# MICAS904-ROSP/DCOM0

# Introduction to Communications Theory

# Part on ”Modulation and Demodulation”

Philippe Ciblat

Telecom Paris, Institut Polytechnique de Paris, Palaiseau, France

# Roadmap

<table>
<tr><th>MICAS904</th><th>ROSP</th></tr>
<tr><td colspan="2">TX (Modulation)<br>Channel modeling<br>RX (Demodulation, Detection)<br>joint with DCOM0 (5TH)</td></tr>
<tr><td colspan="2">Detection, Performances<br>joint with DCOM0 (4TH)</td></tr>
<tr><td>Channel coding (6TH)</td><td>Channel coding (4TH)<br>DCOM0 end</td></tr>
<tr><td colspan="2">ISI management<br>(ZF, MMSE, DFE, OFDM)<br>joint with DCOM1 (4TH)</td></tr>
<tr><td>MICAS904 end</td><td>Blind equalization<br>Synchronization<br>MIMO decoding<br>Nonlinearity mitigation<br>DCOM1 end</td></tr>
</table>

# Outline

- Section 1: Digital Communication scheme
- Section 2: A toy example
- Section 3: Baseband and carrier signals
- Section 4: Propagation channel
- Section 5: Transmitter (Modulation)
- Section 6: Receiver (Demodulation)
  - Matched filter + sampler
  - Nyquist filter

# Section 1: Digital Communication scheme

# Introduction

- Except audio broadcasting (radio), current communication systems are **digital**
  - 2G, 3G, 4G, DVBT, Wifi, Bluetooth
  - ADSL, Fiber
  - MP3, DVD
- Channels: copper twisted pair, powerline, wireless, optical fiber, ...
- Sources: analog (voice) or digital (data)

# If analog source, ...

- **Sampling** (no information loss)

**Nyquist-Shannon Theorem**

Let $t \mapsto x(t)$ be a continuous-time signal of bandwidth $B$. $x(t)$ is perfectly characterized by the sequence $\{x(nT)\}_n$ where $T$ is the sampling period satisfying $1/T \geq B$.

- **Quantization** (information loss)

**Example**

Let us consider voice signal

| Quality | Bandwidth | Sampling | Quantization |
|---|---|---|---|
| 2G | [300Hz, 3400 Hz] | 8kHz | 8 bits |
| Hifi | [20Hz, 20kHz] | 44kHz | 16 bits |

# What is digital?

**Analog system:** $s(t)$ analog source

$$\textit{transmit signal} \ : \ x(t) = f(s(t))$$

- + Pros: low complexity
- – Cons: data transmission, multiple access, performance, limited information processing

**Digital system:** $s_n$ digital source (composed by 0 and 1)

$$\textit{transmit signal} \ : \ x(t) = f(s_n)$$

# Design parameters

- Data rate ........................................ $D_b$ bits/s
- Bandwidth ........................................ $B$ Hz
- Error probability ........................................ $P_e$
- Transmit power (SNR) ........................................ $P$ mW or dBm
- Latency ........................................ $L$

## Goal

$$\max D_b \quad \text{with} \quad \min B, P_e, P, L$$

but

- theoretical limits (information theory)
- physical constraints (propagation, complexity)

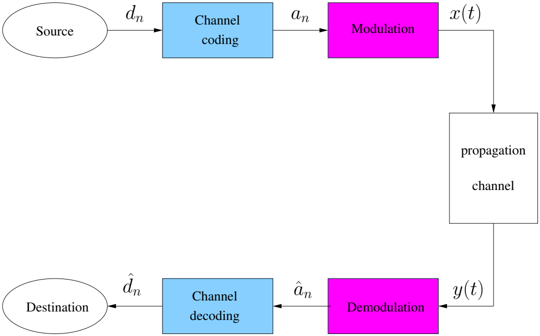
Practical case: depends on Quality of Service (QoS)

- 2G/3G: target $L$ with fixed $D_b$ and variable $P_e$
- ADSL: $\max D_b$ with target $P_e$ and fixed $B$ and $P$

