Unifying Quantitative, Semi-quantitative and Qualitative Spatial Relation Knowledge Representations Using Mathematical Morphology

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Abstract. One of the powerful features of mathematical morphology lies in its strong algebraic structure, that finds equivalents in set theoretical terms, fuzzy sets theory and logics. Moreover this theory is able to deal with global and structural information since several spatial relationships can be expressed in terms of morphological operations. The aim of this paper is to show that the framework of mathematical morphology allows to represent in a unified way spatial relationships in various settings: a purely quantitative one if objects are precisely defined, a semi-quantitative one if objects are imprecise and represented as spatial fuzzy sets, and a qualitative one, for reasoning in a logical framework about space.

Keywords: mathematical morphology, spatial relationships, spatial reasoning.

1 Introduction

One of the powerful features of mathematical morphology lies in its strong algebraic structure, that finds equivalents in set theoretical terms, fuzzy sets theory and logics. Moreover this theory is able to deal with local information, based on the concept of structuring element, but also with more global and structural information since several spatial relationships can be expressed in terms of morphological operations (mainly dilations). We consider here topological relationships (which include part-whole relationships such as inclusion, exclusion, adjacency, etc.) and metric relationships (distances and directional relative position), the interest of these relations being highlighted in very different types of works (vision, GIS, cognitive psychology, artificial intelligence, etc.). The aim of this paper is to show that the framework of mathematical morphology allows to represent in a unified way spatial relationships in various settings: a purely quantitative one if objects are precisely defined, a semi-quantitative one if objects are imprecise and represented as spatial fuzzy sets, and a qualitative one, for

T. Asano et al. (Eds): Geometry, Morphology, . . . 2002, LNCS 2616, pp. 153–164, 2003.

reasoning in a logical framework about space. The proposed framework, briefly presented in Section 2, allows us to address three questions. We first consider the problem of defining and computing spatial relationships between two objects, in both the crisp and fuzzy cases (Section 3). Then in Section 4 we propose a way to represent spatial knowledge in the spatial domain. Finally in Section 5 we show that spatial relationships can be expressed in the framework of normal modal logics, using morphological operations applied on logical formulas. This can be useful for symbolic (purely qualitative) spatial reasoning. These threse types of problems are further developed in [5].

2 Basic Morphological Operations, Fuzzy and Logical Extensions

2.1 Classical Morphology

Let us first recall the definitions of dilation and erosion of a set X by a structuring element B in a space $\mathcal{S}$ (e.g. $\mathbb{R}^n$, or $\mathbb{Z}^n$ for discrete spaces like images), denoted respectively by $D_B(X)$ and $E_B(X)$ [23]:

$$D_B(X) = \{x \in \mathcal{S} \mid B_x \cap X \neq \emptyset\}, \tag{1}$$

$$E_B(X) = \{x \in \mathcal{S} \mid B_x \subseteq X\}, \tag{2}$$

where B_x denotes the translation of B at point x. In these equations, B defines a neighborhood that is considered at each point. It can also be seen as a relationship between points. ¿From these two fundamental operations, a lot of others can be built [23].

2.2 Fuzzy Mathematical Morphology

Several definitions of mathematical morphology on fuzzy sets with fuzzy structuring elements have been proposed in the literature (see e.g. [8, 24, 11]). Here we use the approach using t-norms and t-conorms as fuzzy intersection and fuzzy union. However, what follows applies as well if other definitions are used. Erosion and dilation of a fuzzy set μ by a fuzzy structuring element ν, both defined in a space $\mathcal{S}$, are respectively defined as:

$$E_\nu(\mu)(x) = \inf_{y \in \mathcal{S}} T[c(\nu(y-x)), \mu(y)], \tag{3}$$

$$D_\nu(\mu)(x) = \sup_{y \in \mathcal{S}} t[\nu(y-x), \mu(y)], \tag{4}$$

where t is a t-norm, c a fuzzy complementation, and T is the t-conorm associated to t with respect to c. These definitions guarantee that most properties of morphological operators are preserved [8, 21].

2.3 Morpho-logics

Now, we express morphological operations in a symbolic framework, using logical formulas. Let us consider a language generated by a finite set of propositional symbols and the usual connectives. Kripke's semantics is used. The set of all worlds is denoted by Ω. The set of worlds where a formula φ is satisfied is $Mod(\varphi) = \{\omega \in \Omega \mid \omega \models \varphi\}$.

