Static Analysis of Worst-Case Stack Cache Behavior

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Real-Time Systems

Strict timing **guarantees**

- Critical tasks have to be completed in time

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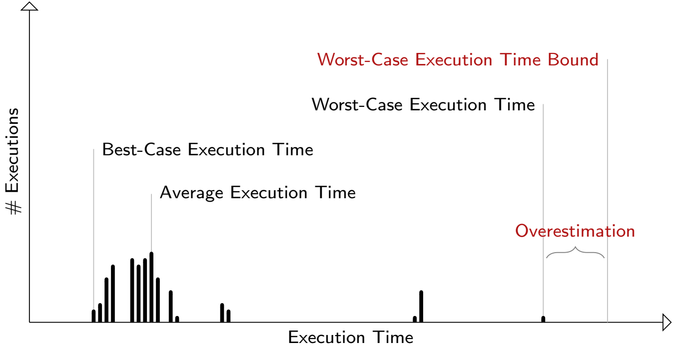
- Critical tasks have to be completed in time
- Bound *Worst-Case Execution Time* (WCET)



WCET Analysis

Bound longest possible execution time of a program

- Covering all potential execution paths
- Covering all potential program inputs
- Covering all potential hardware states

WCET Analysis

Bound longest possible execution time of a program

- Covering all potential execution paths
- Covering all potential program inputs
- Covering all potential **hardware states**
  - Processor pipeline
  - Branch predictors
  - Data and instruction caches
  - Main memory

Example: Miss/Hit Classification

Initial cache state*

0x100	0x200
0x101	0x103

*Cache configuration
2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement

Example: Miss/Hit Classification

Initial cache state*

0x100	0x200
0x101	0x103

`lw [0x100]`

0x100	0x200
0x101	0x103

Classified as hit

*Cache configuration
2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement

Example: Miss/Hit Classification

Initial cache state*

0x100	0x200
0x101	0x103

`lw [0x100]`

0x100	0x200
0x101	0x103

Classified as hit

`lw [0x105]`

0x100	0x200
0x105	0x101

Classified as miss

*Cache configuration
2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement

Example: Miss/Hit Classification

Initial cache state*

0x100	0x200
0x101	0x103

`lw [0x100]`

0x100	0x200
0x101	0x103

Classified as hit

`lw [0x105]`

0x100	0x200
0x105	0x101

Classified as miss

`lw [??]`

??	??
??	??

Classification unclear

*Cache configuration
2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement

Example: Miss/Hit Classification

Initial cache state*

0x100	0x200
0x101	0x103

`lw [0x100]`

0x100	0x200
0x101	0x103

Classified as hit

`lw [0x105]`

0x100	0x200
0x105	0x101

Classified as miss

`lw [??]`

??	??
??	??

Classification unclear

Main challenge

The abstract cache state of the analysis depends on the precise address and order of the executed memory accesses.

*Cache configuration
2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement

Context-Sensitivity

Miss/hit classification requires

- Precise information to disambiguate addresses
- High levels of context-sensitivity
- High levels of virtual loop unrolling
- Analysis effort is multiplied accordingly

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- Precise information to disambiguate addresses
- High levels of context-sensitivity
- High levels of virtual loop unrolling
- Analysis effort is multiplied accordingly

Main problem

Subsequent phases of WCET analysis suffer from high complexity due to this virtual code duplication.

Alternative Solution

Predictable caching

- Dedicated caches designed for analyzability/predictability
- Easy to analyze
- Simple hardware design
- Requiring no/little information on accesses addresses

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In this work

Time-predictable caching of stack data using a *stack cache*.

What is a Stack Cache?

