Satellite Image Mining : Indexing and Retrieval

Master AIC (Apprentissage, Information et Contenu) and D&K (Data & Knowledge) – Université Paris Saclay

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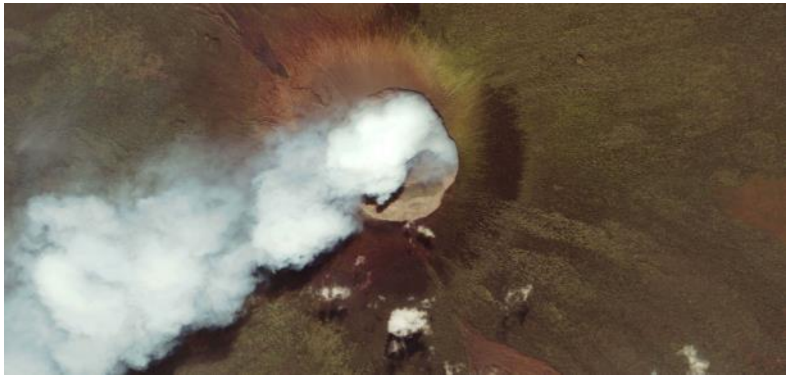
Content

Part I - Remote Sensing and Remote sensing images

Why? How? For Whom?

Why do we need Remote Sensing

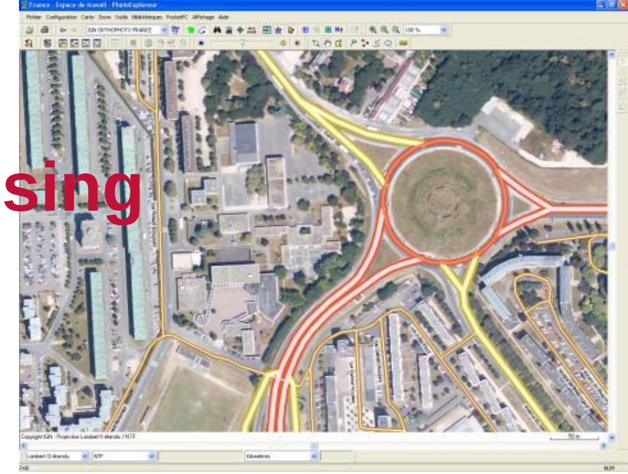
- **Environnement:**
  - Meteorology: short-term weather prediction
  - Climate: long-term monitoring
  - GMES = Global Monitoring for Environment and Security: survey of natural and man-made catastrophies
    - volcanos
    - earthquake, tsunamis, floods
    - Industrial hazards
    - Marine pollution



Why do we need Remote Sensing

- **Agriculture :**
  - Survey and evaluation of crop & farming production
  - Fish & Aquaculture resources management
  - Forestry resources planning
  - Water management, dams, watering
  - Desertification & urban pressure

Why do we need Remote Sensing



- **Town & country planning:**
  - Mapping and inventories
  - Constructions & public work: railways, airports, harbours, dams, …
  - Cities and Mega-cities management
  - Management of moving populations, displacements, installation
  - Climatic impact management
  - Crisis management: fires, floods, …

Why do we need Remote Sensing

- **Defence & Security applications:**
  - Military deployment preparation
  - Military mission debriefing
  - Intelligence and survey of national/foreign territory

