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Fuzzy Models of Spatial Relations, Application to Spatial Reasoning

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8.1 Introduction

Spatial relations are an important component of image content, that proved to be useful for recognition of individual objects and for image understanding. Indeed, spatial relations provide structural information about the scene, which is often more stable that individual object characteristics, can help disambiguating objects of similar appearance, and is often available as prior knowledge. A typical example is anatomy, where relations between anatomical structures are described in anatomical textbooks or dedicated web sites, and can be used to drive the interpretation of medical images. This will be illustrated on magnetic resonance images (MRI) of the brain, for segmenting and recognizing internal brain structures. This is a typical example where shape and appearance information may not be sufficient for recognition, in particular in pathological cases, while using structural knowledge is relevant and helps solving the problem. Similar examples can be found in understanding aerial and satellite images.

One important characteristic of spatial relations is that they often have a clear intuitive meaning in natural language, but crisp mathematical models are often to restrictive, not robust enough, and do no model the intrinsic imprecision attached to the linguistic descriptions of the relations. Fuzzy models are better suited, and allow accounting for imprecision both in the relations and in the objects. This was already mentioned in [21].

My work on fuzzy models of spatial relations was initiated while I was visiting Lotfi Zadeh's lab in Berkeley, where I spent a few months in 1995 and 1997, enjoying the stimulating environment and fruitful discussions, with researchers from different fields of fuzzy sets theory.

The main approach I proposed to model fuzzy spatial relations relies on mathematical morphology [32], because of its strong algebraic framework, which allows developing consistent models in different settings (from purely quantitative ones on sets, to purely qualitative ones in various logics), the fuzzy sets setting being a midway [8]. Another feature is that different types of representations can be proposed, expressing relations as numbers, fuzzy numbers, intervals, distributions, or fuzzy regions of space.

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8.2 Mathematical Morphology to Model Spatial Relations

Spatial relations include set theoretical and topological relations (inclusion, intersection, connection, adjacency...), metric ones (distances, relative direction...), and more complex ones ("between", "along", "parallel to", "aligned"...). Note that this classification extends the one of [21] and the hierarchy proposed in [25]. These relations can be binary, ternary (such as "between"), or n-ary (alignment of a series of objects for instance). Some of them can be crisply defined when objects are crisp (such as the Hausdorff distance between two well defined objects), and need to be extended to the fuzzy case (i.e. when the objects are imprecisely defined or known). Other ones are intrinsically vague ("close to", "to the right of", "between"...), and are then best modeled using fuzzy sets.

The main idea in the proposed models is to make use of morphological operations, in particular dilations, using appropriate structuring elements. This idea comes from the fact that several relations in the crisp case can be converted into algebraic expressions involving set theoretical and morphological operations, which are then easy to extend to the fuzzy case, using fuzzy mathematical morphology. Let us give a few examples:

- adjacency between two crisp objects can be expressed by the fact that the two objects do not intersect, but as soon as a dilation is applied to one of them, intersection occurs. This is translated by a conjunction (using a t-norm) of a degree of non intersection of two fuzzy sets and a degree of intersection of the dilation of one fuzzy set and the other;

- the minimal distance between two crisp objects is equivalent to the minimal size of the dilation that has to be applied to one object so that it meets the other. Again this easily extends to the fuzzy case by using fuzzy dilations.

Direct algebraic expressions have been proposed for vague relations, using similar operations. For instance the region of space which is to the right of another object is defined as the dilation of this object by a fuzzy structuring element representing the semantics of the relation (high membership functions in the horizontal direction, which decrease when going away of this direction). This is illustrated in Figure 8.1. If ν denotes the fuzzy set representing the spatial relation, and μ the reference object (fuzzy set in the spatial domain $\mathscr{S}$), then the degree of satisfaction of the relation is given by the dilation of μ by the structuring element ν: $\forall x \in \mathscr{S},\ \delta_\nu(\mu)(x) = \sup_{y\in\mathscr{S}} C[\nu(y-x), \mu(y)]$, where C denotes a fuzzy conjunction, and more specifically a t-norm. Details on mathematical morphology and the associated properties can be found in [9, 13, 27]. Assessing to which degree another object is to the right of the reference object is then performed by comparing it to the dilation result, for instance using a fuzzy pattern matching approach [19]. Details can be found in [4]. This is an example where we have a direct representation of the relation in the spatial domain, from which we can derive evaluations as numbers, intervals, distributions, etc. An example of spatial representation of the relation "between" is displayed in Figure 8.1 too. Here the reference objects are the two lungs segmented in a medical image (illustrated on one slide). The region between the lungs was used in [26] to guide the segmentation of the heart in non-contrasted 3D CT images.

