Analytical Skeleton from a Cylinders & Disks Decomposition

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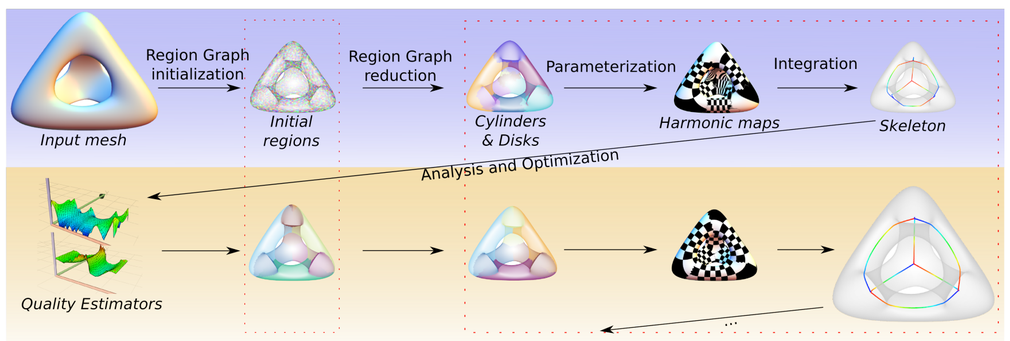
Abstract

We present a new curve skeleton designed for surface modeling and processing. This skeleton is defined as the geometrical integration of a piecewise harmonic parameterization defined over a disk-cylinder surface decomposition. This decomposition is computed using a progressive Region Graph reduction based on both geometric and topological criteria. The skeleton has an analytical form with regularity inherited from the surface one. This form offers well-defined surface-skeleton and skeleton-surface projection. A large repository of *skeletal* shape modeling and processing methods can be derived from it: we illustrate this with an inset surface modeling tool and a new signal filtering operator.

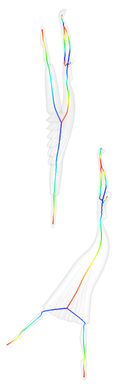
Overview



Input : Manifold triangle mesh.

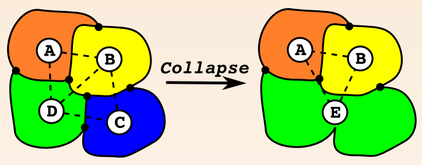
1. Topological **cylinders & disks decomposition**
2. Harmonic cylinder & disks **parameterization**
3. **Skeletonization** by geometrical cylinder integration
4. Connection of the bones
5. *Optional: improved segmentation from skeleton; go to 2.*

Output : Parametric curve skeleton



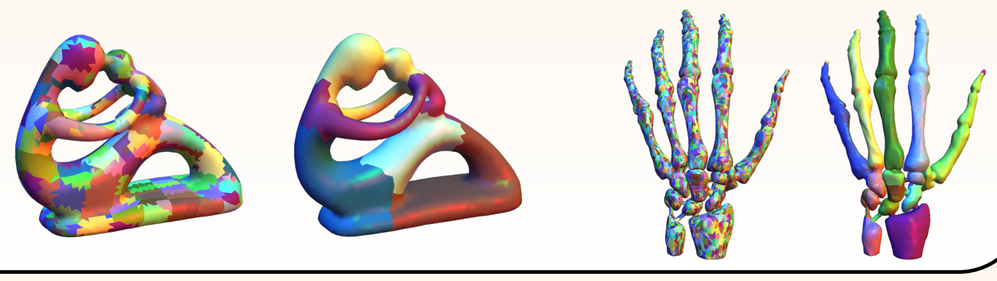
Cylinders & Disks Decomposition

Region Graph = dual graph of the input segmentation
Edge collapse in Region Graph = Regions merging

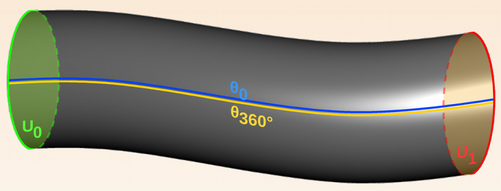


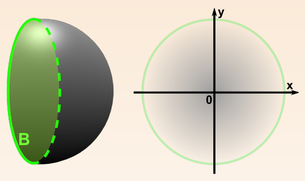
Topology			Class	e_T
R^A	R^B	$R^A \cup R^B$		
Cyl.	Disk	Disk	*Fill Hole*	**0**
Disk	Disk	Cyl.	*Create Cylinder*	**1**
Disk	Disk	Disk	*Grow Disk*	**2**
Cyl.	Disk	Cyl.	*Grow Cylinder*	**3**
Cyl.	Cyl.	Cyl.	*Merge Cylinders*	**4**
Other cases			*Skip*	∞

* Bottom-up segmentation by progressive, error-driven Region Graph edge collapse.
* Error metric from dominant integer topological classes, independently ordered by geometric terms defined for each topological class.
* Edge-collapses discarded if their topological term is ∞, or by geometric tests.