Dedicated cache for stack data

- Simple ring buffer (*FIFO replacement*)
- All stack accesses are guaranteed hits (no need to analyze them)
- Dedicated stack control instructions (need to be analyzed)
  - `sres` *x*: reserve *x* blocks on the stack
  - `sfree` *x*: free *x* blocks on the stack
  - `sens` *x*: ensure that at least *x* blocks are cached
- Intuitively: a cache window following the stack top

Example: Stack Cache

```
(1) function A()       function B()       function C()
(2) sres 2             sres 3             sres 2
(3) call B()           call C()           sfree 2
(4) sens 2             sens 3
(5) call C()           call C()
(6) sens 2             sens 3
(7) sfree 2            sfree 3
```

Logical stack

Stack cache*

*Cache configuration: 4 blocks

Example: Stack Cache

```
(1) function A()
(2) sres 2 ←
(3) call B()
(4) sens 2
(5) call C()
(6) sens 2
(7) sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2
sfree 2
```

Logical stack

A	A							

Stack cache*

A	A		

*Cache configuration: 4 blocks

Example: Stack Cache

```
function A()
sres 2
call B() ←
sens 2
call C()
sens 2
sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2
sfree 2
```

Logical stack

A	A							

Stack cache*

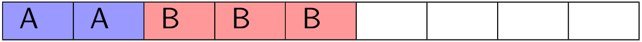
A	A		

*Cache configuration: 4 blocks

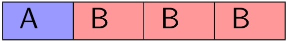
Example: Stack Cache

```
(1) function A()        function B()      function C()
(2) sres 2              sres 3 ←          sres 2
(3) call B()            call C()          sfree 2
(4) sens 2              sens 3
(5) call C()            call C()
(6) sens 2              sens 3
(7) sfree 2             sfree 3
```

Logical stack



Stack cache*



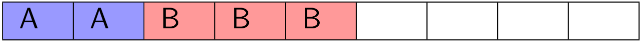
spill 1 block

*Cache configuration: 4 blocks

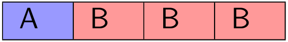
Example: Stack Cache

```
function A()        function B()        function C()
sres 2              sres 3              sres 2
call B()            call C() ←          sfree 2
sens 2              sens 3
call C()            call C()
sens 2              sens 3
sfree 2             sfree 3
```

Logical stack



Stack cache*



*Cache configuration: 4 blocks

Example: Stack Cache

```
function A()      function B()      function C()
sres 2            sres 3            sres 2 ←
call B()          call C()          sfree 2
sens 2            sens 3
call C()          call C()
sens 2            sens 3
sfree 2           sfree 3
```

Logical stack

A	A	B	B	B	C	C		

Stack cache*

B	B	C	C

spill 2 blocks

*Cache configuration: 4 blocks

Example: Stack Cache

```
function A()
sres 2
call B()
sens 2
call C()
sens 2
sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2
sfree 2 ←
```

Logical stack

A	A	B	B	B				

Stack cache*

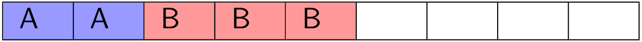
B	B		

*Cache configuration: 4 blocks

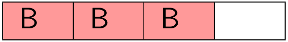
Example: Stack Cache

(1) `function A()`	`function B()`	`function C()`
(2) `sres 2`	`sres 3`	`sres 2`
(3) `call B()`	`call C()`	`sfree 2`
(4) `sens 2`	`sens 3` ←	
(5) `call C()`	`call C()`	
(6) `sens 2`	`sens 3`	
(7) `sfree 2`	`sfree 3`	

Logical stack



Stack cache*



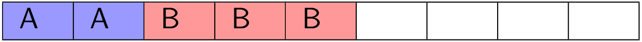
fill 1 block

*Cache configuration: 4 blocks

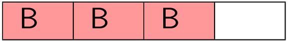
Example: Stack Cache

```
(1) function A()      function B()      function C()
(2) sres 2            sres 3            sres 2
(3) call B()          call C()          sfree 2
(4) sens 2            sens 3
(5) call C()          call C() ←
(6) sens 2            sens 3
(7) sfree 2           sfree 3
```

