A spatial relation ontology using mathematical morphology and description logics for spatial reasoning

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Abstract. In several domains, such as medical image interpretation, spatial relations play a crucial role since they are less prone to variability than properties of structures themselves. Moreover, they constitute an important part of available knowledge. In this paper, we suggest a new way to handle imprecise spatial information in a description logic framework based on mathematical morphology with fuzzy interpretations in concrete domains. Fuzzy representations of concepts contribute to bridge the gap between ontological representations and real data, and the mathematical morphology setting is a core feature of our approach to merge the advantages of ontologies, concrete domain representations and manipulations, and the reasoning power of description logics. The benefits of our logical framework are illustrated on a medical image interpretation task.

1 Introduction

In image interpretation and computer vision, spatial relations between objects and spatial reasoning are of prime importance for recognition and interpretation tasks. Nevertheless, although spatial reasoning has been largely studied in artificial intelligence mainly using qualitative representations based on logical formalisms, there is still a gap with the quantitative representations used in image interpretation.

Description logics (DLs) equipped with concrete domains [15] are a widely accepted way to integrate *concrete and quantitative qualities* of real world objects with conceptual knowledge and as a consequence to combine qualitative and quantitative reasoning useful for real-world applications. In [13], we have proposed a fuzzy spatial ontology, operational for image interpretation, based on the expressive means of description logics with fuzzy concrete domains. In this ontology, the link between abstract concepts and the representation of these concepts in the concrete domains was explicitely built. In this paper, we propose to extend our previous work by the proposition of a new framework which goes deeper in the exploitation of concrete domains. In this framework, the combination of description logics with fuzzy concrete domains and mathematical morphology provides new mechanisms to derive useful concrete representations of concepts and new reasoning tools. The paper is organized as follows. In Section 2, we briefly present how mathematical morphology can be used to derive fuzzy representations of spatial relations. In Section 3, we present the main concepts of the fuzzy spatial relation ontology detailed in [13]. We describe our new framework and its properties in Section 4 and we illustrate the benefits of this framework for image interpretation tasks in Section 5.

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2 Fuzzy representation of spatial relations using mathematical morphology

In this Section, we briefly summarize our previous work on spatial relations. Their representation combines two important features: (i) modeling of imprecision using fuzzy sets, and (ii) computation through mathematical morphology operators.

Imprecision has to be taken into account to model vagueness, inherent to many spatial relations, and to gain in robustness in the representations. For instance, modeling in a mathematical way vague expressions such as "close to", "to the right of" can be appropriately performed in the fuzzy set framework, without defining crisp thresholds on distance or angle values.

Another advantage of fuzzy representations is that they lead to a very generic modeling of different spatial relations, at different levels of granularity. A membership function representing a spatial relation concept provides the semantics of the concept and constitutes an efficient way to convert symbolic notions into numeric ones. Moreover, the membership functions we use have simple shapes (such as trapezoidal ones) that are generic enough to be applicable in many different domains. For each specific domain, the parameters of the membership functions can be appropriately adapted, using prior knowledge or learning procedures. Let us take the example of "close to". This relation is modeled as a fuzzy interval on the real line (see Section 4). This generic model can then be instantiated, through an adequate choice of the parameters, for each particular application (for instance, the meaning of "close to" is obviously not the same if we consider towns observed in satellite imaging, or parts of the human body in medical imaging).

Another important feature of our approach is the use of fuzzy mathematical morphology operators [7] for defining spatial relations. Based on only one operation, the morphological dilation (and its dual: the erosion), relations such as adjacency, distances, relative directional position, or even more complex relations such as "between", could be modeled mathematically with nice properties and behaviors [4, 5].

In this paper, we show how these morphological expressions can be integrated in a spatial relation ontology and in description logics.

3 A spatial relation ontology

With the aim of image interpretation, we proposed in [13] an ontology of spatial relations. The semantic interpretation of images can benefit from representations of useful concepts and the links between them as ontologies. The proposed ontology of spatial relations is intended to guide image interpretation and the recognition of the structures it contains using structural information on the spatial arrangement of these structures. As a formal language, we have chosen de-