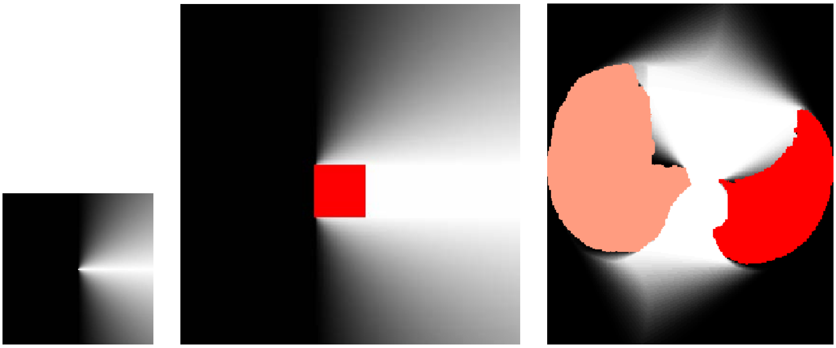

Fig. 8.1. Left: fuzzy structuring element representing the semantics of "right" in the spatial domain. Membership values are represented by grey levels (0 = black, 1 = white). Middle: region to the right of the red square. The membership at each point is the degree of satisfaction of the relation at that point. Right: region "between" the lungs.

A review on fuzzy spatial relations can be found in [7], and one on relative directions in [15], while technical details, along with examples, on several original proposals based on mathematical morphology can be found e.g. in [4, 5, 10, 14, 18, 29, 33, 34].

The main advantages of this approach is that the obtained definitions have good formal properties, provide results that fit the intuitive meaning of the relations, and are robust to the parameters defining the fuzzy structuring elements (in the sense that a fine tuning is not necessary). Moreover, having a common framework for defining several types of relations allows for their combination (see Section 8.4).

8.3 Instantiation of Spatial Relations Models in Various Settings: Towards Spatial Reasoning

Mathematical morphology, in particular its part dealing with deterministic increasing operators, relies on the algebraic framework of complete lattices [22, 31, 32]. Examples of such lattices are the powerset of a set, endowed with inclusion, functions, with the usual partial ordering, partitions, fuzzy sets, formulas in propositional logics, graphs and hypergraphs... A direct consequence is that the proposed spatial relations can be expressed in all these settings [8, 12]. In particular, it is interesting to have a symbolic expression of relations, in a logical framework (for instance by considering a formula as the logical representation of a spatial entity, whose models can be sets or fuzzy sets), to benefit from the reasoning tools inherited from the logics. This applies in propositional logics, but also in modal logics, where a structuring element gives rise to an accessibility relation, and the modalities $\Box$ and $\Diamond$ correspond to erosion and dilation, respectively [6]. Spatial relations can then be directly expressed in this logic. For instance, if φ and ψ are formulas representing two spatial entities, saying that the first one is a non tangential part of the second one can be simply expressed by: $\Diamond\varphi \to \psi$, or equivalently $\varphi \to \Box\psi$. Similarly in description logics, dilation and erosion are considered as binary predicates [24], and interesting links can be established with concept lattices [2] for reasoning purpose. Finally, the developed morphological framework deals with the two components of spatial reasoning (representation of spatial entities and their relations, and reasoning on them), as illustrated next.

8.4 Spatial Reasoning: Example of Model-Based Recognition and Image Understanding Based on Spatial Relations

Spatial reasoning can be defined as the domain of spatial knowledge representation, in particular spatial relations between spatial entities, and of reasoning on these entities and relations. This field has been largely developed in artificial intelligence, in particular using qualitative representations based on logical formalisms. In image interpretation and computer vision it is much less developed and is mainly based on quantitative representations. Our work has shown that semi-quantitative formalisms, using fuzzy sets, have many advantages. A typical example in this domain concerns model-based structure recognition in images, where the model represents spatial entities and relationships between them. For both spatial knowledge representation and reasoning, spatial relationships then constitute an important part of the knowledge we have to handle. Imprecision is often attached to spatial reasoning in images, and can occur at different levels, from knowledge to the type of question we want to answer. The reasoning component includes fusion of heterogeneous spatial knowledge, decision making, inference, recognition. Two types of questions are raised when dealing with spatial relationships:

1. given two objects (possibly fuzzy), assess the degree to which a relation is satisfied;
2. given one reference object, define the area of space in which a relation to this reference is satisfied (to some degree).

