MULTIPLE HYPOTHESIS TRACKING IN CLUTTERED CONDITION

Nicolas Chenouard¹, Isabelle Bloch², Jean-Christophe Olivo-Marin¹

¹Institut Pasteur, Unité d’Analyse d’Images Quantitative; CNRS URA 2582, Paris France
²TELECOM ParisTech, CNRS UMR 5141 LTCI, Paris, France

ABSTRACT

Multiple hypothesis tracking (MHT) is a preferred technique for solving the data association problem in modern multiple targets tracking systems. However its computational cost is generally considered prohibitive for tracking numerous objects in cluttered environments due to numerous targets and spurious measurements. We present in this paper a new MHT formulation in which target perceivability is modeled whereby automatic early track termination and false measurements exclusion reduce the problem complexity and improve the method robustness to clutter. Moreover we propose a MHT implementation exploiting the tree structure of the potential tracks to take full advantages of recent parallel computing technologies. We provide experimental results showing that both the track model and algorithmic design make the algorithm fast and robust even in highly complex situations such as tracking numerous particles in fluorescent microscopy images.

Index Terms— Particle tracking, Multiple Hypothesis Tracking (MHT), bioimaging, Hidden Markov Model

1. INTRODUCTION

The principle of Multiple Hypothesis Tracking (MHT) [1] is to delay the association task between a set of measurements and a set of tracks to an ulterior time when the decision is made easier by the knowledge of future frames. In practice it relies on building all the possible associations between tracks and measurements for a number of successive frames and comparing them. Since the temporal information is well exploited, the MHT is generally accepted as the preferred method for solving the data association problem in modern multiple target tracking systems. Despite many improvements [2], its use was rapidly abandoned for tracking a high number of targets in dense clutter conditions, such as biological targets tracking in fluorescent microscopy images, because of its computational cost. This cost is known to increase exponentially with the number of measurements, and is generally considered prohibitive for such a complex task.

We introduce here an approach that should change this view since we propose a MHT algorithm that is efficient even when tracking numerous particles in dense clutter thanks to our algorithmic design and the use of advanced computing technologies. We propose a scheme whereby fully exploiting the tree structure of the potential tracks makes enumerating all the possible associations and selecting the best one in an extremely efficient way. We use a massive, but exact, associations pruning that is based on a branch and bound scheme. This technique relies on an association score choice which decreases with the additions of tracks to the association, giving a hard decision threshold for pruning. Moreover our algorithmic design allows massively parallel processing which makes the different steps of the algorithm very fast on parallel computing architectures which have become widespread in recent years.

In applications such as bioimaging, frequent targets appearance and disappearance combined with the high density of clutter can lead to continuation of tracks with spurious measurements and false tracks construction. Hence, in this case it is essential to model the target perceivability, which is its capability to generate measurements. We define a two states Hidden Markov Model (HMM) following the model proposed in [3]. It allows the early detection of the end of a track, the initiation of tracks corresponding to real targets only and automatically discarding spurious measurements thanks to the knowledge of future frames.

A proposition of a Probabilistic MHT (PMHT) [4] algorithm using a Markovian state of targets is presented in [5]. However, the PMHT approach relaxes some fundamental MHT principles in order to reduce complexity: its main feature is the candidate tracks averaging over the last steps of the algorithm very fast on parallel computing architectures.

In high clutter density and numerous targets conditions we argue that the competition principle between tracks is a key advantage for discarding false measurements and detecting tracks initiation and termination events. Hence in contrast with the PMHT approach we want to impose that tracks compete, and we therefore focus on the original definition of the MHT algorithm.

In Section 2 we present the track perceivability model which is central to the method. Then Section 3 details our fast MHT implementation and in the last section we give evidence of feasibility and benefits of the proposed MHT in challenging time-lapse microscopy images of biological interest.