Constructor	Syntax	Semantics
atomic concept	A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
individual	a	$a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
Top	$\top$	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$
Bottom	$\bot$	$\bot^{\mathcal{I}} = \emptyset^{\mathcal{I}}$
atomic role	r	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
negation	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
existential restriction	$\exists r.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}} : (x,y) \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
universal restriction	$\forall r.C$	$\{x \in \Delta^{\mathcal{I}} \mid \forall y \in \Delta^{\mathcal{I}} : (x,y) \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}$
value restriction	$\ni r.\{a\}$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}} : (x,y) \in R^{\mathcal{I}} \Rightarrow y = a^{\mathcal{I}}\}$
number restriction	$(\geq nR)$	$\{x \in \Delta^{\mathcal{I}} \mid \ \lvert\{y \mid (x,y) \in R^{\mathcal{I}}\}\rvert \geq n\}$
	$(\leq nR)$	$\{x \in \Delta^{\mathcal{I}} \mid \ \lvert\{y \mid (x,y) \in R^{\mathcal{I}}\}\rvert \leq n\}$
subsumption	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
concept definition	$C \doteq D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$
concept assertion	$a : C$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$(a,b) : R$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$

Table 1. Description logics syntax and interpretation ($\Delta^{\mathcal{I}}$ denotes an abstract domain of the interpretation $\mathcal{I}$).

scription logics since it is compact and expressive and it is the basis of most ontological languages, in particular of the OWL language. We rely on the standard notations of DL, summarized in Table 1.

One important entity of our ontology, as proposed in [13], is the concept ***SpatialObject*** (SpatialObject $\sqsubseteq \top$). Moreover, as mentioned in [14], the nature of spatial relations is twofold: they are concepts with their own properties but they are also links between concepts. For instance, the assertion "X **is to the right of** Y" can be interpreted and represented in two different ways: (i) as an "abstract" relation between X and Y that is either true or false; (ii) as a physical spatial configuration between the two spatial objects X and Y. As a consequence, we use a process of reification of spatial relations as in [14]. A spatial relation is not considered in our ontology as a role (property) between two spatial objects but as a concept on its own (***SpatialRelation***), which is one of the main features of our approach. The notations used in the following are the classical notations of DL syntax and semantics.

- A ***SpatialRelation*** is subsumed by the general concept ***Relation***. It is defined according to a ***ReferenceSystem***.
  SpatialRelation $\sqsubseteq$ Relation $\sqcap \ni$ type.{Spatial} $\sqcap \exists$ hasReferenceSystem.ReferenceSystem
  SpatialRelation subsumes ***TopologicalRelation*** and ***MetricRelation*** which itself subsumes ***DirectionalRelation*** and ***DistanceRelation***. For ***BinarySpatialRelation***, we can also specify *inverse spatial relations* and properties such as *reflexivity, irreflexivity, symmetry, antisymmetry, asymmetry* useful for qualitative spatial reasoning as shown in [14].
- We define the concept ***SpatialRelationWith*** which refers to the set of spatial relations which are defined according to at least one or more reference spatial objects RO.
  SpatialRelationWith $\doteq$ SpatialRelation $\sqcap$
  $\exists$ hasRO.SpatialObject $\sqcap \geq 1$ hasRO
- We define the concept ***SpatiallyRelatedObject*** which refers to the set of spatial objects which have at least one spatial relation (hasSR) with another spatial object. This concept is useful to describe spatial configurations.
  SpatiallyRelatedObject $\doteq$ SpatialObject $\sqcap$
  $\exists$ hasSR.SpatialRelationWith $\sqcap \geq 1$ hasSR
- At last, the concept ***DefinedSpatialRelation*** represents the set of spatial relations for which target and reference objects are defined.
  DefinedSpatialRelation $\doteq$ SpatialRelation $\sqcap$
  $\exists$ hasRO.SpatialObject $\sqcap \geq 1$ hasRO $\sqcap \exists$ hasTargetObject.SpatialObject $\sqcap = 1$ hasTargetObject

An original feature of this ontology is that it is enriched by fuzzy representations of concepts, which define their semantics, and allow establishing the link between these concepts (which are often expressed in linguistic terms) and the information that can be extracted from images. This contributes to reducing the semantic gap and it constitutes a new methodological approach to guide semantic image interpretation. We make use of concrete domains towards this aim. In our previous work, the integration of the fuzzy models was performed by linking concepts of the spatial relation ontology to their corresponding physical fuzzy representation in the image domain. In this paper, we propose a more formal integration, detailed next.

4 Morphological fuzzy description logics

In this section, we introduce mathematical morphology as a spatial reasoning tool. In particular we show how morphological operators can be used to define a specific description logics with fuzzy concrete domains. The main objective is to provide a foundation to reason about qualitative and quantitative spatial relations.

