

# ***Morphologie mathématique - IV***

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# *Segmentation morphologique*

- Critères pour la segmentation d'images :
  - simplicité
  - régularité
  - fidélité
- Paradigmes morphologiques :
  - zones plates
  - bassins versants et ligne de partage des eaux

# *Zones plates : opérateurs connexes*

Exemples (Salembier, 1998) :

Original



Entropy Operator



# *Zones plates : opérateurs connexes*

Exemples (Salembier, 1998) :



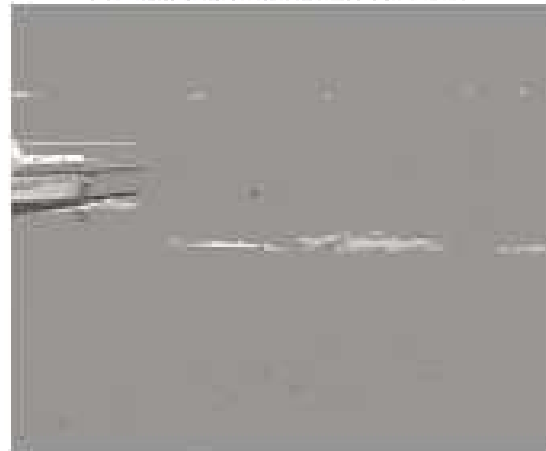
Original frame



Objects with  
translation (0,0)

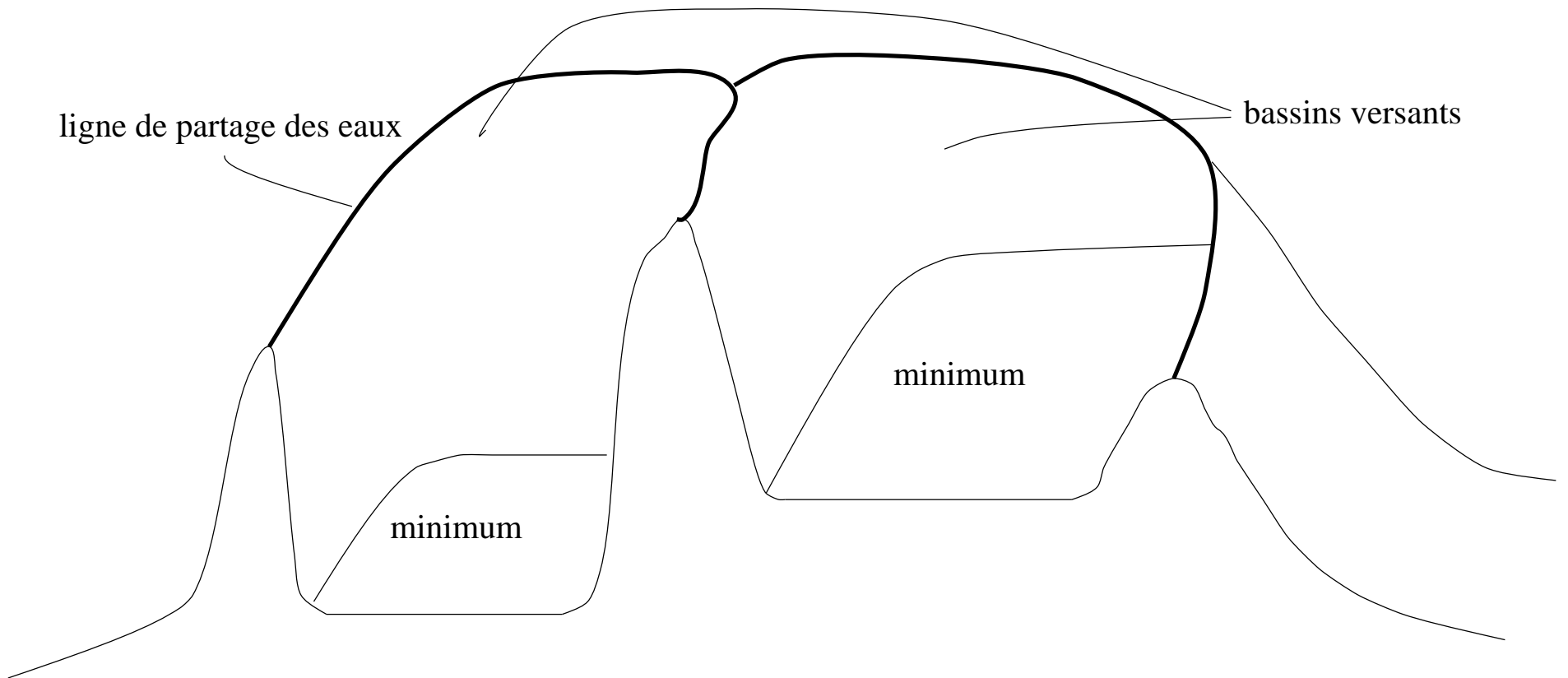


Objects with  
translation (2,0)

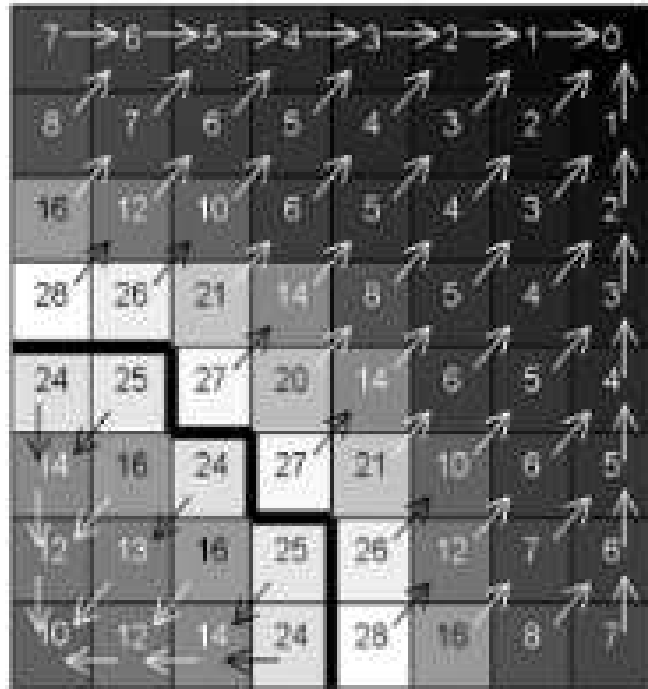


Remaining Objects

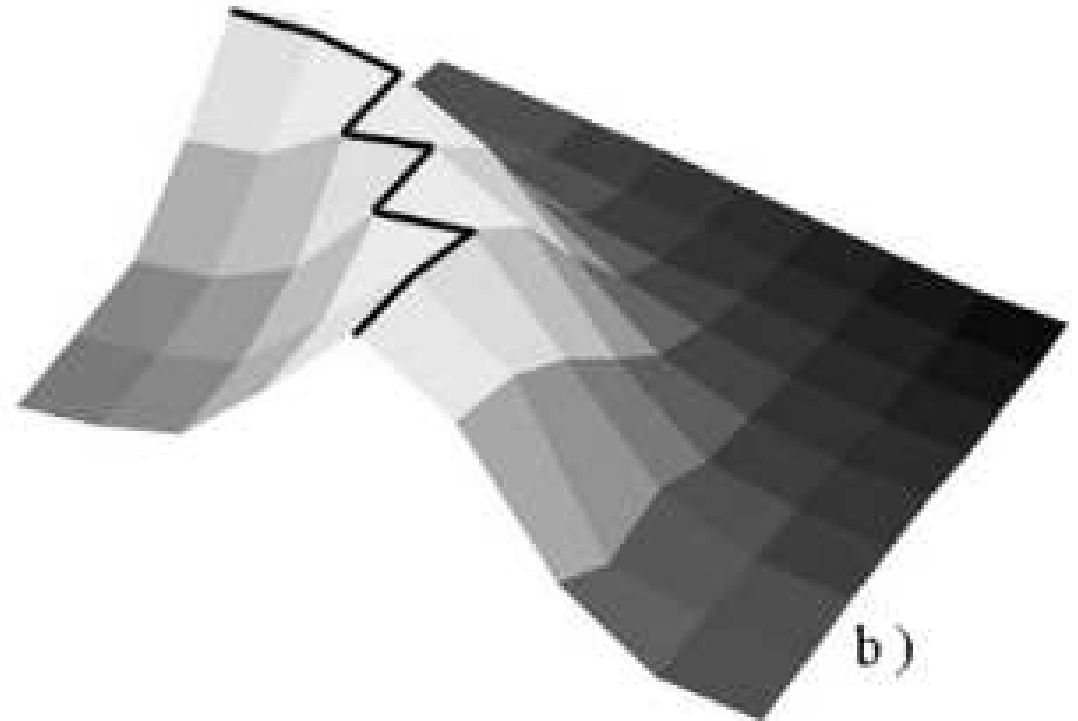
# *Ligne de partage des eaux (LPE)*



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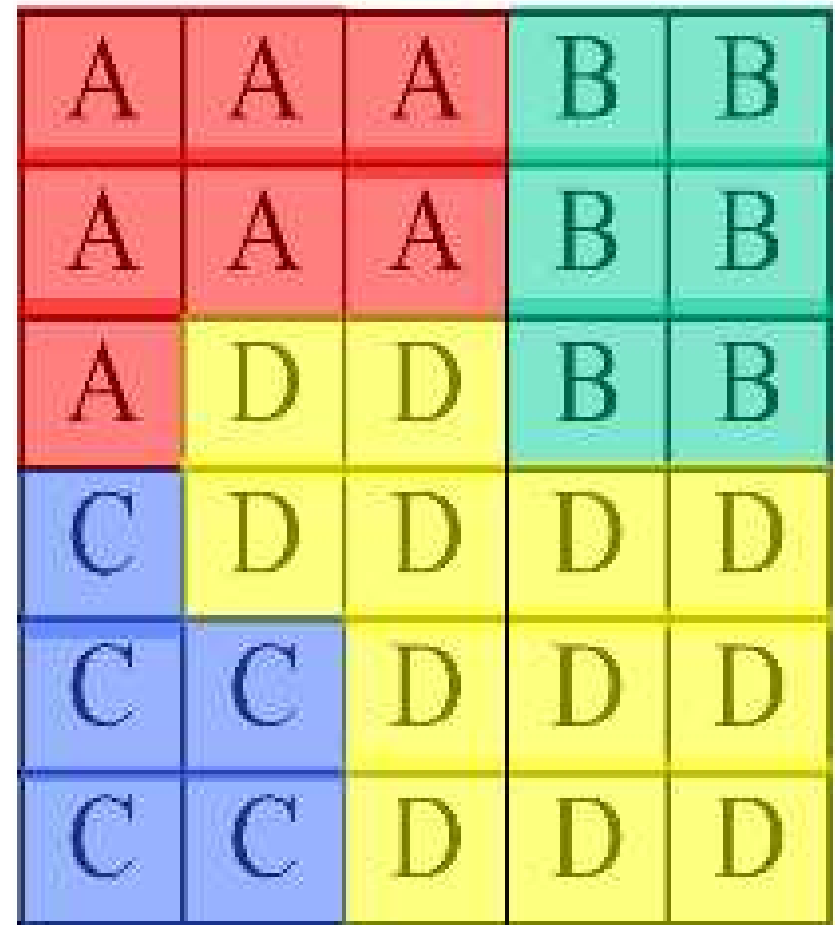
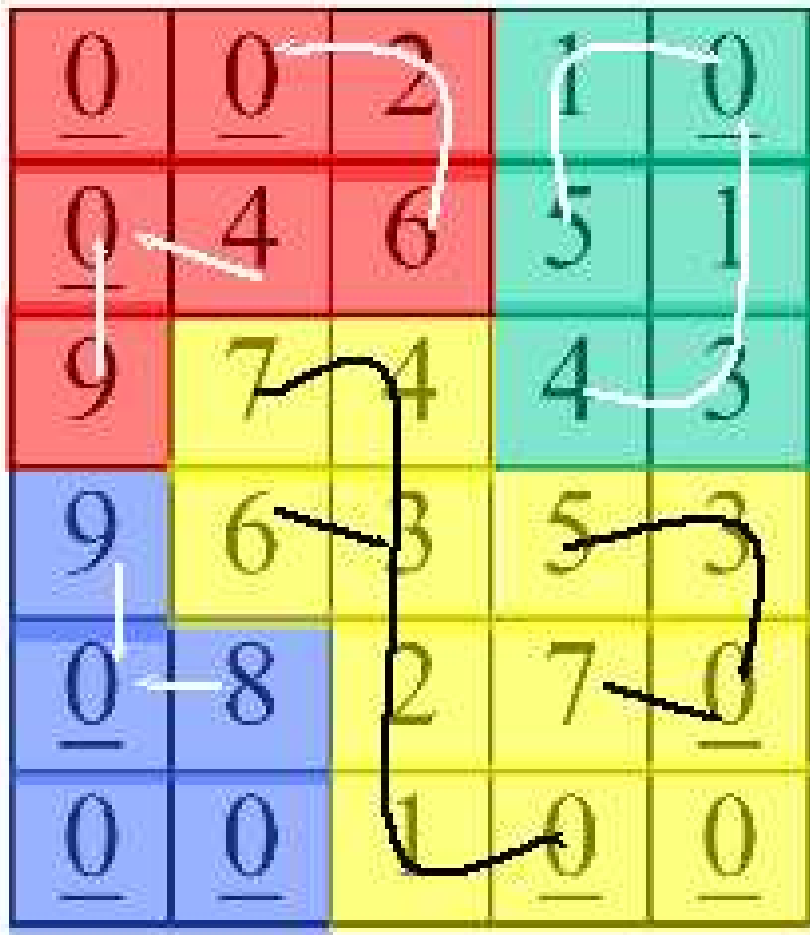


a)



b)

# *LPE par conditions locales*



Plusieurs solutions possibles

# Ligne de partage des eaux : définition

Plus grande pente :

$$Desc(x) = \max\left\{\frac{f(x) - f(y)}{d(x, y)}, y \in V(x)\right\}$$

Dénivelé d'un chemin  $\pi = (x_0, \dots, x_n)$  :

$$T_f(\pi) = \sum_{i=1}^n d(x_{i-1}, x_i) Cout(x_{i-1}, x_i)$$

avec

$$Cout(x, y) = \begin{cases} Desc(x) & \text{si } f(x) > f(y) \\ Desc(y) & \text{si } f(y) > f(x) \\ (Desc(x) + Desc(y))/2 & \text{si } f(y) = f(x) \end{cases}$$



