

SHANNON-KOTELNIKOV MAPPINGS FOR SOFTCAST-BASED JOINT SOURCE-CHANNEL VIDEO CODING

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ABSTRACT

This paper introduces Shannon-Kotelnikov (SK) mapping in the SoftCast joint source-channel video coding scheme. On bandwidth constrained channels, the performance of SoftCast saturates, due to the large amount of data (chunks) dropped to match the bandwidth requirements. Using SK mapping, it is possible to increase the number of chunks that may be transmitted without increasing the bandwidth requirements. The resulting scheme has an increased number of design parameters for which we present a transmission-power constrained optimization. This extends range of channel SNRs over which the PSNR gracefully increases and improves the end-to-end performance at medium to high SNRs. The price to be paid is a performance degradation at low SNRs.

1. INTRODUCTION

The SoftCast-inspired video coders (SoftCast, WaveCast, or D-Cast [1, 2, 3]) are recently developed joint source-channel (JSC) video coding solutions, which have the potential for dramatically improve the quality of the received video in challenging conditions, such as video broadcast over wireless networks. SoftCast replaces nonlinear parts of classical source coders and transmission devices (quantization, entropy coding, channel coding) by linear operators, such as full-frame 2D or full-GoP 3D transforms, power allocation schemes, as well as analog modulations. The joint source coding and transmission process may then be modeled as a linear operator. Thus, the received quality improves gracefully with the quality of the channel, without requiring any adaptation of the coding parameters.

For bandwidth-constrained channels, SoftCast has to limit the number of chunks (groups of transformed and scaled pixels) transmitted per GoP: the chunks containing most energy are transmitted, the remaining ones being dropped. As a consequence, video quality increases until a given channel signal-to-noise ratio (C-SNR) is reached, where the quality saturates at a level that depends on the proportion of transmitted chunks.

The main idea of this paper is to use bandwidth-reducing Shannon-Kotelnikov (SK) mappings [4, 5, 6] to increase the number of chunks transmitted over bandwidth-constrained channels. SK mappings (SKM) are $N:1$ bandwidth-reducing or $1:M$ bandwidth-expanding non-linear mappings. Here, $2:1$ mappings are used to encode several pairs of chunks prior to transmission. Chunks with the most energy are transmitted as with the plain SoftCast. Chunks with less energy are SK-mapped. More information about medium-energy chunks is thus transmitted, which may increase the video quality for users with good channel conditions, without degrading significantly the video quality of users with less good channels.

The rest of this paper briefly overviews the principles of SofCast and SKM in Section 2. The proposed approach to improve SoftCast

with SKM is detailed in Section 3. A particular attention is paid to the resource allocation between uncoded and SK-mapped chunks. Experimental results are detailed in Section 4, before drawing some conclusions in Section 5.

2. BACKGROUND

2.1. SoftCast

The basic idea of SoftCast is to encode the video content with linear operators. Energy compaction is performed via a full-GoP 3D-DCT. Entropy coding, which is highly non-linear, is avoided. Transformed coefficients are grouped into chunks $\mathbf{x}_i \in \mathbb{R}^N$, $i = 1, \dots, n_T$, where N is the number of transformed coefficients per chunk, λ_i is the variance of the coefficients of the i -th chunk, and n_T is the number of chunks per GoP.

An optimized chunk scaling is then performed to minimize the expected reconstruction error at receiver $\mathbf{y}_i = g_i \mathbf{x}_i$. Even though the scaling is linear, the adaptation to the channel bandwidth requires transmitting only n_C chunks per GoP, the remaining $n_T - n_C$ being dropped. This means that, even when the C-SNR is high, the reconstructed video quality is limited by the value of n_C .

Resilience to channel fades is obtained by giving up temporal prediction and ensuring that all packets contribute equally to the quality of the decoded video. For that purpose, SoftCast employs an Hadamard transform of the kept chunks. The physical layer of SoftCast uses, *e.g.*, a classical OFDM framework with some total transmission power constraint. The real-valued samples at the output of the Hadamard transform are then used to modulate the I and Q components of all carriers. In the original SoftCast, LMMSE estimation of the chunks is performed from the channel outputs. Here, inverse Hadamard transform is considered prior to LMMSE estimation of the chunks. This has the advantage of averaging the noise introduced on each carrier and to simplify LMMSE estimation which may be performed chunk per chunk. As a consequence, the part of the transmission scheme between the direct and inverse Hadamard transforms can be viewed as several parallel Gaussian channels with identical noise variance.