Logical stack



Stack cache*



*Cache configuration: 4 blocks

Example: Stack Cache

function A()	function B()	function C()
sres 2	sres 3	sres 2 ←
call B()	call C()	sfree 2
sens 2	sens 3	
call C()	call C()	
sens 2	sens 3	
sfree 2	sfree 3	

Logical stack

A	A	B	B	B	C	C		

Stack cache*

B	B	C	C

spill 1 block

*Cache configuration: 4 blocks

Example: Stack Cache

```
(1) function A()
(2) sres 2
(3) call B()
(4) sens 2
(5) call C()
(6) sens 2
(7) sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2
sfree 2 ←
```

Logical stack

A	A	B	B	B				

Stack cache*

B	B		

*Cache configuration: 4 blocks

Example: Stack Cache

```
(1) function A()
(2) sres 2
(3) call B()
(4) sens 2
(5) call C()
(6) sens 2
(7) sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3 ←
sfree 3
```

```
function C()
sres 2
sfree 2
```

Logical stack

A	A	B	B	B				

Stack cache*

B	B	B	

fill 1 block

*Cache configuration: 4 blocks

Example: Stack Cache

```
(1) function A()        function B()        function C()
(2) sres 2              sres 3              sres 2
(3) call B()            call C()            sfree 2
(4) sens 2              sens 3
(5) call C()            call C()
(6) sens 2              sens 3
(7) sfree 2             sfree 3 ←
```

Logical stack

Stack cache*

*Cache configuration: 4 blocks

Example: Stack Cache

```
(1) function A()        function B()      function C()
(2) sres 2              sres 3            sres 2
(3) call B()            call C()          sfree 2
(4) sens 2 ←            sens 3
(5) call C()            call C()
(6) sens 2              sens 3
(7) sfree 2             sfree 3
```

Logical stack

A	A							

Stack cache*

A	A		

fill 2 blocks

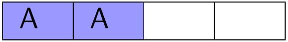
*Cache configuration: 4 blocks

Example: Stack Cache

```
(1) function A()      function B()      function C()
(2) sres 2            sres 3            sres 2
(3) call B()          call C()          sfree 2
(4) sens 2            sens 3
(5) call C() ←        call C()
(6) sens 2            sens 3
(7) sfree 2           sfree 3
```

Logical stack

Stack cache*



*Cache configuration: 4 blocks

Example: Stack Cache

```
(1) function A()
(2) sres 2
(3) call B()
(4) sens 2
(5) call C()
(6) sens 2
(7) sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2 ←
sfree 2
```

Logical stack

A	A	C	C					

Stack cache*

A	A	C	C

*Cache configuration: 4 blocks

Example: Stack Cache

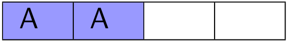
```
(1) function A()
(2) sres 2
(3) call B()
(4) sens 2
(5) call C()
(6) sens 2
(7) sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2
sfree 2 ←
```

Logical stack

Stack cache*



*Cache configuration: 4 blocks

Example: Stack Cache

```
(1) function A()
(2) sres 2
(3) call B()
(4) sens 2
(5) call C()
(6) sens 2 ←
(7) sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2
sfree 2
```

Logical stack

A	A							

Stack cache*

A	A		

*Cache configuration: 4 blocks

Example: Stack Cache

(1) `function A()`	`function B()`	`function C()`
(2) `sres 2`	`sres 3`	`sres 2`
(3) `call B()`	`call C()`	`sfree 2`
(4) `sens 2`	`sens 3`	
(5) `call C()`	`call C()`	
(6) `sens 2`	`sens 3`	
(7) `sfree 2` ←	`sfree 3`	

Logical stack

Stack cache*

*Cache configuration: 4 blocks

Stack Cache Analysis

Two analysis problems

- Bound the maximum amount of *spilling* at `sres`-instructions
- Bound the maximum amount of *filling* at `sens`-instructions
- Other instructions have no impact (`sfree`, loads, stores)

Stack Cache Analysis

Two analysis problems

- Bound the maximum amount of *spilling* at `sres`-instructions
- Bound the maximum amount of *filling* at `sens`-instructions
- Other instructions have no impact (`sfree`, loads, stores)

Main task

Determine the maximum/minimum occupancy-level of the stack cache before `sres`/`sens`-instructions respectively.*

*Assuming `sres`/`sfree` at function entry/exit and `sens` after function calls.

