# Green Coordinates for Triquad Cages in 3D – Additional material

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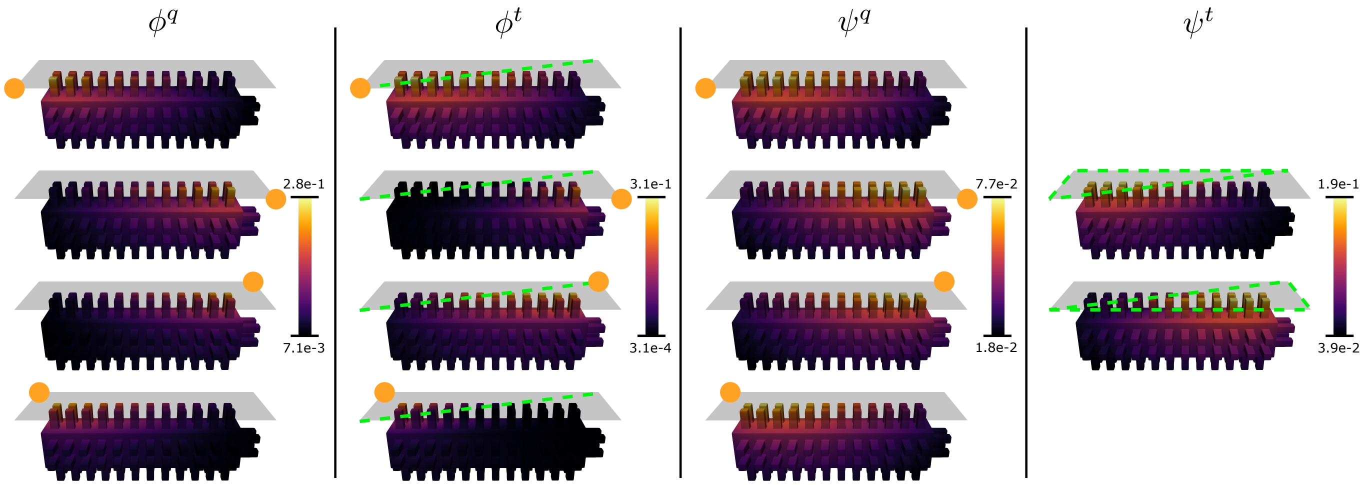


Fig. 1. We compare our coordinates ($\phi^q, \psi^q$) with traditional (triangle-based) Green coordinates ($\phi^t, \psi^t$), each time with respect to orange query point. Cutting the quad into two triangles make asymmetric artifacts appear in the traditional formulation.

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## 1 COORDINATES VISUALIZATION

We show color-coded visualizations of our coordinates in Fig. 1, and compare with traditional (triangle-based) Green coordinates [Lipman et al. 2008].

The main differences that can be observed in this figure are:

- Contrary to traditional Green coordinates, our coordinates do not display asymmetric artifacts resulting from an arbitrary split of the quad into two triangles.
- We use 4 "normal-related" $\psi^q$ coordinates per input quad (one for each quad corner), while traditional Green coordinates contain one $\psi^t$ coordinate per triangle (two $\psi^t$ coordinates per quad after cutting the quad into two triangles).

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*A short note on scale invariance.* Note that the ranges of those coordinates differ drastically. We adapted the color ramp for better visualization (though the range is the same for all subfigures in the same column).

A simple scale analysis reveals that the various types of coordinates scale differently with respect to input scaling. Looking at our final expression for our coordinates (see Eq. (1)), one can see that scaling the input mesh and cage by $\lambda$ will make the (traditional, triangle-based) $\psi^t$ coordinates scale by $\lambda$, while our new coordinate $\psi^q$ will be scaled by $\lambda^{-1}$ and the coordinate $\phi$ is scale-invariant:

$$f(\eta) = \sum_{i\in\mathcal{V}} \underbrace{\phi_i(\eta)}_{\text{scales in } 1} \overbrace{v'_i}^{\text{scales in } \lambda} + \sum_{t\in\mathcal{T}} \underbrace{\psi_t(\eta)}_{\text{scales in } \lambda} \overbrace{\sigma_t \; n'_t}^{\text{scales in } 1} + \sum_{q\in Q} \sum_{k=0}^{3} \underbrace{\psi^q_k(\eta)}_{\text{scales in } \lambda^{-1}} \overbrace{\sigma^k_q N^{q\prime}_k}^{\text{scales in } \lambda^2} \quad (1)$$

While, mathematically, this has no impact on the resulting computations and our formulation is insensitive to uniform scaling of the rest-pose cage and mesh, it has practical consequences on the robustness of the computations, due to floating point rounding.

As explained in the manuscript, we recommend scaling the rest pose configuration in the unit sphere, as we never observed instabilities of the computations at this scale.

## REFERENCES

Yaron Lipman, David Levin, and Daniel Cohen-Or. 2008. Green coordinates. *ACM ToG* 27, 3 (2008), 1–10.