2. BAYESIAN FRAMEWORK AND TARGET PERCEIVABILITY MODEL

The first step of the tracking procedure consists in detecting targets position in the whole sequence. Detections are then linked to form target trajectories. For this we adopt a Bayesian framework in which we aim at building the set of tracks that maximizes the likelihood $P(\Theta|Z^n)$ of the associations between tracks and measurements from the sequence of $n$ images. We denote respectively $Z^n$ and $\Theta^n$ the vector of measurements and a feasible association, both from time 1 up to time $k$. $Z(k) = \{z_i(k)\}_{i=1,m_k}$ is the set of $m_k$ independent measurements in frame $k$. Each measurement is a vector of coordinates $z_i(k) = [x_i(k), y_i(k), z_i(k)]^T$ that are given by the detection procedure, and other optional measured features such
as the local intensity. The set of tracks is composed of \( n \) elements \( \Theta_l = \{ \theta_j \}_{j=1}^n \) whose likelihood \( L(\Theta_l) \) is written in the following way:

\[
L(\Theta_l) \triangleq P(\Theta_l|Z^l) = P(\Theta^l, Z^l)/P(Z^l) \propto P(\Theta^l, Z^l),
\]

where we can re-write:

\[
P(\Theta^l, Z^l) = \prod_{k=1}^l P(Z_0(k)) \prod_{j=1}^n P(\theta^k_j, z^k_j),
\]

when assuming that targets have independent movements and independent measurement processes. In Equation (2) \( Z_0(k) \) is the set of unassigned measurements at time \( k \), that are false detections coming from the sensor noise, and which distribution hence depends on the acquisition device and the detection procedure. The couple \((\theta^k_j, z^k_j)\) corresponds to the set of positions of the target \( j \) and its associated measurements during the time interval \( I_j \) of its presence in the images. When a track is not assigned to a measurement from \( Z(k) \) we associate it to a virtual measurement which is predicted by the dynamic filter. This feature models the temporarily disappearance of a target.

We define a two states HMM for target capability to generate measurements: a target is perceivable (state \( s^1 \)) if it can be detected, otherwise it is non perceivable (state \( s^0 \)). In state \( s^0 \) the object can have physically disappeared, bleached to a low level of intensity or left the surveillance volume. Hence it will not produce any measurement in future frames and the corresponding track should be ended. We define fixed transition probabilities between the states: let \( \pi_{ij} \) be the transition probability between state \( i \) and state \( j \). In the following we consider that a non perceivable target cannot become perceivable again, hence \( \pi_{01} = 0 \). In Equation (2) we write the probability of association \((\theta^k_j, z^k_j)\) up to time \( k \) as:

\[
p(\theta^k_j, z^k_j) = p(\theta^k_j, z^k_j, s^0_j(k)) + p(\theta^k_j, z^k_j, s^1_j(k)),
\]

hence explicitly accounting for the target perceivability. Using the perceivability concept in the estimation task of tracks favors perceivable tracks, hence increasing the robustness of the tracking procedure to spurious measurements.

We derive the following probability for each perceivability state \( s^1_j(k) \) \((i \in \{0, 1\})\) at time \( k \):

\[
p(\theta^k_j, z^k_j, s^1_j(k)) \triangleq \xi^1_j(k)
\]

\[
= p(\theta^k_j, s^1_j(k)) p(z^k_j | \theta^k_j, s^1_j(k)) p(\theta^k_j) p(s^1_j(k)) p(z^k_j | \theta^k_j, s^1_j(k))
\]

\[
= p(\theta^k_j, z^k_j, s^1_j(k)) p(\theta^k_j, z^k_j, s^1_j(k)),
\]

which shows that \( p(\theta^k_j, z^k_j, s^1_j(k)) \) can be evaluated by updating the previous probability \( p(\theta^k_j, z^k_j, s^1_j(k)) \), hence leading to the computation of \( p(\theta^k_j, z^k_j) \) by the iterative estimation of \( p(\theta^k_j, z^k_j) \) with \( r = 1...k \). By applying the Bayes’ rule we derive the probability of target perceivability at time \( k \) as follows:

\[
\lambda^1_j(k) \triangleq p(s^1_j(k)|z^k_j, \theta^k_j) = \frac{\pi_{0j} \xi^l_j(k-1) + \pi_{1j} \xi^1_j(k-1)}{\xi^0_j(k-1) + \xi^1_j(k-1)}.
\]