With these hypotheses, at the receiver side, one has $\tilde{\mathbf{y}}_i = g_i \mathbf{x}_i + \mathbf{n}_i$, where the components of \mathbf{n}_i are realizations of iid zero-mean Gaussian variables with variance σ^2 . The LMMSE estimate of \mathbf{x}_i from $\tilde{\mathbf{y}}_i$ is $\hat{\mathbf{x}}_i = \alpha \tilde{\mathbf{y}}_i$, with $\alpha = \lambda_i g_i / (\lambda_i g_i^2 + \sigma^2)$. When σ^2 is small compared to $\lambda_i g_i^2$, *i.e.*, the C-SNR is large enough, one may use the approximation $\alpha = 1/g_i$. In this case, the total distortion, *i.e.*, the sum of the variance of the chunk coefficients reconstruction error is

$$D_T = E \left(\frac{1}{N} \sum_{i=1}^{n_T} \|\hat{\mathbf{x}}_i - \mathbf{x}_i\|^2 \right) = \sum_{i=1}^{n_C} \frac{\sigma^2}{g_i^2} + \sum_{j=n_C+1}^{n_T} \lambda_j. \quad (1)$$

The last term in (1) is the contribution to D_T of the $n_T - n_C$ dropped chunks, which limits the performance of SoftCast when the channel is clear.

For a given value of n_C , the g_i s that minimize (1) under a total transmission power constraint $\sum_i g_i^2 \lambda_i \leq P$ are shown in [1] to be

$$g_i = \sqrt{\frac{P}{\sqrt{\lambda_i} \sum_{j=1}^{n_C} \sqrt{\lambda_j}}}.$$

2.2. Shannon-Kotelnikov mappings

SKMs are lossy JSC coding tools proposed in [4, 5, 6] for transmitting memoryless sources over AWGN channels. An SKM maps amplitude-continuous, time-discrete source samples directly onto the channel using space-filling curves or surfaces. The source and channel spaces can have different dimensions, thereby achieving either compression or error control, depending on whether the source bandwidth is smaller or larger than the channel bandwidth. In [6], a general theory for 1: N and M :1 dimension-changing SKMs is presented. These schemes show high spectral efficiency and provide both graceful degradation and improvement for imperfect channel state information at the transmitter.

SKMs constitute thus good candidates to match the bandwidth requirements of the transmission of data generated by JSC video coders and actual channel bandwidth: instead of dropping chunks, they may be encoded using bandwidth-reducing SKM. Nevertheless, this raises the problem of the optimum design of the characteristics of the SKM, *i.e.*, determining the proportion of SK-mapped chunks, their scaling, the power allocation between plain and SK-mapped chunks.

In what follow, the considered SKM are 2 : 1 bandwidth-reducing double intertwined Archimedes' spirals with parametric equation

$$\begin{cases} f_1(\theta) &= \frac{\Delta}{\pi} \theta \cos \theta \\ f_2(\theta) &= \frac{\Delta}{\pi} |\theta| \sin \theta \end{cases}, \theta \in \mathbb{R}, \quad (2)$$

where $(f_1(\theta), f_2(\theta))^T$ is a point of the spiral and Δ is the distance between the two neighboring arms of the spiral. Consider a realization $\mathbf{x} = (x_1, x_2)^T$ of some random pair $\mathbf{X} = (X_1, X_2)^T$, where X_1 and X_2 are independent, zero-mean variables with variance λ_1 and λ_2 , with $\lambda_1 \approx \lambda_2$. The 2 : 1 SK mapping of the pair \mathbf{x} aims at determining the point $\mathbf{x}(\tilde{\theta}) = (f_1(\tilde{\theta}), f_2(\tilde{\theta}))^T$ lying the closest to \mathbf{x} on the spiral. The angle $\tilde{\theta}$ from the origin to $\mathbf{x}(\tilde{\theta})$ is then used to evaluate the scalar $z(\tilde{\theta}) = a\tilde{\theta}^2 \text{sgn}(\tilde{\theta})$, where sgn is the sign function. Taking $a = 0.16\Delta$ provides a good approximation of the curvilinear abscissa of $\mathbf{x}(\tilde{\theta})$ on the spiral. Then, $z(\tilde{\theta})$ has to be scaled by some parameter γ to match the power constraint of the channel to get

$$y_\Delta(\mathbf{x}) = \gamma z, \quad (3)$$

which represents the SK-mapped \mathbf{x} . With the previous assumptions, the variance of z may be approximated as

$$\sigma_z^2 = 2 \left(0.16\Delta \frac{\pi^2}{\Delta^2} (\lambda_1 + \lambda_2) \right)^2. \quad (4)$$

