

Abductive Reasoning Using Tableau Methods for High-Level Image Interpretation

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Abstract. Image interpretation is a dynamic research domain involving not only the detection of objects in a scene but also the semantic description considering context information in the whole scene. Image interpretation problem can be formalized as an abductive reasoning problem, i.e. an inference to the best explanation using a background knowledge. In this work, we present a framework using a tableau method for generating and selecting potential explanations of the given image when the background knowledge is encoded using a description that is able to handle spatial relations.

1 Introduction

High-level semantics extraction from an image is an important research area in artificial intelligence. Many related fields like image annotation, activity recognition and decision-support systems take advantage of semantic content. Scene understanding, which translates low level signal information into meaningful semantic information, belongs to one of the fundamental abilities of human beings. In this work, beyond a single object understanding based on low level features such as colors and forms, we focus on a complex description which relies on context information like spatial relations as well as prior knowledge on the application domain. Our aim is to extract high-level semantic information from a given image and translate it at a linguistic level. Concretely, we are interested in the interpretation of cerebral images with tumors. The high-level information corresponds to the presence of diverse types of pathologies as well as descriptions of brain structures and spatial relations among them in a brain image. For instance, according to different levels of anatomical prior knowledge on brain pathology, two possible descriptions of the image in Figure 1 could be:

- an abnormal structure is present in the brain,
- a peripheral non-enhanced tumor is present in the right hemisphere¹.

¹ We use the classical “left is right” convention for display. The “right” structure is on the left side in Figure 1 (i.e. on the right side of the brain).

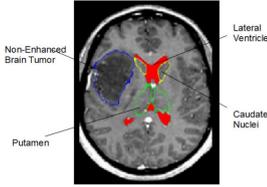


Fig. 1. A slice of a pathological brain volume (MRI acquisition), where some structures are annotated.

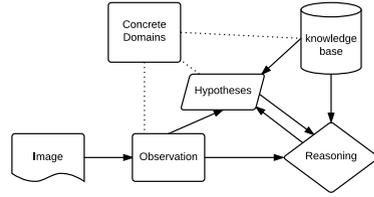


Fig. 2. A general schema of image interpretation task in this work.

In the context of this work, the decision process is modeled as an abductive reasoning [1] using Description Logics. Abductive reasoning is a backward-chaining inference, consisting in generating hypotheses and finding the “best” explanation of a given observation. New knowledge should be added in order to positively entail the observation. Image interpretation can be expressed as an abductive reasoning mechanism. Figure 2 shows the major components of our framework. The main components encompass an observation of a given image, a prior knowledge base of the application domain and the reasoning service for the purpose of image interpretation. The given image is translated into symbolic representations in terms of logical formulas by segmentation and recognition of objects using image processing tools. The recognized structures are represented as individuals of concepts, and spatial relationships are computed and represented as role individuals. The future work will involve concrete domains. Concrete domains [6], considered as a real world model (e.g. image space) linked with abstract terminologies, is as well a useful part which benefits from complementary information of abstract level of knowledge in the image representation. Hypotheses are formulated with the help of the reasoning process taking both the observation and the background knowledge into account. The relations between the hypothesis and the reasoning are in two directions. One is backward-chaining for generating potential hypotheses. The other is forward-chaining reasoning to select satisfiable and preferred explanations.

To achieve our goal, we need to answer the following questions:

- *How to model the prior knowledge and formalize an appropriate representation in a given application domain? (Section 2)*
- *How to generate hypotheses to explain the observed scene? (Section 3 and 4)*
- *How to define a criterion to choose the “best” explanation in our case? (Section 3 and 4)*

2 Background and Related Work

Description Logics (DLs) are a family of knowledge representation formalisms [4]. We use $\mathcal{ALCHL}_{\mathcal{R}^+}$ including inverse roles, symmetric roles and transitive role

axioms [11] in this paper. The role axioms are represented in a restricted form such as $r \equiv s^-$ (inverse roles) and $r \circ r \sqsubseteq r$ (transitive role axioms). A more complete overview of Description Logics can be found in [4].