4.1 Description of the formalism

The proposed framework is based on extensions of the basic description logics incorporating concrete domains $\mathcal{ALC}(\mathcal{D})$ [15]. In particular, as in [8], we integrate *fuzzy concrete information* into description logic concepts using fuzzy concrete domains. We first briefly recall the definition of concrete domains and we introduce their use in description logics.

Definition [8] A *concrete domain* $\mathcal{D}$ is a pair $(\Delta_{\mathcal{D}}, \Phi_{\mathcal{D}})$ where $\Delta_{\mathcal{D}}$ is a set and $\Phi_{\mathcal{D}}$ a set of predicates names on $\Delta_{\mathcal{D}}$. A *fuzzy concrete domain* is a pair $(\Delta_{\mathcal{D}}, \Phi_{\mathcal{D}})$, where $\Delta_{\mathcal{D}}$ is an interpretation domain and $\Phi_{\mathcal{D}}$ a set of fuzzy predicates d with a predefined arity n and an interpretation $d^{\mathcal{D}} : \Delta_{\mathcal{D}}^n \rightarrow [0,1]$, which is a $n-$ary fuzzy relation over $\Delta_{\mathcal{D}}$.

Role and concept terms. Let C, R_a, R_c, I_a, I_c be non empty and pair-wise disjoint sets of *concept names, abstract role names, concrete role names, abstract individual names, concrete individual names*. R_a also contains a non-empty set F_a of *abstract features names* and R_c contains a non-empty set F_c of *concrete features names*. These features are functional roles. A composition of features (denoted $f_1, f_2, ...$) is called a feature chain. In addition to the concept and terms constructors described in Table 1, we have the following constructs with $P \in \Phi_{\mathcal{D}}$ a predicate name associated with an arity n and a predicate $P^{\mathcal{D}} \subseteq \Delta_{\mathcal{D}}^n$, and $u_1, ..., u_n$,$v_1, ..., v_m$ are features chains:

- Predicate exists restriction: $\exists u_1, ..., u_n.P$ interpreted by : $\{a \in \Delta^{\mathcal{I}} \mid \exists x_1, ...x_n \in \Delta_{\mathcal{D}} : (u_1^{\mathcal{I}}(a) = x_1) \wedge ... \wedge (u_n^{\mathcal{I}}(a) = x_n) \wedge (x_1, ...x_n) \in P^{\mathcal{D}}\}$;
- Role forming predicate restriction [12]: $\exists\,(u_1, ..., u_n)\,(v_1, ..., v_m)\,.P$ interpreted by:
  $\{(x,y) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid \exists r_1, \ldots, r_n, s_1, \ldots, s_m \in \Delta_{\mathcal{D}} :$
  $u_1^{\mathcal{I}}(x) = r_1, \ldots, u_n^{\mathcal{I}}(x) = r_n, v_1^{\mathcal{I}}(y) = s_1, \ldots, v_m^{\mathcal{I}}(y) = s_m$
  and $(r_1, \ldots, r_n, s_1, \ldots, s_m) \in P^{\mathcal{D}}\}$.

Terminology and assertions. A Tbox $\mathcal{T}$ is a finite set of terminological axioms ($A \doteq D$ and $A \sqsubseteq D$) and an Abox $\mathcal{A}$ is a finite set of assertions ($a : C$ (concept membership), $(a,b) : R$ (role filler),

$(a, x) : f$ (feature filler) and $(x_1, ..., x_n) : P$ (concrete domain predicate membership)).

In our framework, we instantiate the description logics $\mathcal{ALCF}(\mathcal{D})$ (standard $\mathcal{ALC}$ extended with functional roles and concrete domains) with the fuzzy concrete domain $S = (\Delta_\mathcal{S}, \Phi_\mathcal{S})$. $\Delta_\mathcal{S} = \mathcal{S}$ is a 3D space (the image space) where fuzzy concrete objects are defined. $\mathcal{S}$ is typically $\mathbb{Z}^2$ or $\mathbb{Z}^3$ for 2D or 3D images. Let $\mathcal{F}$ the set of fuzzy sets defined over the spatial domain $\mathcal{S}$. In our framework, $\Phi_\mathcal{S}$ contains:

- The unary predicates $\bot_S$ and $\top_S$ denoting $\emptyset$ and Δ_S.
- The names of two unary fuzzy predicates μ and ν which associate to a spatial object concept X and to a spatial relation concept R the interpretations μ_X and ν_R in $\mathcal{S}$ ($\mu_X \in \mathcal{F}, \nu_R \in \mathcal{F}$). For each point $x \in \mathcal{S}$, $\mu_X(x)$ represents the degree to which x belongs to the spatial representation of the object X in the image and ν_R represents the fuzzy structuring element defined on $\mathcal{S}$ which represents the fuzzy spatial relation R in the image space (as mentioned in Section 2, relations are defined using mathematical morphology operators, with an appropriate structuring element).
- The names of two binary fuzzy predicates δ and ε with $\delta_{\nu_R}^{\mu_X}$ the fuzzy dilation and $\varepsilon_{\nu_R}^{\mu_X}$ the fuzzy erosion of the spatial fuzzy set μ_X by the structuring element ν_R. They are defined in [7] by :
  - $\forall x \in \mathcal{S}, \varepsilon_{\nu_R}(\mu_X)(x) = \varepsilon_{\nu_R}^{\mu_X}(x) = \inf_{y \in \mathcal{S}} T(1 - \nu_R(y - x), \mu_X(y))$ where T is a t-conorm (fuzzy union) [9];
  - $\forall x \in \mathcal{S}, \delta_{\nu_R}(\mu_X)(x) = \delta_{\nu_R}^{\mu_X}(x) = \sup_{y \in \mathcal{S}} t(\nu_R(x - y), \mu_X(y))$ where t is a t-norm (fuzzy intersection) [9].

  Here dual definitions of these operators are chosen for their properties, as will be seen later. Duality is intended with respect to the complementation c defined as $c(\alpha) = 1 - \alpha$ but other complementations can be used as well.
- Names for composite fuzzy predicates consisting of composition of elementary binary predicates.

We now illustrate how these fuzzy concrete domain predicates are used to represent spatial relations and to support spatial inference. As in [11], we assume that each abstract spatial relation concept and each abstract spatial object concept is associated with its fuzzy representation in the concrete domain by the concrete feature hasForFuzzyRepresentation, denoted hasFR (it is a concrete feature because each abstract concept has only one fuzzy spatial represention in the image space).

- SpatialObject $\doteq \exists$ hasFR.$\mathcal{F}$. It defines a SpatialObject as a concept which has a spatial existence in image represented by a spatial fuzzy set.
- In the same way, we have : SpatialRelation $\doteq$ Relation $\sqcap \exists$ hasFR.$\mathcal{F}$.

Then, the following constructs can be used to define the other concepts of the ontology:

- $\exists$ hasFR.μ_X restricts the concrete region associated with the object X to the specific spatial fuzzy set μ_X,
- $\exists$ hasFR.ν_R restricts the concrete region associated with the relation R to the specific fuzzy structuring element ν_R,
- $\exists$ hasFR.$\delta_{\nu_R}^{\mu_X}$ restricts the concrete region associated to the spatial relation R to a referent object X, denoted R_X, to the spatial fuzzy set obtained by the dilatation of μ_X by ν_R,
- each concept R_X can then be defined by: R_X $\doteq$ SpatialRelation $\sqcap \exists$ hasRO.X $\sqsubseteq$ SpatialRelationWith and R_X $\equiv$ SpatialRelation $\sqcap \exists$ (hasFR,hasRO.hasFR).λ where λ is a binary fuzzy predicate built with the mathematical fuzzy operators δ and ε. For a relation R which has a referent object X, (hasFR,hasRO.hasFR).$\delta \equiv$ hasFR. $\delta_{\nu_R}^{\mu_X}$.
- $C \doteq$SpatialObject$\sqcap$hasSR.R_X denotes the set of spatial objects which have a spatial relation of type R with the referent object X and we have the following axioms: $C \sqsubseteq \exists$relationTo.X and $C \sqsubseteq$ SpatiallyRelatedObject.

Admissibility of $S = (\Delta_S, \Phi_S)$. A concrete domain $\mathcal{D}$ is called admissible iff (i) the set of its predicate names is closed under negation and contains a name $\top_\mathcal{D}$ for $\Delta_\mathcal{D}$, and (ii) the satisfiability problem for finite conjunctions of predicates is decidable. Here condition (i) is satisfied and (ii) remains to be proved.