The underlying idea for constructing morphological operations on logical formulas is to consider set interpretations of formulas and worlds. Since in classical propositional logics, the set of formulas is isomorphic to 2^{Ω}, up to the logical equivalence, we can identify φ with $Mod(\varphi)$, and then apply set-theoretic morphological operations. We recall that $Mod(\varphi \vee \psi) = Mod(\varphi) \cup Mod(\psi)$, $Mod(\varphi \wedge \psi) = Mod(\varphi) \cap Mod(\psi)$, and $Mod(\varphi) \subseteq Mod(\psi)$ iff $\varphi \models \psi$.

Using these equivalences, dilation and erosion of a formula φ are defined as [7]:

$$Mod(D_B(\varphi)) = \{\omega \in \Omega \mid B(\omega) \cap Mod(\varphi) \neq \emptyset\}, \tag{5}$$

$$Mod(E_B(\varphi)) = \{\omega \in \Omega \mid B(\omega) \models \varphi\}, \tag{6}$$

where $B(\omega) \models \varphi$ means $\forall \omega' \in B(\omega), \omega' \models \varphi$.

The structuring element B represents a relationship between worlds and defines a "neighborhood" of worlds. It can be for instance defined as a ball of a distance between worlds [18]. The condition for dilation expresses that the set of worlds in relation to ω should be consistent with φ, i.e.: $\exists \omega' \in B(\omega),\ \omega' \models \varphi$. The condition for erosion is stronger and expresses that φ should be satisfied in all worlds in relation to ω.

Now we consider the framework of normal modal logics [10] and use an accessibility relation as relation between worlds. We define an accessibility relation from any structuring element B (or the converse) as: $R(\omega, \omega')$ iff $\omega' \in B(\omega)$. Let us now consider the two modal operators $\Box$ and $\Diamond$ defined from the accessibility relation as [10]:

$$\mathcal{M}, \omega \models \Box\varphi \text{ iff } \forall \omega' \in \Omega,\ R(\omega, \omega') \Rightarrow \mathcal{M}, \omega' \models \varphi, \tag{7}$$

$$\mathcal{M}, \omega \models \Diamond\varphi \text{ iff } \exists \omega' \in \Omega,\ R(\omega, \omega') \text{ and } \mathcal{M}, \omega' \models \varphi, \tag{8}$$

where $\mathcal{M}$ denotes a standard model related to R. Equation 7 can be rewritten as:

$$\omega \models \Box\varphi \Leftrightarrow B(\omega) \models \varphi, \tag{9}$$

which exactly corresponds to the definition of erosion of a formula, and Equation 8 can be rewritten as:

$$\omega \models \Diamond\varphi \Leftrightarrow B(\omega) \cap Mod(\varphi) \neq \emptyset, \tag{10}$$

which exactly corresponds to a dilation. This shows that we can define modal operators derived from an accessibility relation as erosion and dilation with a structuring element:

$$\Box\varphi \equiv E_B(\varphi), \tag{11}$$

$$\Diamond\varphi \equiv D_B(\varphi). \tag{12}$$

The modal logic constructed from erosion and dilation has a number of theorems and rules of inference, detailed in [4, 6], which increase its reasoning power. All these definitions and properties extend to the fuzzy case, if we consider fuzzy formulas, for which $Mod(\varphi)$ is a fuzzy set of Ω. A fuzzy structuring element can be interpreted as a fuzzy relation between worlds. Its usefulness will appear for expressing intrinsically vague spatial relationships such as directional relative position.

3 Computing Spatial Relationships from Mathematical Morphology: Quantitative and Semi-quantitative Setting

In this Section we consider the problem of defining and computing spatial relationships between two objects. We consider the general case of a 3D space $\mathcal{S}$, where objects can have any shape and any topology, and consider both topological and metric relationships [17, 13]. We distinguish also between relationships that are mathematically well defined (such as set relationships, adjacency, distances) and relationships that are intrinsically vague, like relative directional position, for which fuzzy definitions are appropriate. If the objects are imprecise, as is often the case if they are extracted from images, then the semi-quantitative framework of fuzzy sets proved to be useful for their representation, as spatial fuzzy sets (i.e. fuzzy sets defined in the space $\mathcal{S}$), and both types of relations have then to be extended to the fuzzy case. Results can also be semi-quantitative, and provided in the form of intervals or fuzzy numbers.