The knowledge base used in our framework is built with three blocks: terminologies (TBox), role axioms (RBox) and assertions (ABox) ($\mathcal{K} = \{\mathcal{T}, \mathcal{R}, \mathcal{A}\}$). An example of a knowledge base referring to brain anatomy is as follows, where *Lvl* and *Lvr* denote left and right lateral ventricles, and left and right caudate nuclei are denoted by *Cnl* and *Cnr*. The general knowledge is represented in the TBox, which describes basic axioms of the background knowledge. The ABox represents the assertions, involving the facts in the observation (such as information extracted from an image). The complete knowledge base is given as follows:

$$\begin{aligned}
 \text{TBox} = \{ & \text{Hemisphere} \sqsubseteq \exists \text{isPartOf} . \text{Brain} \\
 & \text{BrainStructure} \sqsubseteq \exists \text{isPartOf} . \text{Brain} \\
 & \text{BrainDisease} \sqsubseteq \exists \text{isPartOf} . \text{Brain} \sqcap \neg \text{BrainStructure} \\
 & \text{Tumor} \sqsubseteq \text{BrainDisease} \\
 & \text{Lvl} \sqsubseteq \text{BrainStructure} \sqcap \exists (\text{rightOf} \sqcap \text{closeTo}) . \text{Cnl} \\
 & \text{Lvr} \sqsubseteq \text{BrainStructure} \sqcap \exists (\text{leftOf} \sqcap \text{closeTo}) . \text{Cnr} \\
 & \text{Cnl} \sqsubseteq \text{BrainStructure} \\
 & \text{Cnr} \sqsubseteq \text{BrainStructure} \\
 & \text{PeripheralHemisphere} \sqsubseteq \text{Hemisphere} \\
 & \text{CentralHemisphere} \sqsubseteq \text{Hemisphere} \sqcap \neg \text{PeripheralHemisphere} \\
 & \text{PeripheralTumor} \sqsubseteq \text{Tumor} \sqcap \exists \text{isPartOf} . \text{PeripheralHemisphere} \sqcap \exists \text{farFrom} . (\text{Lvl} \sqcup \text{Lvr}) \\
 & \text{SmallDeformingTumor} \sqsubseteq \text{Tumor} \sqcap \exists \text{closeTo} . (\text{Cnl} \sqcup \text{Cnr}) \} \\
 \\
 \text{RBox} = \{ & \text{rightOf} \equiv \text{leftOf}^- \\
 & \text{above} \equiv \text{below}^- \\
 & \text{closeTo} \equiv \text{closeTo}^- \\
 & \text{farFrom} \equiv \text{farFrom}^- \\
 & \text{isPartOf} \circ \text{isPartOf} \sqsubseteq \text{isPartOf} \\
 & \text{hasPart} \circ \text{hasPart} \sqsubseteq \text{hasPart} \\
 & \text{isPartOf} \equiv \text{hasPart}^- \} \\
 \\
 \text{ABox} = \{ & a : \text{Cnl} \\
 & b : \text{unknown} \\
 & c : \text{Brain} \\
 & \langle a, b \rangle : \text{leftOf}, \text{closeTo} \\
 & \langle b, c \rangle : \text{isPartOf} \}
 \end{aligned}$$

This knowledge base example demonstrates a practical way to represent brain anatomy. For instance, $\text{Lvl} \sqsubseteq \text{BrainStructure} \sqcap \exists (\text{rightOf} \sqcap \text{closeTo}) . \text{Cnl}$ expresses that the left lateral ventricle belongs to the brain structure which is on the right of and close to the left caudate nucleus. In the RBox, inverse relations ($\text{rightOf} \equiv \text{leftOf}^-$) and transitive relations ($\text{hasPart} \circ \text{hasPart} \sqsubseteq \text{hasPart}$) are used to represent spatial relation properties. In the ABox, a, b, c are individuals corresponding to observed objects in the image. $a : \text{Cnl}$ is a concept assertion and $\langle b, c \rangle : \text{isPartOf}$ is a role assertion, expressing that b is a part of c .

High level image interpretation is important in image analysis, for various tasks such as image annotation [17], event detection [14] and diagnostic problems [2, 3]. Image interpretation combines image processing with artificial intelligence techniques to derive reasonable semantics.