The measurement probability \( p(z^k_j | \theta^k_j, s^1_j(k)) \) can take four forms, depending on whether it is a real detection and depending of the target perceivability. If the target is perceivable the measurement probability is computed as:

\[
p(z^k_j | \theta^k_j, z^k_j, s^1_j(k)) = \begin{cases} P_D P_C \sigma_j(k) & \text{if } z^k_j \in Z(k) \\
1 - P_D P_C & \text{if } z^k_j \notin Z(k)
\end{cases},
\]

where \( P_D \) and \( P_C \) are respectively the probability of detecting a target and the probability that the target position falls in the search gate given by the kinetic model. In Equation (5) \( \sigma_j(k) \) is the probability that the measurement \( z^k_j \) originates from the target \( t_j \) under the assumption that \( t_j \) exists up to the time \( k \). This probability is computed with an estimation technique of the target state derived from the Kalman filter. We use the Interacting Multiple Models (IMM) filter [7] that is able to accurately model various types of movement by maintaining an adaptive mixture of kinetic models that self adapts to abrupt motion changes.

In contrast, a non perceivable target does not generate a real measurement. Hence if the measurement is real we compute its probability as the probability of a false detection, and if it is virtual we do not have to consider it since both the measurement and the track do not exist. The corresponding probabilities are:

\[
p(z^k_j | \theta^k_j, z^k_j, s^0_j(k)) = \begin{cases} P_{FD} & \text{if } z^k_j \in Z(k) \\
1 & \text{if } z^k_j \notin Z(k)
\end{cases},
\]

where \( P_{FD} \) is the probability that the measurement \( z^k_j \) is a false detection. Usually in tracking applications a uniform spatial distribution in the images is assumed for false detections, while their number \( n_0(k) = |Z_0(k)| \) is modeled as a random value sampled in a Poisson process. We propose instead to use an exponential distribution for \( n_0(k) \) with a mean \( \lambda_V \), which is the expected number of false detections in the surveillance volume \( V \). Resulting probabilities are given by:

\[
p(Z_0(k)) = \lambda_V^{-1} e^{-\lambda_V^{-1} n_0(k)} V^{-1}
\]

\[
= \lambda_V^{-1} \prod_{i=1}^{m_k} (e^{-\lambda_V^{-1} V^{-1}})^{1-\delta_i(k)},
\]

where \( \delta_i(k) = 1 \) if the measurement \( z^k_i \) is assigned to a track and \( \delta_i(k) = 0 \) otherwise. Equation (7) involves the product of an independent contribution \( e^{-\lambda_V^{-1} V^{-1}} \) for each non assigned measurement to the probability \( p(Z_0(k)) \). By doing so we are able to integrate the perceivability concept in tracks likelihood when setting \( P_{FD} \) to a proper value. More specifically, when the probability of perceivability of an object is zero we should consider that it does not exist anymore. Hence a real measurement associated to such a track should participate to the tracks likelihood (Eq. (1)) in the same way as a non assigned measurement:

\[
\forall z^k_j \in Z(k) \ /\\n\lim_{p(s^1_j(k)|\theta^k_j, z^k_j) \to 0} p(z^k_j | \theta^k_j, z^k_j) = e^{-\lambda_V^{-1} V^{-1}}.
\]

which implies:

\[
p(z^k_j | \theta^k_j, z^k_j, s^0_j(k)) = e^{-\lambda_V^{-1} V^{-1}}.
\]

From Equations (6) and (9) we therefore set \( P_{FD} = e^{-\lambda_V^{-1} V^{-1}} \).