# A few systems

| System | $D_b$ | $B$ | $P_e$ | Spectral efficiency |
|---|---|---|---|---|
| DVB | 10Mbits/s | 8MHz | $10^{-11}$ | 1,25 bits/s/Hz |
| 2G | 13kbits/s | 25kHz | $10^{-2}$ | 0,5 bits/s/Hz |
| ADSL | 500kbits/s | 1MHz | $10^{-7}$ | 0,5 bits/s/Hz |

# Transceiver/Receiver structure




## Question ?

How to design

- Modulation/demodulation boxes
- Coding/decoding boxes

... depending on propagation channel

# Section 2: A toy example

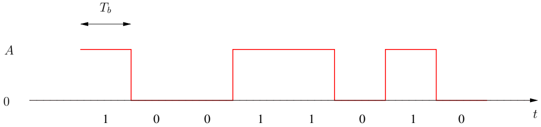
# The “old” optical fiber

Goal:

- Sending a bit stream $a_n \in \{0, 1\}$ at data rate $D_b$ bits/s
- Data $a_n$ will be sent at time $nT_b$ with $T_b = 1/D_b$ s

How?

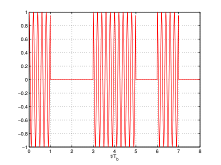
- $x(t) = 0$ if $a_n = 0$ within $[nT_b, (n+1)T_b) \Rightarrow$ **No light**
- $x(t) = A$ if $a_n = 1$ within $[nT_b, (n+1)T_b) \Rightarrow$ **Light**




## but

Light has a color
($\sim$ wavelength)

$$x_c(t) = x(t)\cos(2\pi f_0 t)$$



# Mathematical framework

- Each data has a shape. Here, the rectangular function
- Each shape is multiplied by an amplitude. Here, either $A$ or 0
- Each data is shifted at the right time

$$x(t) = \sum_n s_n g(t - nT_s)$$

with

- $g(t)$ **shaping filter**. Here, $g(t)$ rectangular function
- $s_n$ **symbol** sequence. Here $s_n = Aa_n$
- $T_s$ **symbol period**. Here, $T_s = T_b$

Finally

$$x_c(t) = x(t)\cos(2\pi f_0 t)$$

# Degrees of freedom

- carrier frequency $f_0$
  - impact on propagation condition
  - impact on data rate (see later)

- shaping filter $g(t)$
  - impact on bandwidth

    $$S_x(f) \propto |G(f)|^2$$

    with $G(f)$ Fourier Transform of $g(t)$
  - impact on receiver complexity and performance (see later)

- symbol $s_n$
  - impact on data rate: multi-level
  - impact on performance (see later)

- symbol period $T_s$
  - impact on data rate
  - impact on bandwidth (through the choice of $g(t)$)

# Section 3: Baseband/carrier signals

# Questions

$$x_c(t) = x(t)\cos(2\pi f_0 t)$$

with

- $x_c(t)$: carrier signal
- $x(t)$: baseband signal $\Rightarrow$ (complex) envelope

**Q1: Is there another way to translate the signal?** $x(t) \rightarrow x_c(t)$

- YES
- I/Q modulator
- **Complex-valued signal**

**Q2: How retrieving** $x(t)$ **from** $x_c(t)$**?**

- I/Q demodulator

# Mathematical framework

Instead of using only cos, we can use simultaneously cos and sin

$$\begin{aligned} x_c(t) &= x_p(t)\cos(2\pi f_0 t) - x_q(t)\sin(2\pi f_0 t) \\ &= \Re\left((x_p(t) + i x_q(t))e^{2i\pi f_0 t}\right) \end{aligned}$$

with

- $x_p(t)$ a baseband real-valued signal of bandwidth $B$: **In-phase**
- $x_q(t)$ another real-valued signal of bandwidth $B$: **Quadrature**

**We may have two streams in baseband for one carrier signal!**

## Complex envelope

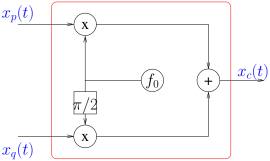
The baseband signal can be represented by the so-called complex envelope

$$x(t) = \frac{1}{\sqrt{2}}(x_p(t) + i x_q(t))$$

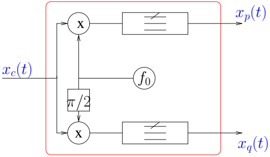
# Mathematical framework (cont'd)

