P_w} \right)$$

When CSIT, fair subcarrier assignement 2

$$\max_{\text{OFDMA allocation}} \sum_{k=1}^{K} \omega_k R_k \qquad \max_{\text{OFDMA allocation}} \min_k R_k$$

$\rightsquigarrow$ Subcarrier n assigned to user k iff it maximizes for this n $\omega_k \log_2(1 + |H_k(n)|^2 P_k(n)/P_w)$

$\rightsquigarrow$ Users definitively offer the same rate at the expense of a low sum rate



Section 4 : Resource allocation

Principle

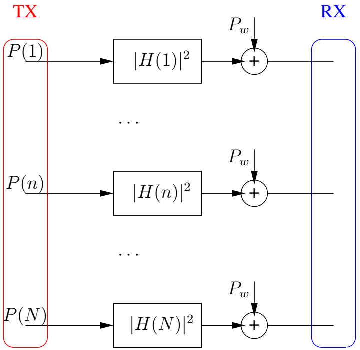
Play with

- power
- subcarrier assignement
- modulation and coding scheme
- ...

to improve the rate, the energy consumption, the latency, ...

In context of multi-user/multi-cell/... interference-disturbed communications, try to mitigate the interference degradation

Single-user example: the waterfilling [1948]



- Data rate maximization
- Power constraint:

$$\sum_{n=1}^{N} P(n) = P_{\max}$$

  with maximum power $P_{\max}$.
- Perfect CSIT

Problem: maximum capacity?

$$[P(1)^*, \cdots, P(N)^*] = \arg \max_{P(1),\cdots,P(N)} \sum_{n=1}^{N} \log_2 \left(1 + |H(n)|^2 \frac{P(n)}{P_w}\right)$$

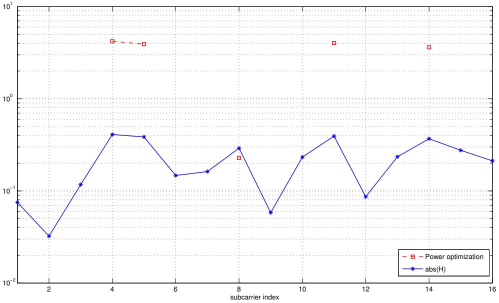
Convex optimization problem ($\Rightarrow$ KKT conditions)

Corresponding algorithm

$$P(n)^* = \left(\nu - \frac{P_w}{|H(n)|^2}\right)^+$$

with

- ν chosen s.t. $\sum_{n=1}^{N} P(n)^* = P_{\max}$.
- $(\bullet)^+ = \max(0, \bullet)$.

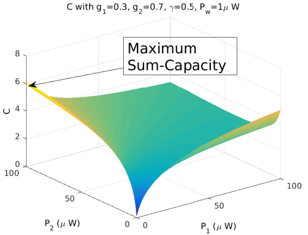


Multi-user context

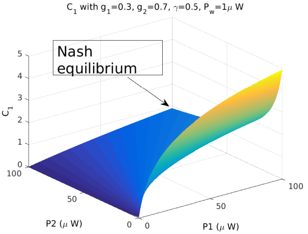
Exemple : uplink or multi-cell interference γ with $(P_1, P_2) \in [0, P_{\max}]^2$

$$\begin{cases} y_1 = h_1 x_1 + \gamma h_2 x_2 + w_1 & \Rightarrow \quad R_1 = \log_2\left(1 + \frac{|h_1|^2 P_1}{\gamma^2 |h_2|^2 P_2 + P_w}\right) \\ y_2 = \gamma h_1 x_1 + h_2 x_2 + w_2 & \Rightarrow \quad R_2 = \log_2\left(1 + \frac{|h_2|^2 P_2}{\gamma^2 |h_1|^2 P_1 + P_w}\right) \end{cases}$$

"Social" optimization
maximization of $R = R_1 + R_2$
If $|h_2| > |h_1|$, $P_1^* = 0$ et $P_2^* = P_{\max}$