# *Ligne de partage des eaux : définition*

Distance topographique

$$T_f(x, y) = \inf\{T_f(\pi), \pi = (x_0 = x, x_1, \dots, x_n = y)\}$$

(vaut 0 sur un plateau)

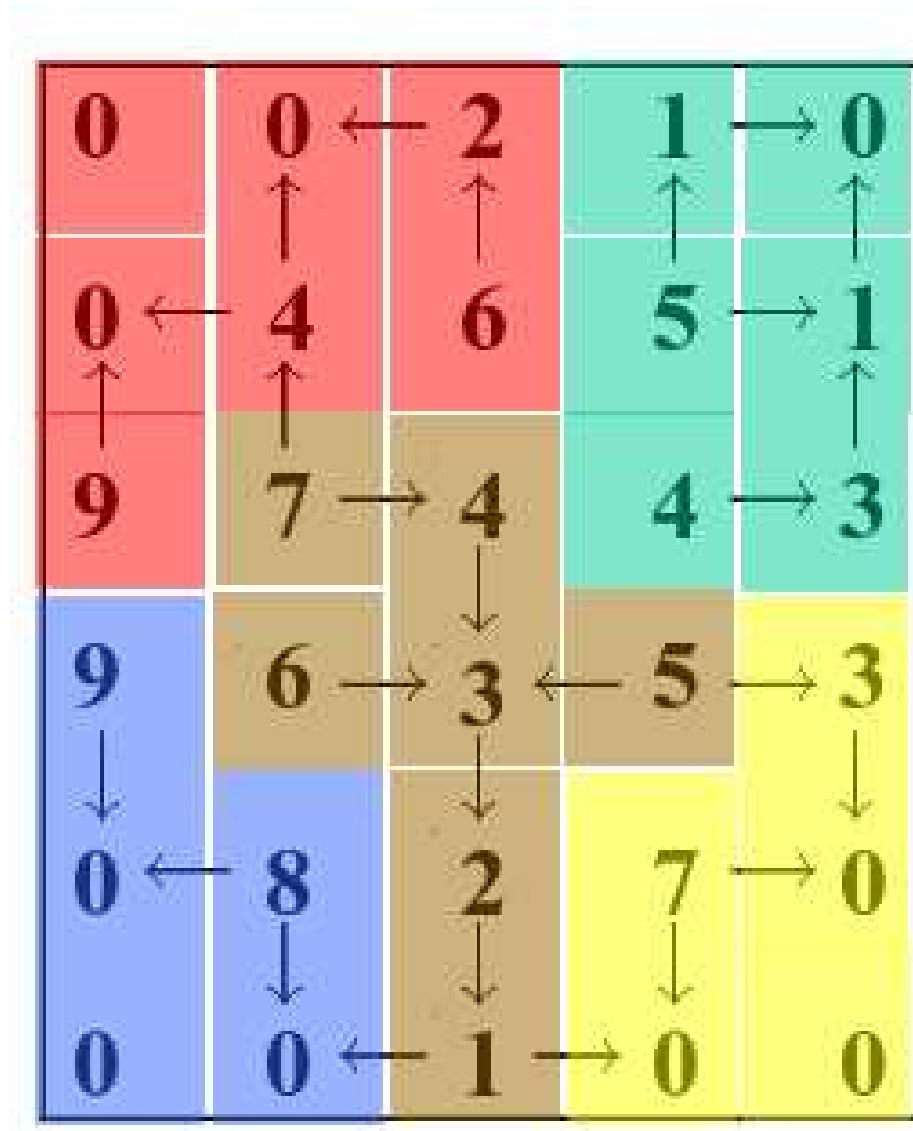
Bassin versant associé au minimum régional  $M_i$  :

$$BV(M_i) = \{x, \forall j \neq i, T_f(x, M_i) + f(M_i) < T_f(x, M_j) + f(M_j)\}$$

Ligne de partage des eaux :

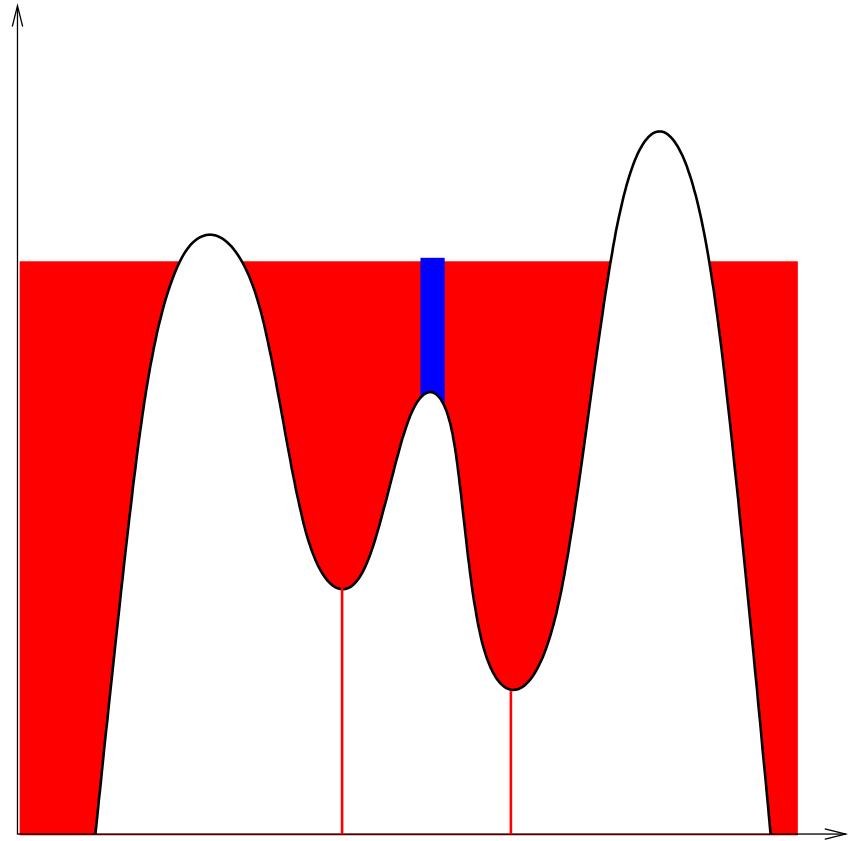
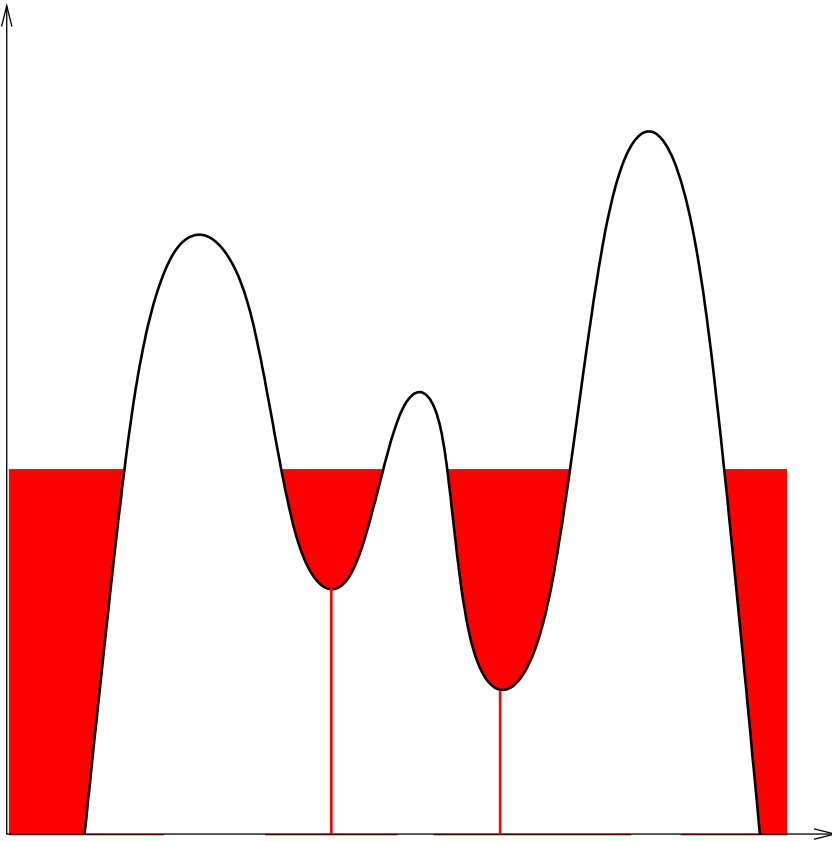
$$LPE(f) = [\cup_i BV(M_i)]^C$$

# *LPE par distance topographique*



Solution unique, lignes pas forcément fines.

# *Approche par immersion*



# *Approche par immersion*

A	A	W	B	B
A	A	W	B	B
A	W	B	B	B
W	B	B	W	D
C	W	B	W	D
C	C	W	D	D

Lignes par forcément fines.

# Construction de la ligne de partage des eaux

$f$  telle que  $f(x) \in [h_{\min}, h_{\max}]$ ,  $f^h = \{x, f(x) \leq h\}$

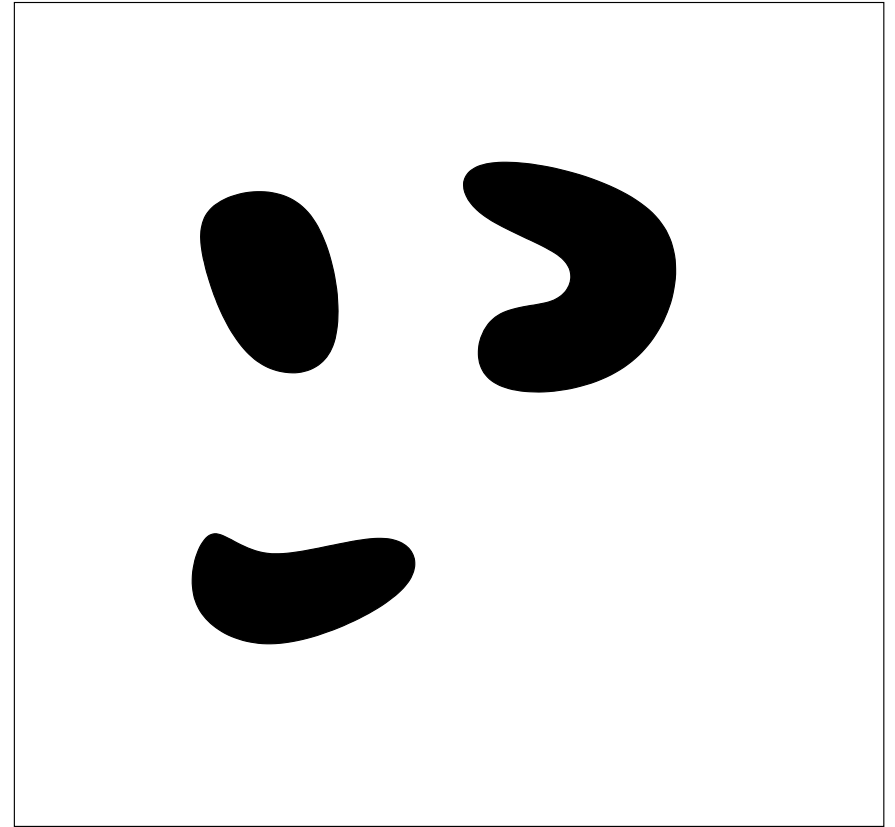
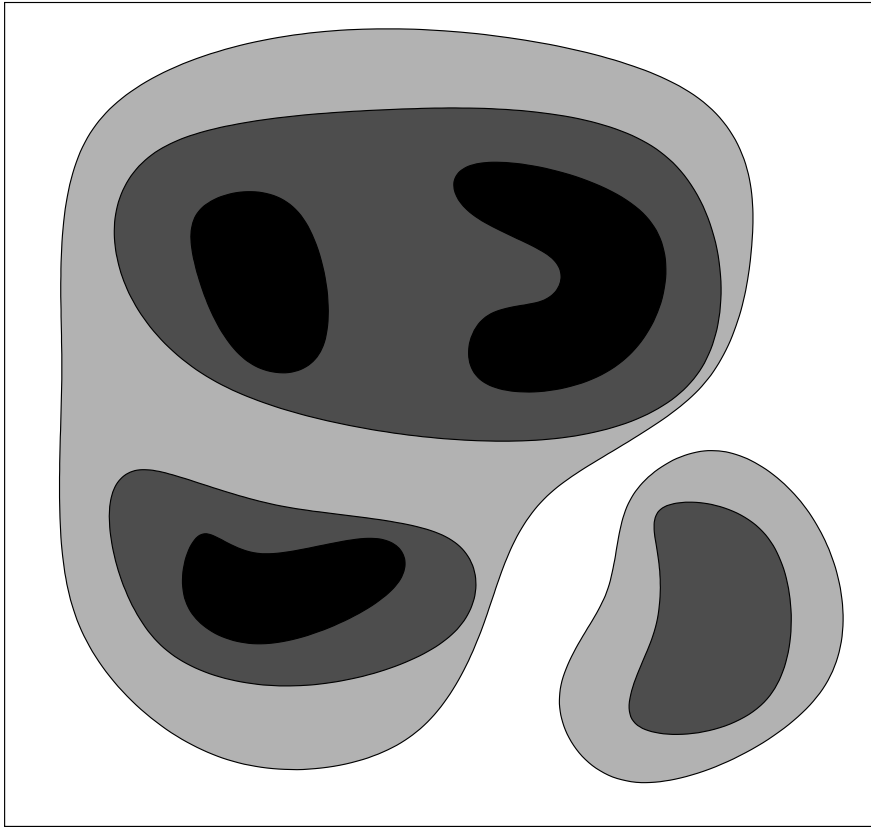
$$X_{h_{\min}} = f^{h_{\min}}$$