Image interpretation task was regarded as an abduction problem in [2, 9, 15]. In [15], DL-safe rules were proposed to map high level concepts and occurrence objects in the scene and their relationships. The rules ensure the expressivity

and preserve the decidability of the reasoning. However, only the concept defined in the rules can be inferred using this formalism. In [2], the image interpretation was formulated as a concept abduction problem. The DL is \mathcal{EL} . The knowledge base is processed using formal concept analysis and the abductive reasoning utilizes morphological operators. In [9], a probabilistic model is integrated into the abductive reasoning in order to facilitate the preference selection.

The tableau method was first adapted in Description Logics formalisms for a market matchmaking problem [7]. Colucci *et al.* modeled this problem as a concept abduction in the DL \mathcal{ALN} [7], where the observations are the demand and the supply is treated as the explanation for the meet of the request. The tableau method has also been studied by Halland *et al.* in [10] for a TBox abduction problem. For a TBox abduction problem, a TBox axiom in the form $\phi = C \sqsubseteq D$ is an explanation enforcing the entailment of the observation, which is also in the form of a TBox subsumption form. Similar to the tableau method for the concept abduction, if the disjunction of two concepts A_1 and $\neg A_2$ can create a clash of the tableau, then $A_2 \sqsubseteq A_1$ is considered as a potential explanation.

Klarman *et al.* [13] present a tableau method for ABox abduction in \mathcal{ALC} . This method integrates first-order logic reasoning techniques. First, the background knowledge and the observation are transformed into first-order syntax. Then, a tableau in the context of the first-order logic is built and solutions are selected in the open branches. The results are transformed into Description Logic from the first-order logic in the end. In [8], Du *et al.* introduced a tractable approach to ABox abduction, called the query abduction problem. However, the potential hypotheses are restricted to atomic concepts and roles in the TBox.

Another ingredient in abductive reasoning is the selection of the “best” explanation. As a set of syntactical candidates generated using the tableau method, the selection relies on explicit restrictions for choosing the “best” explanation. Restrictions concern filtering out inappropriate hypotheses, for instance, inconsistent hypotheses (\mathcal{H}_1 such that $\mathcal{K} \cup \mathcal{H}_1 \models \perp$) and independent hypotheses (\mathcal{H}_1 entails the observation independently without background knowledge, such that $\mathcal{H}_1 \models \mathcal{O}$). These types of hypotheses need to be removed. In addition, minimality criteria are required to select the “best” among the filtered candidates. Though the desired candidates are selected, the solutions can be infinite. Therefore, defining minimality criteria is an important manner to find a preference among all the potential hypotheses. Biennu discussed a set of basic minimality criteria for abductive reasoning in DLs in [5] such as semantic minimality and cardinal minimality.

3 Abductive Reasoning Using Tableau Method

In this section, we will introduce how abduction is applied to image interpretation from two aspects (generation of hypotheses and selection of a preferred explanation).

Definition 1 (Concept Abduction). *Let \mathcal{L} be a DL, $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ be a knowledge base in \mathcal{L} , C, D two concepts in \mathcal{L} and suppose that they are satisfiable with*

respect to \mathcal{K} . The logical formalism of abduction in DLs is represented as follows: given an observation concept \mathcal{O} , a hypothesis is a concept \mathcal{H} such that $\mathcal{K} \models \mathcal{H} \sqsubseteq \mathcal{O}$.

As all observed objects in the ABox can be formulated by an appropriate concept, our problem is modeled as a concept abduction. $\mathcal{K} \models \mathcal{H} \sqsubseteq \mathcal{O}$. \mathcal{H} is an explanation of the given observation \mathcal{O} if \mathcal{H} is subsumed by \mathcal{O} w.r.t. \mathcal{K} . The subsumption problem can be converted into a test of satisfiability which requires to prove that $\mathcal{H} \sqcap \neg\mathcal{O}$ is unsatisfiable. According to the strategy proposed by Aliseda [1], a potential hypothesis \mathcal{H} is the concept which makes the tableau of $\mathcal{H} \sqcap \neg\mathcal{O}$ closed as a consequence.