In order to answer these questions and address both representation and reasoning issues, we rely on three different frameworks and their combination: mathematical morphology [32], fuzzy set theory [35], and formal logics and the attached reasoning and inference power. The association of these three frameworks for spatial reasoning allows answering two important requirements: expressiveness and completeness with respect to the types of spatial information we want to represent [1].

As an illustration, let us consider the example where we would like to segment and recognize brain structures in a 3D MRI image, based on an anatomical model, which includes structures and their spatial relations. The usual description is given in a linguistic form, from which we can derive computational models relying on ontologies and graphs, and where spatial relations can be learned from examples [3, 11, 17, 20, 23, 24]. The model is then used for guiding the recognition. We summarize here a few approaches we have developed. Details can be found in the mentioned references.

The first approach uses a model graph and the image to segment is represented as a graph too, for instance from an over-segmentation of the image. The segmentation and recognition process is then formalized as a graph matching problem [16, 30].

In the second approach, a sequential segmentation of the internal brain structures is performed [11, 17]. The segmentation and the recognition are achieved at the same time. Each segmentation uses the spatial information encoded in the model, and more specifically the spatial relations to the previously segmented structures. This information allows restricting the search domain around the structure, in which a deformable model yields the segmentation result. The reasoning can rely on an ontology, where spatial relations are expressed based on morphological operators [23, 24], and on logical formalisms [2, 6]. In this approach, there is no initial segmentation of the image, but it raises questions on the order of segmentation of the different objects and on how to avoid the propagation of potential errors. These questions have been addressed in [20] by optimizing a segmentation path in the graph, based on saliency and structural information, and by allowing backtracking on the defined path to avoid error propagation.

Another approach was proposed in [28], which is global and uses a constraint network encoding all spatial relations that should be satisfied by the structures. Each anatomical structure is linked with a region of space which satisfies all constraints

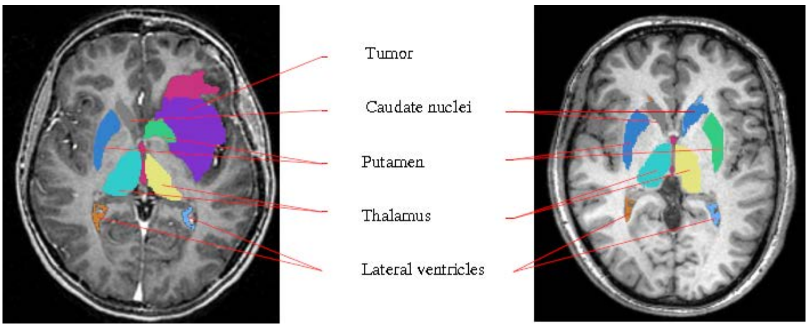

Fig. 8.2. Segmentation and recognition of a few brain structures from 3D MRI in a pathological case (left) and in a normal one (right), obtained with the sequential method. Thanks to the spatial relations, which remain stable even in presence of a pathology, the tumor does not prevent the correct segmentation of the normal structures, even if they are strongly deformed. Only one slide is displayed, but the relations are modeled and computed in 3D and the segmentation is performed in 3D too. (PhD thesis of Geoffroy Fouquier [20].)

in the network. Hence the problem is expressed as a constraint satisfaction problem (CSP). Since it is hard to solve this problem directly, only the bounds of the domain of each variable (i.e. structure to be segmented) are modified by the process and sequentially reduced using specifically designed propagators derived from the spatial constraints. In the reduced domain around the structure, the final segmentation is obtained by a minimal surface algorithm.

A typical segmentation and recognition result is shown in Figure 8.2 for a pathological case and a normal one, obtained with the sequential method. Another example is illustrated in 3D in Figure 8.3, where results have been obtained with the global CSP method.