### 3. Fast MHT Design

**3.1. MHT Scheme**

The MHT technique aims at building iteratively the association \( \Theta^l(k) \) that maximizes the likelihood \( P(\Theta^l(k)|Z^l(k)) \) instead of maximizing \( P(\Theta^l|Z^l) \) as instantaneous association algorithms do.
Fig. 1. Proposed MHT flow chart

At frame $k$, the previous processing steps provide the set of tracks $\Theta_{k-1}$ and we have to extend the tracks during $d + 1$ frames with measurements from the set $Z_{k:k+d}$. For each frame we adopt a four steps procedure summarized in Figure 1.

First we build independently for each track $\theta_{k-1} \in \Theta_{k-1}$ the set of potential tracks $\Gamma_{\theta_{k-1}}^{1:k+d}$ up to frame $k + d$ formed by the possible associations of $\theta_{k-1}$ with detections from $Z_{k:k+d}$. The second step consists in dividing the global association problem into a set of smaller tasks by clustering the association trees that are concurrent for at least one measurement. We denote $\Gamma_{\theta_{j}}^{\star k} + d$ the set of potential tracks forming the cluster $c_{j}$. Then in the association selection step for each cluster $c_{j}$ we build a subset of potential tracks $\Theta_{c_{j}}^{\star k+d} \subset \Gamma_{c_{j}}^{\star k+d}$ which has the highest likelihood $L(\Theta_{c_{j}}^{\star k})$. In the final step the best association is built by merging the association found for each cluster: $\Theta = \bigcup_{j} \Theta_{c_{j}}^{\star k+d}$. On this basis, validated tracks are either continued or ended while a number of new tracks are validated.

In the following we give a detailed presentation of steps 1 and 3 only as they are central to our method.

### 3.2. Potential tracks formation

For each track $\theta_{j}^{k-1} \in \Theta_{k-1}$ we independently enumerate potential tracks built by associating it with measurements from $Z_{k:k+d}, \theta_{j}^{k-1}$ can be associated only to a subset of $p$ measurements from $Z(k)$ that fall into the track search gate. These associations allow us to form a set $\Gamma_{\theta_{j}}^{1:k+d} = \{\theta_{j}^{k}, z_{i}\}_{i=1..p}$ of potential tracks. $\theta_{j}^{k}$ is the potential track that is built by associating the track $\theta_{j}^{k-1}$ with the detection $z_{i} \in Z(k)$. We iteratively repeat the association process for every set $\Gamma_{\theta_{j}}^{t}$ with $t = k..k + d - 1$.

We model the formation of potential tracks $\Gamma_{\theta_{j}}^{k+d}$ as the construction of trees of feasible associations since a track $\theta_{j}^{k-1}$ may give birth to a set of potential tracks $\Gamma_{\theta_{j}}^{t}$ which in turn may create potential tracks and so on. Each track $\theta_{j}^{k-1}$ is the root of a tree of nodes $\Gamma_{\theta_{j}}^{k:k+d}$. Moreover we create a tree from each detection in $Z_{k:k+d}$ in order to model the possibility for new targets to appear. Nodes in $\Gamma_{\theta_{j}}^{k:k+d}$ without any link to a node at the next level of the tree constitute the possible tracks of continuation for $\theta_{j}^{k-1}$.

During the nodes formation process we label the potential tracks according to their probability of perceivability. A track $\theta_{j}^{t}$ at frame $t$ is confirmed if $\exists t' \leq t$ such that $\lambda_{t'}^{t}(t') \geq p_{c}$ and terminated if $\lambda_{t'}^{t}(t) \leq p_{v}$. Details on the computation of the confirmation and termination thresholds, $p_{c}$ and $p_{v}$, will be provided elsewhere. Since a terminated track has a low probability of perceivability it is useless to continue it. Hence, we do not associate any more detections to a terminated track. Similarly, in the association selection procedure we will consider only confirmed potential tracks. Both techniques reduce significantly the size of the association problem.

We have taken advantage of the tree structure of the procedure to implement a recursive track construction procedure that is massively parallel: each time a node is created, it launches in a parallel way a node creation procedure for each measurement it can be associated with at the next step, and so on. By doing so, multithreading computing technologies make the construction of thousands of potential tracks very fast as shown in the Experiments section.