In the forward-chaining inference such as deduction, the corresponding axioms of the TBox are integrated in the tableau method using the normalization process [4]. The more general concept (in the right of a subsumption relation) can be obtained if the more specific concept is satisfied (in the left side of a subsumption relation). In Colucci’s method, the authors employ this replacement strategy. In other words, the more specific concept cannot be inferred from the more general concept. For instance, a concept D can be inferred by getting a concept C with the axiom $C \sqsubseteq D$ in a deductive way since a model of the concept C is also a model of D . However, this is not suitable for a backward-chaining inference, which intends to find a concept C as a hypothesis for D . A possible solution is to add the internalized concept (see Definition 2) in the tableau.

Definition 2 (Internalized concept [4]). Let \mathcal{T} be a TBox and a set of axioms formulated as $C_i \sqsubseteq D_i$. The internalized concept of the TBox is defined as follows:

$$C_{\mathcal{T}} \equiv \sqcap_{(C_i \sqsubseteq D_i \in \mathcal{T})} (\neg C_i \sqcup D_i)$$

For example, the internalized concept of the axiom $LVI \sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNI$ is $\neg LVI \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNI)$.

If $C_i \sqsubseteq D_i$, then $\top \sqsubseteq \neg C_i \sqcup D_i$ and $C_{\mathcal{T}} \equiv \top$. As a consequence, all interpretations of the TBox \mathcal{T} are equivalent to interpretations of the internalized concept $C_{\mathcal{T}}$. Therefore, every interpretation element belongs to $C_{\mathcal{T}}^{\mathcal{I}}$ and $C \equiv C \sqcap C_{\mathcal{T}}$ is proved.

We reformulate the subsumption checking in terms of satisfiability: the concept $\mathcal{H} \sqcap \neg\mathcal{O}$ is not satisfiable w.r.t. \mathcal{T} , where \mathcal{H} is an explanation, \mathcal{O} is an observation, \mathcal{T} is a TBox. This problem can be reduced by testing the satisfiability of a concept $\mathcal{H} \sqcap \neg\mathcal{O} \sqcap C_{\mathcal{T}}$, where $C_{\mathcal{T}}$ is the internalized concept of \mathcal{T} . The concept \mathcal{H} that causes unsatisfiability of $\mathcal{H} \sqcap \neg\mathcal{O} \sqcap C_{\mathcal{T}}$ is a potential hypothesis, i.e. the tableau built from this concept is closed. We follow this strategy and propose an extension of the work by Colucci *et al.* in [7].

Each interpretation element in the tableau has now four label functions: $\mathbf{T}(x)$, $\mathbf{F}(x)$, $\mathbf{T}(\langle x, y \rangle)$, $\mathbf{F}(\langle x, y \rangle)$, where x, y are interpretation elements in $\Delta^{\mathcal{I}}$. They are defined as follows:

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$ be a knowledge base, x, y interpretation elements, C, D two concepts and r, s two roles in the given DL, we have:

- $\mathbf{T}(x)$ represents a set of concepts such that x is one possible interpretation element: $C \in \mathbf{T}(x)$ iff $x \in C^{\mathcal{I}}$.
- $\mathbf{F}(x)$ represents a set of concepts such that x is not one possible interpretation element: $D \in \mathbf{F}(x)$ iff $x \notin D^{\mathcal{I}}$.
- $\mathbf{T}(\langle x, y \rangle)$ represents a set of roles between x and y : $r \in \mathbf{T}(\langle x, y \rangle)$ iff $\langle x, y \rangle \in r^{\mathcal{I}}$.
- $\mathbf{F}(\langle x, y \rangle)$ represents a set of unsatisfiable roles between x and y : $s \in \mathbf{F}(\langle x, y \rangle)$ iff $\langle x, y \rangle \notin s^{\mathcal{I}}$.

In the initialization step, the root node of the tableau is initialized with the concept $C_{\mathcal{T}} \sqcap \neg \mathcal{O}$. As $C_{\mathcal{T}} \sqcap \neg \mathcal{O}$ belongs to $\mathbf{T}(1)$, we add its negation to $\mathbf{F}(1)$. This technique avoids adding the negation before selecting concepts to generate contradictions in the tableau. We can prove the equivalence between $C \in \mathbf{T}(x)$ and $\neg C \in \mathbf{F}(x)$. Suppose that for $x \in \Delta^{\mathcal{I}}$, x is an interpretation element of a concept C , and x is also an interpretation individual of the concept $\neg C$. As a consequence, x is an interpretation of the concept $C \sqcap \neg C \equiv \perp$. There is no such interpretation. Thus, if $x \in C^{\mathcal{I}}$, then $x \notin (\neg C)^{\mathcal{I}}$, and conversely, $x \in C^{\mathcal{I}}$ is proved when $x \notin (\neg C)^{\mathcal{I}}$.