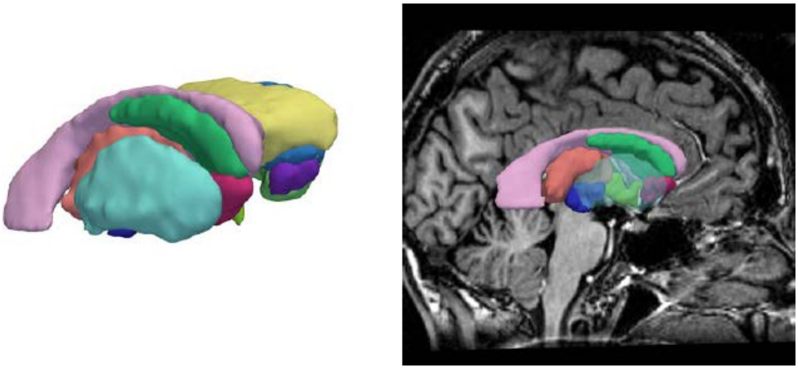

Fig. 8.3. 3D view of segmentation and recognition results obtained with the global CSP method for the following structures: caudate nuclei, putamen, lateral ventricles, thalami, third ventricle, accumbens nuclei and sub-thalami. (PhD thesis of Olivier Nempont [28].)

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References

1. Aiello, M.: Spatial Reasoning, Theory and Practice. PhD thesis, University of Amsterdam (February 2002)
2. Atif, J., Hudelot, C., Bloch, I.: Abduction in description logics using formal concept analysis and mathematical morphology: application to image interpretation. In: 8th International Conference on Concept Lattices and Their Applications (CLA 2011), Nancy, Paris, pp. 405–408 (October 2011)

3. Atif, J., Hudelot, C., Fouquier, G., Bloch, I., Angelini, E.D.: From Generic Knowledge to Specific Reasoning for Medical Image Interpretation using Graph-based Representations. In: International Joint Conference on Artificial Intelligence, IJCAI 2007, Hyderabad, India, pp. 224–229 (January 2007)
4. Bloch, I.: Fuzzy Relative Position between Objects in Image Processing: a Morphological Approach. IEEE Transactions on Pattern Analysis and Machine Intelligence 21(7), 657–664 (1999)
5. Bloch, I.: On Fuzzy Distances and their Use in Image Processing under Imprecision. Pattern Recognition 32(11), 1873–1895 (1999)
6. Bloch, I.: Modal Logics based on Mathematical Morphology for Spatial Reasoning. Journal of Applied Non Classical Logics 12(3-4), 399–424 (2002)
7. Bloch, I.: Fuzzy Spatial Relationships for Image Processing and Interpretation: A Review. Image and Vision Computing 23(2), 89–110 (2005)
8. Bloch, I.: Spatial Reasoning under Imprecision using Fuzzy Set Theory, Formal Logics and Mathematical Morphology. International Journal of Approximate Reasoning 41, 77–95 (2006)
9. Bloch, I.: Duality vs. Adjunction for Fuzzy Mathematical Morphology and General Form of Fuzzy Erosions and Dilations. Fuzzy Sets and Systems 160, 1858–1867 (2009)
10. Bloch, I., Colliot, O., Cesar, R.M.: On the Ternary Spatial Relation Between. IEEE Transactions on Systems, Man, and Cybernetics SMC-B 36(2), 312–327 (2006)
11. Bloch, I., Géraud, T., Maître, H.: Representation and Fusion of Heterogeneous Fuzzy Information in the 3D Space for Model-Based Structural Recognition - Application to 3D Brain Imaging. Artificial Intelligence 148, 141–175 (2003)
12. Bloch, I., Heijmans, H.J.A.M., Ronse, C.: Mathematical Morphology. In: Aiello, M., Pratt-Hartman, I., van Benthem, J. (eds.) Handbook of Spatial Logics, ch. 13, pp. 857–947. Springer. Dordrecht (2007)
13. Bloch, I., Maître, H.: Fuzzy Mathematical Morphologies: A Comparative Study. Pattern Recognition 28(9), 1341–1387 (1995)
14. Bloch, I., Maître, H., Anvari, M.: Fuzzy Adjacency between Image Objects. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 5(6), 615–653 (1997)
15. Bloch, I., Ralescu, A.: Directional Relative Position between Objects in Image Processing: A Comparison between Fuzzy Approaches. Pattern Recognition 36, 1563–1582 (2003)
16. Cesar, R.M., Bengoetxea, E., Bloch, I., Larrañaga, P.: Inexact Graph Matching for Model-Based Recognition: Evaluaton and Comparison of Optimization Algorithms. Pattern Recognition 38, 2099–2113 (2005)
17. Colliot, O., Camara, O., Bloch, I.: Integration of Fuzzy Spatial Relations in Deformable Models - Application to Brain MRI Segmentation. Pattern Recognition 39, 1401–1414 (2006)
18. Colliot, O., Tuzikov, A.V., Cesar, R.M., Bloch, I.: Approximate Reflectional Symmetries of Fuzzy Objects with an Application in Model-Based Object Recognition. Fuzzy Sets and Systems 147, 141–163 (2004)
19. Dubois, D., Prade, H., Testemale, C.: Weighted Fuzzy Pattern Matching. Fuzzy Sets and Systems 28, 313–331 (1988)
20. Fouquier, G., Atif, J., Bloch, I.: Sequential model-based segmentation and recognition of image structures driven by visual features and spatial relations. Computer Vision and Image Understanding 116(1), 146–165 (2012)
21. Freeman, J.: The Modelling of Spatial Relations. Computer Graphics and Image Processing 4(2), 156–171 (1975)