Assuming that $y_\Delta(\mathbf{x})$ is transmitted on a AWGN channel with noise variance σ^2 and that the channel output is r , an MMSE estimation of \mathbf{x} may be performed from r , as in [7]. One may also resort to a less-complex suboptimal LMMSE estimate. Estimating θ as $\hat{\theta} = \sqrt{\frac{|r|}{a\gamma}} \text{sgn} r$ allows to get the following approximation of the variance

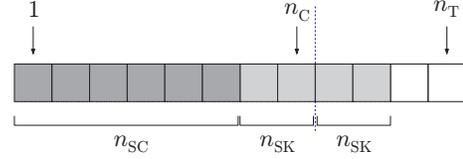


Fig. 1. Proposed approach: location of the $2n_{SK}$ SK-mapped chunks of the per-component reconstruction error between \mathbf{x} and $\mathbf{x}(\hat{\theta}) = (f_1(\hat{\theta}), f_2(\hat{\theta}))^T$ (see [5] for more details):

$$\sigma_r^2 = \frac{1}{2} \left(\frac{\Delta^2}{12} + \frac{\sigma^2}{\gamma^2} \right). \quad (5)$$

3. PROPOSED APPROACH

The main idea of this work is to use SKMs together with SoftCast, with the goal of improving the global power/distortion trade-off. Instead of dropping $n_T - n_C$ chunks as in plain SoftCast, SKMs are used to transmit n_{SK} additional chunks with $n_{SK} < n_C$.

More precisely, assuming that chunks have been sorted by decreasing energy, those indexed from 1 to $n_{SC} = n_C - n_{SK}$ are transmitted as with plain SoftCast. The following $2n_{SK}$ chunks (from $n_{SC} + 1$ to $n_{SC} + 2n_{SK} = n_C + n_{SK}$) are combined into n_{SK} pairs which are SK-mapped. The remaining chunks $n_T - n_C - n_{SK}$ are discarded. With this technique, the number of transmitted symbols is Nn_C independently from the number of SK-mapped chunks. The plain SoftCast scheme is obtained taking $n_{SK} = 0$.

The problem of optimal power allocation is made more difficult by the presence of $2n_{SK}$ SK-mapped chunks. The remainder of this section formulates this problem and provides an optimal solution.

We transmit n_{SC} plain chunks, *i.e.*, non SK-mapped, using scaling factors g_i , and n_{SK} SK-mapped pairs of chunks. More precisely, considering the i -th chunk \mathbf{x}_i , the transmitted symbols are the elements of n_C N -dimensional vectors $\mathbf{y}_i = g_i \mathbf{x}_i$, $i = 1, \dots, n_{SC}$ and $\mathbf{y}_i = \gamma_{i-n_{SC}} \mathbf{z}_{i-n_{SC}}$, $i = n_{SC} + 1 \dots n_{SC} + n_{SK}$, where g_i and γ_j are suitable scaling factors for the plain and SK-mapped pairs of chunks, and

$$\mathbf{z}_j = \text{SM}(\mathbf{x}_{n_{SC}+2j-1}, \mathbf{x}_{n_{SC}+2j}). \quad (6)$$

is the vector containing the SK-mapped pairs of coefficients in the chunks $\mathbf{x}_{n_{SC}+2j-1}$ and $\mathbf{x}_{n_{SC}+2j}$, which is performed as in Section 2.2.

The system parameters are n_C , n_{SK} , g_i , γ_j , Δ_j , and the reference variance of the channel noise σ^2 . These parameters are tuned to minimize a total distortion under a total transmission power constraint, which are now evaluated.

3.1. Distortion evaluation

The total distortion at receiver side of a given GOP is the sum of the average distortions of each chunk:

$$D_T = \sum_{i=1}^{n_T} D_i = \sum_{i=1}^{n_{SC}} D_i + \sum_{j=1}^{n_{SK}} \bar{D}_j + \sum_{\ell=n_T-n_{SC}-2n_{SK}}^{n_T} \lambda_\ell \quad (7)$$

where D_i is the distortion of a plain chunk, \bar{D}_j the distortion of a SK-mapped chunk, and the last term in (7) is the distortion associated to last discarded chunks as in (1). As in plain SoftCast,

$D_i = \frac{\sigma^2}{g_i^2}$, see (1) where σ^2 is the receiver noise variance. Moreover,

$$\bar{D}_j = \frac{\Delta_j^2}{12} + \frac{\sigma^2}{\gamma_j^2} \quad (8)$$

is directly deduced from (5), where one has taken into account the fact that two chunks are jointly encoded.