Terminology

Occupancy
Number of cache blocks utilized at a given program point.

Displacement
Number of cache blocks spilled to main memory at a function call.

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Occupancy
Number of cache blocks utilized at a given program point.

Displacement
Number of cache blocks spilled to main memory at a function call.

Observation
Knowing an occupancy bound at function entry, the occupancy at a program point within that function can be bounded using the displacement of function calls on all paths to the program point.

Example: Displacement

(1) `function A()`	`function B()`	`function C()`
(2) `sres 2`	`sres 3`	`sres 2`
(3) `call B()`	`call C()`	`sfree 2`
(4) `sens 2`	`sens 3`	
(5) `call C()`	`call C()`	
(6) `sens 2`	`sens 3`	
(7) `sfree 2`	`sfree 3`	

Displacement at `call C()`: 2

Example: Displacement

```
function A()
sres 2
call B()
sens 2
call C()
sens 2
sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2
sfree 2
```

Displacement at `call C()`: 2

Displacement at `call B()`: $3 + 2 = 5$

Example: Occupancy

```
(1) function A()      function B()      function C()
(2) sres 2            sres 3            sres 2
(3) call B()          call C()          sfree 2
(4) sens 2            sens 3
(5) call C()          call C()
(6) sens 2            sens 3
(7) sfree 2           sfree 3
```

Occupancy at C()

$A()_3 \rightarrow B()_3 \rightarrow C()$: 4 $\rightarrow$ spill 2 blocks

Example: Occupancy

```
(1) function A()        function B()        function C()
(2) sres 2              sres 3              sres 2
(3) call B()            call C()            sfree 2
(4) sens 2              sens 3
(5) call C()            call C()
(6) sens 2              sens 3
(7) sfree 2             sfree 3
```

Occupancy at C()

$A()_3 \rightarrow B()_3 \rightarrow C()$: 4 $\rightarrow$ spill 2 blocks

$A()_3 \rightarrow B()_5 \rightarrow C()$: 3 $\rightarrow$ spill 1 blocks

Example: Occupancy

```
(1) function A()
(2) sres 2
(3) call B()
(4) sens 2
(5) call C()
(6) sens 2
(7) sfree 2
```

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

```
function C()
sres 2
sfree 2
```

Occupancy at `C()`

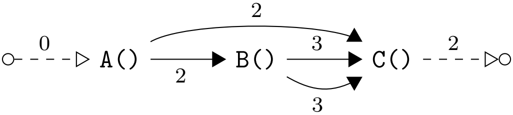
$A()_3 \rightarrow B()_3 \rightarrow C()$: 4 $\rightarrow$ spill 2 blocks
$A()_3 \rightarrow B()_5 \rightarrow C()$: 3 $\rightarrow$ spill 1 blocks
$A()_5 \rightarrow C()$: 2 $\rightarrow$ no spilling

Bounding the Displacement

Search path with shortest/longest tail on a weighted call graph

- Edge weights
  The number of blocks reserved in the calling function
- Edges to artificial sink node
  Capturing paths through functions without a call

Example



Function	Minimum	Maximum
A()	4	7
B()	5	5
C()	2	2

Handling Recursion

Unbounded longest paths/tails

- Modeled using Integer Linear Programming (ILP)
- Recursion bounded by (user-supplied) ILP constraints
- Model nested calls as flow in/out of call graph nodes