Assuming $B/2 < f_0$, we have

I/Q modulator

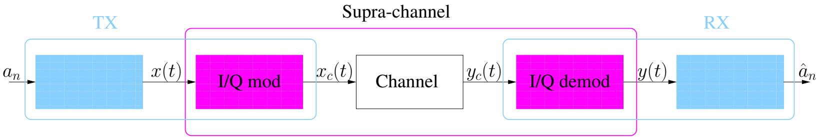



I/Q demodulator




In practice, we work with complex envelope

- smaller bandwidth $B$ instead of $2f_0 + B$
- no cos and sin disturbing terms



# A few wireless systems

When $f_0$ increases

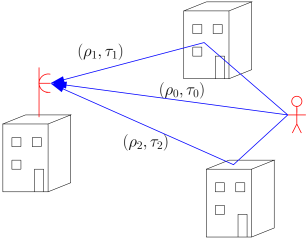
- propagation degrades ($1/f^2$)
- antenna size decreases ($1/f$)
- bandwidth $B$ may increase

| System | $f_0$ | $B$ | Antenna size |
|---|---|---|---|
| Intercont. | $\sim$ 10MHz (HF) | 100kHz | 100m |
| DVBT | 600MHz (UHF) | $\sim$ 1 MHz | 1m |
| 2G | 900MHz | $\sim$ 1 MHz | 10cm |
| Wifi | 5.4 GHz | 10MHz | $\sim$ 1cm |
| Satellite | 11GHz | 100MHz | – |
| Personal Network | 60GHz | – | – |

# Section 4: Propagation channel

# Multipath channel

- typical wireless channel
- valid also for ADSL and optical fiber (low SNR)




$$\begin{aligned} y(t) &= \sum_k \rho_k x(t - \tau_k) + w(t) \\ &= c(t) \star x(t) + w(t) \end{aligned}$$

with noise $w(t)$

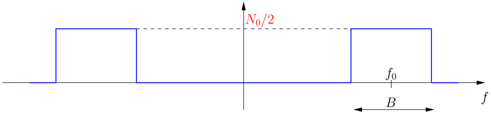
- **Dispersion time:** $T_d = \max_k \tau_k$
- **Coherence bandwidth:** $B_c = \min_f \arg\max_\delta \{\|C(f) - C(f+\delta)\| < \varepsilon\}$

$$B_c = \mathcal{O}(1/T_d)$$

# Noise property

Let $w_c(t)$ be the (random) noise at carrier level

- $w_c(t)$ is zero-mean (real-valued) Gaussian variable
- $w_c(t)$ is stationary ($\mathbb{E}[w_c(t)^2]$ independent of $t$)
- $w_c(t)$ is <u>almost</u> white




$$P = \int S_w(f)df = N_0 B$$

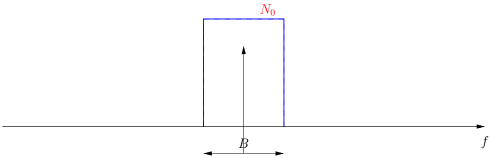
**What's happened for complex envelope $w(t)$?**

# Noise property (cont'd)

$$w(t) = \frac{1}{\sqrt{2}}(w_p(t) + iw_q(t))$$

with

1. $w_p(t)$ and $w_q(t)$ zero-mean (real-valued) stationary Gaussian variable with the same spectrum




2. $w_p(t)$ and $w_q(t)$ are independent

# Model: Gaussian channel

- Short multipaths ($T_d$) compared to symbol period ($T_s$)
- Holds for Hertzian beams
- Holds for Satellite
- Holds also for very low data rate transmission

$$y(t) = x(t) + w(t)$$

# Model: Frequency-Selective channel

- Holds for cellular systems (2G with $T_d = 4T_s$)
- Holds for Local Area Network (Wifi with $T_d = 16T_s$)
- Holds for ADSL ($T_d = 100T_s$)
- Holds also for Optical fiber (the so-called chromatic dispersion)

$$y(t) = c(t) \star x(t) + w(t)$$

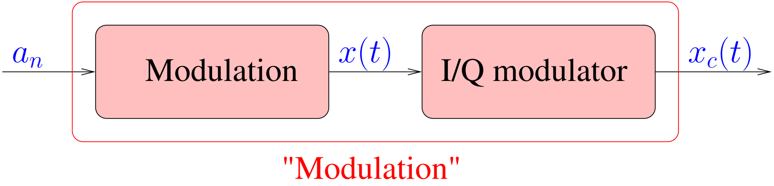
$\Rightarrow$ InterSymbol Interference (ISI)

