Digital Information Processing

“Nonlinear mitigation”

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Outline

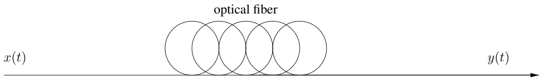
- Section 1: System model
- Section 2: Back propagation approach
- Section 3: Volterra series approach

Section 1 : System model

Problem statment

For the sake of simplicity

- 1 polarization stream (straightforward generalization)
- 1 span (straightforward generalization)



Increasing the data rate on optical fiber per wavelength

Multi-level constellation

$\Rightarrow$ *M*-QAM

$\Rightarrow$ Optical SNR has to be high enough

$\Rightarrow$ Input power has to be increased

Problem statment (cont'd)

When input power has to be increased ⇒ **Kerr effect**

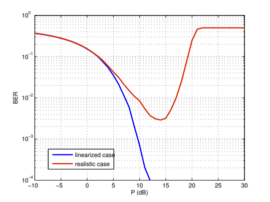
- Light refraction index depends on the power

$$n = n_0 + n(P)$$

- Nonlinear effect appears

⇒ **optical fiber is not a linear filter anymore**

- Error floor on BER
- Nonlinear processing for pushing down the floor



Input/Output relationship

NonLinear Schrödinger Equation (NLSE)

$$i\frac{\partial A}{\partial z} + i\frac{\alpha}{2}A - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \underbrace{\gamma_{NL}|A|^2 A}_{\text{nonlinear term}} = 0$$

with

- $x(t) = A(t, 0)$ and $y(t) = A(t, L)$
- L the fiber/span length
- α the attenuation
- β_2 the dispersion (CD)
- γ_{NL} the nonlinear coefficient

No closed-form solution without further simplifications (α or β_2)

[see lecture on Information Theory]

Solutions

First solution: based on NLSE

- Counterwise solution of NLSE $\rightarrow$ **back propagation** [Ip2008]
- Complicate operations

Second solution: based on Volterra series (VS)

- VS models quite well optical fiber propagation [Brandt1997,Bononi2002]
- *Third-order* inverse Volterra series [Lui2012,Amari2014]

Third solution: based on Neural Network (NN)

- not done in optical communications yet

Section 2: Back propagation approach

Inverse NLSE

We remind

$$\frac{\partial A}{\partial z} = \left[i\gamma_{NL}|A|^2 + \left(-\frac{\alpha}{2} - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} \right) \right] A$$

with $A(t,0) = x(t)$ and $A(t,L) = y(t)$

Remark

$\alpha \neq 0,\ \beta_2 = 0,\ \gamma = 0$	$A(t,L) = e^{-\alpha L/2} A(t,0)$
$\alpha = 0,\ \beta_2 \neq 0,\ \gamma = 0$	$A(f,L) = e^{2i\pi^2 \beta_2 f^2 L} A(f,0)$
$\alpha = 0,\ \beta_2 = 0,\ \gamma \neq 0$	$A(t,L) = e^{i\gamma_{NL} L \lvert A(t,0)\rvert^2} A(t,0)$

Input/Output permutation by opposite sign $\Rightarrow$ Back propagation

$x(t)$ obtained by solving Inverse NLSE with initial condition $y(t)$

$$\frac{\partial A}{\partial z} = \left[-i\gamma_{NL}|A|^2 + \left(\frac{\alpha}{2} + i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} \right) \right] A$$

Inverse NLSE

We remind

$$\frac{\partial A}{\partial z} = \left[i\gamma_{NL}|A|^2 + \left(-\frac{\alpha}{2} - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} \right) \right] A$$

with $A(t,0) = x(t)$ and $A(t,L) = y(t)$

Remark

$\alpha \neq 0, \beta_2 = 0, \gamma = 0$	$A(t,0) = e^{+\alpha L/2} A(t,L)$
$\alpha = 0, \beta_2 \neq 0, \gamma = 0$	$A(f,0) = e^{-2i\pi^2 \beta_2 f^2 L} A(f,L)$
$\alpha = 0, \beta_2 = 0, \gamma \neq 0$	$A(t,0) = e^{-i\gamma_{NL} L \|A(t,L)\|^2} A(t,L)$

Input/Output permutation by opposite sign $\Rightarrow$ Back propagation

$x(t)$ obtained by solving Inverse NLSE with initial condition $y(t)$

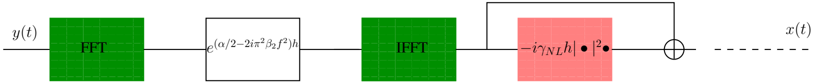
$$\frac{\partial A}{\partial z} = \left[-i\gamma_{NL}|A|^2 + \left(\frac{\alpha}{2} + i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} \right) \right] A$$