3.2. Transmission power evaluation

The transmission power of the i -th plain chunk is $g_i^2 \lambda_i$; similarly, the power for the j -th SK-mapped chunk z_j is $\gamma_j^2 \sigma_{z_j}^2$. Using (4), one gets

$$\sigma_{z_j}^2 = E[\|z_j\|^2] = 2 \left[0.16 \frac{\pi^2}{\Delta_j} (\lambda_{n_{\text{SC}}+2j-1} + \lambda_{n_{\text{SC}}+2j}) \right]^2$$

Defining $t_j = \sqrt{2} \cdot 0.16 \pi^2 (\lambda_{n_{\text{SC}}+2j-1} + \lambda_{n_{\text{SC}}+2j})$, one deduces the transmission power constraint of the SK-SoftCast scheme

$$\sum_{i=1}^{n_{\text{SC}}} g_i^2 \lambda_i + \sum_{j=1}^{n_{\text{SK}}} \frac{\gamma_j^2 t_j^2}{\Delta_j^2} \leq P. \quad (9)$$

3.3. Constrained optimization

The parameters of the proposed SK-SoftCast scheme have to be tuned to minimize the distortion (7) under the transmission power constraint (9). The problem being quite complex, we assume first that n_{C} , n_{SK} , and σ^2 are fixed and perform a constrained optimization of the other parameters. The value of n_{C} has to be chosen so that the transmitted data fit the available bandwidth. As will be seen in Section 4, the values of n_{SK} and σ^2 should be tuned based on the distribution of the channel quality of the receivers. For the distortion optimization, one focuses on the first two terms in (7).

The transmission power constrained distortion optimization problem may be solved introducing the following Lagrangian deduced from (7), (8), and (9)

$$\begin{aligned} \mathcal{L}(\mathbf{g}, \boldsymbol{\gamma}, \boldsymbol{\Delta}, \mu) &= \sum_{i=1}^{n_{\text{SC}}} \frac{\sigma^2}{g_i^2} + \sum_{j=1}^{n_{\text{SK}}} \left(\frac{\Delta_j^2}{12} + \frac{\sigma^2}{\gamma_j^2} \right) \\ &+ \mu \left[\sum_{i=1}^{n_{\text{SC}}} g_i^2 \lambda_i + \sum_{j=1}^{n_{\text{SK}}} \frac{\gamma_j^2 t_j^2}{\Delta_j^2} - P \right], \end{aligned} \quad (10)$$

where μ is the Lagrange multiplier.

Deriving \mathcal{L} with respect to g_i , one deduces the optimal scaling factor for plain chunks

$$g_i = \left(\frac{\sigma^2}{\mu \lambda_i} \right)^{1/4}. \quad (11)$$

Then, deriving \mathcal{L} with respect to γ_j , one obtains the optimal scaling factor for SK-mapped chunks

$$\gamma_j = \left(\frac{\Delta_j^2 \sigma^2}{\mu t_j^2} \right)^{1/4} \quad (12)$$

Deriving \mathcal{L} with respect to Δ_j , one gets the optimal distance between spiral arms for each pair of SK-mapped chunks

$$\Delta_j = (12 \mu t_j^2 \gamma_j^2)^{1/4}. \quad (13)$$

Combining (12) and (13), one deduces $\Delta_j^2/12 = \sigma^2/\gamma_j^2$, i.e., for each SK-mapped pair of chunks, the power allocation is optimal when the two contributions to the distortion \bar{D}_j (quantization and channel noise) are identical. Moreover, using the same equations, one may express the optimal values of γ_j and Δ_j as functions of μ and other known parameters only

$$\Delta_j = (12 \sqrt{\mu} t_j \sigma)^{1/3}, \quad \gamma_j = \left(\frac{12 \sigma^4}{\mu t_j^2} \right)^{1/6}. \quad (14)$$

To derive the value of μ in (10), one uses the total transmission power constraint (9), to show that μ has to satisfy

$$P = a \mu^{-1/2} + b \mu^{-2/3} \quad (15)$$

where

$$a = \sigma \sum_{i=1}^{n_{\text{SC}}} \lambda_i^{1/2}, \quad b = \left(\frac{\sigma^2}{12} \right)^{1/3} \sum_{j=1}^{n_{\text{SK}}} t_j^{2/3}. \quad (16)$$

Then μ requires a numerical solution of (15), where $a > 0$ and $b \geq 0$. The solution of (15) exists and is unique and positive, since (15) is the sum of two strictly decreasing functions with image \mathbb{R}^+ . From μ , one determines g_i using (11). Then Δ_j and γ_j are obtained from (14).