## Remark

- Channel type (ISI?) is modified according to data rate
- The higher the rate is, the stronger the ISI is ($T_d \gg T_s$)

# Section 5: Transmitter (Modulation)

# Question




How associating bits $a_n$ with analog (baseband) signal $x(t)$?

# Binary modulation

- Waveform: $x_0(t)$ if bit '0' and $x_1(t)$ if bit '1'
- Binary linear modulation

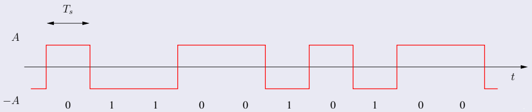
$$x_0(t) = Ag(t) \quad \text{and} \quad x_1(t) = -Ag(t)$$

with symbols $-A$ and $A$, and the shaping filter $g(t)$

If the symbol period is $T_s$, then

$$x(t) = \sum_k s_k g(t - kT_s) \quad \text{with} \quad s_k \in \{-A, A\}$$

## Example ($g(t)$ rectangular function)



# Multi-level modulation

- Bandwidth of $x(t)$ ($B$) identical of that of $g(t)$ ($1/T_p$):
  - If $T_p \gg T_s$, InterSymbol Interference (see rectangular case)
  - If $T_p \ll T_s$, bandwidth is wasted (signal oscillates at $1/T_s$)

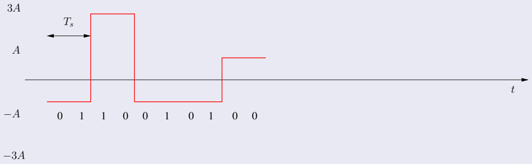
$$T_s \approx T_p$$

- Spectral efficiency is 1bit/s/Hz in binary modulation

**Multi-level modulation:** one symbol contains more than one bit

## Exemple ($M = 4$)

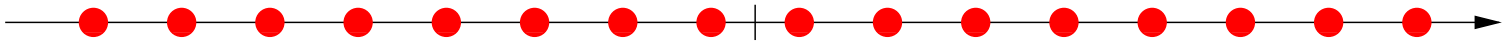
$$'00' \mapsto A \quad '01' \mapsto -A \quad '10' \mapsto 3A \quad '11' \mapsto -3A$$



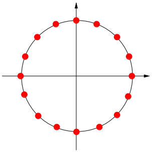
# Constellations

Constellation = set of possible symbols

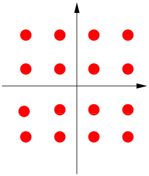
- Pulse Amplitude Modulation (PAM)



- Phase Shift Keying (PSK)

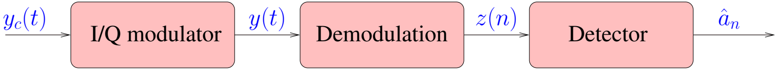


- Quadrature Amplitude Modulation (QAM)



# Section 6: Receiver (Demodulation)

# Question




**Two main boxes:**

- How coming back to discrete-time signal: demodulation
- How detecting optimally the transmit bits (from $z(n)$): detector

### Goal

Describing and justifying the demodulation

# A mathematical tool: signal space

Let $L^2$ be the space of energy-bounded function

$$L^2 = \left\{ f \text{ s.t. } \int |f(t)|^2 dt < +\infty \right\}$$

**$L^2$ is an infinite-dimensional vectorial space**

## Properties

- $L^2$ has an inner product

$$< f_1(t)|f_2(t) >= \int f_1(t)\overline{f_2(t)}dt$$

  - leads to "orthogonality" principle: $< f_1(t)|f_2(t) >= 0$
  - leads to a norm: $\|f(t)\| = \sqrt{< f(t)|f(t) >}$
- $L^2$ has an infinite-dimensional orthonormal (otn) basis: $\{\Psi_m(t)\}_m$

$$\forall f \in L^2, \exists\{\beta_m\}_m, f(t) = \sum_m \beta_m \Psi_m(t) \text{ with } \beta_m =< f(t)|\Psi_m(t) >$$