### 3.3. Association hypothesis selection up to frame $k + d$

The association hypothesis selection procedure consists in finding a subset of potential tracks $\Theta_{k:k+d} \subset \Gamma_{\theta_{j}}^{k:k+d}$ maximizing the likelihood $L(\Theta_{k:k+d})$. The number of feasible associations is dramatically increased by the presence of false detections that should remain not associated to any track, hence enumerating all the associations is unfeasible in a short time. Fortunately the unicity principle of association between tracks and detections imposes numerous tracks incompatibilities that can be exploited to reduce the number of considered associations. A usual way to solve the issue is to formulate it as a standard convex optimization problem with a set of constraints given by tracks incompatibilities. We have however preferred to develop our own solver that fully takes advantage of the tree structure of the tracks formation process in two ways: (1) building only a very restricted number of associations $\Theta_{k:k+d}$ by pruning huge sets of solutions, (2) massively parallel computing.

For the sake of clarity we rewrite the likelihood Equation (2) of the association $\Theta_{k:k+d}$ as:

$$L(\Theta) = \prod_{t=1..k+d} p(Z_{0}(t)) \prod_{j} f(\theta_{j}^{k+d}) = F_{\Theta}(\theta_{k:k+d})$$

where $f(\theta_{j}^{k+d}) = p(\theta_{j}^{k+d}, z_{j}^{k+d})$ denotes the joint probability of the associations selected for the track $\theta_{j}$, $\mathcal{F}(\Theta_{k:k+d}) = \prod_{j} f(\theta_{j}^{k+d})$ the product of all track probabilities, and $\mathcal{F}_{\Theta}(\theta_{k:k+d}) = \prod_{t=1..k+d} p(Z_{0}(t))$ the probability of measurements considered as spurious detections.

We begin by back-propagating in the potential tracks trees the probabilities $f(\theta_{j}^{k+d})$ such that each node knows the probability $f_{\star}$ that is the greatest among the subtree of tracks originating from it. By doing so we are able to know beforehand the greatest track probability we would be able to achieve by going through a node when exploring a tracks tree.

The proposed association hypothesis selection procedure proceeds by recursive exploration of trees and relies on an efficient branch and bound technique to build as few as possible hypotheses.

First a track $\theta_{1}$ is selected in the first tree. Hence for now the set of tracks is $\Theta = \{\theta_{1}\}$ and the incomplete product of track likelihoods is initialized as: $\mathcal{F}(\Theta) = f(\theta_{1})$. The second tree is then considered. The compatibility of its root node $\theta_{2}^{k-1}$ is checked against $\Theta$: if the measurement associated to the node is already used by any track in $\Theta$ the hypothesis is abandoned since a measurement can be associated to one track at most.

By using a track originating from the node $\theta_{2}^{k-1}$ the greatest achievable likelihood is $f_{\star}(\theta_{2}^{k-1})$. So we compute $\mathcal{F}(\Theta, \theta_{2}^{k-1}) = \mathcal{F}(\Theta) \times f(\theta_{2}^{k-1})$, the best likelihoods product when using a track originating from this node. By definition, $\mathcal{F}_{\Theta}(\theta_{2}^{k-1}, \theta_{1}) \leq 1$, so the product $\mathcal{F}(\Theta)$ decreases each time a track is added to the set $\Theta$. We therefore ensure that for any track $\theta_{j}$ selected in a tree to extend $\Theta$, we get $\mathcal{F}(\Theta \cup \theta_{j}) \leq \mathcal{F}(\Theta)$. Moreover,
\[ 0 \leq F_0(\Theta) \leq 1 \] by definition, which implies that: \( L(\Theta) \leq F(\Theta) \).
In summary, when considering a node the following set of inequalities is valid:

\[
L(\Theta) \leq F(\Theta \cup \theta_i) \leq F(\Theta)
\]  
(11)

Let \( L^* \) be the greatest likelihood found until now, and \( \Theta^{\star k+d} \) the corresponding set of track. When considering a node \( \theta^k_{j_z i} \), if we observe \( F^+(\Theta, \theta^k_{j_z i}) \leq L^* \), we then deduce from Equation (11) that \( L(\Theta) \leq L^* \) for any combination of a track originating from \( \theta^k_{j_z i} \) with tracks of other trees that could be added to complete \( \Theta \). In this case the association process is stopped since the likelihood of any set of tracks based on \( \Theta \) and \( \theta^k_{j_z i} \) cannot achieve a likelihood greater than the likelihood of \( \Theta^{\star k+d} \).