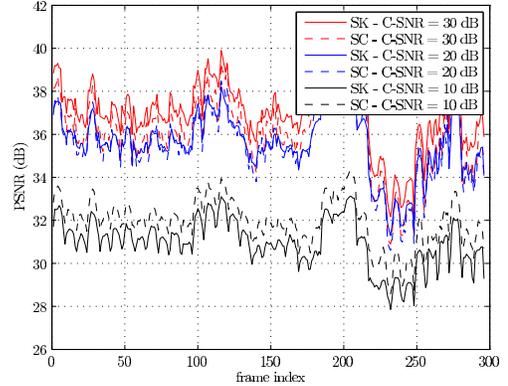


Fig. 2. PSNR of Foreman as a function of the frame index of SK-SoftCast (SK) and SoftCast (SC) for various C-SNRs

4. EXPERIMENTAL RESULTS

The performance of the proposed SK-SoftCast scheme has been compared to the reference SoftCast proposed in [1]. Two video sequences, foreman.cif and Kimono1_1920x1080_24 from the HEVC test set have been considered. Only their luminance has been encoded. For both sequences, GoPs of 8 pictures are considered. The chunk size is 36×44 and $n_{\text{C}} = 128$ for foreman whereas the chunk size is 30×40 and $n_{\text{C}} = 512$ for Kimono. This corresponds for SoftCast to the transmission of 25% of the chunks for Foreman and to 3.7% of the chunks for Kimono. The transmission power is taken as $P = 1$. The value of σ^2 depends on the C-SNR, which is usually unknown at the transmitter and may vary among receivers in case of broadcast applications. It has thus to be adjusted considering some reference C-SNR.

Figure 2 shows the PSNR as a function of the frame index for Foreman encoded with SK-SoftCast (SK) and SoftCast (SC) for various C-SNRs. The reference SNR is chosen as 20dB and $n_{\text{SK}} = 64$

pairs of chunks have been SK-mapped. One observes that at a real C-SNR of 10dB, SK experiences a degradation in PSNR of about 1.5dB, while between 20dB and 30dB, the performance improvement reaches 2dB. This improvement is observed for all frames. This tendency is confirmed by Figure 3(a). SK performs better than SC when the C-SNR is above 17dB. Similar results are observed for the HD sequence Kimono (Fig. 3(b)).

The evolution of the PSNR gain on Foreman for SK compared to SC is represented in Figure 4(a) as a function of the C-SNR for $n_{SK} = 32$, $n_{SK} = 64$, and $n_{SK} = 96$, all at a target C-SNR of 20dB. One sees that increasing n_{SK} increases the PSNR gains at high C-SNRs, but the price to be paid is an increased loss at low C-SNRs. The value $n_{SK} = 64$ seems to be a good compromise, as shown by the averaged PSNR gain evaluated over the C-SNR interval $[0, 40]$ dB. Figure 4(b) presents similar results for five different design SNRs, namely 10dB, 15dB, 20dB, 25dB, and 30dB, all for $n_{SK} = 64$. Each of the five configurations outperforms the other when the actual C-SNR is around 17dB, which is somewhat surprising, as one would expect that the transition is around the design SNR. This is due to the bad quality of the SKM distortion model (4) at low C-SNR, see the next experiment. Finally, we also observe that in the $[0, 40]$ dB interval, the mean design SNR of 20dB provides the best average performances.

To better understand the previous results, the evolution of the SNR at receiver as a function of the C-SNR for a zero-mean unit-variance Gaussian source is considered. This source generates pairs of samples transmitted over a Gaussian channel allowing the transmission of a single sample per channel use. The OPTA curve of this setup [5] is

$$\text{SNR}_{dB} = 10 \log_{10} \left(\sqrt{1 + 10^{C-\text{SNR}_{dB}/10}} \right). \quad (17)$$

The theoretical (Th) curves are obtained from (5), where σ^2 depends on the actual C-SNR and Δ and γ have to be optimized considering some reference C-SNR. Three reference C-SNRs are considered, namely, 10dB, 20dB, and 30dB. The corresponding results obtained with the SK scheme are represented by the SK curves. Finally, the performance of a system where only half of the samples are transmitted is also provided (SC curve). This would correspond to the behavior of SoftCast when all chunks have the same variance, and only half of them are transmitted.

