# Industrial Organization 04 

Product differentiation

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## Outline

(1) Introduction: different forms of product differentiation
(2) Models of horizontal differentiation:

- The Hotelling model (exogenous locations, then endogenous locations on a linear city)
- The Salop model (circular city, equilibrium with free entry)
(3) Model of vertical differentiation


## Introduction

Michael Porter (Competitive advantage, 1986)
Competitive advantage stems from the many discrete activities a firm performs in designing, producing, marketing, delivering and supporting its product. Each of these activities can contribute to a firm's relative cost position and create a basis for differentiation.


A competitive advantage should be

- Significant and rare
- Defendable


## Introduction

Three forms of competitive advantage:

- Differentiation
- Costs
- A combination of the two

Two ways to gain a competitive advantage through differentiation:

- By creating a real difference between products
- By influencing consumer preferences (e.g., through advertising)


## Various forms of differentiation

"Horizontal" differentiation:

- Different varieties
- Consumers have different tastes
- If the products are all sold at the same price, consumers choose different varieties
"Vertical" differentiation:
- Different qualities
- Consumers agree on the ranking of the goods in terms of quality
- If the products are all sold at the same price, consumers all choose the product with the "highest" quality

Lancaster's approach:

- A good as a bundle of characteristics


## Sources of differentiation

Different possibilities of differentiation:

- The product (shape, style, design, reliability, etc.), the service (ordering, delivery, installation, etc.), the staff, the point of sale, the brand image, etc.
- Notion of "positioning" in marketing. Brand strategy. The consumer must be able to identify the characteristics of the product in relation with their needs


## Differentiation strategies

We are going to examine the following questions:

- How does differentiation affect competition among firms?
- When differentiation is endogenous, what is the equilibrium? Do firms actually choose to differentiate themselves?
- Are the products offered in equilibrium close or far apart?
- What is the effect of differentiation on entry of new players?


## The Hotelling model (1929)

- A "street" or a "space of tastes" represented by the interval $[0,1]$
- A mass 1 of consumers are distributed uniformly along this interval
- Two firms, 1 and 2 , sell a good (the same good) on this street
- Firms compete in prices
- Marginal cost of production $c$
- Consumers buy 0 or 1 unit of the good
- Utility of the good for a consumer: $v$
- $\rightarrow$ But there is a transportation cost the consumer pays to get the good: $t$ per unit of distance, quadratic function
- So, the net utility takes the form $v-p-\operatorname{transportation~cost~}$


## Exogenous location of firms

Firm 1 is at $x_{1}=0$ and firm 2 at $x_{2}=1$

Firm 1
Firm 2

$$
x_{1}=0
$$

$$
x_{2}=1
$$

## The marginal consumer

Method for calculating demand: determine the indifferent (marginal) consumer

Definition of the marginal consumer
In the presence of consumers with heterogeneous tastes, the marginal consumer is the consumer who is indifferent between two possible choices

Here: the marginal consumer is indifferent between buying from firm 1 and buying from firm 2

Where is the marginal consumer approximately located?

## Marginal consumer and demand of each firm

## Demand of firm 2

## Demand of firm 1

Firm 1
Firm 2
marginal
consumer
$\tilde{x} \in[0,1]$

## Demand functions

- The marginal consumer $\tilde{x}$ is defined by

$$
v-\left(p_{1}+(\bar{x}-0)^{2} t\right)=v-\left(p_{2}+(1-\bar{x})^{2} t\right)
$$

SO,

$$
\tilde{x}=\frac{1}{2}+\frac{p_{2}-p_{1}}{2 t}
$$

- From this, we can derive the demand of firm 1 (if prices are not too different)

$$
D_{1}\left(p_{1}, p_{2}\right)=\tilde{x}=\frac{1}{2}+\frac{p_{2}-p_{1}}{2 t}
$$

- The demand of firm 2 is $D_{2}=1-\tilde{x}=1-D_{1}$


## Profits and reaction functions

We start by defining the:
Profit functions

$$
\pi_{i}=\left(p_{i}-c\right)\left(\frac{1}{2}+\frac{p_{j}-p_{i}}{2 t}\right)
$$