22. Heijmans, H.J.A.M., Ronse, C.: The Algebraic Basis of Mathematical Morphology – Part I: Dilations and Erosions. Computer Vision, Graphics and Image Processing 50, 245–295 (1990)
23. Hudelot, C., Atif, J., Bloch, I.: Fuzzy Spatial Relation Ontology for Image Interpretation. Fuzzy Sets and Systems 159, 1929–1951 (2008)
24. Hudelot, C., Atif, J., Bloch, I.: A Spatial Relation Ontology Using Mathematical Morphology and Description Logics for Spatial Reasoning. In: ECAI 2008 Workshop on Spatial and Temporal Reasoning, Patras, Greece, pp. 21–25 (July 2008)
25. Kuipers, B.J., Levitt, T.S.: Navigation and Mapping in Large-Scale Space. AI Magazine 9(2), 25–43 (1988)
26. Moreno, A., Takemura, C.M., Colliot, O., Camara, O., Bloch, I.: Using Anatomical Knowledge Expressed as Fuzzy Constraints to Segment the Heart in CT images. Pattern Recognition 41, 2525–2540 (2008)
27. Mike, N., Kerre, E.E.: Classical and Fuzzy Approaches towards Mathematical Morphology. In: Kerre, E.E., Nachtegael, M. (eds.) Fuzzy Techniques in Image Processing, ch. 1. STUDFUZZ, pp. 3–57. Physica-Verlag, Heidelberg (2000)
28. Nempont, O., Atif, J., Angelini, E.D., Bloch, I.: Structure Segmentation and Recognition in Images Guided by Structural Constraint Propagation. In: European Conference on Artificial Intelligence ECAI, Patras, Greece, pp. 621–625 (July 2008)
29. Nempont, O., Atif, J., Angelini, E.D., Bloch, I.: A New Fuzzy Connectivity Measure for Fuzzy Sets and Associated Fuzzy Attribute Openings. Journal of Mathematical Imaging and Vision 34, 107–136 (2009)
30. Perchant, A., Bloch, I.: Fuzzy Morphisms between Graphs. Fuzzy Sets and Systems 128(2), 149–168 (2002)
31. Ronse, C., Heijmans, H.J.A.M.: The Algebraic Basis of Mathematical Morphology – Part II: Openings and Closings. Computer Vision, Graphics and Image Processing 54, 74–97 (1991)
32. Serra, J.: Image Analysis and Mathematical Morphology. Academic Press, New-York (1982)
33. Takemura, C.M., Cesar, R.M., Bloch, I.: Modeling and measuring the spatial relation "along": regions, contours and fuzzy sets. Pattern Recognition 45, 757–766 (2011)
34. Vanegas, M.C.: Spatial relations and spatial reasoning for the interpretation of earth observation images using a structural model. PhD thesis, Télécom ParisTech (January 2011)
35. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)