We assume that the concepts are expressed in a negation normal form (NNF). For a concept $C \in \mathcal{ALC}$, $\neg C$ in the NNF is denoted by \bar{C} . The expansion rules used in our work are:

1. Conjunction

T if $C \sqcap D \in \mathbf{T}(x)$, we add C and D in $\mathbf{T}(x)$.

F if $C \sqcup D \in \mathbf{F}(x)$, we add C and D in $\mathbf{F}(x)$.

2. Disjunction

T if $C \sqcup D \in \mathbf{T}(x)$, the branch is divided into two $(\mathbf{T}(x_1), \mathbf{T}(x_2))$. $\mathbf{T}(x_1) = \mathbf{T}(x) \cup \{C\}$ and $\mathbf{T}(x_2) = \mathbf{T}(x) \cup \{D\}$

F if $C \sqcap D \in \mathbf{F}(x)$, the branch is divided into two $(\mathbf{F}(x_1), \mathbf{F}(x_2))$. $\mathbf{F}(x_1) = \mathbf{F}(x) \cup \{C\}$ and $\mathbf{F}(x_2) = \mathbf{F}(x) \cup \{D\}$

3. Existential restriction

T if $\exists r.C \in \mathbf{T}(x)$ and there does not exist a y such that $r \in \mathbf{T}(\langle x, y \rangle)$ and $C \in \mathbf{T}(y)$, we create a new interpretation element y and then add r in $\mathbf{T}(\langle x, y \rangle)$, and C in $\mathbf{T}(y)$.

F if $\forall r.C \in \mathbf{F}(x)$ and there does not exist a y such that $r \in \mathbf{T}(\langle x, y \rangle)$ and $C \in \mathbf{F}(y)$, we create a new interpretation element y and then add r in $\mathbf{T}(\langle x, y \rangle)$, and C in $\mathbf{F}(y)$.

4. Universal restriction

T if $\forall r.C \in \mathbf{T}(x)$ and for all y such that $r \in \mathbf{T}(\langle x, y \rangle)$ and $C \notin \mathbf{T}(y)$, we add C in $\mathbf{T}(y)$.

F if $\exists r.C \in \mathbf{F}(x)$ and for all y such that $r \in \mathbf{T}(\langle x, y \rangle)$ and $C \notin \mathbf{F}(y)$, we add C in $\mathbf{F}(y)$.

5. Replacement of axioms in \mathcal{T}

T if $A \in \mathbf{T}(x)$ and $A \equiv C \in \mathcal{T}$, we add C in $\mathbf{T}(x)$.

- T** if $\neg A \in \mathbf{T}(x)$ and $A \equiv C \in \mathcal{T}$, we add \overline{C} in $\mathbf{T}(x)$.
F if $\neg A \in \mathbf{F}(x)$ and $A \equiv C \in \mathcal{T}$, we add \overline{C} in $\mathbf{F}(x)$.
F if $A \in \mathbf{F}(x)$ and $A \equiv C \in \mathcal{T}$, we add C in $\mathbf{F}(x)$.
6. r^- -rule
T if $r \in \mathbf{T}(\langle x, y \rangle)$, then $r^- \in \mathbf{T}(\langle y, x \rangle)$.
F if $r \in \mathbf{F}(\langle x, y \rangle)$, then $r^- \in \mathbf{F}(\langle y, x \rangle)$.
7. $\forall r_{trans}$ -rule
T if $\forall r.C \in \mathbf{T}(x)$ and r is a transitive role, then for all y such that $r \in \mathbf{T}(\langle x, y \rangle)$, $\forall r.C \in \mathbf{T}(y)$.
F if $\exists r.C \in \mathbf{F}(x)$ and r is a transitive role, then for all y such that $r \in \mathbf{T}(\langle x, y \rangle)$, $\exists r.C \in \mathbf{F}(y)$.
8. r_{\sqcap} -rule
T if $r \sqcap s \in \mathbf{T}(\langle x, y \rangle)$, we add r and s in $\mathbf{T}(\langle x, y \rangle)$.
F if $r \sqcup s \in \mathbf{F}(\langle x, y \rangle)$, we add r and s in $\mathbf{F}(\langle x, y \rangle)$.