3.1 Set Relationships

Computing set relationships, like inclusion, intersection, etc. if the objects are precisely defined does not call for specific developments. If the objects are imprecise, stating if they intersect or not, or if one is included in the other, becomes a matter of degree. A degree of inclusion can be defined as an infimum of a t-conorm (as for erosion). A degree of intersection μ_{int} can be defined using a supremum of a t-norm (as for fuzzy dilation) or using the fuzzy volume of the t-norm in order to take more spatial information into account. The degree of non-intersection is then simply defined by $\mu_{\neg int} = 1 - \mu_{int}$. The interpretations in terms of erosion and dilation allow to include set relationships in the same mathematical morphology framework as the other relations.

3.2 Adjacency

Adjacency has a large interest in image processing and pattern recognition, since it denotes an important relationship between image objects or regions. For any two subsets X and Y in the digital space $\mathbb{Z}^n$, the adjacency of X and Y can be expressed in terms of morphological dilation, as:

$$X \cap Y = \emptyset \ \ and \ \ D_B(X) \cap Y \neq \emptyset, \ \ D_B(Y) \cap X \neq \emptyset, \tag{13}$$

where B denotes the elementary structuring element associated to the chosen digital connectivity. This structuring element is usually symmetrical, which means that the two conditions $D_B(X) \cap Y \neq \emptyset$ and $D_B(Y) \cap X \neq \emptyset$ are equivalent, so only one needs to be checked.

Adjacency between fuzzy sets can be defined by translating this expression into fuzzy terms, by using fuzzy dilation. The binary concept becomes then a degree of adjacency between fuzzy sets μ and ν:

$$\mu_{adj}(\mu, \nu) = t[\mu_{\neg int}(\mu, \nu), \mu_{int}[D_B(\mu), \nu], \mu_{int}[D_B(\nu), \mu]]. \tag{14}$$

This definition represents a conjunctive combination of a degree of non-intersection $\mu_{\neg int}$ between μ and ν and a degree of intersection μ_{int} between one fuzzy set and the dilation of the other.

This definition is symmetrical, reduces to the binary definition if μ, ν and B are binary, and is invariant with respect to geometrical transformations.

3.3 Distances

The importance of distances in image processing is well established. Their extensions to fuzzy sets (e.g. [25]) can be useful for several aspects of image processing under imprecision. Mathematical morphology allows to define distances between fuzzy sets that combine spatial information and membership comparison. In the binary case, there exist strong links between mathematical morphology (in particular dilation) and distances (from a point to a set, and several distances between two sets), and this can also be exploited in the fuzzy case. The advantage is that distances are then expressed in set theoretical terms, and are therefore easier to extend with nice properties than usual analytical expressions. Here we present the case of Hausdorff distance. The binary equation defining the Hausdorff distance:

$$d_H(X, Y) = \max[\sup_{x \in X} d(x, Y), \sup_{y \in Y} d(y, X)] \tag{15}$$

can be expressed in morphological terms as:

$$d_H(X, Y) = \inf\{n, X \subseteq D^n(Y) \text{ and } Y \subseteq D^n(X)\}. \tag{16}$$

A distance distribution, expressing the degree to which the distance between μ and μ' is less than n is obtained by translating this equation into fuzzy terms:

$$\Delta_H(\mu, \mu')(n) = t[\inf_{x \in \mathcal{S}} T[D^n_\nu(\mu)(x), c(\mu'(x))], \inf_{x \in \mathcal{S}} T[D^n_\nu(\mu')(x), c(\mu(x))]], \tag{17}$$

where c is a complementation, t a t-norm and T a t-conorm.

A distance density, expressing the degree to which the distance is equal to n, can be derived implicitly from this distance distribution. A direct definition of a distance density can be obtained from:

$$d_H(X,Y) = 0 \Leftrightarrow X = Y, \tag{18}$$

and for $n > 0$

$$d_H(X,Y) = \\ n \Leftrightarrow X \subseteq D^n(Y) \text{ and } Y \subseteq D^n(X) \text{ and } \left(X \not\subset D^{n-1}(Y) \text{ or } Y \not\subset D^{n-1}(X)\right). \tag{19}$$