$$X_{h+1} = \text{MinReg}_{h+1}(f) \cup \text{ZI}_{f^{h+1}}(X_h)$$

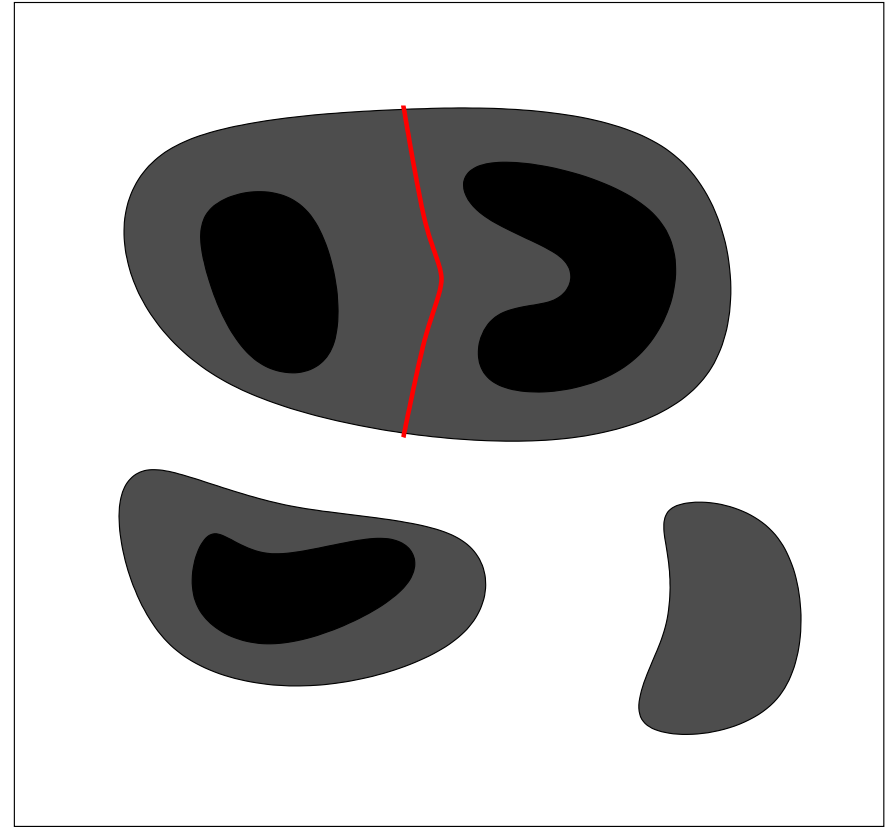
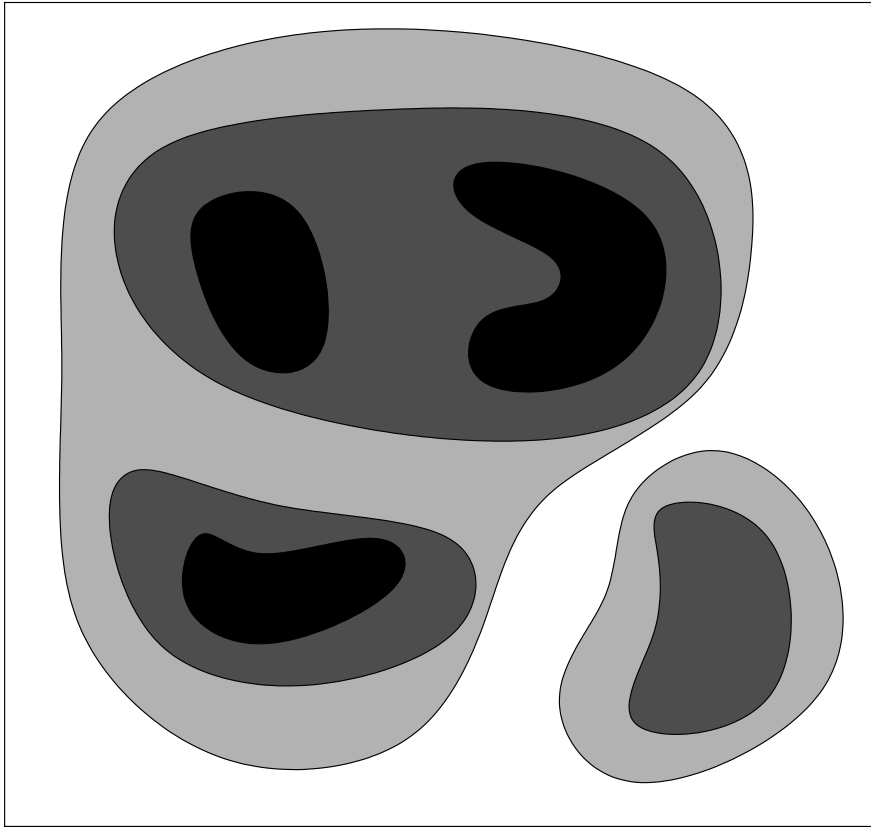
$$BV = X_{h_{\max}}$$

$$\text{LPE}(f) = X_{h_{\max}}^C$$

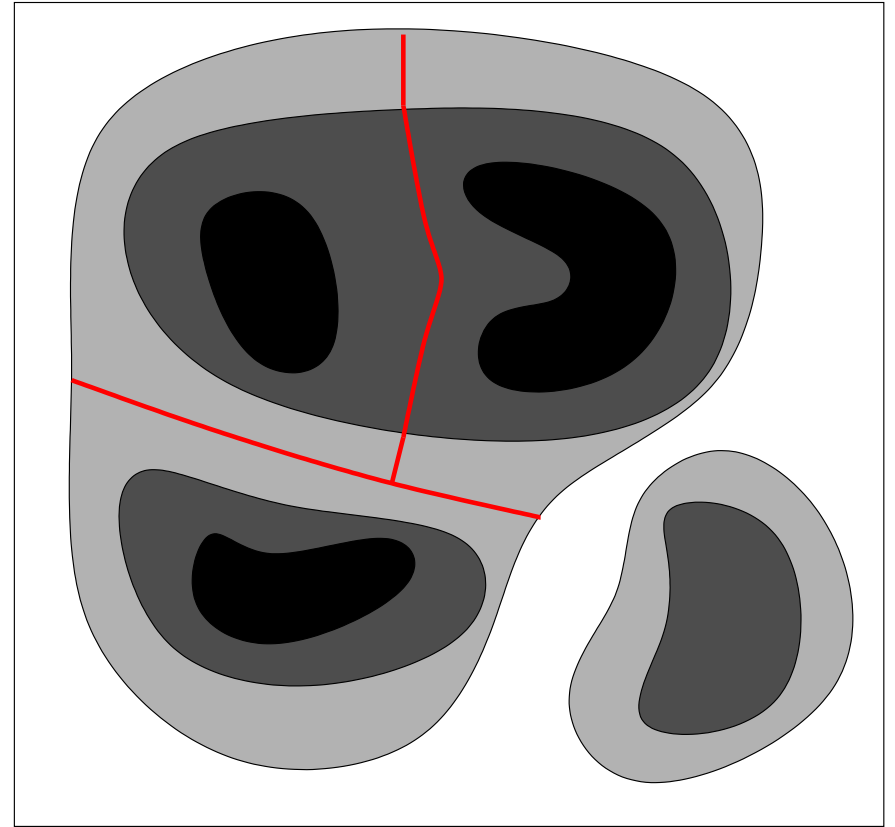
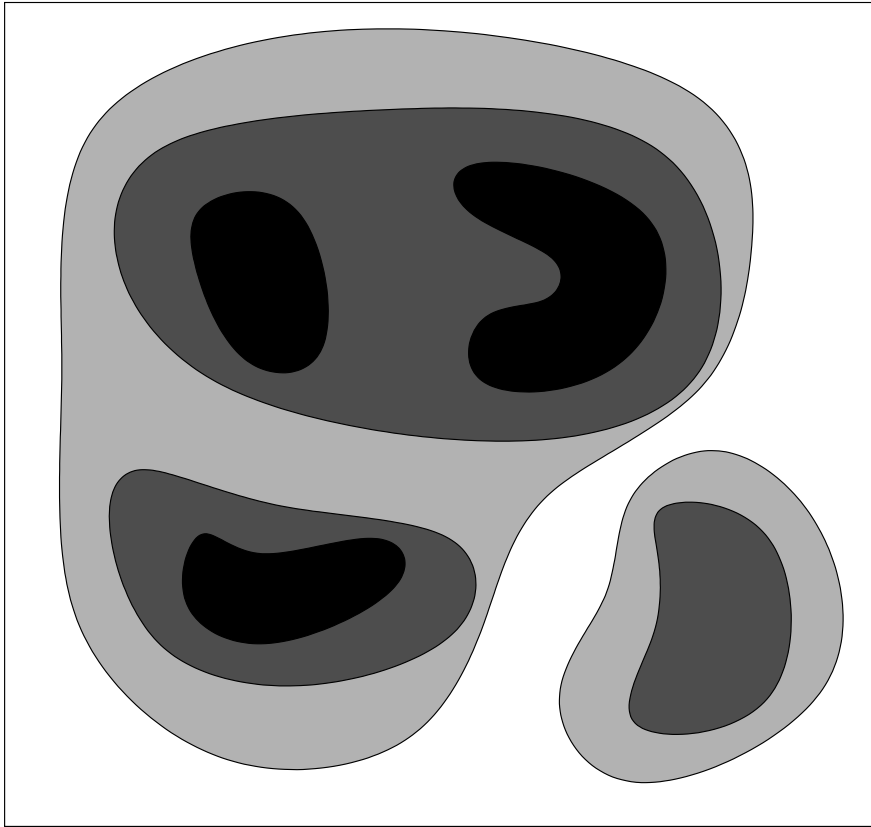
# *Illustration de l'algorithme*



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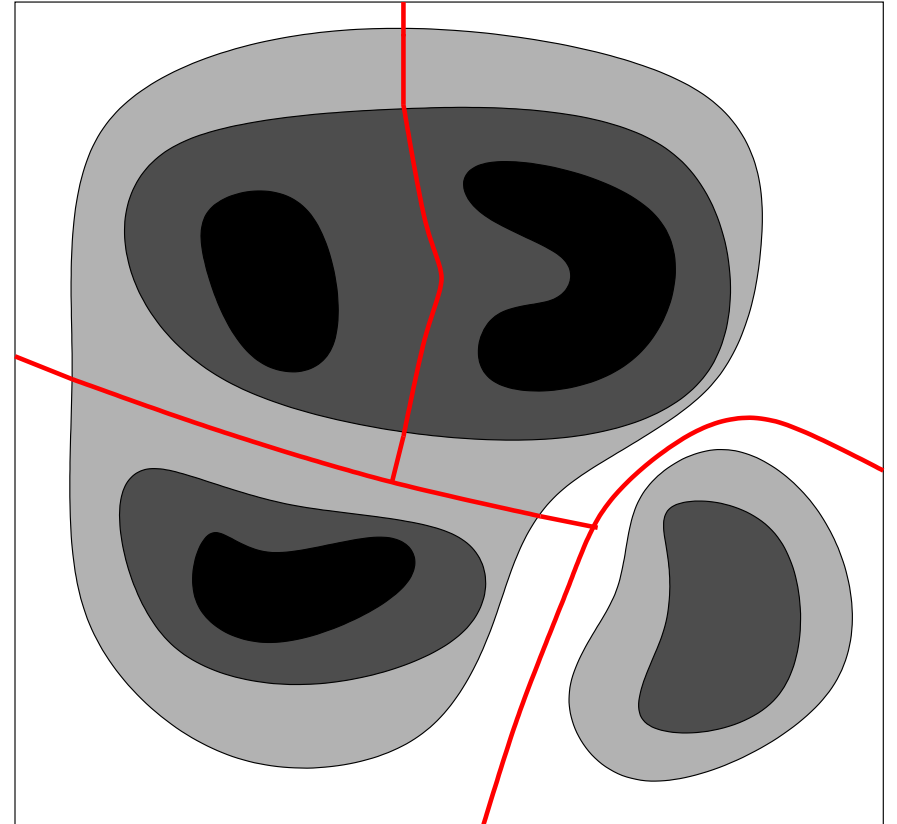
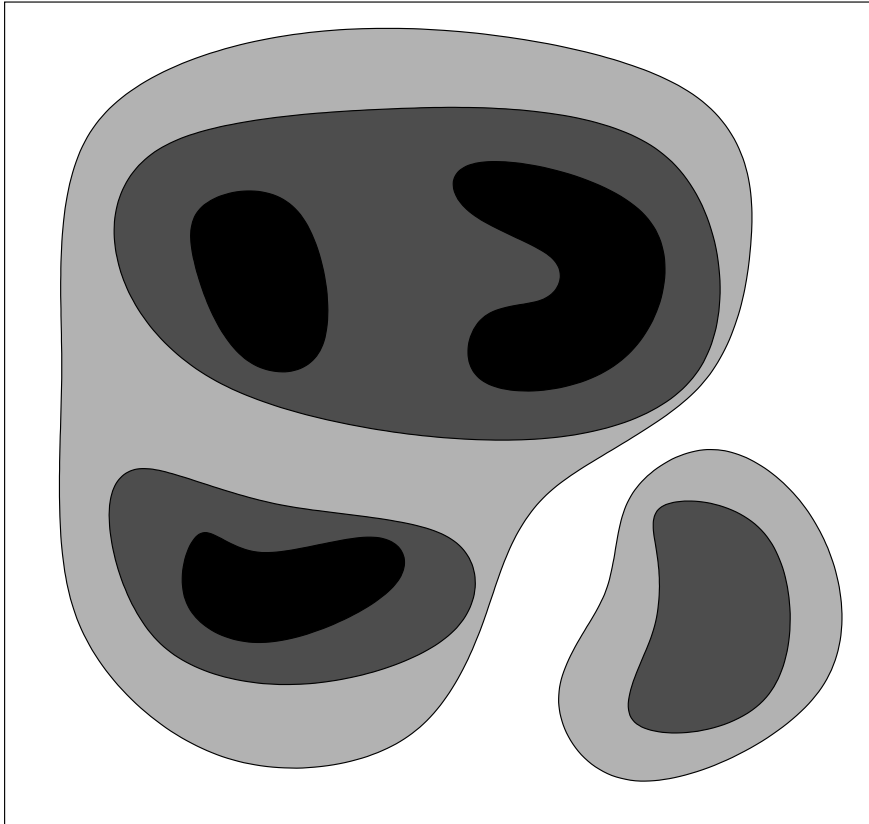