Parameterization



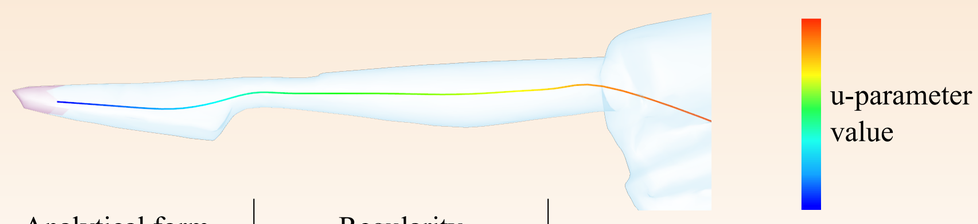


$$\begin{cases} u(v) = 0 & \forall v \in U_0 \\ u(v) = 1 & \forall v \in U_1 \\ \triangle u(v) = 0 & \forall v \notin U_0 \cup U_1 \end{cases} \quad \begin{cases} \theta(v) = 0 & \forall v \in \theta_0 \\ \theta(v) = 360 & \forall v \in \theta_{360} \\ \triangle \theta(v) = 0 & \forall v \notin \theta_0 \cup \theta_{360} \end{cases}$$

$$\begin{cases} x(v) = \cos(\theta(v)) & \forall v \in B \\ y(v) = \sin(\theta(v)) & \forall v \in B \\ \triangle x(v) = 0 & \forall v \notin B \\ \triangle y(v) = 0 & \forall v \notin B \end{cases}$$

* Harmonic map for each region
* Discrete laplacian using cotangent weights
* Solved in the least squares sense

Skeletonization



Analytical form by integration

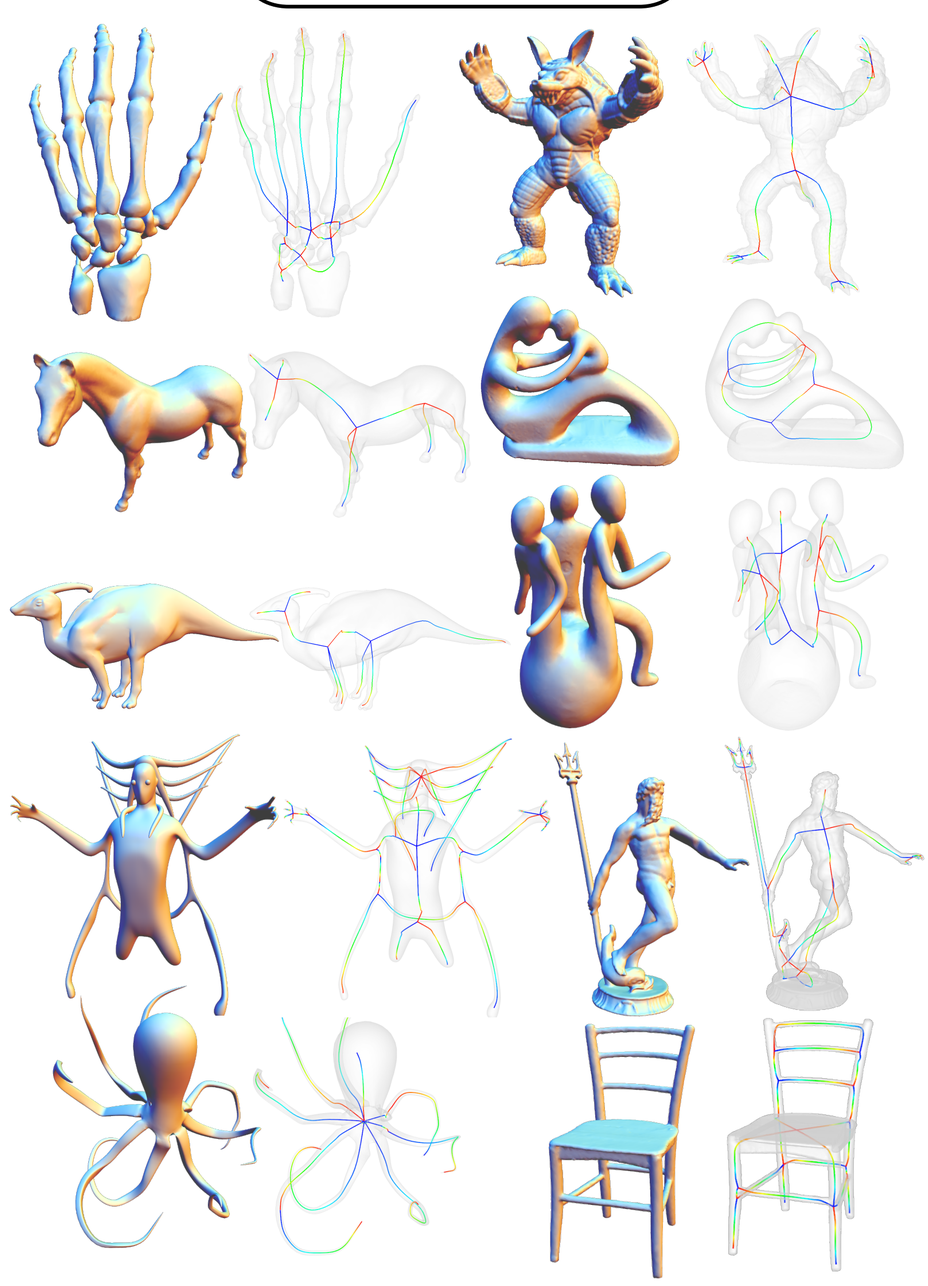
$$b(u) = \frac{\int_0^{2\pi} p(u,\theta)\,d\theta}{\int_0^{2\pi} d\theta}$$