- Firm $i$ maximizes its profit $\pi_{i}$, taking the rival's price $p_{j}$ as given
- The first order condition gives the optimal price for firm $i$ as a function of the rival's price $p_{j}$
$\rightarrow$ It is the reaction function of firm $i$ (or the best-response function)
Reaction function of firm $i$ :

$$
p_{i}=R_{i}\left(p_{j}\right)=\frac{c+t+p_{j}}{2}
$$

## Nash equilibrium

The Nash equilibrium corresponds to the intersection of the reaction functions
We have

$$
p_{i}=R_{i}\left(R_{j}\left(p_{i}\right)\right)
$$

where

$$
p_{i}=R_{i}\left(p_{j}\right)=\frac{c+t+p_{j}}{2}
$$

What is the equilibrium price?
Equilibrium price

$$
p^{\star}=c+t
$$

## Equilibrium with exogenous location of firms

Equilibrium price

$$
p^{\star}=c+t
$$

$\rightarrow$ The equilibrium price increases with $t$

Conclusion
When firms' locations are fixed, an increase in the level of differentiation (measured by $t$ ) increases firms' market power

## When firms choose where to locate

$\rightarrow$ We say that location decisions are endogenous ( $\neq$ exogenous)

We study a two-stage game where:
(1) Firms choose their locations
(2) Then, given their locations, firms set prices

We look for the subgame perfect equilibrium. We use backward induction:

- First, we look for the equilibrium of the price competition (last) stage
- Next, we solve for the equilibrium locations in the first stage
- Assuming that firms expect equilibrium prices to prevail in the second stage (subgame perfection)


## Endogenous locations

Firm 1 is located at $x_{1}$ of the left end and firm 2 at $x_{2}$ of the right end

Firm 1

$x_{1} \geq 0$

Firm 2

$$
x_{2} \geq 0
$$

## Stage 2: choice of prices

We first determine each firm's demand for given prices

- The marginal consumer $\tilde{x}$ is given by:

$$
v-\left(p_{1}+\left(\widetilde{x}-x_{1}\right)^{2} t\right)=v-\left(p_{2}+\left(1-x_{2}-\widetilde{x}\right)^{2} t\right)
$$

so, the marginal consumer is located at

$$
\tilde{x}=x_{1}+\frac{1-x_{1}-x_{2}}{2}+\frac{p_{2}-p_{1}}{2 t\left(1-x_{1}-x_{2}\right)}
$$

- If $0 \leq \tilde{x} \leq 1$, the demands for firm 1 and firm 2 are $D_{1}=\tilde{x}$ and $D_{2}=1-\tilde{x}$


## Stage 2: choice of prices

Next, we determine the reaction functions:

- The profit function of firm $i(i=1,2)$ is:

$$
\pi_{i}=\left(p_{i}-c\right)\left(x_{i}+\frac{1-x_{1}-x_{2}}{2}+\frac{p_{j}-p_{i}}{2 t\left(1-x_{1}-x_{2}\right)}\right)
$$

- Firm $i$ maximizes its profit with respect to $p_{i}$, taking its rival's price $p_{j}$ as given
- We find the two first-order conditions for firm 1 and firm 2, which gives two reaction functions


## Stage 2: choice of prices

The intersection of the reaction functions gives the equilibrium prices

The equilibrium prices at stage 2 are

$$
\begin{aligned}
& p_{1}^{*}=c+t\left(1-x_{1}-x_{2}\right)\left(1+\frac{x_{1}-x_{2}}{3}\right) \\
& p_{2}^{*}=c+t\left(1-x_{1}-x_{2}\right)\left(1+\frac{x_{2}-x_{1}}{3}\right)
\end{aligned}
$$

## Stage 1: choice of location

- At stage 1, firm 1 (for example) chooses her location taking the location of firm 2 as given
- She anticipates the equilibrium price of stage 2
- Therefore, her profit maximization problem is:

$$
\max _{x_{1}}\left(p_{1}^{*}\left(x_{1}, x_{2}\right)-c\right) D_{1}\left(x_{1}, x_{2}, p_{1}^{*}\left(x_{1}, x_{2}\right), p_{2}^{*}\left(x_{1}, x_{2}\right)\right)
$$

- We compute the first-order condition $\rightarrow$ it is the total derivative of the profit function $\pi_{1}$ with respect to $x_{1}$
- Indeed, the location choice affects profit in two different ways:
(1) Direct effect: $\pi_{1}$ depends on $x_{1}$
(2) Indirect effects: $\pi_{1}$ depends on $p_{1}$ and $p_{2}$, which in turn depend on $x_{1}$


## Stage 1: choice of location

- We can ignore the effect of $x_{1}$ to $\pi_{1}$ because (it is the "envelop theorem")

$$
\left.\frac{\partial \pi_{1}}{\partial p_{1}} \frac{\partial p_{1}^{*}}{\partial x_{1}}\right|_{p_{1}=p_{1}^{\star}, p_{2}=p_{2}^{\star}}=0
$$