If the node \( \theta^{k-1}_{j_z} \) satisfies both the compatibility test with \( \Theta \) and the bounding test on \( F^+(\Theta, \theta^{k-1}_{j_z}) \) we repeat the exploration procedure for the nodes it is linked to at the next level. This process is applied recursively until the end of a branch is reached. In this case a \( \Theta \) duplicate is extended with the track \( \theta_{j_z} \) with which we ended: \( \Theta = \Theta \cup \theta_{j_z} \) and another tree is selected.

\( \Theta \) is complete if it contains a potential track for every non terminated track in \( \Theta^{\star k-1} \), but still it is eventually extended with other potential tracks that correspond to the appearance of a target. If the association likelihood \( L(\Theta) \) is greater than \( L^* \) the corresponding associations is stored as the best one until then: \( \Theta^{\star k+d} = \Theta \) and \( L^* = L(\Theta) \).

The proposed procedure takes advantage of the tree structure of the potential tracks to exclude huge sets of associations each time a node is branched to the current hypothesis, which significantly speeds up the construction of the feasible associations \( \Theta^{\star k+d} \). It is worth noting that these techniques ensure exact, instead of heuristic, pruning so we end up with certainty with the optimal solution \( \Theta^{\star k+d} \).

Thanks to the proposed algorithmic scheme we have designed a recursive implementation of the whole association selection procedure: each time a node is valid for association extension the branch and bound technique is started for each of its following nodes. In practice we launch a new thread to perform each of these tasks, so thousands of associations are built and compared in parallel, which significantly reduces the computation time on parallel computing architectures.

### 4. EXPERIMENTS

A recent biological study [8] has focused on the transport of Golgi units (vesicles) in the ovocytes of the Drosophila melanogaster thanks to disk scanning confocal microscopy imaging. In order to detect GFP labeled vesicles we first applied wavelet-based technique [9] adapted to the low SNR condition (PSNR \( \approx 5 \)). From these detections 111 trajectories were built by the proposed MHT technique with a depth of 4. As shown in Figure 4, trajectories are long and stop as soon as targets disappear, which reflects on the ability of the method to track vesicles during long times and to detect their appearance/disappearance despite the high level of clutter: we measured up to 25% of spurious detections. The trajectories show motions that are diverse and changing, revealing the successful integration of the IMM filter in the algorithm. The processing of the whole sequence of 150 images took only 18 seconds on a Mac pro 8-core 2.8 GHz thanks to the parallel computing implementation that takes full advantage of the eight cores of the cpu. Result movies are available on line at http://bioimageanalysis.org/2436/.

We provide in [10] performance assessment results with biological data and comparisons with a number of reference particle tracking algorithms. We show that the proposed MHT procedure provides reliable tracks of hundreds of particles in a short time, which is impossible with other techniques due to the extremely noisy conditions.

---

**Fig. 2.** Fluorescent units tracking with the proposed MHT algorithm.

### 5. CONCLUSION

In this paper we have proposed a new MHT formulation in which the robustness to spurious measurements is increased by the use of a Markov model of target perceivability. A new MHT implementation that exploits the tree structure of the potential tracks to take advantage of recent parallel computing technologies has been presented. The combination of track model with the algorithmic design makes the MHT technique fast even for high density of targets in a cluttered condition, as shown on a fluorescent microscopy example. Future works will include both refinements of Markov states definition depending on optical properties of the targets and further implementation optimizations such as heuristic pruning and larger scale parallel computing.

### 6. REFERENCES