Individual optimization
game theory with (R_1, R_2)
Nash eq. if $P_1^* = P_2^* = P_{\max}$



Problem 1: Power allocation with target SNR [1992]

K TX and K RX with interference

$$y_k = h_{k,k}x_k + \sum_{n\neq k} h_{k,n}x_n + w_k$$

with $P_k = \mathbb{E}[|x_k|^2]$

Problem statment: find $\{P_n\}$ satisfying target SNR β_k for user k, $\forall k$

$$\frac{|h_{k,k}|^2 P_k}{\sum_{n\neq k} |h_{k,n}|^2 P_n + P_w} \geq \beta_k, \ \forall k$$

$$\Updownarrow$$

Problem statment: find $\mathbf{p} = [P_1, \cdots, P_K]^{\mathrm{T}}$ s.t.

$$(\mathbf{Id}_K - \mathbf{F})\mathbf{p} = \mathbf{b}$$

with $\mathbf{F} = \{\bar{\delta}_{k,n}|h_{k,n}|^2\beta_k/|h_{k,k}|^2\}_{k,n}$, $\mathbf{b} = P_w[\beta_1/|h_{1,1}|^2, \cdots, \beta_K/|h_{K,K}|^2]^{\mathrm{T}}$

Problem 1: cont'd

- $\mathbf{F}$ non negative matrix (NNM)
- $\mathbf{b}$ and $\mathbf{p}$ non-negative vectors

NNM theory $\Leftrightarrow$ Perron-Frobenius theorem

Result

If $\mathbf{F}$ primitive ($\exists m$ s.t. $\mathbf{F}^m > 0$), then following propositions are equivalent

- The eigenvalue with the largest absolute value of $\mathbf{F}$ lies in $(0, 1)$
- $(\mathbf{Id}_K - \mathbf{F})^{-1}$ exists and is strictly positive
- $\mathbf{p}^\star = (\mathbf{Id}_K - \mathbf{F})^{-1}\mathbf{b}$ is the positive solution of problem statment

  Hint: $(\mathbf{Id}_K - \mathbf{F})^{-1} = \sum_\ell F^\ell$

- Problem is sometimes not feasible (see toy example 1 –slide 3–)
- Can be overcome by orthogonalization or receiver improvement

Problem 1: example

We would like to solve the problem

$$\left\{ \begin{array}{l} \frac{|h_1|^2 P_1}{\gamma^2 |h_2|^2 P_2 + P_w} = \beta \\ \frac{|h_2|^2 P_2}{\gamma^2 |h_1|^2 P_1 + P_w} = \beta \end{array} \right.$$

1. Find constraints on γ, β for existence of P_1 and P_2
2. Find closed-form expressions for P_1 and P_2 wrt γ, β, h_1, h_2 et P_w
3. Analyse impact of each parameter
4. Numerical illustrations : calculate P_1 and P_2 for $\gamma = 1/\sqrt{6}$ and $\gamma = 1/\sqrt{2}$ when $\beta = 1$, $h_1 = h_2 = 1$ and $P_w = 1\mu W$

Problem 2: Iterative power allocation [1995]

Uplink scheme with power allocation and BTS assignement (for K users and K BTS)
At BTS ℓ, we have for user k

$$y_\ell = h_{\ell,k}x_k + \sum_{n\neq k} h_{\ell,n}x_n + w_\ell$$

We have at time 1

- initial power allocation $\mathbf{p}^{(1)} = [p_1^{(1)}, \cdots, p_K^{(1)}]^{\mathrm{T}}$
- initial BTS assignement: $c_k^{(1)}$ provide the index of BTS for user k

At BTS ℓ

$$\frac{|h_{\ell,k}|^2 P_k}{\sum_{n\neq k} |h_{\ell,n}|^2 P_n + P_w} = \beta_k \Leftrightarrow \underbrace{P_k}_{\text{required at BTS } \ell} = \frac{\beta_k(\sum_{n\neq k} |h_{\ell,n}|^2 P_n + P_w)}{|h_{\ell,k}|^2}$$