The theoretical curves are very close to the optimal performance theoretically attainable (OPTA) when the C-SNR is close to the reference SNR. The actual performance of the SK scheme matches much better the theoretical curves at high C-SNR and when the reference C-SNR is high than at low C-SNR. The discrepancy at low C-SNR is due to the use of an LMMSE estimator, which effect is not taken into account in (5). This explains the results obtained in Figure 4(b). In all cases, one sees that using SK-mappings is beneficial compared to a solution where only half of the samples are transmitted only when the C-SNR is above some threshold that depends on the reference C-SNR. This explains the PSNR loss in Figures 3 and 4 when the C-SNR is low.

5. CONCLUSIONS

This paper introduces Shannon-Kotelnikov mappings in the SoftCast JSC video coding scheme. With SoftCast, a large proportion of the encoded chunks have to be dropped to match the bandwidth constraint, when it is stringent. In the proposed SK-SoftCast scheme, part of the chunks are SK-mapped, which increases the amount of transmitted chunks, without increasing the bandwidth requirements.

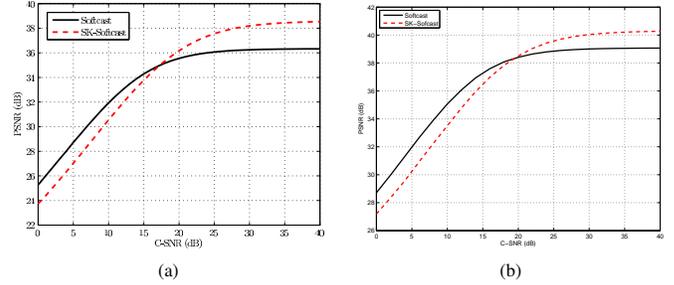


Fig. 3. PSNR of Foreman (a) and Kimono (b) as a function of the C-SNR for SK-SoftCast and plain SoftCast

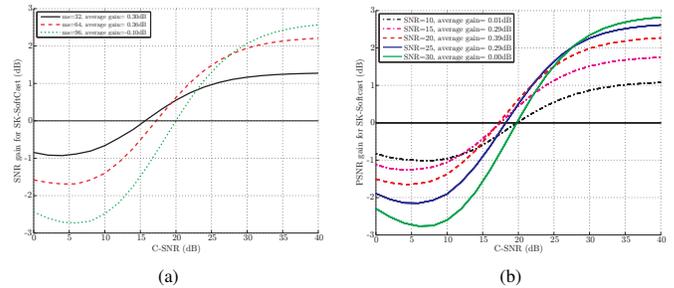


Fig. 4. Improvement in average PSNR provided by SK compared to SC for Foreman as a function of the C-SNR, for various n_{SK} (a) and for various target C-SNRs (b)

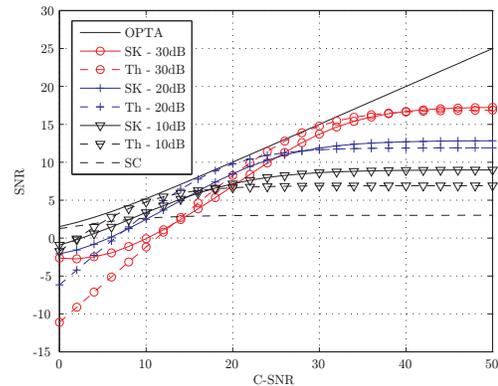


Fig. 5. Performance on synthetic source samples of the SK distortion model, the SK actual performance, compared to the OPTA, and to the transmission of half of the samples

SK-SoftCast increases the range of C-SNRs over which the PSNR gracefully increases, improves the performance at medium to high SNRs, where SoftCast saturates. The price to be paid is a decrease of the performance at low SNRs. This is mainly due to the fact that at low SNR, LMMSE estimation of SK-mapped chunks is suboptimal compared to the more time-consuming MMSE estimate, see [7]. Moreover, the SK-mapping distortion model is better at medium to high SNRs, than at low SNRs, which complicates optimization.

This work opens several research directions, related to the optimization of the number of chunks to encode using SKMs, to the target SNR to consider. For that purpose, when considering broadcast applications, the characteristics of the channels seen by the various receivers may be useful to optimize these parameters. Beside that, one may try to optimize the way SK-mappings are used, considering the content of the GoPs.

6. REFERENCES

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