**Any function is described by complex-valued coefficients**

# A signal subspace

Let $E$ be a subspace of $L^2$ generated by the functions $\{f_m(t)\}_{m=1,\cdots,M}$

$$E = \text{span}(\{f_m(t)\}_{m=1,\cdots,M}) = \left\{ \sum_{m=1}^{M} \alpha_m f_m(t) \text{ for any complex } \alpha_m \right\}$$

## Property

This subspace has a finite dimension and a finite otn basis

$$D = \dim_{\mathbb{C}} E \quad \text{and} \quad E = \text{span}\{\Phi_\ell(t)\}_{\ell \in \{1,\cdots,D\}}$$

For instance, let $f(t)$ be a function in $E$

$$f(t) = \sum_{\ell=1}^{D} s^{(\ell)} \Phi_\ell(t) \text{ with } s^{(\ell)} = < f(t)|\Phi_\ell > \in \mathbb{C}$$

- $\mathbf{s} = [s^{(1)}, \cdots, s^{(D)}]^{\mathrm{T}}$ corresponds to the analog signal $f(t)$
- Usually, we prefer to work with **s** (which will carry **information**)

# Exhaustive demodulator

$$y(t) = \sum_k s_k h(t - kT_s) + w(t)$$

with any symbol $s_k$ and any filter $h(t)$

## Question

**How sampling without information loss?**

- Nyquist-Shannon Theorem: sampling at $f_e > B$. Then $y(n/f_e)$ contains all the information on $y(t)$

  **Actually information ($\{s_k\}$) is only a part of $y(t)$**

- Exhaustive demodulator based on subspace principle

- Information $\{s_k\}$ belongs to the subspace $E$

$$E = \text{span}(\{h(t - kT_s)\}_k)$$

- Noise $w(t)$ belongs to $E$ and $E^\perp$ (orthogonal of $E$)

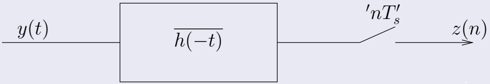
$$w(t) = w_E(t) + w_{E^\perp}(t) \quad (w_E(t) \text{ and } w_{E^\perp}(t) \text{ independent})$$

**Consequently, projection on $E$ contains any information on $\{s_k\}$ in $y(t)$**

# Exhaustive demodulator (cont'd)

## Projection on $E$

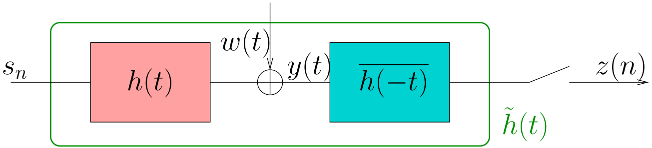
$$\begin{aligned} z(n) &= < y(t) | h(t - nT_s) > \\ &= \int y(\tau)\overline{h(\tau - nT_s)}d\tau \\ &= \overline{h(-t)} \star y(t)_{|t=nT_s} \end{aligned}$$




**Projection = Matched filter + Sampling**

**Remark:** Sampling at $T_s$ and not at $T_e$

# Input/output discrete-time model




$$z(n) = \sum_{\ell} \tilde{h}(\ell T_s) s_{n-\ell} + w(n)$$

with

- $\tilde{h}(t) = \overline{h(-t)} \star h(t)$
- $w(n) = \overline{h(-t)} \star w(t)_{|t=nT_s}$ zero-mean complex-valued stationary Gaussian with spectrum

$$S_w(e^{2i\pi f}) = N_0 \tilde{h}(e^{2i\pi f}) = N_0 |h(e^{2i\pi f})|^2$$

# Orthogonal basis case

What's happened when $\{h(t-kT_s)\}_k$ is an **otn** basis $\Leftrightarrow$ **No ISI**

$$z(n) = s_n + w(n)$$

## Equivalent proposition

- $\{h(t-kT_s)\}_k$ **otn** basis
- $\tilde{h}(t)$ **Nyquist** filter

$$\tilde{h}(\ell T_s) = \delta_{\ell,0} \Leftrightarrow \sum_k \tilde{H}\left(f - \frac{k}{T_s}\right) = T_s$$