Problem 2: cont'd

We choose for user k the BTS leading to its smallest power at time $m+1$ assuming the other users play with their power of time m, then we get the power for user k at time $m+1$.
Therefore given the power at time m, we have at time $m+1$

$$\left\{ \begin{array}{l} c_k^{(m+1)} = \arg\min_\ell \frac{\beta_k(\sum_{n\neq k}|h_{\ell,n}|^2 P_n^{(m)} + P_w)}{|h_{\ell,k}|^2} \\ P_k^{(m+1)} = \min_\ell \frac{\beta_k(\sum_{n\neq k}|h_{\ell,n}|^2 P_n^{(m)} + P_w)}{|h_{\ell,k}|^2} \end{array} \right.$$

Standard interference function

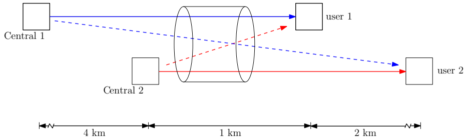
$$f : \mathbf{p}^{(m)} \mapsto \mathbf{p}^{(m+1)}$$

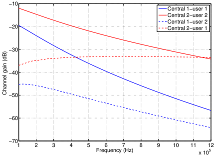
- positivity: $f(\mathbf{p}) > 0$ for any $\mathbf{p} \geq 0$
- monotonicity: $f(\mathbf{p}) \geq f(\mathbf{p}')$ if $\mathbf{p} \geq \mathbf{p}'$
- scalability: $f(\alpha\mathbf{p}) \leq \alpha f(\mathbf{p})$ for any $\alpha > 1$

If f has a fixed point $\mathbf{p}^\star$, then the algorithm converges to it

Problem 3: Sum-rate maximization [2010]

Context: Interference between twister pairs in ADSL





Optimization problem

$$\max_{P_1(1),\cdots,P_1(N),P_2(1),\cdots,P_2(N)} \omega\left(\sum_{n=1}^{N} R_1(n)\right) + (1-\omega)\left(\sum_{n=1}^{N} R_2(n)\right)$$

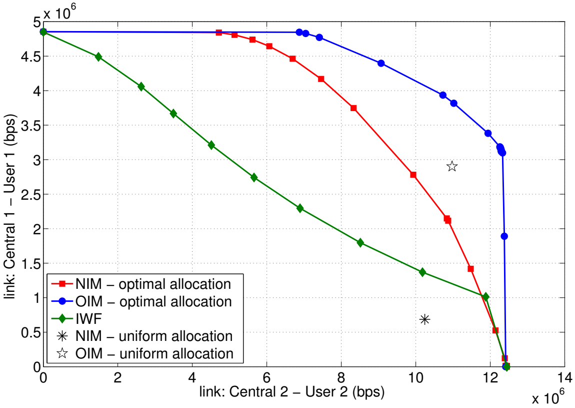
with

$$R_k(n) = \log_2\left(1 + \frac{|h_{k,k}(n)|^2 P_k(n)}{|h_{k,k'}(n)|^2 P_{k'}(n) + P_w}\right)$$

under constraints $\sum_{n=1}^{N} P_1(n) \leq P_{\max}$ and $\sum_{n=1}^{N} P_2(n) \leq P_{\max}$

Problem 3: cont'd

- NIM : Interference seen as "noise"
- OIM : Interference optimally managed



Conclusion

- Multi-user communications are a crucial issue
- Extension to Network Communications
- We omit to discuss about
  - no CSIT available for doing resource allocation
  - numerous problems : actually one problem per configuration
  - numerous mathematical techniques: convex optimization, operational research, game theory, deep learning, ...
- Here, just some preliminary results on resource allocation
- Resource allocation analyzed deeply in
  - ▷ Master IP Paris “Information Processing: Machine Learning, Communications, and Security” (M2 MICAS)

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