Ensure Analysis

Bound the maximum filling at `sens`-instructions

1. Pre-compute the *maximum* displacement at function calls
2. Perform a function-local data-flow analysis
   - Propagate the minimum occupancy to `sens`-instructions
   - Assume a full stack cache at function entry
   - Adjust occupancy at `sens`-instructions
   - Adjust occupancy at call sites using maximum displacement
3. Bound worst-case filling using the minimum occupancy

Ensure Analysis

Bound the maximum filling at `sens`-instructions

1. Pre-compute the *maximum* displacement at function calls
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   - Propagate the minimum occupancy to `sens`-instructions
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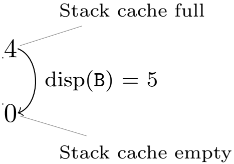
   - Adjust occupancy at `sens`-instructions
   - Adjust occupancy at call sites using maximum displacement
3. Bound worst-case filling using the minimum occupancy

Observation

This analysis is context-insensitive.

Example: Ensure Analysis

function A()
sres 2
call B()
sens 2
call C()
sens 2
sfree 2



Example: Ensure Analysis

function A()
sres 2
call B()
sens 2 .... 0 → 2 Fill 2 blocks
call C()
sens 2
sfree 2

Example: Ensure Analysis

function A()
sres 2
call B()
sens 2 2
call C() — disp(C) = 2
sens 2 2
sfree 2

Example: Ensure Analysis

```
function A()
sres 2
call B()
sens 2
call C()          2
sens 2            2   No filling
sfree 2
```

Reserve Analysis

Bound the maximum spilling at `sres`-instructions

1. Pre-compute the *minimum* displacement at function calls
2. Perform a function-local data-flow analysis
   - Propagate the worst-case occupancy to function calls
   - Assume a full stack cache at function entry
   - Adjust occupancy at `sens`-instructions
   - Adjust occupancy at call sites using minimum displacement
3. Perform an inter-procedural analysis on the call graph
   - Construct the *Spill Cost Graph*
   - Iteratively discover new stack cache contexts
   - Derive new contexts by either
     3.1 Adding the amount of locally reserved blocks
     3.2 Propagating the worst-case occupancy

Reserve Analysis

Bound the maximum spilling at `sres`-instructions

1. Pre-compute the *minimum* displacement at function calls
2. Perform a function-local data-flow analysis
   - Propagate the worst-case occupancy to function calls
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   - Adjust occupancy at `sens`-instructions
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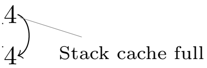
   - Construct the *Spill Cost Graph*
   - Iteratively discover new stack cache contexts
   - Derive new contexts by either
     - 3.1 Adding the amount of locally reserved blocks
     - 3.2 Propagating the worst-case occupancy

Observation

The analysis only uses the call graph to handle context-sensitivity.

Example: Worst-Case Occupancy

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```



Example: Worst-Case Occupancy

```
function B()
sres 3
call C()
sens 3
call C()
sens 3
sfree 3
```

4

2

$\text{disp}(\texttt{C}) = 2$

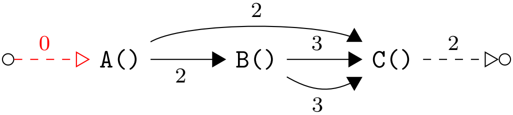
Example: Worst-Case Occupancy

```
function B()
sres 3
call C()        2
sens 3          ↓ Fill 1 blocks
call C()        3
sens 3
sfree 3
```

Example: Worst-Case Occupancy

function B()
sres 3 4
call C()
sens 3 3
call C()
sens 3
sfree 3

Example: Spill Cost Graph

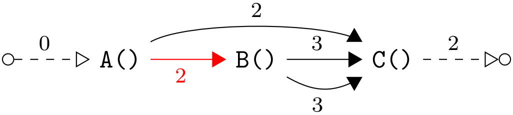


(a) Call graph

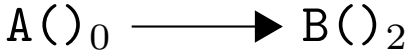
A()$_0$

(b) Spill cost graph (construction)