Regularity

$$\begin{tikzcd} M \arrow[rr, "p"] \arrow[dr, bend right, "\mu"'] \arrow[ddr, bend right, "\lambda"'] & & \mathbb{R}^3 \\ & \mathbb{R}^2 \arrow[ur, bend right, "\hat{p}"'] \arrow[d, "\tilde{\lambda}"] & \\ & \mathbb{R}^2 \arrow[uur, bend right, "\tilde{p}"'] & \end{tikzcd}$$

* skeleton regularity inherited from the surface
* spline representation

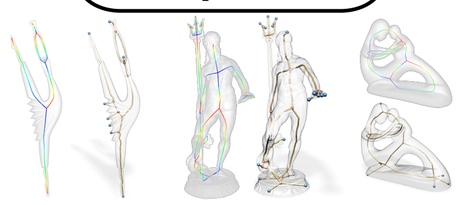
Resulting Skeletons



Properties

* **Analytical** and mathematically **smooth** skeleton
* Parametric skeleton ⇔ surface projection
* Interactive skeleton editing

Comparison

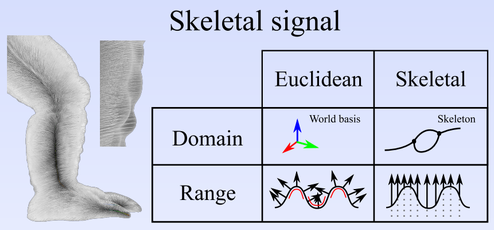


Comparison with a contraction skeleton [1]

New Applications

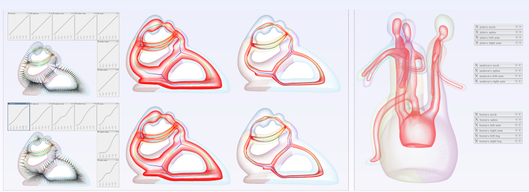
* Signal processing and editing
* Inset surfaces and volume editing
* Generalized cylinders reconstruction
* Adaptive FFD cage creation
* Detection of constriction lines
* Quad-dominant meshing

Skeletal signal

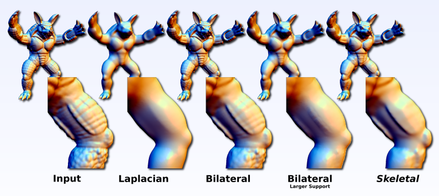


Normal variation cannot model mid and large scale features

Inset surfaces and volume editing



New bilateral mesh filter



References

[1] AU O. k.-c. , TAI C.-L. , CHU H.-K. , COHEN-OR D., LEE T.-Y.: Skeleton extraction by mesh contraction. *ACM Transactions on Graphics 27*, 3 (2008)

[2] SHAMIR A.: A survey on mesh segmentation techniques. *Computer Graphics Forum 27*, 6 (2008), 1539-1556.

[3] JONES T. R., DURAND F., DESBRUN M.: Non-iterative, feature-preserving mesh smoothing. In *ACM SIGGRAPH* (2003)