The contradiction is classified into two types: homogeneous clash and heterogeneous clash.

Definition 3 (Clash [7]). *A clash in a branch can be divided into two categories:*

1. *A branch is defined as a homogeneous clash if:*
 - $\perp \in \mathbf{T}(x)$ or $\top \in \mathbf{F}(x)$.
 - $\{A, \neg A\} \in \mathbf{T}(x)$ or $\{A, \neg A\} \in \mathbf{F}(x)$.
2. *A branch is defined as a heterogeneous clash if:*
 - $\{A \text{ or } \neg A\} \in \mathbf{T}(x) \cap \mathbf{F}(x)$.

We illustrate this procedure on the brain MR image with a tumor (Figure 1) using the knowledge base described in Section 2. In this example, the observation is a concept for an unknown object considering the background knowledge: $\mathcal{O} \equiv \exists(\text{leftOf}^- \sqcap \text{closeTo}^-).CNl \sqcap \exists \text{isPartOf}.Brain^2$.

$$Lvl \sqsubseteq BrainStructure \sqcap \exists(\text{rightOf} \sqcap \text{closeTo}).CNl$$

$$SmallDeformingTumor \sqsubseteq Tumor \sqcap \exists \text{closeTo}.(CNl \sqcup CNr).$$

By applying expansion rules, the construction process of the tableau is shown in Figure 3. We explain only the first part of the development of the tableau procedure. The hypotheses are generated from open branches. In this example, we have two sets of concepts for the expanded part:

$$H_1 = \{Lvl, SmallDeformingTumor\}$$

$$H_2 = \{Lvl, \forall \text{closeTo}.\neg CNr, \forall \text{closeTo}.CNl\}.$$

The concepts in these two sets are basic elements to build a hypothesis \mathcal{H} . We assume that the second part of the tableau is closed. Therefore, a hypothesis \mathcal{H} is a conjunction of one concept from each set H_i . To avoid redundancy, we take the minimum hitting set in order to construct hypotheses from the candidate sets.

² We use image processing tools to recognize some known structures and to compute their spatial relationships. The description in logical formalism is given manually.

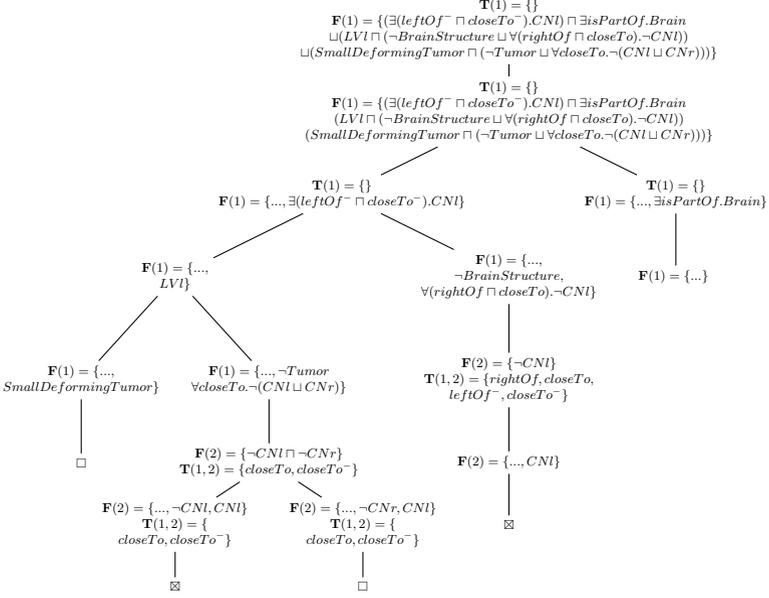


Fig. 3. The process of constructing the tableau by applying expansion rules.

Definition 4 (Hitting set). Let $\{S_1, \dots, S_n\}$ be a collection of sets. A hitting set T is a subset $T \subseteq \cup_{i=1}^n S_i$ such that T contains at least one element of each set in the collection $T \cap S_i \neq \emptyset$ ($1 \leq i \leq n$). The minimal hitting set is a hitting set T_m if \nexists hitting set T' such that $T' \subset T_m$.