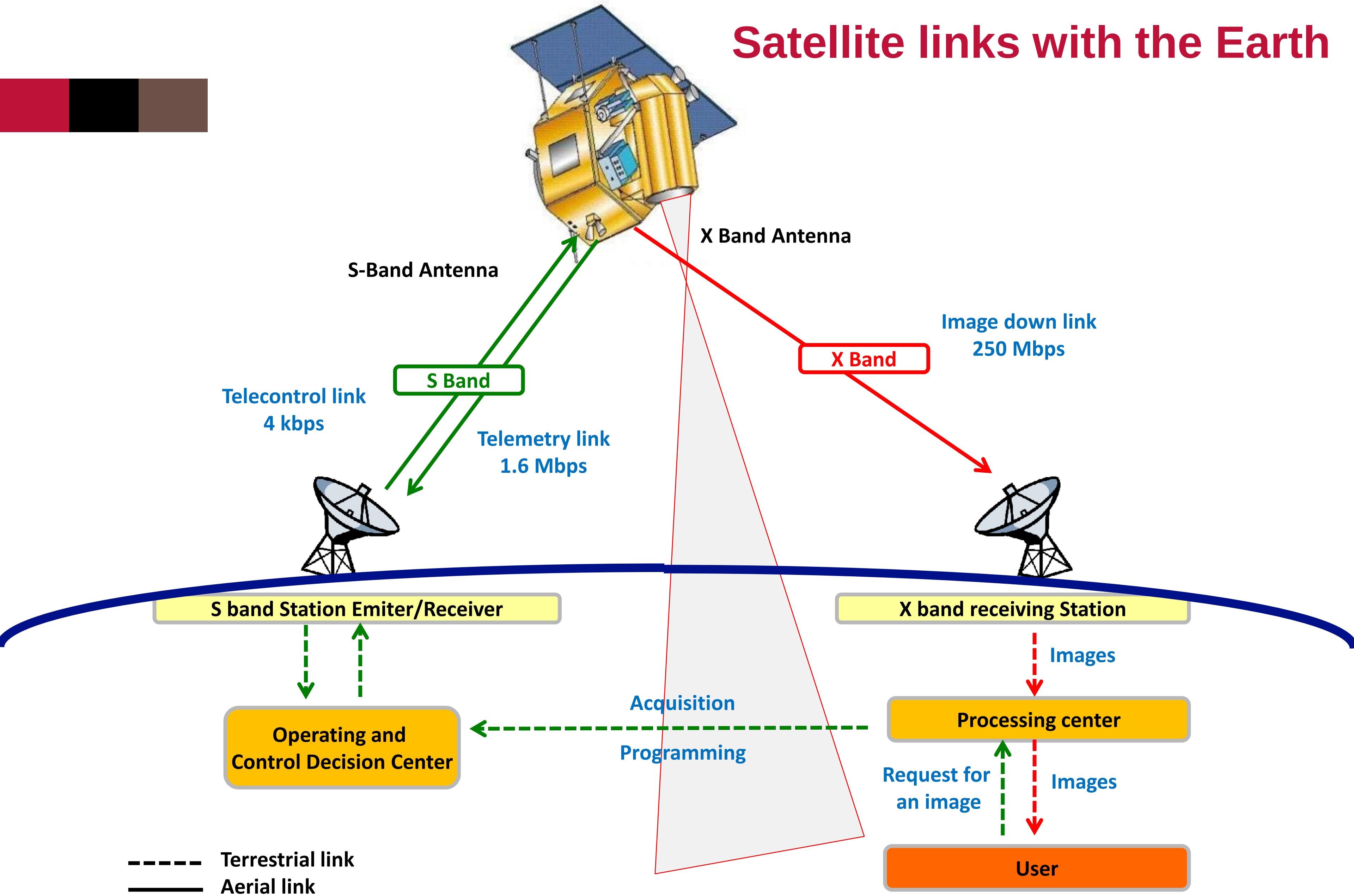
How to prepare a remote sensing program

How is prepared a remote sensing program

- **Conceive the sensor:** application, customers
- **Determine which satellite / which launcher**
- **Conceive the ground-station and the data management process :** economical, social and technical issues