# *Illustration de l'algorithme*

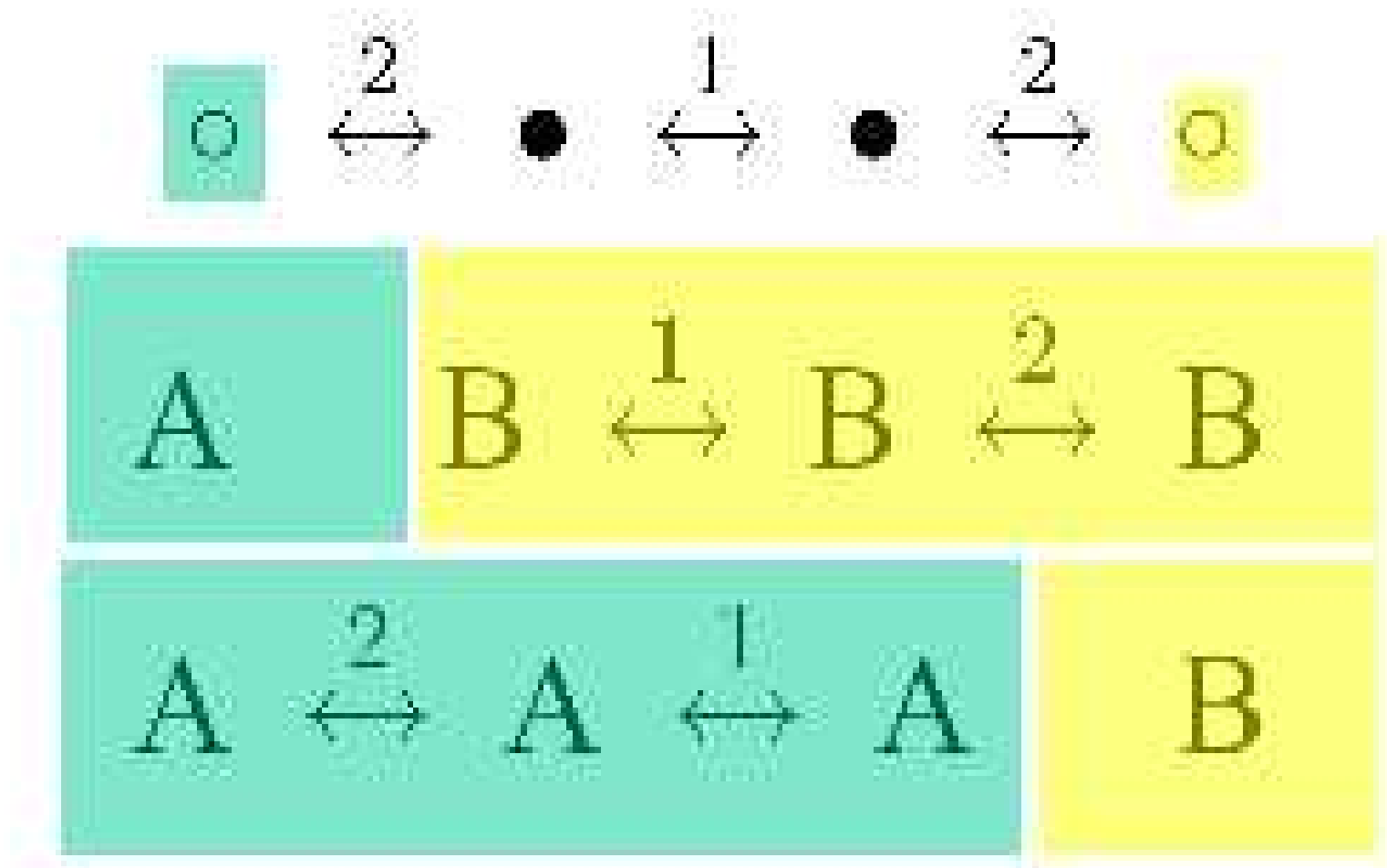




# *Illustration de l'algorithme*

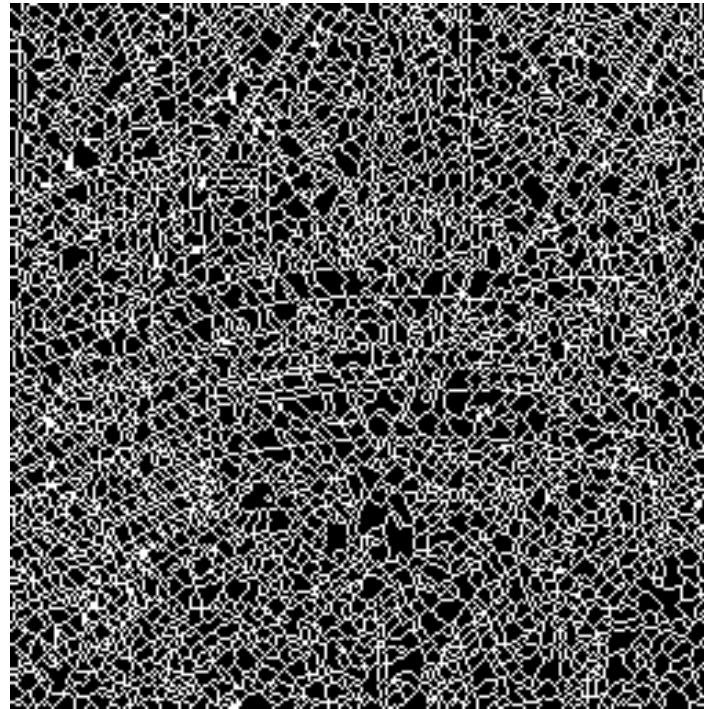


# Minimum spanning forest

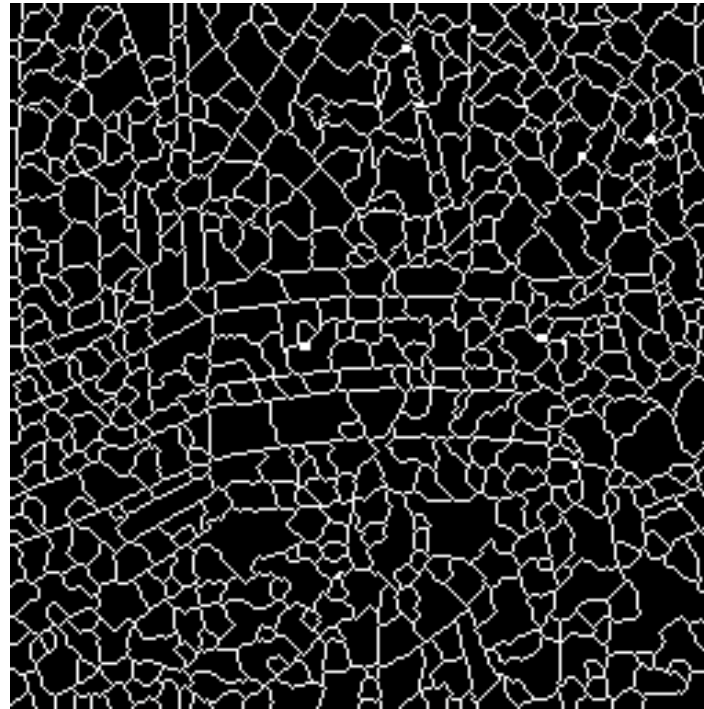


(MSF  $\Rightarrow$  LPE par chemin le plus court, mais pas la réciproque)

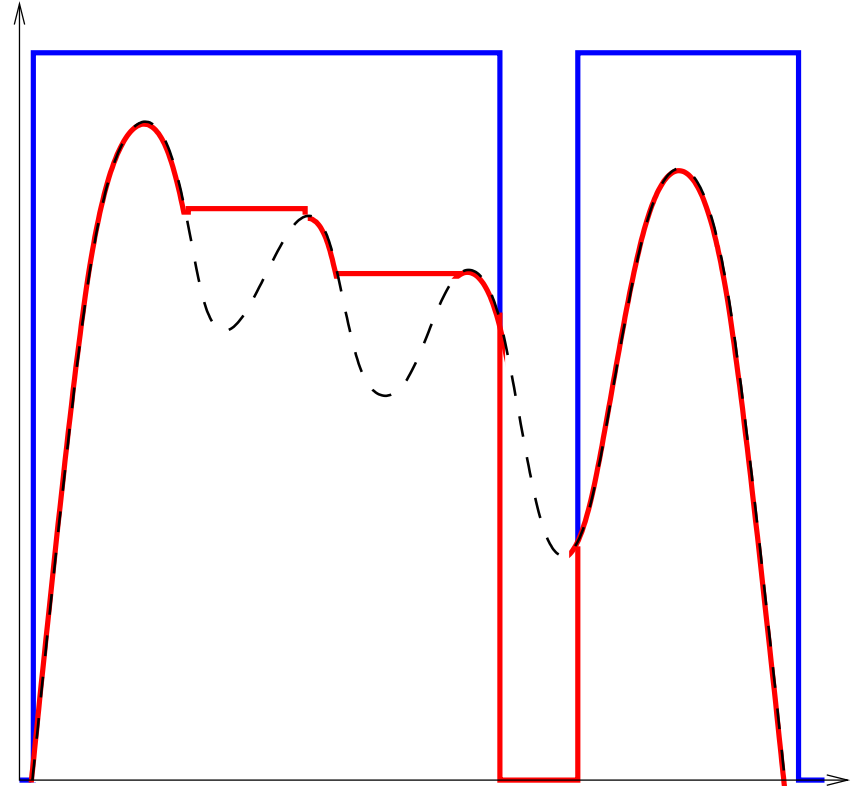
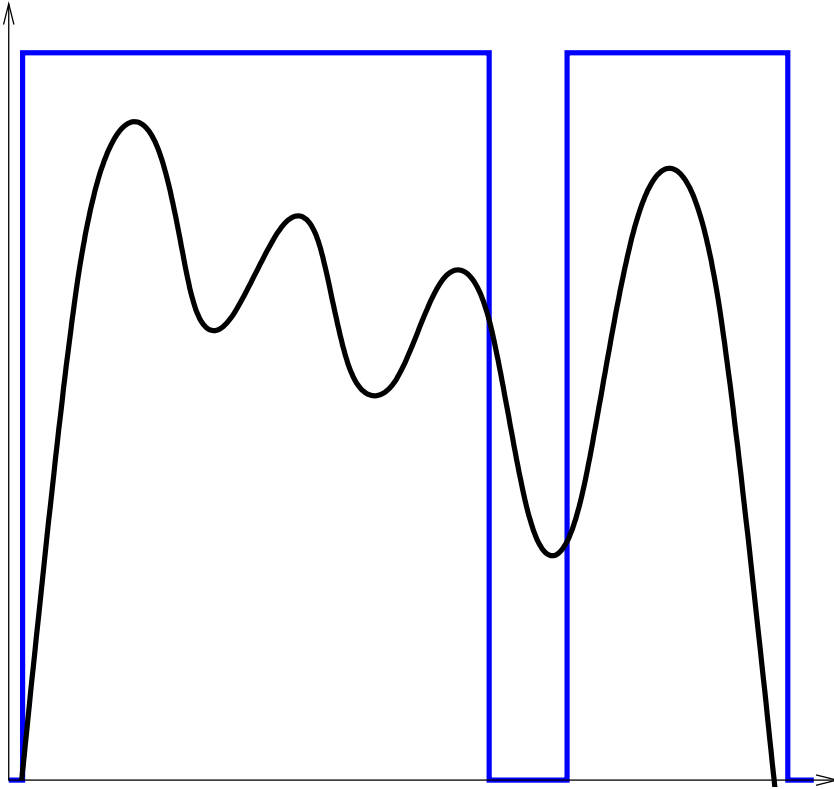
# *Ligne de partage des eaux et sur-segmentation*



# *Ligne de partage des eaux et sur-segmentation*



# *Erosion géodésique pour imposer des marqueurs*



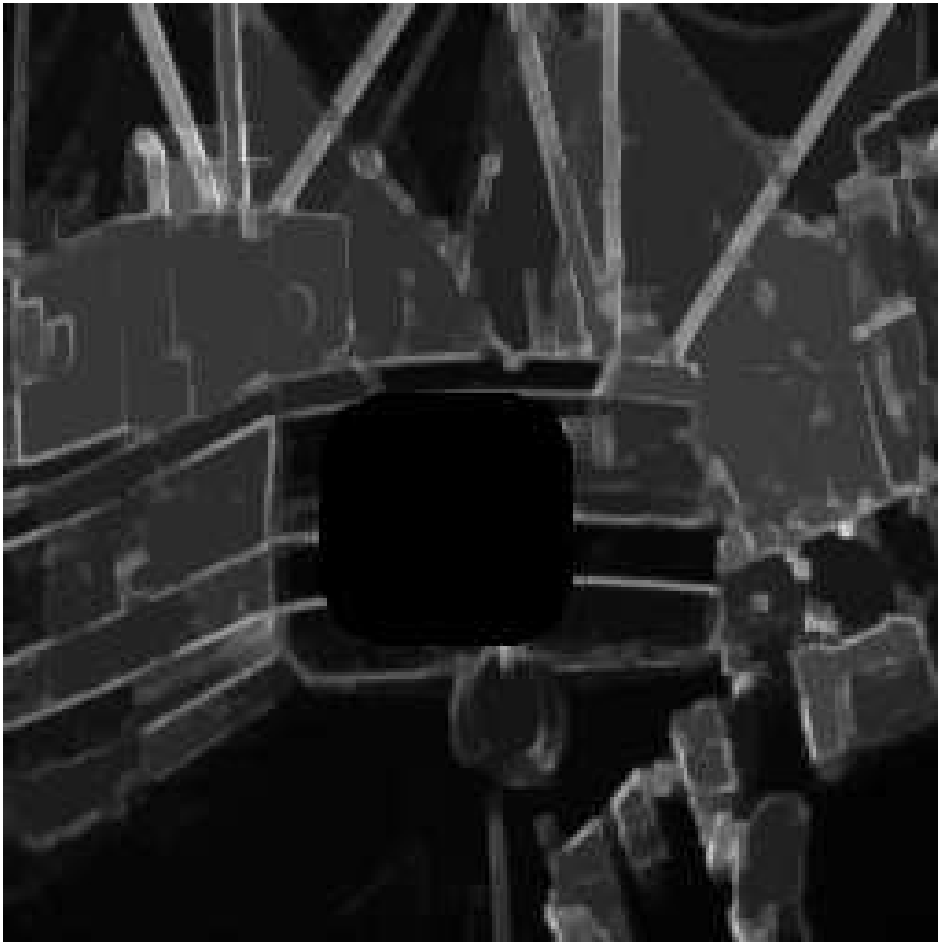


# *Ligne de partage des eaux contrainte par des marqueurs*

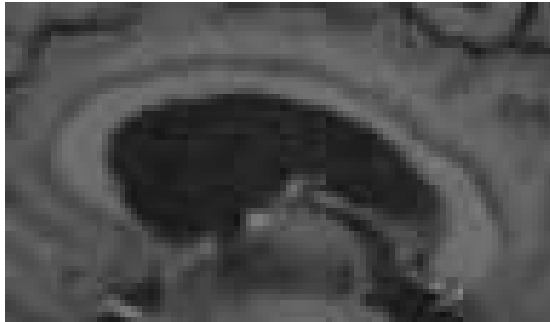
$f$  : fonction sur laquelle on veut appliquer la ligne de partage des eaux