- $h(t)$ **square-root Nyquist**

$$\tilde{h}(t) = \overline{h(-t)} \star h(t) \Leftrightarrow H(f) = \sqrt{\tilde{H}(f)}$$

In practice, $h(t)$ square-root Nyquist iff

- Gaussian channel .......no ISI provided by propagation channel
- $g(t)$ square-root Nyquist ........no ISI provided by shaping filter

# Nyquist filter

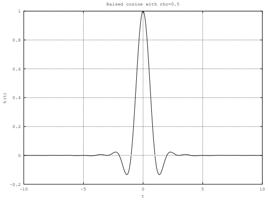
## Main property

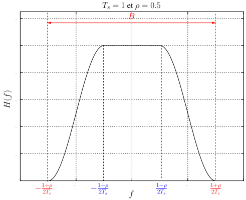
If $h(t)$ square-root Nyquist, then

$$B > \frac{1}{T_s}$$

**Examples:**

- $h(t)$ rectangular $\Leftrightarrow$ $\tilde{h}(t)$ triangular
- $h(t)$ square-root raised cosine (srrc) $\Leftrightarrow$ $\tilde{h}(t)$ raised cosine




with roll-off $\rho$ ($\rho = 0.22$ in 3G, $\rho = 0.05$ in DVB-S2, $\rho = 0.01$ in WDM-Nyquist)

# Consequence on noise

If $h(t)$ square-root Nyquist, then $w(n)$ **white** noise

- $w(n) = w_R(n) + iw_I(n)$
- $w_R(n)$ and $w_I(n)$ independent
- $\mathbb{E}[w_R(n)^2] = \mathbb{E}[w_I(n)^2] = N_0/2$
- $\mathbb{E}[|w(n)|^2] = N_0$ and $\mathbb{E}[w(n)^2] = 0$

## Probability density function (pdf)

$$
\begin{aligned}
p_w(x) &= p_{w_R,w_I}(x_R, x_I) = p_{w_R}(x_R)p_{w_I}(x_I) \\
&= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{x_R^2}{N_0}} \times \frac{1}{\sqrt{\pi N_0}} e^{-\frac{x_I^2}{N_0}} = \frac{1}{\pi N_0} e^{-\frac{x_R^2+x_I^2}{N_0}} \\
&= \frac{1}{\pi N_0} e^{-\frac{|x|^2}{N_0}}
\end{aligned}
$$

# Non-orthogonal basis case

What's happened when $\{h(t - kT_s)\}_k$ is a **non-otn** basis

- ISI
- Colored noise

## Equivalent model

By using whitening filter $f$, we have

$$y(n) = f \star z(n) = \sum_{\ell=0}^{L} h(\ell) s_{n-\ell} + w(n)$$

with $w(n)$ white Gaussian noise

# Conclusion

Sequence of slides stops here but it remains to do

- Detector
  - → recovering $s_n$ from $z(n)$ in no-ISI case
  - → recovering $s_n$ from $z(n)$ in ISI case
- Performances
- Channel coding/decoding
- Channel impulse response estimation (blind equalization)
- Synchronization

# References


[Tse2005] D. Tse and P. Viswanath, Fundamentals of wireless communications, 2005

[Goldsmith2005] A. Goldsmith, Wireless Communications, 2005

[Proakis2000] J. Proakis, Digital Communications, 2000

[Benedetto1999] S. Benedetto and E. Biglieri, Principles of digital transmission with wireless applications, 1999

[Viterbi1979] A. Viterbi and J. Omura, Principles of digital communications and coding, 1979

[Gallager2008] R. Gallager, Principles of digital communications, 2008

[Wozencraft1965] J. Wozencraft and I. Jacob, Principles of communications engineering, 1965

[Barry2004] J. Barry, D. Messerschmitt and E. Lee, Digital communications, 2004

[Sklar2001] B. Sklar, Digital communications : fundamentals ans applications, 2001

[Ziemer2001] R. Ziemer and R. Peterson, Introduction to digital communication, 2001