- So, we have

$$
\frac{d \pi_{1}}{d x_{1}}=\left(p_{1}^{*}\left(x_{1}, x_{2}\right)-c\right)(\underbrace{\frac{\partial D_{1}}{\partial x_{1}}}_{\text {direct effect }(+)}+\underbrace{\frac{\partial D_{1}}{\partial p_{2}} \frac{\partial p_{2}^{*}}{\partial x_{1}}}_{\text {indirect effect (-) }})
$$

## Stage 1: choice of location

In a game with several stages, we potentially have

- Direct effects: when variables chosen in the first stages directly affect the profit functions
- Indirect effects (or strategic effects): when variables chosen in the first stages affect the strategies defined in later stages, which in turn affect the profit functions


## Here:

- The direct effect (+) is a demand effect
- The indirect or strategic effect (-) is the intensification of competition


## Stage 1: choice of location

We find that the strategic effect (-) always dominates the direct effect of the market share (+)

What are the firms' differentiation strategies in equilibrium?

Perfect equilibrium of the game
In equilibrium, firms choose maximum differentiation

In a context of price competition, firms have strong incentives to differentiate themselves in order to soften competition

Even though they also have an incentive to "imitate" their rivals in order to capture their market share

## Comparison with social optimum

How do firms' location choices compare to the social optimum?
Do firms differentiate themselves too much or too little?
The socially optimal locations are those that minimize production and transportation costs. These costs are minimized when the two firms are located at $1 / 4$ and at $3 / 4$

Therefore, there is too much differentiation

## If prices are exogenous

Assume that prices are exogenous (fixed)
What are the firms' locations in equilibrium?
In equilibrium, the two firms produce the same (average) variety $\rightarrow$ there is minimal differentiation

## The example of television

If we apply these results to competition between TV channels on the audience market...
... we find that channels financed (purely) by subscription fees should be more differentiated than channels financed (purely) by advertising
... and differentiation is minimal for TV channels financed by advertising
Other dimensions of competition that could change this result?

- Differentiation to reduce competition in the quality of TV programs
- Differentiation to prevent viewers from switching to the channel with the least advertising


## Salop model (1979)

Salop model (1979) of a circular city

- A differentiation circle with perimeter equal to 1 (a "circular city")
- A mass 1 of consumers are uniformly distributed around the circle
- $n$ identical firms are also located on the circle $\rightarrow$ we assume that they are uniformly distributed around the circle
- Firms compete in prices
- Marginal cost $c$
- We assume a "free entry" condition and denote by $f$ the fixed cost of entry

How many firms enter in equilibrium? Are there enough entries or on the contrary too few/too many entries?

## The circular city

Firm 1


## Demand functions

- Consider the pricing decision for firm $i$
- Firm $i$ has two close "rivals", which propose the same price $p$ (symmetry assumption)
- The two rivals are both located at a distance of $1 / n$ from firm $i$
- We start by determining the location $\tilde{x}$ of the indifferent consumer between firm $i$ and its rival located $1 / n$ further away:

$$
p_{1}+t \tilde{x}=p+t\left(\frac{1}{n}-\tilde{x}\right)
$$

that is,

$$
\tilde{x}=\frac{1}{2 n}+\frac{p-p_{1}}{2 t}
$$

So, we have

$$
D_{1}\left(p_{1}, p\right)=2 \tilde{x}
$$

## The demand of firm 1



## Price equilibrium

Firm 1's profit is:

$$
\pi_{1}=\left(p_{1}-c\right)\left(\frac{1}{n}+\frac{p-p_{1}}{t}\right)
$$

We solve for firm 1's best response function:

$$
p_{1}^{\star}(p)=\frac{c+p+t / n}{2}
$$

We assume a symmetric equilibrium where $p_{1}=p$, so

$$
p^{\star}=c+\frac{t}{n}
$$

The equilibrium profit is (with a fixed $\operatorname{cost} f$ ):

$$
\pi^{\star}(n)=\frac{t}{n^{2}}-f
$$

## Equilibrium with free entry

How can we find the number of entrants in equilibrium?
$\rightarrow$ There is entry as long as the profit of a new entrant is strictly positive
We find the number of firms $n$ that satisfies the zero-profit condition in order to obtain the number of entrants in equilibrium:

$$
n^{\star}=\sqrt{\frac{t}{f}}
$$

So, the long-run equilibrium price is

$$
p^{\star}=c+\sqrt{t f}
$$

Effect of $f$ ? Effect of $t$ ?

## Comparison with social optimum

It can be shown that from the point of view of social welfare there is too much entry

How can we explain this result?