$g$  : fonction de marquage (sélectionne des minima régionaux)

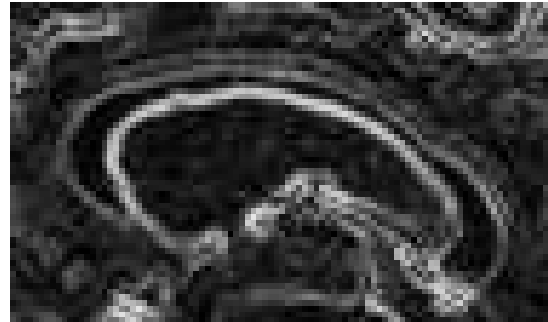
Reconstruction :  $E_{f \wedge g}(g, B_\infty)$  (seulement les minima sélectionnés)



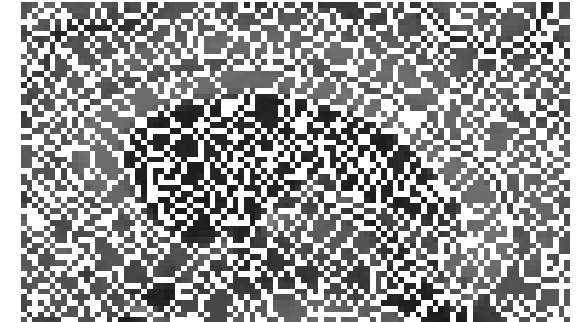
# Exemple



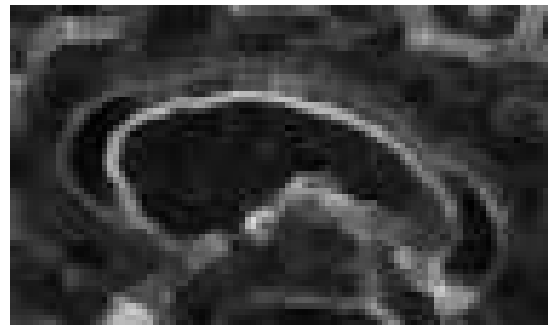
(a)



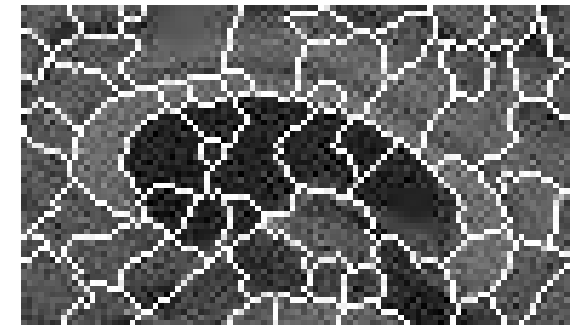
(b)



(c)



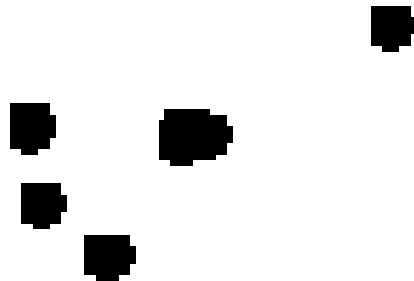
(d)



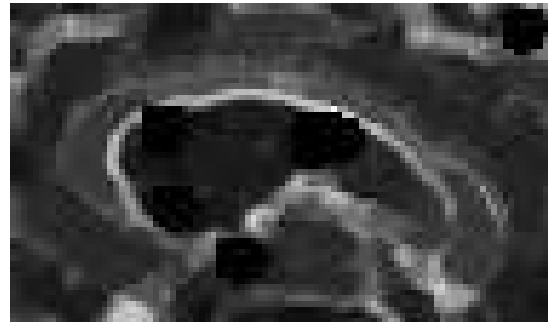
(e)

(a) Image originale, extraite d'une image IRM du cerveau. (b) Gradient morphologique.  
(c) Ligne de partage des eaux superposée à l'image initiale. (d) Fermeture de taille 1 de l'image de gradient. (e) Ligne de partage des eaux du gradient fermé.

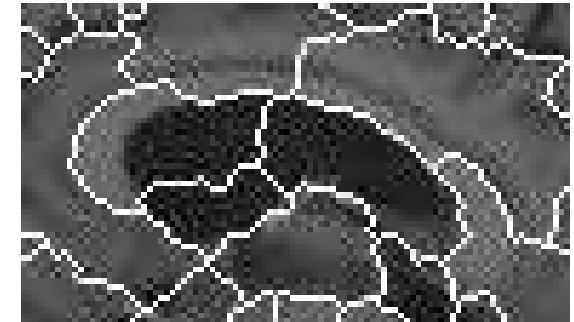
# Exemple



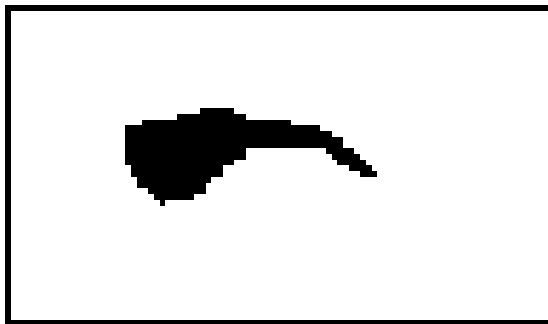
(f)



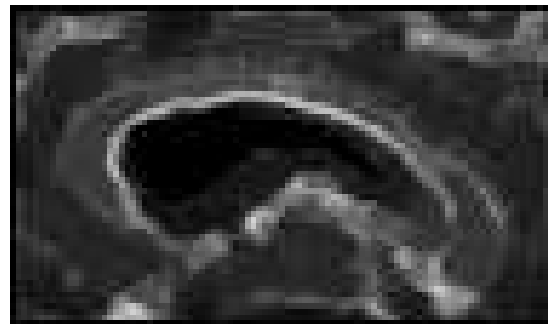
(g)



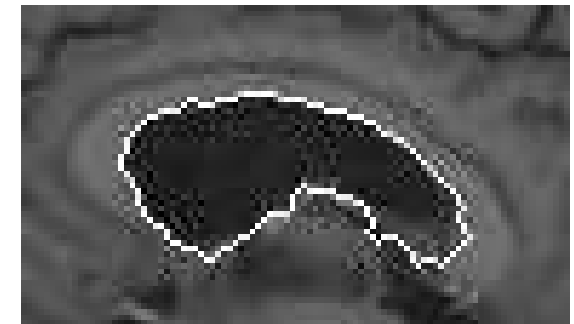
(h)



(i)



(j)

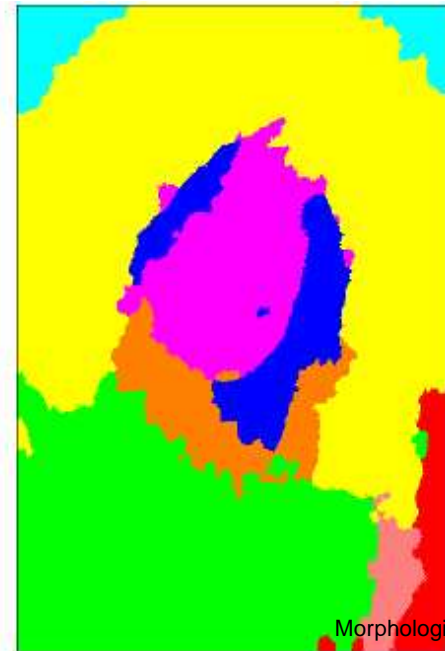


(k)

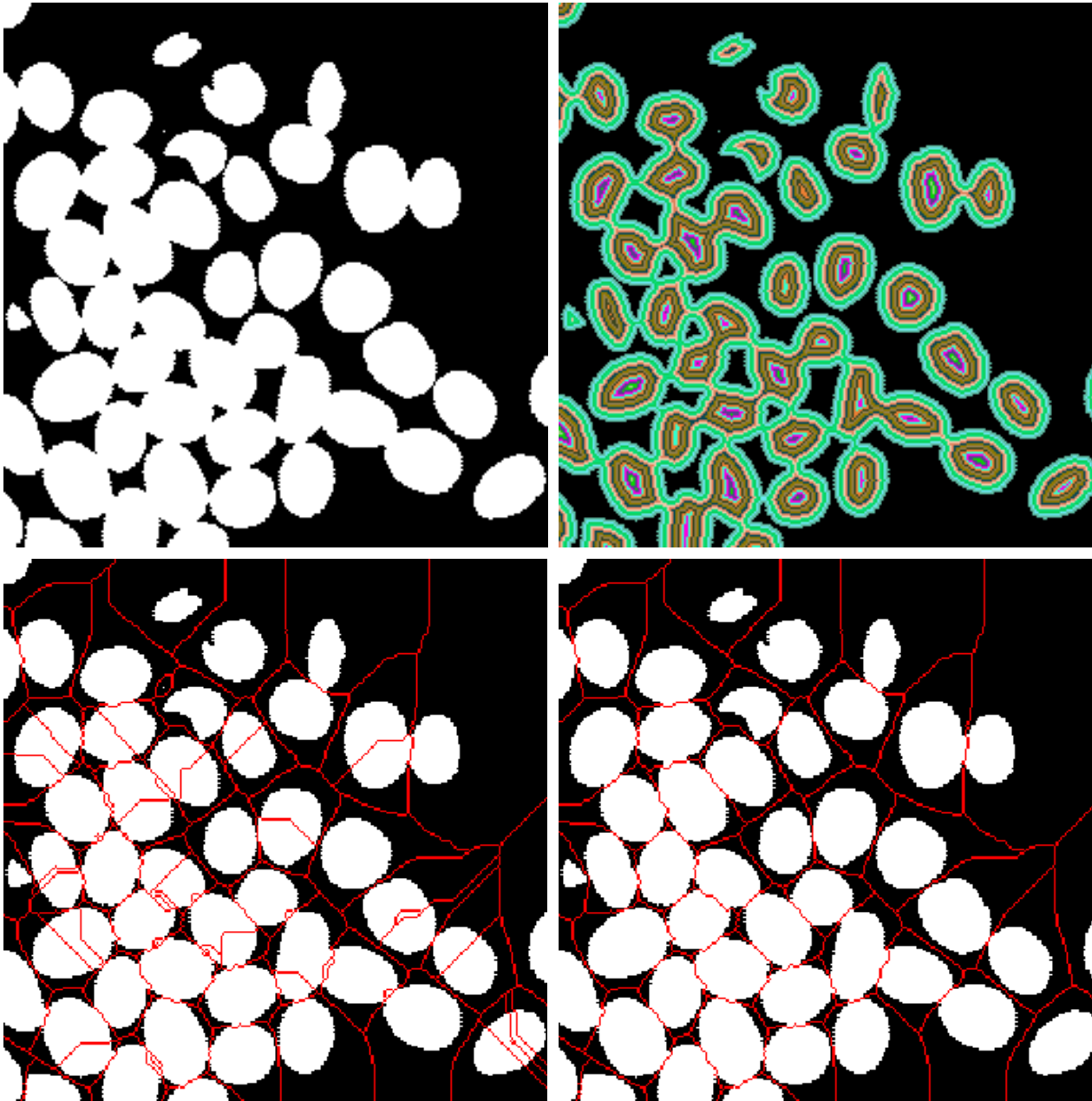
(f) Marqueurs donnés par les minima régionaux de la figure ?? . (g) Reconstruction du gradient. (h) Ligne de partage des eaux. (i) Marqueurs dans les ventricules (partie noire centrale) et au bord de l'image. (j) Gradient reconstruit. (k) Ligne de partage des eaux donnant bien les contours du ventricule.



# Marqueurs interactifs



# *Séparation d'objets binaires connectés*



# ***LPE par minimisation d'énergie***

Minimisation d'une énergie exprimant la fidélité (Boomgard, 2000) :

$$E = \sum_i \int \int_{D_i} (f(D_i) + T_f(x, D_i)) dx$$

$D_i$  = minimum local

$f(D_i)$  = valeur du minimum local

# Régularisation de la LPE

- par fermeture
- watersnakes (Boomgard, 2003) : terme supplémentaire sur la longueur des contours dans l'énergie

$$E = \sum_i \left( \int \int_{D_i} (f(D_i) + T_f(x, D_i)) dx + \beta \int_{\partial D_i} ds \right)$$

- contraintes de géométrie
- inondation "visqueuse" (Vachier et Meyer)