Solving NLSE

Split the span of length L into S slices of length h (small enough)
For each slice

- Solve the linear part: $\tilde{A}(f, z+h) = e^{(\alpha/2 - 2i\pi^2\beta_2 f^2)h} A(f, z)$
- Solve the nonlinear part: $A(t, z+h) = e^{-i\gamma_{NL} h |\tilde{A}(t,z+h)|^2} \tilde{A}(t, z+h)$
  so $A(t, z+h) \approx \tilde{A}(t, z+h) - i\gamma_{NL} h |\tilde{A}(t, z+h)|^2 \tilde{A}(t, z+h)$

Split-Step Fourier Method (SSMF)



Issue

- Solution is structure-agnostic: digital com. not taken into account
- Noise takes part to the Solution: it is even not a ZF!
- Serial processing

Section 3: Volterra series approach

Refresher on Volterra series

- Extension to nonlinear processing of filtering principle
- In real-valued and continuous-time case,

$$\begin{aligned} y(t) &= \int h_1(\tau)x(t-\tau)d\tau + \iint h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 \\ \cdots &+ \underbrace{\int \cdots \int h_n(\tau_1,\cdots,\tau_n)x(t-\tau_1)\cdots x(t-\tau_n)d\tau_1\cdots d\tau_n}_{n\text{-th order term}} + \cdots \end{aligned}$$

- Adaptation to
  - discrete time ($\int \to \sum$)
  - complex-valued (conjugate involved)

Kernels

$(\tau_1,\cdots,\tau_n) \mapsto h_n(\tau_1,\cdots,\tau_n)$ is the n-th order kernel

Operator tool for Volterra series

Volterra series can be represented through "operator tool"

$$y = H_1(x) + H_2(x) + \cdots + H_n(x) + \cdots$$

with

$$x \mapsto H_n(x) = \int \cdots \int h_n(\tau_1, \cdots, \tau_n) x(t - \tau_1) \cdots x(t - \tau_n) d\tau_1 \cdots d\tau_n$$

Operator

- H_n (related to kernel h_n) is the n-th order operator
- $\mathbf{H} = [H_1, H_2, \cdots, H_n, \cdots]$ is the global operator

Inverse Volterra series

We are looking for inverse Volterra series defined by **K** such that

$$\mathbf{K}(\mathbf{H}(x)) = x \Longleftrightarrow \mathbf{KH} = \mathbf{Id}$$

Be careful: non-commutative operator
Actually, too strong constraints, so we just are looking for

$$\mathbf{KH}(x) = \mathbf{P}(x) = x + P_{n+1}(x) + P_{n+2}(x) + \cdots$$

The first n-th order (except the 1-st) are forced to be null

Solution [Schetzen1975]

$$\begin{aligned} K_1 &= H_1^{-1} \\ K_2 &= -K_1 H_2 K_1 \\ K_3 &= -K_1 H_3 K_1 + K_2 + K_2 H_2 K_1 - K_2 (H_1 + H_2) K_1 \end{aligned}$$

Equations for kernels much more complicate

Application to optical fiber

- Volterra series provide approximate solution to NLSE
- **In the frequency domain**, we have [Brandt1997]

$$Y(f) \approx e^{2i\pi^2\beta_2 L f^2} X(f) + Y_{NL}(f) + W(f)$$

with

$$\begin{aligned} Y_{NL}(f) &= e^{2i\pi^2\beta_2 L f^2}(2i\pi)\gamma_{NL} \iiint \delta(f - f_1 + f_2 - f_3) X(f_1) X^*(f_2) X(f_3) \\ &\times \frac{e^{L\left(-\alpha + 2i\pi^2\beta_2(-f^2+f_1^2-f_2^2+f_3^2)\right)} - 1}{-\alpha + 2i\pi^2\beta_2(-f^2+f_1^2-f_2^2+f_3^2)} df_1 df_2 df_3 \end{aligned}$$

Volterra Series Transfer Function (VSTF)
powerful tool for solving NLSE

Application to optical fiber (cont'd)

$$Y = \underbrace{H_1(X)}_{Y_1} + \underbrace{H_3(X)}_{Y_3}$$

where

- input X and output Y of the fiber
- H_1 and H_3 are the first-order and third-order VSTF

$$\begin{aligned} Y_1(f) &= h_1(f)X(f) \\ Y_3(f) &= \iint h_3(f_1, f_2, f - f_1 + f_2)X(f_1)X^*(f_2)X(f - f_1 + f_2)df_1 df_2 \end{aligned}$$

where the kernels h_1 and h_3 are given by

$$\begin{aligned} h_1(f) &= e^{2i\pi^2\beta_2 L f^2} \\ h_3(f_1, f_2, f - f_1 + f_2) &= 2i\pi C h_1(f) \end{aligned}$$

with $C = \gamma_{NL}(1 - e^{-\alpha L})/\alpha$.