Example: Spill Cost Graph

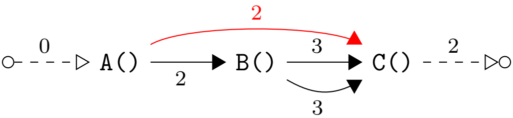


(a) Call graph

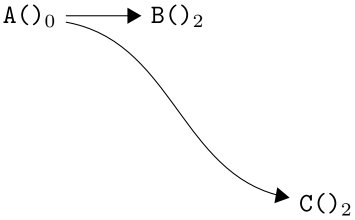


(b) Spill cost graph (construction)

Example: Spill Cost Graph

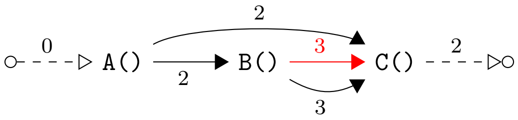


(a) Call graph

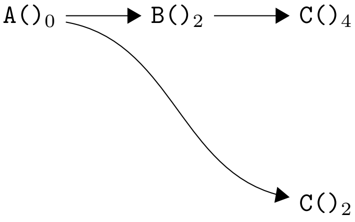


(b) Spill cost graph (construction)

Example: Spill Cost Graph

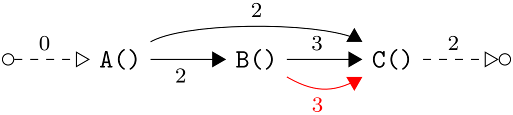


(a) Call graph

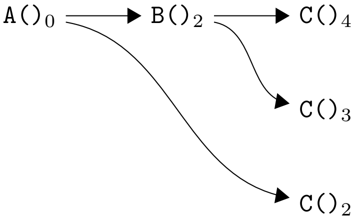


(b) Spill cost graph (construction)

Example: Spill Cost Graph



(a) Call graph

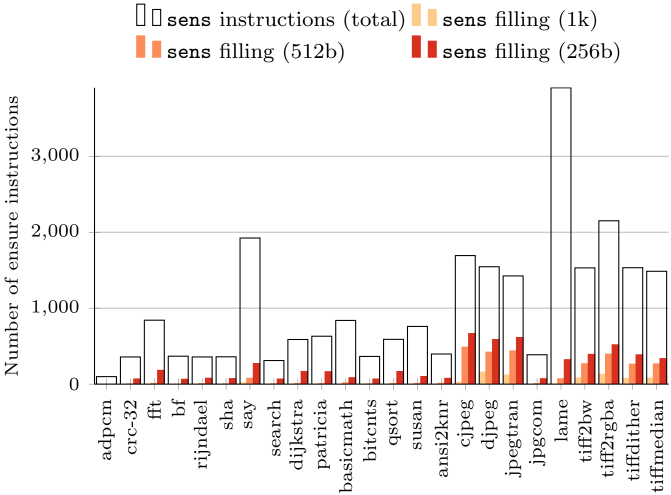


(b) Spill cost graph (construction)

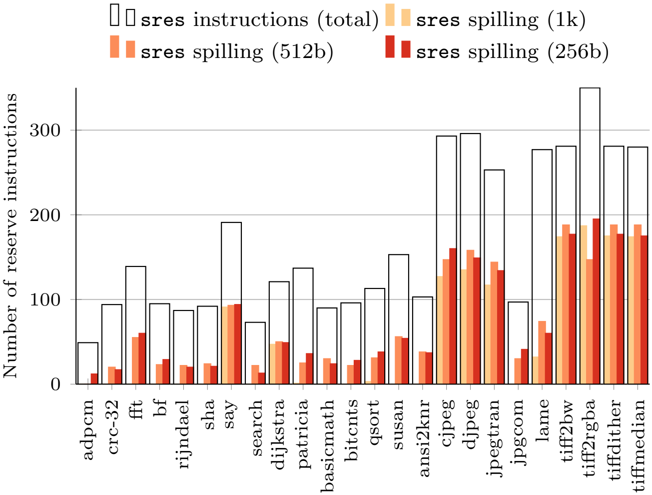
Experimental Setup

- MiBench benchmark suite
- LLVM compiler 3.3 for the Patmos processor
- Stack cache configurations: 256B, 512B, 1kB
- Compile benchmarks and perform stack cache analysis
  - Context-insensitive Ensure Analysis
  - Fully context-sensitive Reserve Analysis