# *Watersnakes : exemples (Boomgard, 2003)*

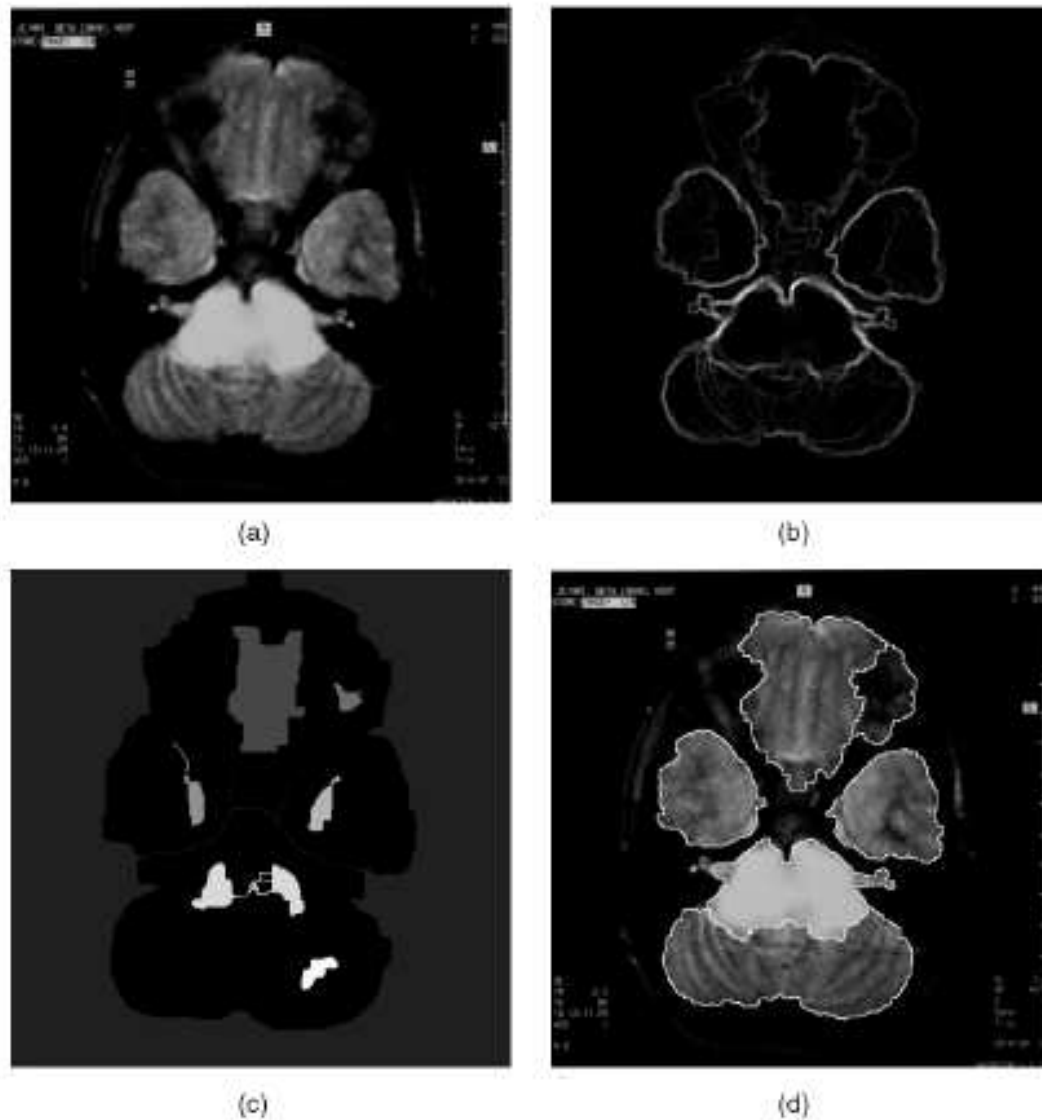


Fig. 6. (a) A brain image. (b) The relief computed from morphological gradient. (c) The markers extracted. (d) The result of the original watershed segmentation, shown for comparison.

# *Watersnakes : exemples (Boomgard, 2003)*

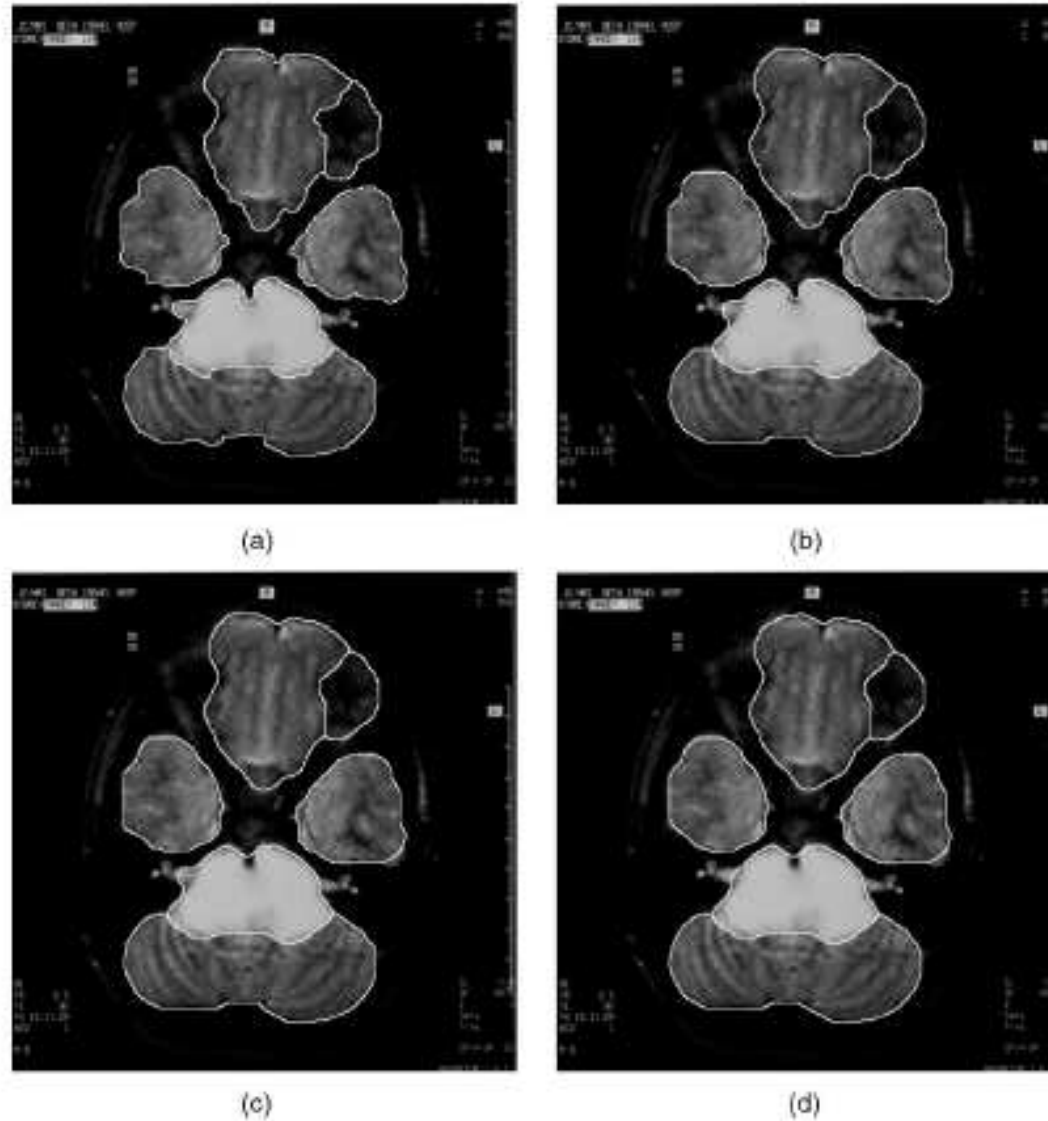


Fig. 7. The segmentation result of the watersnake algorithm based on energy discretization with (a)  $\beta = 10$ , (b)  $\beta = 50$ , (c)  $\beta = 100$ , and (d)  $\beta = 150$ . Note, in comparison with the original watershed segmentation in Fig. 6d, that the results in figure are smoother, but still identify the main objects.



# *Watersnakes : exemples (Boomgard, 2003)*

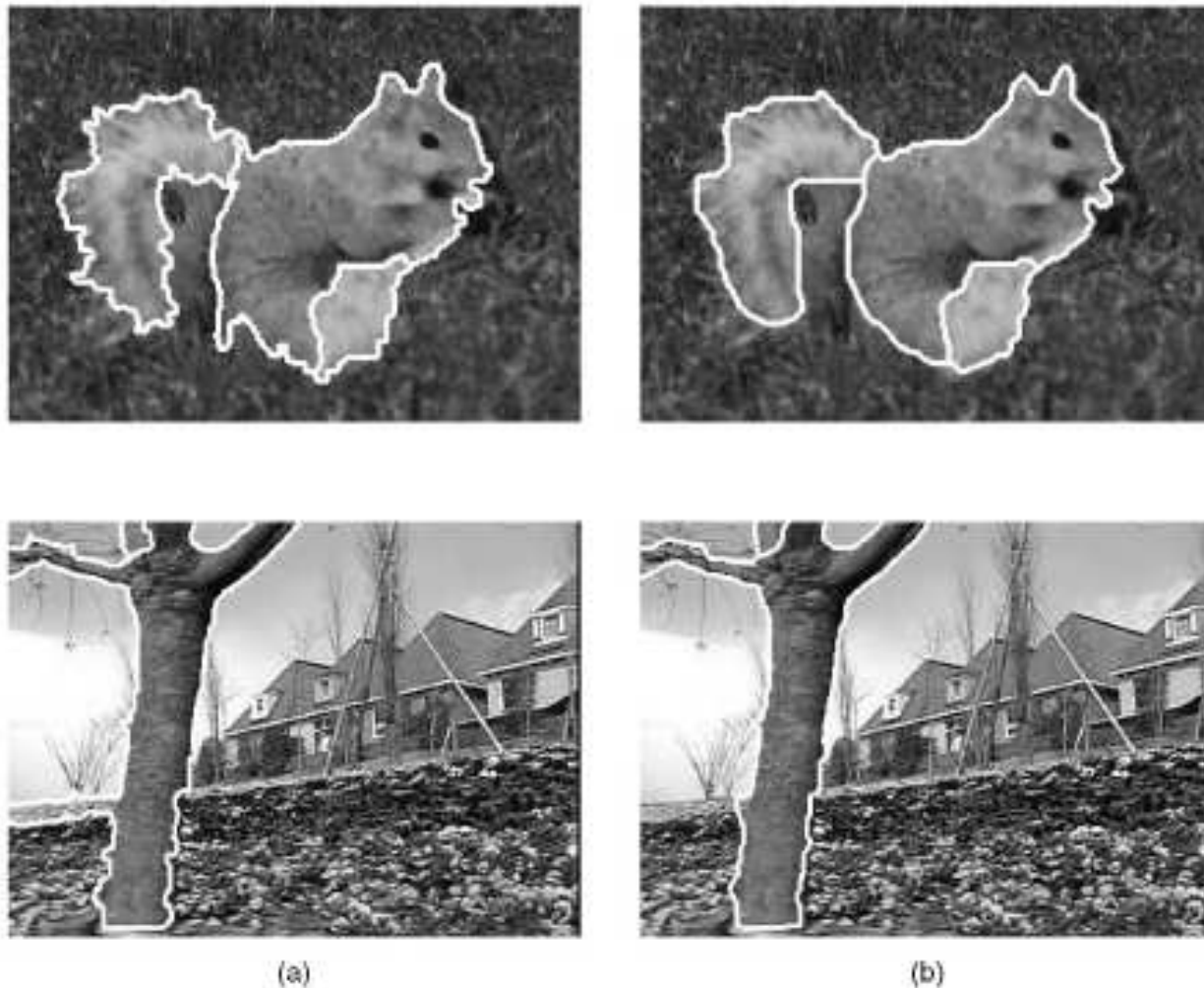
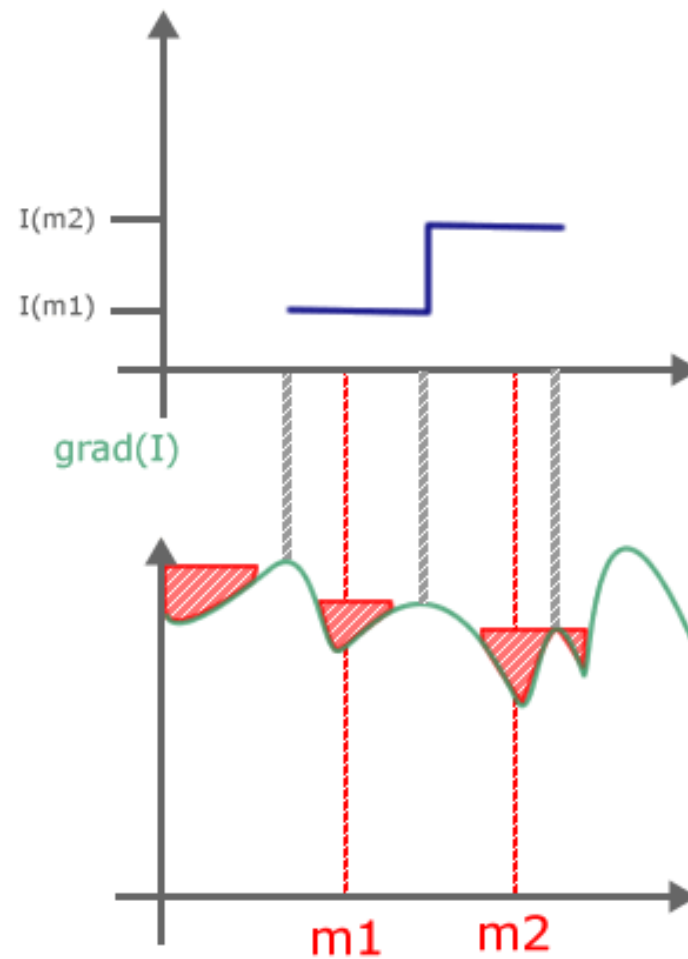


Fig. 8. Segmentation by (a) watershed and (b) watersnake ( $\beta = 50$ ). In the bottom row, the result is shown for the object of interest only.