Volterra based receiver

We propose to make the decision on Z obtained as follows

$$Z = \underbrace{K_1(Y)}_{Z_1} + \underbrace{K_3(Y)}_{Z_3}$$

with K_1 and K_3 the Inverse VSTF operators associated with (H_1, H_3)

Denoting $\Delta\Omega = (f_1 - f)(f_1 - f_2)$, we obtain

$$\begin{aligned} Z(f) &= \underbrace{e^{-2i\pi^2\beta_2 L f^2}}_{k_1(f)} Y(f) \\ &+ \iint \underbrace{\frac{iC}{4\pi^2} e^{-2i\pi^2\beta_2 L f^2} e^{-4i\pi^2\beta_2 L\Delta\Omega}}_{k_3(f_1, f_2, f-f_1+f_2)} Y(f_1)Y^*(f_2)Y(f-f_1+f_2)df_1 df_2 \end{aligned}$$

- Looks complicate and tedious due to double integral
- <u>but</u> practical implementation is possible

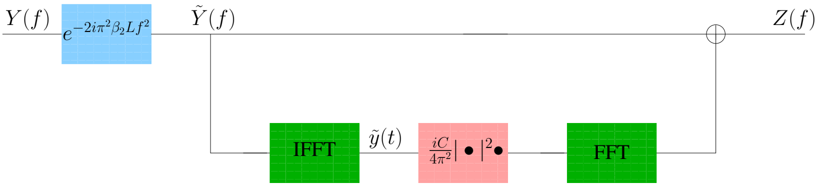
Practical implementation

By working on $\tilde{Y}(f) = e^{-2i\pi^2\beta_2 L f^2} Y(f)$, we easily have

$$K_3(Y) = \frac{iC}{4\pi^2} \iint \tilde{Y}(f_1)\tilde{Y}^*(f_2)\tilde{Y}(f - f_1 + f_2) df_1 df_2$$

Let $\tilde{y}(t)$ be the Inverse Fourier Transform of $\tilde{Y}(f)$, we get

$$\begin{aligned} K_3(Y) &= \frac{iC}{4\pi^2} \int |\tilde{y}(t)|^2 \tilde{y}(t) e^{2i\pi f t} dt \\ &= \frac{iC}{4\pi^2} \mathrm{FT}\left(|\tilde{y}(t)|^2 \tilde{y}(t)\right) \end{aligned}$$

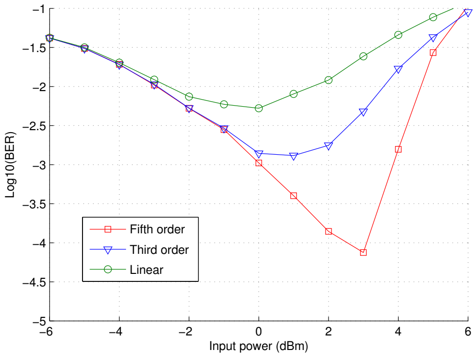


Numerical results: setup

- One polarization
- Data rate is only 200Gb/s
- Four bands of 20GHz each and spaced by a 10GHz guard band
- 16-QAM OFDM with 512 subcarriers on each band
- 20 spans of 100km each
- $\alpha = 0.2\mathrm{dB.km}^{-1}$
- $\beta_2 = 17\mathrm{ps.nm}^{-1}.\mathrm{km}^{-1}$
- $\gamma_{NL} = 0.0014\mathrm{m}^{-1}.\mathrm{W}^{-1}$

Numerical results: BER vs. input power

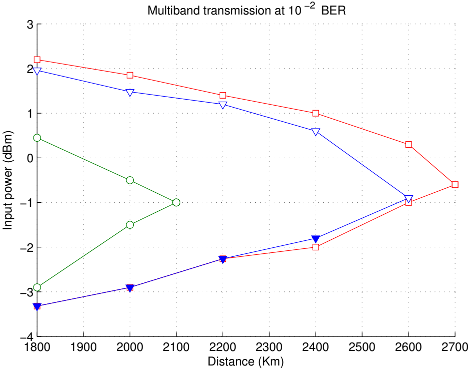
- one active band



Third-order IVSTF outperforms linear processing, and the smallest BER is really improved

Numerical results: input power vs. distance

- four active bands
- target BER at 10^{-2}



Compared to linear processing, the gain is around 500km

Conclusion

- Nonlinear processing provides gain
- Active research since numerous open issues
  - receiver design
  - new waveform (NonLinear Fourier Transform `[Kschischang2014]`)
- Topic close to satellite communications !

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