Translating these equations leads to a definition of the Hausdorff distance between two fuzzy sets μ and μ' as a fuzzy number:

$$\delta_H(\mu,\mu')(0) = t[\inf_{x\in\mathcal{S}} T[\mu(x), c(\mu'(x))], \inf_{x\in\mathcal{S}} T[\mu'(x), c(\mu(x))]], \tag{20}$$

$$\delta_H(\mu,\mu')(n) = t[\inf_{x\in\mathcal{S}} T[D^n_\nu(\mu)(x), c(\mu'(x))], \inf_{x\in\mathcal{S}} T[D^n_\nu(\mu')(x), c(\mu(x))],$$
$$T(\sup_{x\in\mathcal{S}} t[\mu(x), c(D^{n-1}_\nu(\mu')(x))], \sup_{x\in\mathcal{S}} t[\mu'(x), c(D^{n-1}_\nu(\mu)(x))])]. \tag{21}$$

The obtained distance is positive (the support of this fuzzy number is included in $\mathbb{R}^+$). It is symmetrical with respect to μ and μ'. The separability property (i.e. $d(\mu,\nu) = 0 \Leftrightarrow \mu = \nu$) is not always satisfied. However, we have $\delta_H(\mu,\mu')(0) = 1$ implies $\mu = \mu'$ for T being the bounded sum ($T(a,b) = \min(1, a+b)$), while it implies μ and μ' crisp and equal for $T = \max$. The triangular inequality is not satisfied in general.

3.4 Directional Relative Position from Conditional Fuzzy Dilation

Relationships between objects can be partly described in terms of relative position, like "to the left of". Because of the inherent vagueness of such expressions, they may find a better understanding in the framework of fuzzy sets, as fuzzy relationships, even for crisp objects. A few works propose fuzzy approaches for assessing the directional relative position between objects, which is an intrinsically vague relation [2, 15, 16, 19, 20].

The approach used here relies on a fuzzy dilation that provides a map (or fuzzy landscape) where the membership value of each point represents the degree of the satisfaction of the relation to the reference object. This approach has interesting features: it works directly in the image space, without reducing the objects to points or histograms, and it takes the object shape into account.

We consider a (possibly fuzzy) object R in the space $\mathcal{S}$, and denote by $\mu_\alpha(R)$ the fuzzy subset of $\mathcal{S}$ such that points of areas which satisfy to a high degree the relation "to be in the direction $\boldsymbol{u}_\alpha$ with respect to object R" have high membership values, where $\boldsymbol{u}_\alpha$ is a vector making an angle α with respect to a reference axis. We express $\mu_\alpha(R)$ as the fuzzy dilation of μ_R by ν, where ν

is a fuzzy structuring element depending on α: $\mu_\alpha(R) = D_\nu(\mu_R)$ where μ_R is the membership function of the reference object R. This definition applies both to crisp and fuzzy objects and behaves well even in case of objects with highly concave shape. In polar coordinates (but this extends to 3D as well), ν is defined by[1]: $\nu(\rho, \theta) = f(\theta - \alpha)$ and $\nu(0, \theta) = 1$, where $\theta - \alpha$ is defined modulo π and f is a decreasing function, e.g. $f(\beta) = \max[0, \cos\beta]^2$ for $\beta \in [0, \pi]$.

Once we have defined $\mu_\alpha(R)$, we can use it to define the degree to which a given object A is in direction $\boldsymbol{u}_\alpha$ with respect to R. Let us denote by μ_A the membership function of the object A. The evaluation of relative position of A with respect to R is given by a function of $\mu_\alpha(R)(x)$ and $\mu_A(x)$ for all x in $\mathcal{S}$. The histogram of $\mu_\alpha(R)$ conditionally to μ_A is such a function. A summary of the contained information could be more useful in practice, and an appropriate tool for this is the fuzzy pattern matching approach [12]: the matching between two possibility distributions is summarized by two numbers, a necessity degree N (a pessimistic evaluation) and a possibility degree Π (an optimistic evaluation), as often used in the fuzzy set community. The possibility corresponds to a degree of intersection between the fuzzy sets A and $\mu_\alpha(R)$, while the necessity corresponds to a degree of inclusion of A in $\mu_\alpha(R)$. These operations can also be interpreted in terms of fuzzy mathematical morphology, since Π corresponds to a dilation, while N corresponds to an erosion.

