11.8. Exercises

1. *Multi-rate IP access*

We extend the model of section 11.1 to the case of multiple peak rates, corresponding to different offers of an Internet Service Provider. More precisely, assume that N user classes share the same link of capacity C, class i consisting of K_i users with peak rate r_i and offered traffic a_i. Check that equation (11.2) is satisfied for each class.

Now assume that the mean throughput is a fraction η of the peak rate for each class. Give the equation satisfied by η assuming the actual total traffic is equal to the capacity. Calculate η for a link of capacity $C = 100\,\mathrm{Mbit/s}$ shared by two user classes, characterized by the following parameters: $K_1 = 500$, $K_2 = 200$, $r_1 = 1\,\mathrm{Mbit/s}$, $r_2 = 2\,\mathrm{Mbit/s}$ and $a_1 = a_2 = 200\,\mathrm{kbit/s}$.

2. *Mobility and hand-over*

A base station of a 2G mobile network has a capacity of C GSM slots. Calls arrive in the cell according to a Poisson process with intensity λ and have exponential durations with parameter μ; denote by $\alpha = \lambda/\mu$ the associated traffic intensity. A proportion p of the calls arrive from neighboring cells due to mobility: such calls are said to be in *hand-over*.

In order to limit the blocking rate of these calls, the following mechanism of trunk reservation is implemented: when there are n slots left or less, any new incoming call is blocked and lost; only calls in hand-over are accepted in the limit of the cell capacity. Verify that the number of on-going calls is a birth-death process. Deduce the blocking rate of both types of calls. What is the ratio of these blocking rates when $n = 1$? Compute the corresponding value for $C = 20$ slots, $\alpha = 15\,\mathrm{E}$ and $p = 5\%$.

3. *Degraded 2G calls*

We are interested in the blocking rate of calls in a 2G cell, when only a proportion p of the mobiles support the degraded mode, see section 11.2. At which traffic model does this system correspond? Calculate the blocking rate of each type of mobile for $C = 12$ slots, $\alpha = 10\,\mathrm{E}$ and $p = 60\%$. During which fraction of time must a mobile supporting the degraded mode use this mode?

4. *EDGE adaptive modulation*

Consider the data traffic model introduced in section 11.2, taking into account the adaptive modulation of the EDGE standard. More precisely, assume the cell is divided in two areas, in which the maximal transmission rates (on 4 radio slots) are respectively equal to $r_1 = 300\,\mathrm{kbit/s}$ and $r_2 = 100\,\mathrm{kbit/s}$. Proportions p_1 and p_2 of the traffic are respectively in areas 1 and 2, with $p_1 + p_2 = 1$. Denote by A the offered traffic (in $\mathrm{bit/s}$) and by $C = 4m$ the radio capacity in number of slots.

Give the equivalent traffic intensity α in erlangs, that is the mean number of occupied slots in the absence of constraint (i.e. $C = \infty$); deduce the expression of the cell load. Compute this value for $A = 1\,\mathrm{Mbit/s}$, $C = 8$ slots and $p_1 = p_2 = 1/2$.

Now assume that the total number of flows cannot exceed $n = 8m$. Assuming that the slot sharing is perfectly fair, give the mean throughput of each type of mobile and the blocking rate.

5. *Dynamic sharing between voice and data in a 2G network*
A 2G cell has C slots that are dynamically shared between voice and data. Data traffic shares the slots left by voice traffic, in the limit of 4 slots. Specifically, data flows have access to $\min(C - x, 4)$ slots, where x is the number of voice calls. Voice calls and data calls arrive according to independent Poisson processes of respective intensities λ and λ'. The other assumptions are those of section 11.2.

Give the transition rates of the Markov process describing the number of voice calls and the number of data flows. Voice calls are assumed to be in non-degraded mode and the number of data flows is limited to n. Is this process reversible? Give the mean number of slots that can be used by data traffic.

6. *WiFi adaptive modulation*
Consider a WiFi access point shared by two users, whose radio conditions yield the respective physical rates $r_{\mathrm{PHY},1}$ and $r_{\mathrm{PHY},2}$. Calculate the total throughput r of both users when they transfer data simultaneously using TCP.

Now assume that each user transfers a sequence of files of exponential sizes with mean σ, with an exponential idle period with parameter ν between two data transfers. Give the transition graph of the Markov process describing the state of both users. Is the process reversible? Give its stationary distribution, then the mean throughput of each active user. Compute the numerical values for the 802.11g standard, with $r_{\mathrm{PHY},1} = 54\,\mathrm{Mbit/s}$, $r_{\mathrm{PHY},2} = 11\,\mathrm{Mbit/s}$, $\sigma = 1\,\mathrm{MB}$ and $1/\nu = 1\,\mathrm{s}$.