Experiments: Ensure Analysis



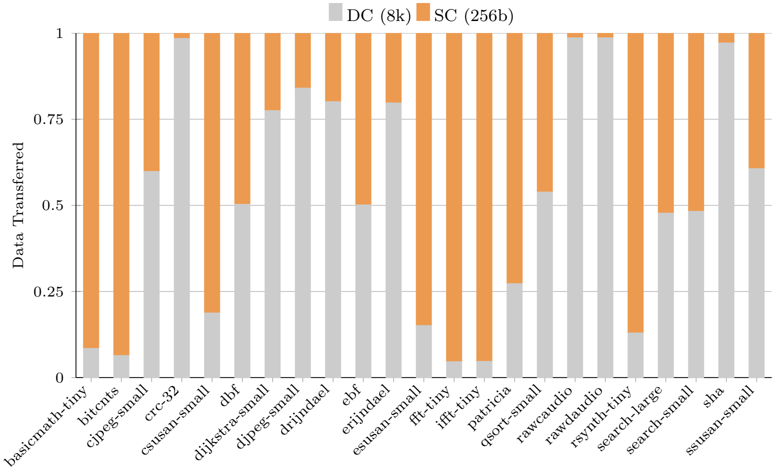
Experiments: Reserve Analysis



Conclusion

- Novel cache design dedicated to stack data
- Analyzable caching strategy
- Does not require knowledge of addresses
- Does not require analysis of individual accesses
- Simple analysis
  - Compute displacement on call graph
  - Perform function-local data-flow analysis
  - Compute Spill Cost Graph on call graph

Why use a Stack Cache?



Normalized data transfer volume between the Patmos CPU and its data caches.

DC . . . data cache SC . . . stack cache

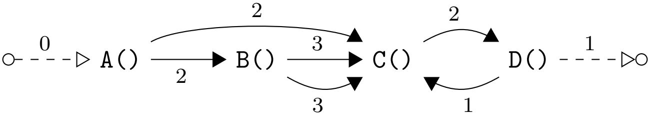
Handling Recursion

Unbounded longest paths/tails

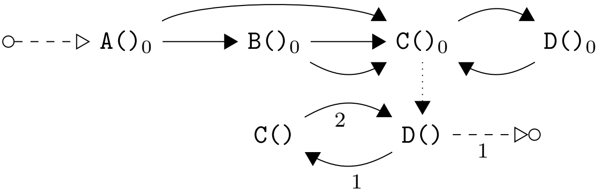
- Requires (user-supplied) bounds of recursion depth
- Modeled using Integer Linear Programming (ILP)
    - Recursion bounded by (user-supplied) ILP constraints
    - Model nested calls as flow in/out of call graph nodes
- Longest tail
    - Duplicate call graph
    - One copy contains zero-weighted edges
    - One copy contains the original edge weights
    - Transition edges from one copy to the other

Example: Displacement with Recursion

Search the path with the longest tail starting at `C()`



(a) Original call graph



(b) Transformed call graph

Extensions

- Generalized placement of stack control instructions
- Well-formed programs
  - Recursive definition based on well-formed paths
  - Matching arguments at first reserve r and last free f
  - The sub-path $r \rightsquigarrow f$ is well-formed
- Required changes
  - Computation of edge weights in weighted call graph (depth first search)
  - Minor modifications to function-local data-flow analysis
  - Representation of contexts within functions
- Pruning of Spill Cost Graph
  - Lossless elimination of contexts (no impact on spill cost)
  - Lossy merging of similar contexts (impact on spill cost)