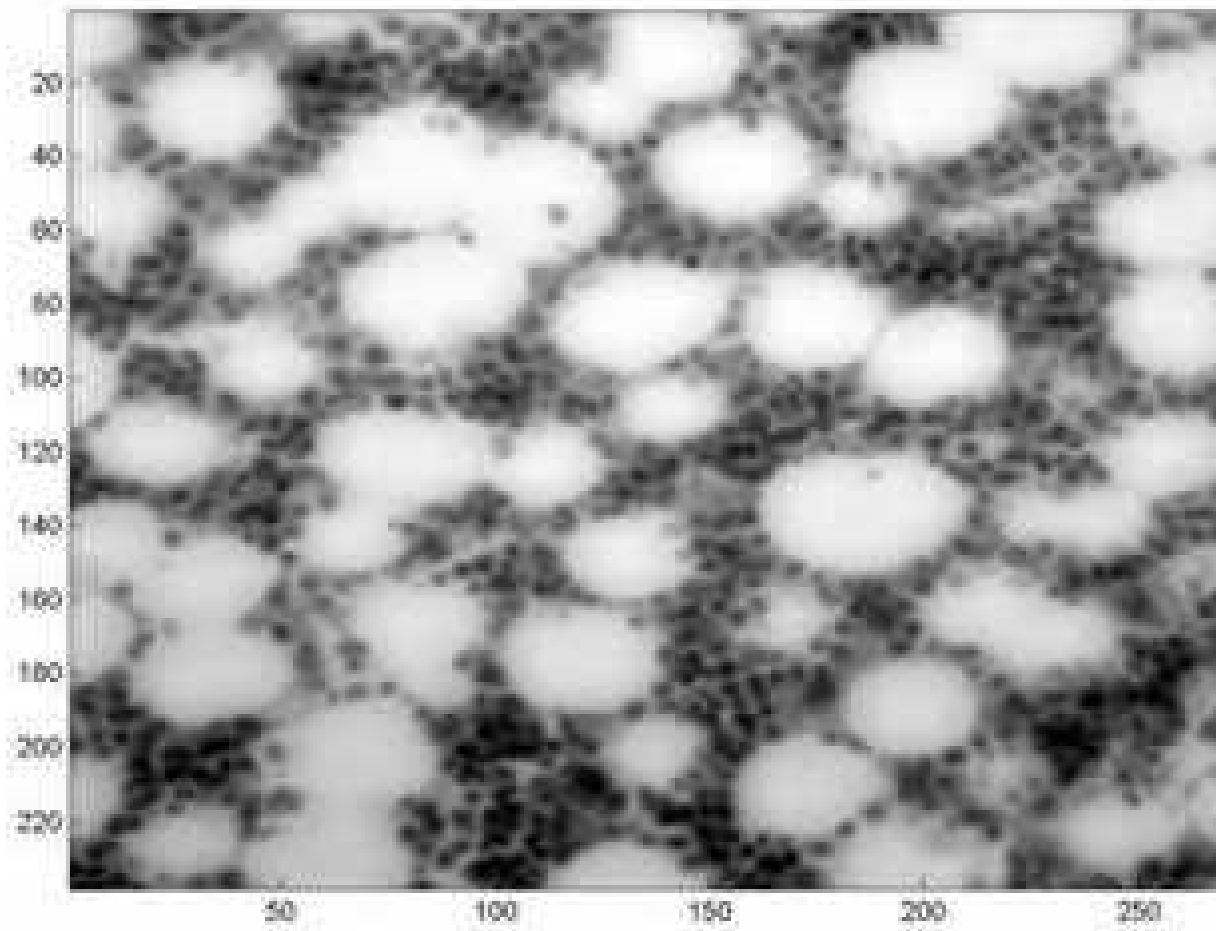
# *LPE hiérarchique*

Remplissage des bassins versants (peut être itéré) :