4.2 Examples for a few relations

Distance relations. To illustrate our approach, we take the example of distance relations. As in [8] we use a trapezoidal function $trz(x; a, b, c, d)$ to define the semantics of "close to": $\mathbb{R}^+ \to [0, 1]$ which represents the degree of membership to the distance relation with $trz(t; a, b, c, d) = 0$ if $t \leq a$ or $t \geq d$; $(t - a)/(b - a)$ if $t \in]a, b[$; $(d - t)/(d - c)$ if $t \in]c, d[$. Note that any other uni-modal membership function having a similar behavior could be used as well. For the Close_To relation, $a = b = 0$. From this membership function, we can define a structuring element ν_{CLOSE_TO}. This structuring element provides a representation in the spatial domain $\mathcal{S}$ [3]:

$$\forall x \in \mathcal{S}, \nu_{CLOSE\_TO}(x) = trz(d(x, O); a, b, c, d) \qquad (1)$$

where $d(x, O)$ is the distance from x to the origin O of $\mathcal{S}$ (Euclidean distance, or a digital distance when working on a discrete space).

We can thus define the abstract spatial relation Close_to by its fuzzy representation in the concrete domain $\mathcal{S}$, i.e. Close_to $\doteq$ DistanceRelation $\sqcap \exists$hasFR.ν_{CLOSE_TO}. Let X $\doteq \exists$ hasFR.μ_X, μ_X being the spatial fuzzy set representing the spatial extent of the object X in the concrete domain (image space). Using the concept-forming predicate operator $\exists f.P$ (see [11]), we can define restrictions for the fuzzy representation of the abstract spatial concept Close_to_X using the dilation operator δ. As a consequence, we have Close_to_X $\doteq$ DistanceRelation $\sqcap \exists$hasFR.$\delta_{\nu_{CLOSE\_TO}}^{\mu_X}$. The value $\delta_{\nu_{CLOSE\_TO}}^{\mu_X}(x)$ represents the degree to which a point x of $\mathcal{S}$ belongs to the fuzzy dilation of the fuzzy spatial representation of X by the structuring element ν_{CLOSE_TO}. Moreover, it restricts the role filler of hasFR to be specific fuzzy spatial sets.

Other distance relations can be defined in a similar way, by adapting the parameters of the trapezoidal function and the definition of the interpretation in terms of dilation. For instance, ν_{FAR_FROM} can be derived from Equation 1 by choosing a trapezoidal function expressing the semantics of this relation, i.e. b is chosen as the smallest distance for which the relation is satisfied with a non-zero degree, c is the largest distance for which the relation is not completely satisfied and $d = +\infty$. We then define Far_from_X $\doteq$ DistanceRelation $\sqcap \exists$hasFR.$(1 - \delta_{1-\nu_{FAR\_FROM}}^{\mu_X})$. Note that any fuzzy complementation c could be used, the most usual one being $c(\alpha) = 1 - \alpha$.

This approach naturally extends to any distance relation expressed as a vague interval, a fuzzy interpretation being provided by $trz(t; a, b, c, d)$. Two fuzzy sets are then defined: $\nu_1(x) = trz(d(x, 0); 0, 0, c, d)$ and $\nu_2(x) = trz(d(x, O); 0, 0, a, b)$. We then define Dist_to_X $\doteq$ DistanceRelation $\sqcap \exists$hasFR.$(\delta_{\nu_1}^{\mu_X} \setminus \delta_{\nu_2}^{\mu_X})$. The first structuring element ν_1 has a semantics of "not farther than the upper bound of the interval" while ν_2 means "not closer than the lower bound of the interval".

For a given relation R, once we have an interpretation in the concrete domain as a structuring element ν_R and an interpretation of R_X, e.g. as $\delta_{\nu_R}^{\mu_X}$ (or other forms involving a dilation as seen above), we can finally define Y_R_X for a target object concept Y (concept DefinedSpatialRelation in Section 3) with an interpretation expressed as a fitting function $fit(\mu_Y, \delta_{\nu_R}^{\mu_X})$, which can return a number, an interval, a fuzzy number, etc. [2, 3]. It can be for instance an interval defined by the degree of inclusion of μ_Y in $\delta_{\nu_R}^{\mu_X}$ and the degree of intersection of these fuzzy sets, as in a fuzzy pattern matching approach [10]. This applies also to the relations that will be detailed next. Note that this requires to add a second concrete domain, for the interpretations of fit functions.