- Private and social incentives are not aligned
- New entrants offer new varieties but also steal customers from their rivals (business stealing)


## Brand proliferation

Consider the circular city model again
Could firms decide to increase the number of products to prevent competitors from entering the market (brand proliferation strategy)?

For instance, in 1972, the top six companies in the US breakfast cereal market held $95 \%$ of the market

Between 1950 and 1972, they launched more than 80 different brands
The FTC charged the 4 companies with abuse of a dominant position (but lost the case)

## Vertical differentiation

In the horizontal differentiation model, firms produce different types of products but offer the same quality

Vertical differentiation necessarily implies asymmetries: there are high quality providers and low quality providers

Is the principle of maximum differentiation still valid?

## Model of vertical differentiation

- Two firms, 1 and 2, produce goods of different qualities: $s_{1}$ and $s_{2}$
- Marginal cost of production $c$
- Production cost of quality is 0
- Firms first choose their product quality, then simultaneously choose their prices (two-stage game)
- All consumers value quality, but at different levels $\rightarrow$ consumers' valuation for quality uniformly distributed on $[\underline{\theta}, \bar{\theta}]$, with

$$
\underline{\theta} \geq 0 \text { and } \bar{\theta}=\underline{\theta}+1
$$

- A consumer with valuation $\theta$ for quality derives the following utility:

$$
U(\theta)=\left\{\begin{array}{rr}
\theta s_{i}-p_{i} & \text { if she buys from firm } i \\
0 & \text { otherwise }
\end{array}\right.
$$

## Model of vertical differentiation

## Other assumptions:

- $s_{2}>s_{1}$ : firm 2 is the high-quality firm, firm 1 the low-quality firm
- We define $\Delta s=s_{2}-s_{1}$ as the difference in quality
- Enough heterogeneity between consumers: $\bar{\theta} \geq 2 \underline{\theta}$ (otherwise the low quality firm is excluded)
- The market is "covered" in equilibrium (i.e., all consumers buy a good):

$$
c+\frac{\bar{\theta}-2 \underline{\theta}}{3}\left(s_{2}-s_{1}\right) \leq s_{1} \underline{\theta}
$$

## Roadmap for solving the model

This model is solved in a similar way as the Hotelling model

We begin by solving for the price competition equilibrium in the second stage
(1) We first determine the marginal consumer
(2) This gives the demand functions for each firm
(3) We then find the best response functions
(1) The intersection of the reaction functions gives the equilibrium prices

Next, we determine the equilibrium quality choices in the first stage, assuming that firms expect the price equilibrium to prevail in the second stage

## Equilibrium prices

The equilibrium prices at the second stage are:

$$
\begin{aligned}
& p_{1}^{\star}=c+\left(\frac{\bar{\theta}-2 \underline{\theta}}{3}\right) \Delta s \\
& p_{2}^{\star}=c+\left(\frac{2 \bar{\theta}-\underline{\theta}}{3}\right) \Delta s
\end{aligned}
$$

Vertical differentiation (like horizontal differentiation) gives firms market power: $p_{1}^{\star}>c$ and $p_{2}^{\star}>c$

The price of the firm with high quality (firm 2) is higher than the price of the low quality firm (firm 1) : $p_{2}^{\star}>p_{1}^{\star}$

The price gap is equal to $p_{2}^{\star}-p_{1}^{\star}=\Delta s / 3 \rightarrow$ it increases with the degree of differentiation between firms

## Quality choices

Assume that $s \in[\underline{s}, \bar{s}]$
Quality choices?
Equilibrium of the game of vertical differentiation
There are two Nash equilibria, such that one firm offers the lowest quality and the other offers the highest quality
$\rightarrow$ same principle of maximum differentiation as in the Hotelling model
If the game is played sequentially, the firm that plays first chooses the high quality

## Take-aways (1)

- Firms try to distinguish themselves from their competitors by developing differentiation strategies that allow them to earn higher profits
- In the Hotelling model with quadratic transportation costs and exogenous locations, firms charge a price equal to marginal cost plus transportation costs
- In the Hotelling model with quadratic transportation costs, when firms choose their locations, they choose to differentiate as much as possible. Differentiation is excessive compared to the social optimum


## Take-aways (2)

- In the Salop model, the long-run equilibrium price (with free entry) is equal to the marginal cost plus the square root of the transportation cost multiplied by the fixed cost of entry. Therefore, the more there is differentiation and the higher the entry costs, the higher the price the firms can charge to consumers.
- There is too much entry compared to the social optimum (brand proliferation strategy).
- Vertical differentiation (e.g., differentiation in quality) also allows firms to charge higher prices.