The inconsistent hypotheses ($\mathcal{K} \cup \mathcal{H} \models \perp$) and irrelevant hypotheses ($\mathcal{H} \models \emptyset$) also need to be removed during the construction process. An exhaustive algorithm (Algorithm 1) is elaborated from the minimal hitting set algorithm [16].

Algorithm 1. Exhaustive search algorithm of selecting hitting sets.

- 1: **input:** A collection of sets $\{S_1, \dots, S_n\}$
 - 2: **output:** A collection of hitting sets \mathcal{H}
 - 3: $\mathcal{H} = \emptyset$
 - 4: Root node initialization.
 - 5: **for** i from 1 to n **do**
 - 6: Create new children nodes for all concepts of S_i in every leaf node
 - 7: An intermediate hypothesis hyp_j is the conjunction of all the concepts in the same branch
 - 8: Delete the branch j if hyp_j is inconsistent w.r.t. the TBox
 - 9: **end for**
 - 10: The conjunction of all concepts in each branch j represents a potential hypothesis \mathcal{H}_j
 - 11: **return:** $\mathcal{H} = \cup \{\mathcal{H}_j\}$
-

In order to choose a preferred solution among the hitting sets, two basic minimality criteria are used in our framework: subsumption criterion and cardinal minimality.

Definition 5 (Subsumption criterion). For an abduction problem $\mathcal{P} = \langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$, \mathcal{H}_i is a $\sqsubseteq_{\text{minimal}}$ explanation if there does not exist an explanation \mathcal{H}_j for \mathcal{P} such that $\mathcal{H}_i \sqsubseteq \mathcal{H}_j$.

Definition 6 (Cardinal minimality criterion). For an abduction problem $\mathcal{P} = \langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$, H is a set of concepts $\{C_1, \dots, C_n\}$ and $\mathcal{H} = C_1 \sqcap \dots \sqcap C_n$. \mathcal{H}_i is a \leq_{minimal} explanation if there does not exist an explanation \mathcal{H}_j for \mathcal{P} such that $|\mathcal{H}_i| \leq |\mathcal{H}_j|$.

In our example, $\mathcal{H}_1 = LVI$ and $\mathcal{H}_2 = SmallDeformingTumor \sqcap \forall closeTo. \neg CNr$ are equally preferred if we choose the subsumption criterion. However, $\mathcal{H}_1 = LVI$ is preferred if we consider the cardinal minimality criterion.

4 Conclusions and Perspectives

We have exploited Description Logics and an associated tableau method for knowledge representation and reasoning in image interpretation. A first model of background knowledge of brain anatomy including spatial information is proposed. At this stage, we have adapted the tableau method for generating preferred hypotheses w.r.t. the TBox.

Several directions will be considered in the future. A first direction is to generate adaptive hypotheses iteratively. We have shown that the tableau method produces a large amount of hypotheses, however, most of them are irrelevant or unsatisfiable. In order to avoid getting these hypotheses, an iterative method will be considered. Instead of adding all internalized concepts into the tableau, only relevant axioms are added to corresponding branches that cause a closure. This action can avoid generating unsatisfiable hypotheses. Since the observation is a conjunction of the concepts, the partial hypotheses in each branch will be ordered according to the minimality criterion. The selection process for the “best” explanation will be directly embedded into the tableau construction process.

Concrete domains are necessary in image interpretation since they provide an interface between abstract logical level and concrete image space, because semantic truth models may not have corresponding regions in concrete domains. For example, a concept $CNI \sqcap \exists rightOf. CNr$ could be verified to be satisfiable w.r.t. to a defined TBox. However, this concept may not have a model in the image space. This aspect will also be studied in the future.

Fuzzy logic is also a useful ingredient in knowledge representation dealing with imprecision and vague information. This aspect has been proved to be important for spatial reasoning by combining fuzzy relations in the concrete domains to Description Logics for image interpretation [12]. Another strategy to integrate fuzzy set theory into knowledge representation is to add fuzzy values to terminological and assertional knowledge at the logical level. This part of the work will allow dealing directly with satisfaction degrees of spatial relations.

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