Directional Relations. As for distances, directional relations can be defined using fuzzy structuring elements. We first define a fuzzy set on an angle space, that provides the semantics of the relation (see [2, 16]). Let us consider the relation In_Direction_α. From the fuzzy set $f_\alpha(\theta)$ in the angle space, we derive a fuzzy structuring element as [2]: $\forall x \in \mathcal{S}, \nu_{IN\_DIRECTION\_\alpha}(x) = f_\alpha(|\angle(Ox, \vec{i}) - \alpha|)$ where $\angle(Ox, \vec{i})$ denotes the angle between Ox and the first coordinate axis, and the angle difference is computed in $[-\pi, \pi]$. For instance the relation Right_of can be derived from $f_0(\theta) = \cos^2(\theta)$ for $\theta \in [-\pi/2, \pi/2]$ and 0 otherwise. Extension to 3D is obtained by defining a direction with two angles. Then we define In_Direction_α _of_X $\doteq$ DirectionalRelation $\sqcap \exists$hasFR.$\delta_{\nu_{IN\_DIRECTION\_\alpha}}^{\mu_X}$.

Adjacency relations. We propose to express adjacency as a distance relation, with a semantics of "very close to". Thus we use ν_{CLOSE_TO}, with parameters of the trapezoidal function set to $a = b = 0$ and c and d taking small values. This allows: (i) expressing this topological relation in the same formalism as the other relations; (ii) computing the interpretation (fuzzy representation) of ADJ_TO_X with a dilation; (iii) incorporating a tolerance in the definition (through the choice of the values of c and d), avoiding to consider it in a too strict way, and thus gaining in robustness. This is motivated by the fact that adjacency may depend on one point only, which often induces a loss of the adjacency property for only tiny changes in the considered objects. Note that strict adjacency is included in the proposed formalism by simply choosing $c = 1$ and $d = 2$ (for digital spaces).

Mereotopological relations. Our formalism leads also to simple definitions of internal or external boundaries of an object. For instance, the concept "external boundary" can be defined as ExtB_X $\doteq$ SpatialObject $\sqcap \exists$hasFR.$(\delta_{\nu_0}^{\mu_X} \setminus \mu_X)$ where ν_0 denotes an elementary structuring element (it can be crisp or fuzzy and defined according to the digital connectivity on $\mathcal{S}$). In a similar way, we define the internal boundary as IntB_X $\doteq$ SpatialObject $\sqcap \exists$ hasFR.$(\mu_X \setminus \varepsilon_{\nu_0}^{\mu_X})$.

Based on these notions, all relations of the mereotopology or of RCC-8 [1, 17, 19] can be expressed in our framework: parthood: P(X,Y) $\doteq$ X $\sqsubseteq$ Y; partial overlap: PO(X,Y) $\doteq$ (X $\sqcap$ Y $\neq \bot$); disconnection: DC(X,Y) $\doteq$ (X $\sqcap$ Y $= \bot$); external connection: EC(X,Y) $\doteq$ (X $\sqcap$ Y $= \bot$) $\sqcap$ X_ADJ_TO_Y; proper part: PP(X,Y) $\doteq$ (X $\sqsubseteq$ Y) $\sqcap$ (ExtB_X $\sqsubseteq$ Y); tangential properpart: TPP(X,Y) $\doteq$ (X $\sqsubseteq$ Y) $\sqcap$ (ExtB_X $\sqcap \neg$Y $\neq \bot$). This shows that the proposed formalism allows integrating in a very simple way spatial relations defined in other works.

More complex relations. Relations such as "between", or "surround", can be expressed in the same formalism, using dilations or compositions of previously defined relations [6]. They are not detailed here since their semantics is somewhat more complex and not independent of the shape of the involved objects. It is then more difficult to define R and R_X independently.

4.3 Properties

As before we denote in a general way by R a spatial relation concept, X a spatial object concept, and R_X the concept "relation R to X". Using the classical partial order on fuzzy sets $\leq$, $(\mathcal{F}, \leq)$ is a complete lattice, hence the appropriate framework for defining mathematical morphology operators. The associated infimum and supremum are denoted by $\wedge$ and $\vee$ (min and max here). The interpretation in the concrete domain of $X_1 \sqcap X_2$ is then $\wedge(\mu_{X_1}, \mu_{X_2})$ and the one of $X_1 \sqcup X_2$ is $\vee(\mu_{X_1}, \mu_{X_2})$. Several interesting properties of description logics can be derived from properties of mathematical morphology (for properties of mathematical morphology see [18] and [7] for the fuzzy case). We summarize here the most important ones:

- dilation commutes with the supremum: $\delta_\nu(\mu_{X_1}) \vee \delta_\nu(\mu_{X_2}) = \delta_\nu(\mu_{X_1} \vee \mu_{X_2})$ and $\delta_{\nu_1}(\mu_X) \vee \delta_{\nu_2}(\mu_X) = \delta_{\nu_1 \vee \nu_2}(\mu_X)$, and therefore we have the following equivalences between concepts: R_$X_1 \sqcup$ R_$X_2 \equiv$ R_($X_1 \sqcup X_2$) and R1_X$\sqcup$ R2_X $\equiv$ R12_X where R12 has for fuzzy representation $\nu_1 \vee \nu_2$;
- for the infimum, we only have: $\delta_\nu(\mu_{X_1}) \wedge \delta_\nu(\mu_{X_2}) \geq \delta_\nu(\mu_{X_1} \wedge \mu_{X_2})$ hence R_($X_1 \sqcap X_2$) $\sqsubseteq$ R_$X_1 \sqcap$ R_X_2 ;
- increasingness: $\mu_{X_1} \leq \mu_{X_2} \Rightarrow \forall \nu \in \mathcal{F}, \delta_\nu(\mu_{X_1}) \leq \delta_\nu(\mu_{X_2})$ and $\nu_1 \leq \nu_2 \Rightarrow \forall \mu_X \in \mathcal{F}, \delta_{\nu_1}(\mu_X) \leq \delta_{\nu_2}(\mu_X)$ hence $X_1 \sqsubseteq X_2 \Rightarrow \forall$ R, R_$X_1 \sqsubseteq$ R_X_2 and $R_1 \sqsubseteq R_2 \Rightarrow \forall$ X, R_1_X $\sqsubseteq$ R_2_X;
- iterativity property: $\delta_{\nu_1}(\delta_{\nu_2}(\mu_X)) = \delta_{\delta_{\nu_1}(\nu_2)}(\mu_X)$ hence R_1_(R_2_X) $\equiv$ (R_1_R_2)_X, where R_1_R_2 is the relation having as fuzzy representation $\delta_{\nu_1}(\nu_2)$;
- extensivity: $\nu(O) = 1 \Rightarrow \forall \mu_X \in \mathcal{F}, \mu_X \leq \delta_\nu(\mu_X)$ hence X $\sqsubseteq$ R_X for any relation defined by a dilation with a structuring element containing the origin of $\mathcal{S}$ (with membership value 1);
- duality: for the chosen definition of fuzzy dilation and erosion, we have $\varepsilon_\nu(\mu_X) = 1 - \delta_\nu(1 - \mu_X)$ (or a similar equation with any complementation c), which induces relations between some relations. For instance the fuzzy representation of IntB_X can be written as: $\mu_X \setminus \varepsilon_{\nu_0}^{\mu_X} = \mu_X \wedge \delta_{\nu_0}^{1-\mu_X} = \delta_{\nu_0}^{1-\mu_X} \setminus (1 - \mu_X)$, hence IntB_X $\equiv$ ExtB_$\neg$X.

These properties provide the basis for inference processes. Other examples use simple operations, such as conjunction and disjunction of relations, in addition to these properties, to derive useful spatial representations of potential areas of target objects, based on knowledge about their relative positions to known reference objects. This will be illustrated in the following section on a concrete example in brain imaging.

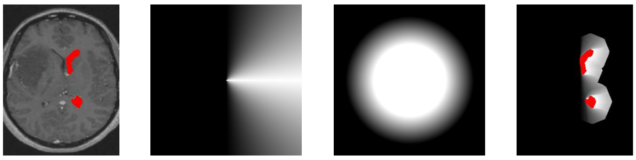
5 Application to medical image interpretation

In this section, we show how our framework can be used to support terminological and spatial reasoning in a cerebral image interpretation application. In particular, our aim is to segment and recognize anatomical structures progressively by using the spatial information between the different structures. We denote respectively LV, RLV and LLV the *Lateral Ventricle*, the *Right Lateral Ventricle* and the *Left Lateral Ventricle*. The other anatomical structures we consider are the *Caudate Nuclei* (denoted CN, RCN, LCN) which are *grey nuclei* (denoted GN) of the brain. We have the following TBox describing anatomical knowledge :