4 Spatial Representations of Spatial Relationships

Now we address a second type of problem, and given a reference object, we define a spatial fuzzy set that represents the area of the space where some relationship to this reference object is satisfied (to some degree). The advantage of these representations is that they map all types of spatial knowledge in the same space, which allows for their fusion and for spatial reasoning (this occurs typically in model-based pattern recognition, where heterogeneous knowledge has to be gathered to guide the recognition). This constitutes a new way to represent spatial knowledge in the spatial domain [3].

For each piece of knowledge, we consider its "natural expression", i.e. the usual form in which it is given or available, and translate it into a spatial fuzzy set in $\mathcal{S}$ having different semantics depending on the type of information (on objects, spatial imprecision, relationships to other objects, etc.).

The numerical representation of membership values assumes that we can assign numbers that represent degrees of satisfaction of a relationship for instance. These numbers can be derived from prior knowledge or learned from examples, but usually there remain some quite arbitrary choices. However, we have to keep in mind that mostly the ranking is important, not the individual numerical values.

[1] This definition of ν is discontinuous at the origin. A continuous function could be obtained by modeling the fact that the direction of a point or of an object closed to the origin is imprecise.

4.1 Set Relationships

Set relationships specify if areas where other objects can be localized are forbidden or possible. The corresponding region of interest has a binary membership function (1 in authorized portions of the space, 0 elsewhere). This extends to the fuzzy case as: $\mu_{\text{set}}(x) = t[\mu_{O^{\text{in}}}(x), 1 - \mu_{O^{\text{out}}}(x)]$, where t is a t-norm, which expresses a conjunction between inclusion constraint in the objects O^{in} and exclusion constraint from the objects O^{out}. The properties of t-norms guarantee that good properties are satisfied.

4.2 Other Topological Relations

Other topological relations (adjacency, etc.) can be treated in a similar way and involve morphological operators. For instance, an object that is a non tangential proper part of μ has to be searched in $E_\nu(\mu)$.

4.3 Distances

Again, morphological expressions of distances, as detailed in Section 3, directly lead to spatial representation of knowledge about distances. Let us assume that we want to determine B, subject to satisfy some distance relationship with an object A. According to the algebraic expressions of distances, dilation of A is an adequate tool for this. For example, if knowledge expresses that $d(A, B) \geq n$, then B should be looked for in $D^{n-1}(A)^C$. Or, if knowledge expresses that B should lay between a distance n_1 and a distance n_2 of A, i.e. the minimum distance should be greater than n_1 and the maximum distance should be less than n_2, then the possible domain for B is reduced to $D^{n_2}(A) \setminus D^{n_1-1}(A)$.

In cases where imprecision has to be taken into account, fuzzy dilations are used, with the corresponding equivalences with fuzzy distances. The extension to approximate distances calls for fuzzy structuring elements. We define them through their membership function ν on $\mathcal{S}$, with a spherical symmetry, where ν only depends on the distance to the center of the structuring element and corresponds to the knowledge expression, as a fuzzy interval for instance [14]. The increasingness of fuzzy dilation with respect to both the set to be dilated and the structuring element guarantees that the obtained expressions have the required properties.

4.4 Relative Directional Position

The definition of directional position between two sets described in Section 3 relies directly on a spatial representation of the degree of satisfaction of the relation to the reference object. Therefore the first step of the proposed approach directly provides the desired representation as the fuzzy set $\mu_\alpha(A)$ in $\mathcal{S}$.

5 Symbolic Representations of Spatial Relationships

In this Section, we use the logical framework presented in Section 2. For spatial reasoning, interpretations can represent spatial entities, like regions of the space. Formulas then represent combinations of such entities, and define regions, objects, etc., which may be not connected. For instance, if a formula φ is a symbolic representation of a region X of the space, it can be interpreted for instance as "the object we are looking at is in X". In an epistemic interpretation, it could represent the belief of an agent that the object is in X. The interest of such representations is also to deal with any kind of spatial entities, without referring to points. If φ represents some knowledge or belief about a region X of the space, then $\Box\varphi$ represents a restriction of X. If we are looking at an object in X, then $\Box\varphi$ is a necessary region for this object. Similarly, $\Diamond\varphi$ represents an extension of X, and a possible region for the object.