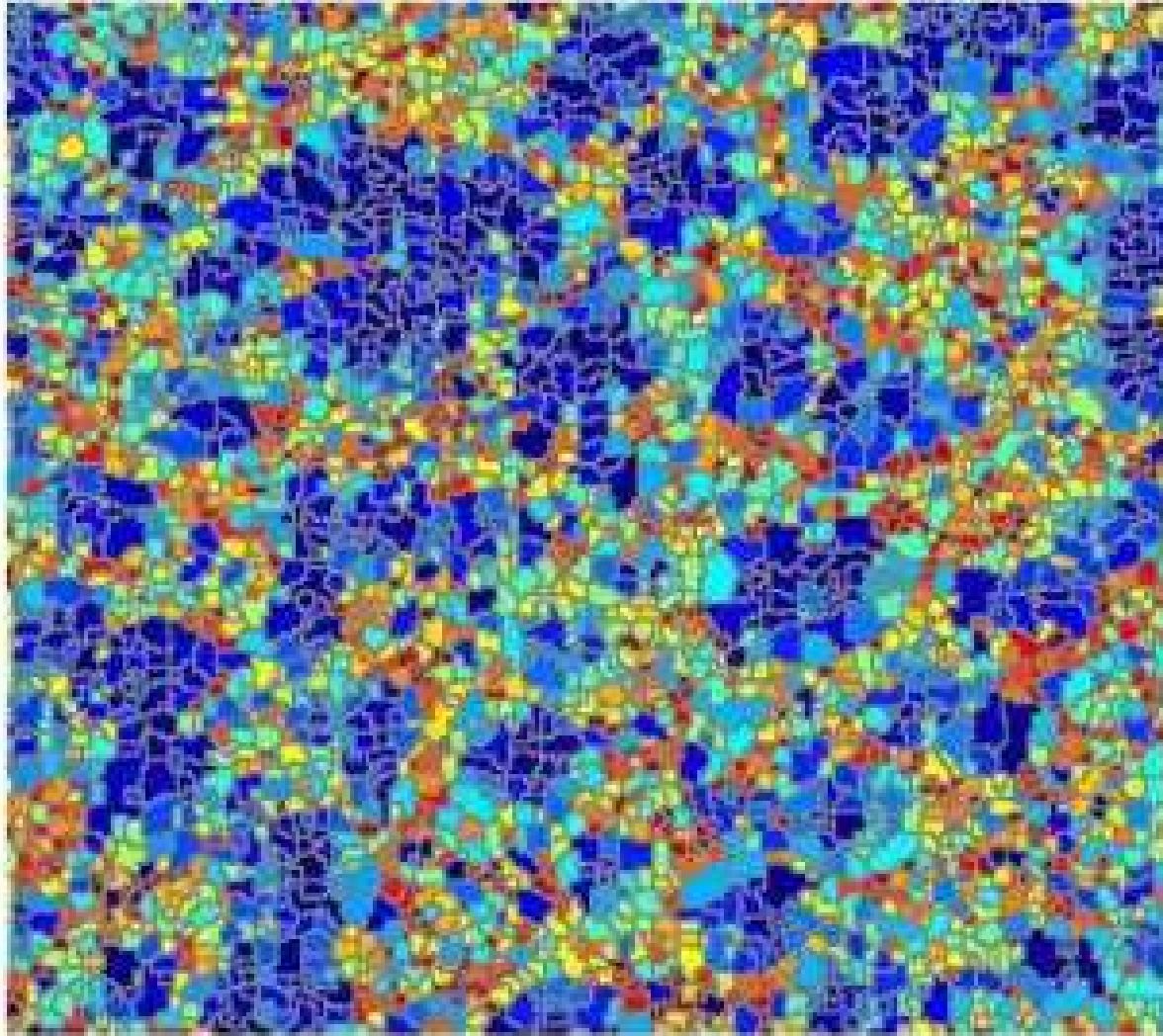


# *LPE hiérarchique*



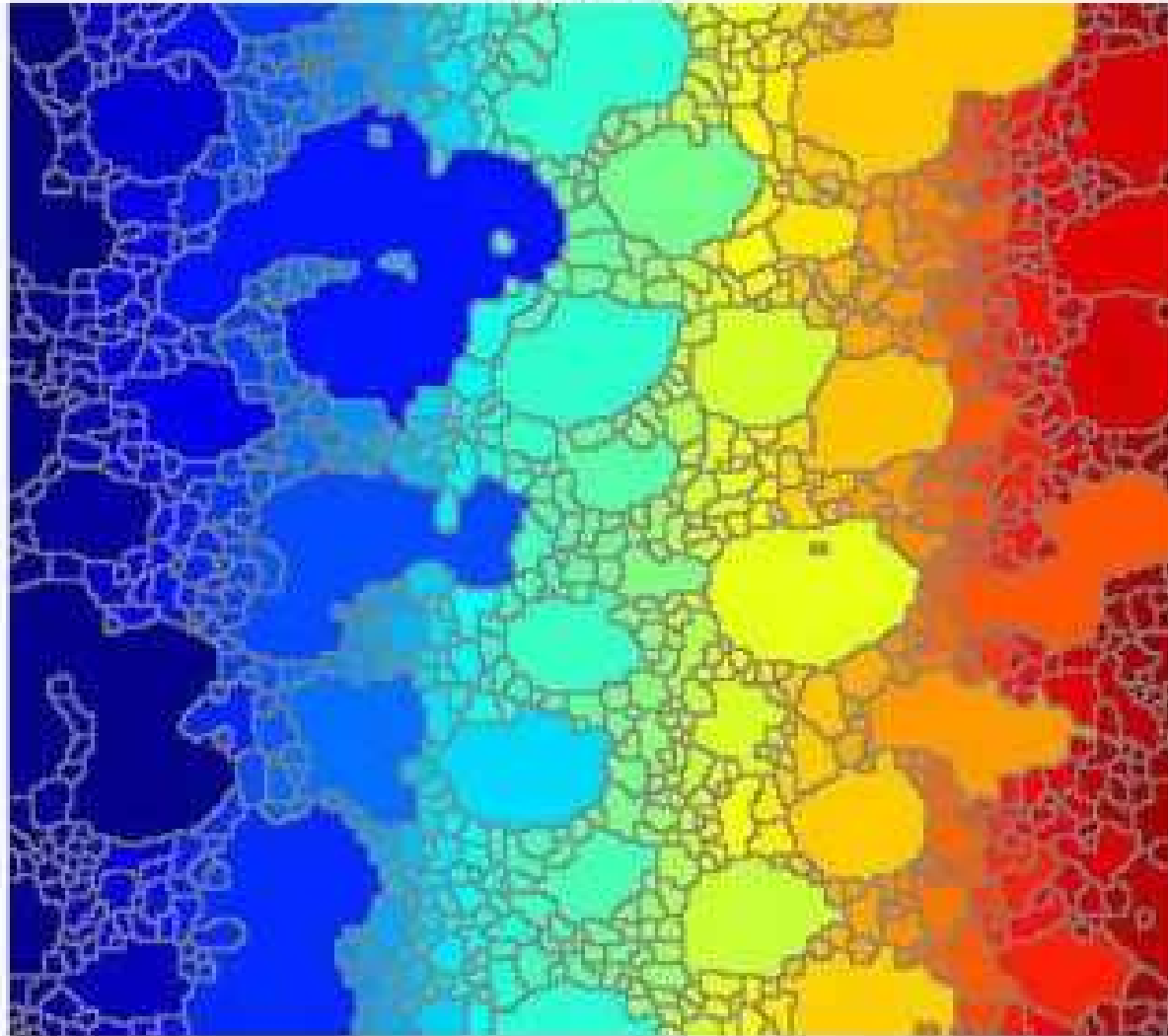
# *LPE hiérarchique*

Watershed

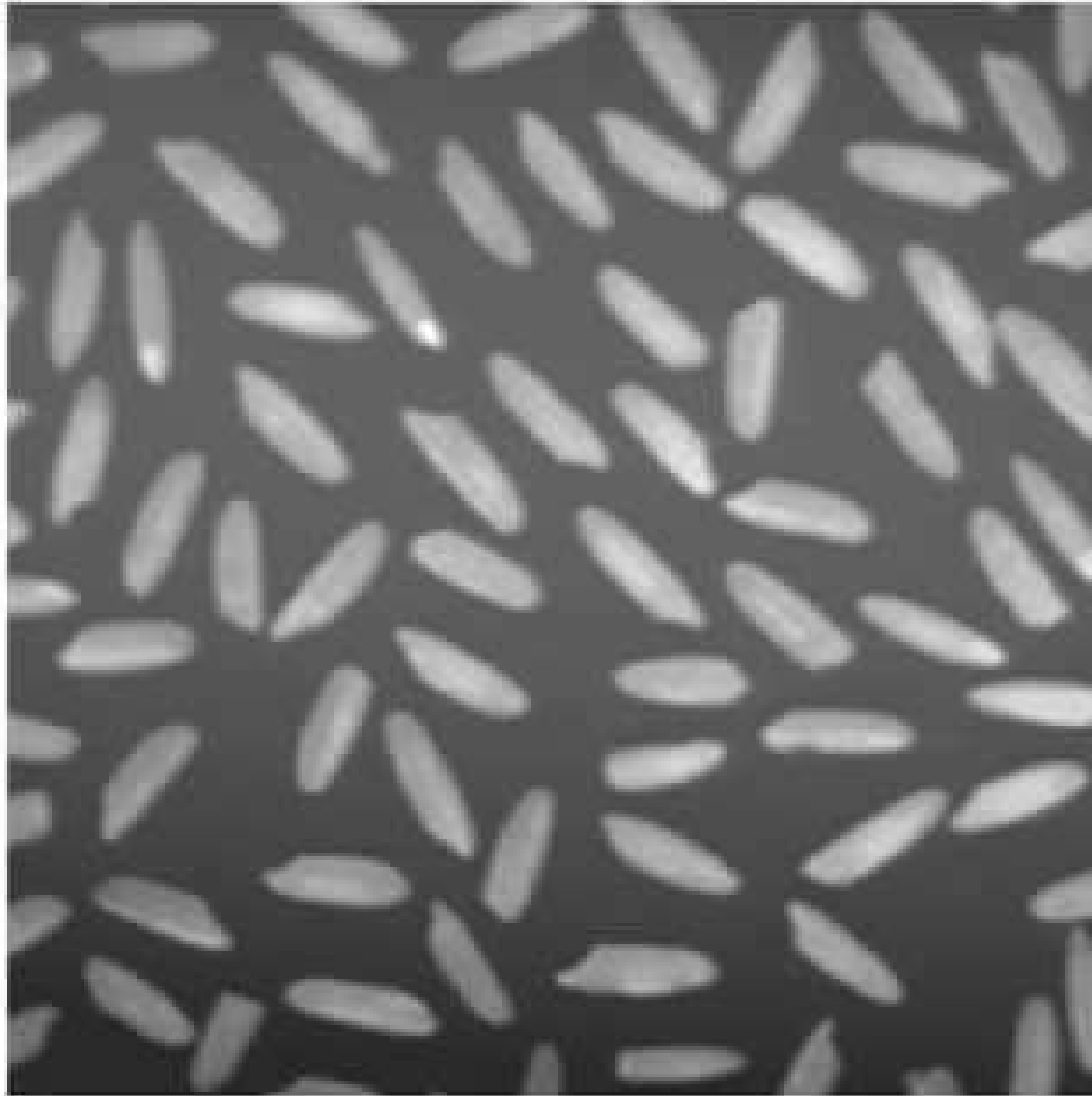


# *LPE hiérarchique*

Hierarchique

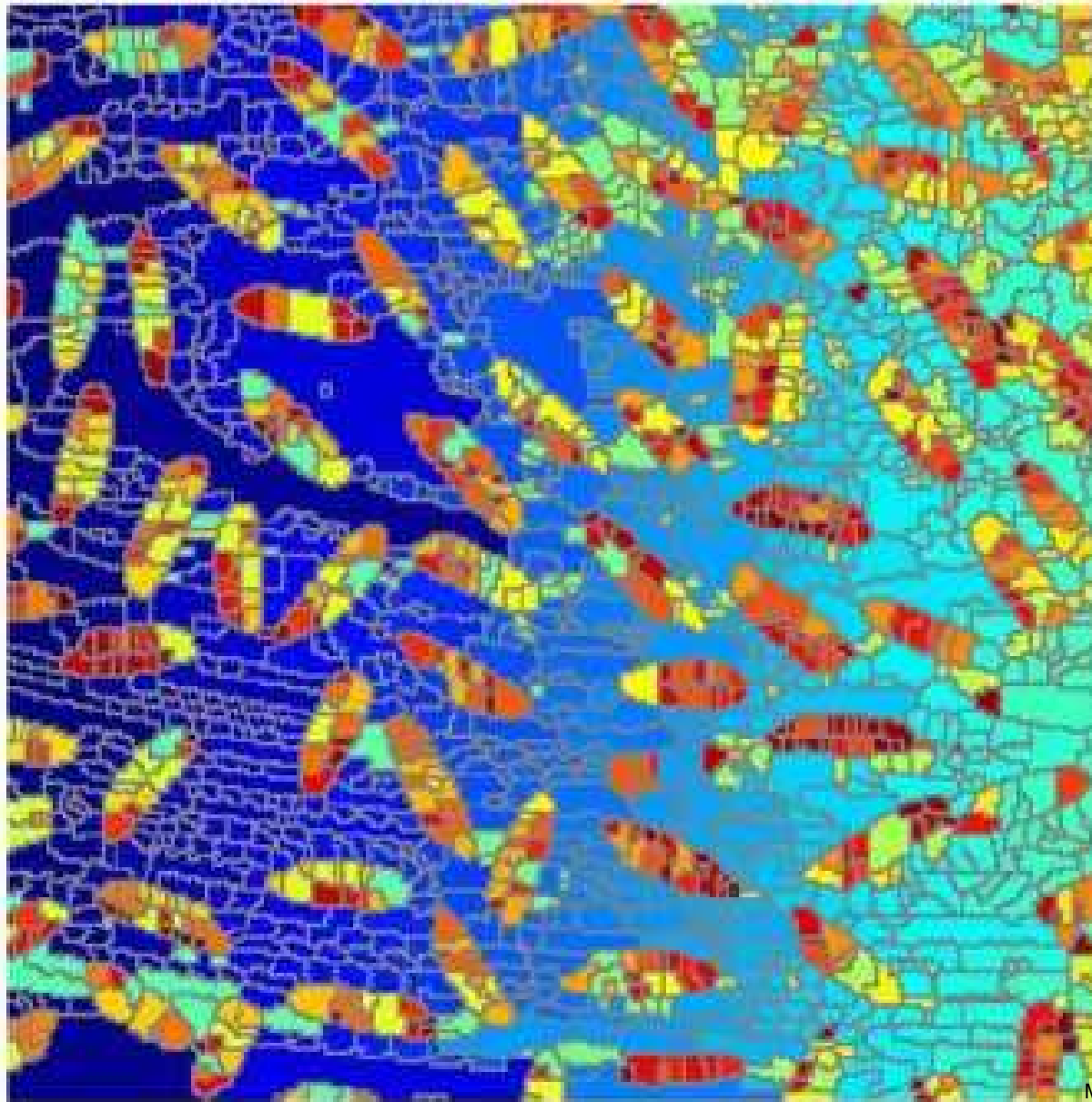


# *LPE hiérarchique*



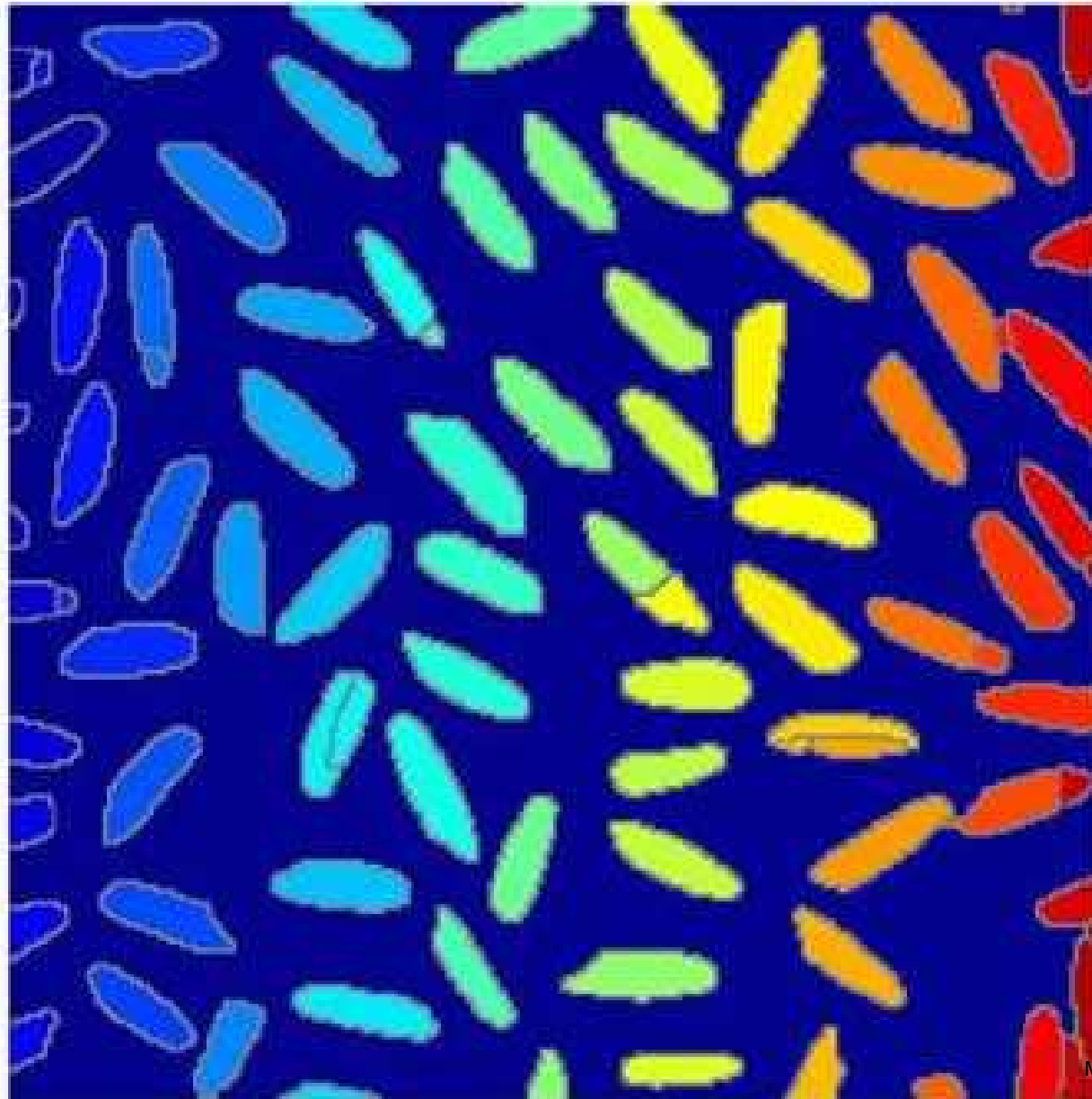
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Watershed

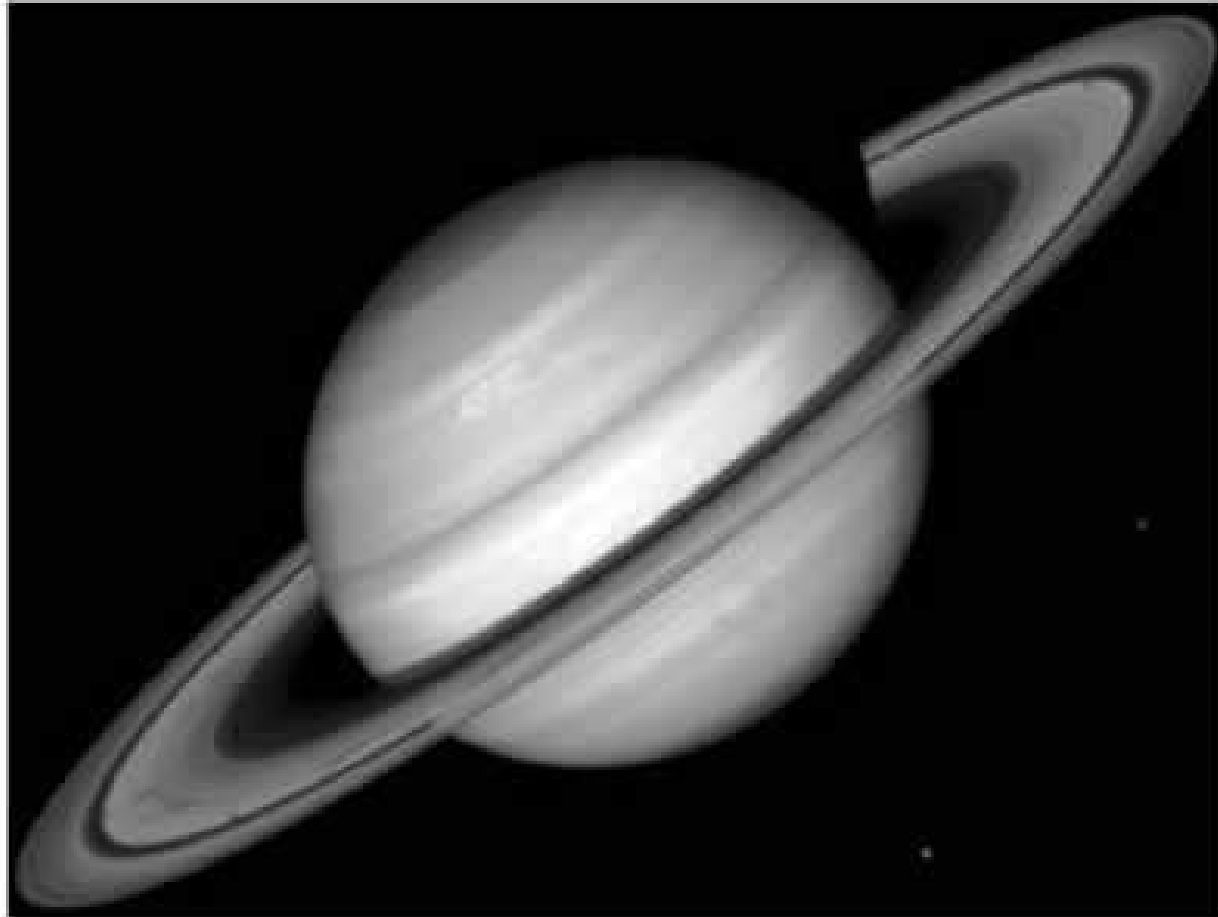


# *LPE hiérarchique*

Hierarchique

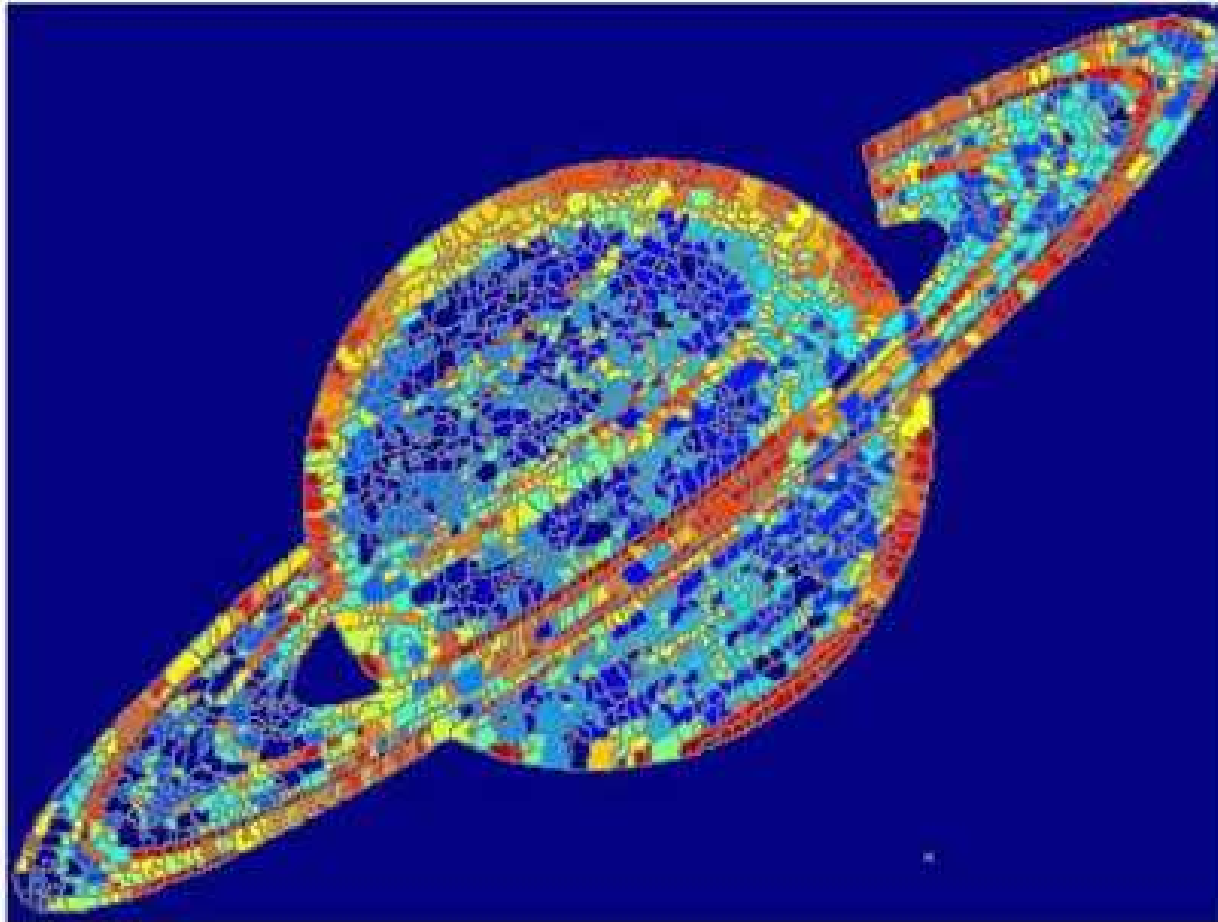


# *LPE hiérarchique*



# *LPE hiérarchique*

Watershed





# *LPE hiérarchique*

Hierarchique