AnatomicalStructure $\sqsubseteq$ SpatialObject
GN $\sqsubseteq$ AnatomicalStructure
RLV $\doteq$ AnatomicalStructure $\sqcap$ $\exists$ hasFR.μ_{RLV}
LLV $\doteq$ AnatomicalStructure $\sqcap$ $\exists$ hasFR.μ_{LLV}
LV $\equiv$ RLV $\sqcup$ LLV
Right_of $\doteq$ DirectionalRelation $\sqcap$ $\exists$ hasFR.$\nu_{IN\_DIRECTION\_0}$
Close_to $\doteq$ DistanceRelation $\sqcap$ $\exists$ hasFR.ν_{CLOSE_TO}
Right_of_RLV $\doteq$ DirectionalRelation $\sqcap$ $\exists$ hasRO.RLV $\sqcap$ $\exists$ hasFR.$\delta^{\mu_{RLV}}_{\nu_{IN\_DIRECTION\_0}}$
Close_To_RLV $\doteq$ DistanceRelation $\sqcap$ $\exists$ hasRO.RLV $\sqcap$ $\exists$ hasFR.$\delta^{\mu_{RLV}}_{\nu_{CLOSE\_TO}}$
RCN $\doteq$ GN $\sqcap\exists$ hasSR.(Right_of_RLV $\sqcap$ Close_To_RLV)
CN $\doteq$ GN $\sqcap\exists$ hasSR.(Close_To_LV)
CN $\equiv$ RCN $\sqcup$ LCN

The role forming predicate also allows defining explicitly dilation or erosion as a role (for instance the dilation that defines the region to the right of the materlat ventricle):
dilate $\doteq$ (hasFR,hasRO.hasFR).δ
Right_Of_RLV $\doteq$ Right_Of $\sqcap\exists$ dilate.RLV

In an object recognition task, we now consider the situation illustrated in Figure 1 where the right lateral ventricle has already been extracted from the image.



(a) (b) (c) (d)

Figure 1. (a) The right ventricle corresponding to the image region S_1 is superimposed on one slide of the original image. (b) Fuzzy structuring element representing the semantics of Right_of in the image. (c) Fuzzy structuring element representing the semantics of Close_To in the image. (d) $\delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}} \wedge \delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}}$.

The situation in Figure 1 corresponds to the following ABox:
c_1 : RLV , (c_1,μ_{S_1}): hasFR
r_1 : Right_of, (r_1,$\nu_{IN\_DIRECTION\_0}$): hasFR
r_2 : Close_to, (r_2,ν_{CLOSE_TO}): hasFR

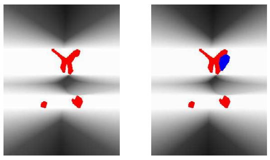
In a first example, our aim is to find some spatial constraints in the image domain on an instance c_2 of the Right Caudate Nucleus, i.e. to find constraints on concrete domains to ensure the satisfiability of the assertion c_2 : RCN , (c_2,μ_{S_2}): hasFR. Using the inference mechanisms of description logics and the properties of our framework, we derive the following spatial contraints :
$\mu_{S_2} \leq \delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}} \wedge \delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}}$.

In a second example, illustrating disjunctions of relations, we are interested in all the instances of Caudate Nuclei in the image. A caudate nucleus is a grey nucleus which is either to the right or to the left of the lateral ventricles (there is one in each brain hemisphere). This information can be represented by the following axioms:
CN $\doteq$ GN $\sqcap\exists$ hasSR.(Right_of_LV $\sqcup$ Left_of_LV)
Using the property of disjunction of relations, we have :
Right_of_LV $\sqcup$ Left_of_LV $\equiv$ SpatialRelation $\sqcap\exists$ hasFR.$\delta^{\mu_{LV}}_{\nu_{RIGHT\_OF} \vee \nu_{LEFT\_OF}}$.

As a consequence, the search space for the caudate nuclei is computed by $\delta_{\nu_{RIGHT\_OF} \vee \nu_{LEFT\_OF}}(\mu_{LV})$, which is equivalent to $\delta_{\nu_{RIGHT\_OF}}(\mu_{LV}) \vee \delta_{\nu_{LEFT\_OF}}(\mu_{LV})$. This fuzzy region is represented in Figure 2.

6 Conclusion

In this paper, we extended our previous work described in [13] by the proposition of a framework for spatial relations and spatial reasoning under imprecision based on description logics with fuzzy interpretations in concrete domains and fuzzy mathematical morphology. This framework enables to integrate qualitative and quantitative information and to derive appropriate representations and reasoning tools for an operational use in image interpretation. Future work aims at addressing questions of satisfiability and admissibility, and at further developing the brain imaging example.



(a) (b)

Figure 2. (a) Fuzzy interpretation of the disjunction of the relations left or right to LV. (b) One of the caudate nuclei is displayed.

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