5.1 Topological Relationships

Let us first consider topological relationships, and two formulas φ and ψ representing two regions X and Y of the space. Note that all what follows holds in both crisp and fuzzy cases. Simple topological relations such as inclusion, exclusion, intersection do not call for more operators than the standard ones of propositional logic. But other relations such that *X is a tangential part of Y* can benefit from the morphological modal operators. Such a relationship can be expressed as:

$$\varphi \to \psi \text{ and } \Diamond\varphi \land \neg\psi \text{ consistent.} \tag{22}$$

Indeed, if X is a tangential part of Y, it is included in Y but its dilation is not. If we also want X to be a proper part, we have to add the condition:

$$\neg\varphi \land \psi \text{ consistent.} \tag{23}$$

Let us now consider adjacency (or external connection). Saying that X is adjacent to Y means that they do not intersect and as soon as one region is dilated, it intersects the other. In symbolic terms, this relation can be expressed as:

$$\varphi \land \phi \text{ inconsistent and } \Diamond\varphi \land \psi \text{ consistent and } \varphi \land \Diamond\psi \text{ consistent.} \tag{24}$$

It could be interesting to link these types of representations with the ones developed in the community of mereotopology, where such relations are defined respectively from parthood and connection predicates [1, 22]. Interestingly enough, erosion is defined from inclusion (i.e. a parthood relationship) and dilation from intersection (i.e. a connection relationship). Some axioms of these domains could be expressed in terms of dilation. For instance from a parthood postulate $P(X,Y)$ between two spatial entities X and Y and from dilation, tangential proper part could be defined as:

$$TPP(X,Y) = P(X,Y) \land \neg P(Y,X) \land \neg P(D(X),Y). \tag{25}$$

5.2 Distances

Again we use expressions of minimum and Hausdorff distances in terms of morphological dilations. The translation into a logical formalism is straightforward. Expressions like $d_{\min}(X, Y) \leq n$ translate into:

$$\Diamond^n \varphi \wedge \psi \text{ consistent and } \Diamond^n \psi \wedge \varphi \text{ consistent.} \tag{26}$$

Similarly for Hausdorff distance, we translate $d_H(X, Y) = n$ by:

$$(\forall m < n, \psi \wedge \neg\Diamond^m \varphi \text{ consistent or } \varphi \wedge \neg\Diamond^m \psi \text{ consistent})$$
$$\text{and } (\psi \rightarrow \Diamond^n \varphi \text{ and } \varphi \rightarrow \Diamond^n \psi). \tag{27}$$

The first condition corresponds to $d_H(X, Y) \geq n$ and the second one to $d_H(X, Y) \leq n$.

Let us consider an example of possible use of these representations for spatial reasoning. If we are looking at an object represented by ψ in an area which is at a distance in $[n_1, n_2]$ of a region represented by φ, this corresponds to a minimum distance greater than n_1 and to a Hausdorff distance less than n_2. Then we have to check the following relation:

$$\psi \rightarrow \neg\Diamond^{n_1} \varphi \wedge \Diamond^{n_2} \varphi. \tag{28}$$

This expresses in a symbolic way an imprecise knowledge about distances represented as an interval. If we consider a fuzzy interval, this extends directly using fuzzy dilation.

These expressions show how we can convert distance information, which is usually defined in an analytical way, into algebraic expressions through mathematical morphology, and then into logical ones through morphological expressions of modal operators.

5.3 Directional Relative Position

Here we rely again on the approach where the reference object is dilated with a particular structuring element defined according to the direction of interest. Let us denote by D^d the dilation corresponding to a directional information in the direction d, and by $\Diamond^d$ the associated modal operator. Expressing that an object represented by ψ has to be in direction d with respect to a region represented by φ amounts to check the following relation: $\psi \rightarrow \Diamond^d \varphi$. In the fuzzy case, this relation can hold to some degree. This formulation directly inherits the properties of directional relative position defined from dilation, such as invariance with respect to geometrical transformations.

6 Conclusion

The spatial arrangement of objects in images provides important information for recognition and interpretation tasks, in particular when the objects are embedded in a complex environment like in medical or remote sensing images. Such information can be expressed in different ways varying from purely quantitative and precise ones to purely qualitative and symbolic ones. We have shown in this paper that mathematical morphology provides a unified and consistent framework to express different types of spatial relationships and to answer different questions about them, with good properties. Due to the strong algebraic structure of this framework, it applies to objects represented as sets, as fuzzy sets, and as logical formulas as well. This establishes links between theories that were so far disconnected. Applications of this work concern model-based pattern recognition, spatial knowledge representation issues, and spatial reasoning. First results have already been obtained using this framework in brain imaging [14] and mobile robotics [9]. Illustrations are also whown in [3, 5].

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