➔ 15 to 20 years

Satellite links with the Earth

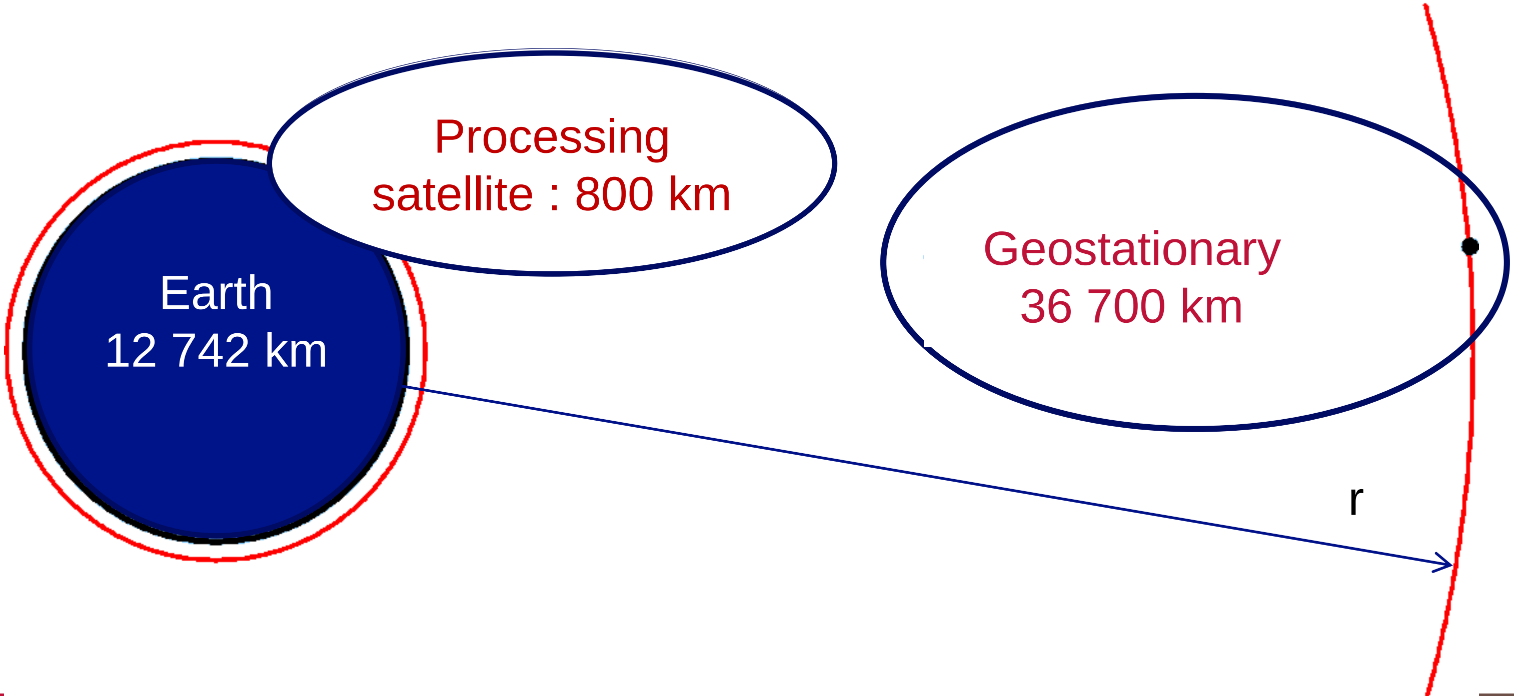


Satellite : orbit choice

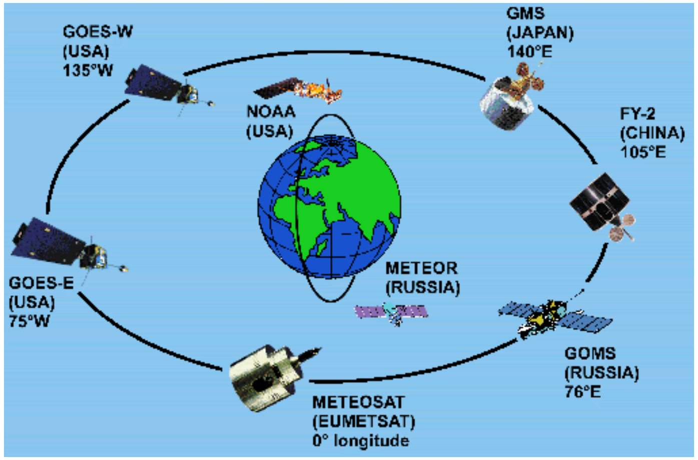
$$\vec{F} = -\frac{\mu\, m}{r^2}\frac{\vec{r}}{r}$$

- **Mecanics laws:**
  - Newton = centripetal force
  - Satellite speed = driving force

  ➔ elliptical or circular trajectory (Kepler)

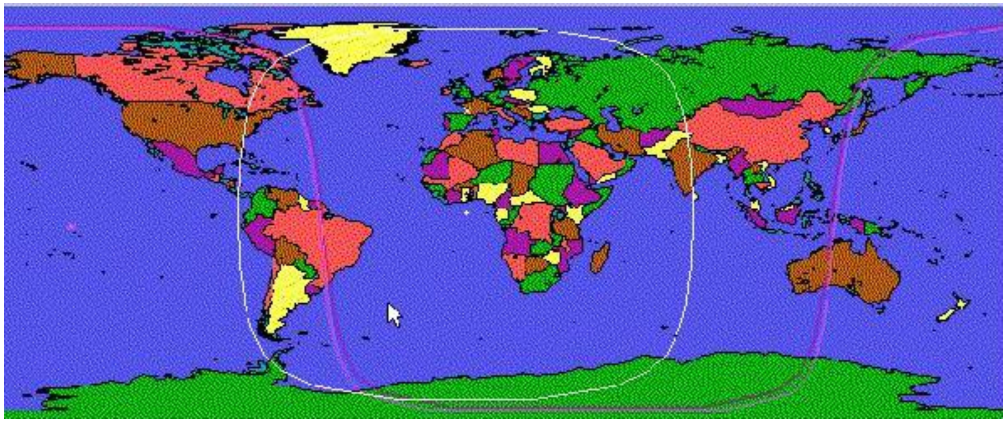


Orbit choice



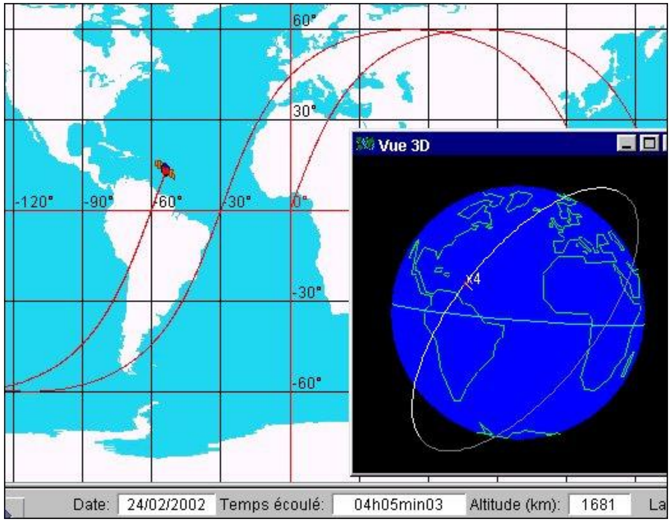
- **1) Geostationary**
  - Always in the Equator Plane
  - Always at vertical of the same point on the Equator
  - Altitude ~ 36 700 km
  - Field of view: ~1/3 Earth: always the same

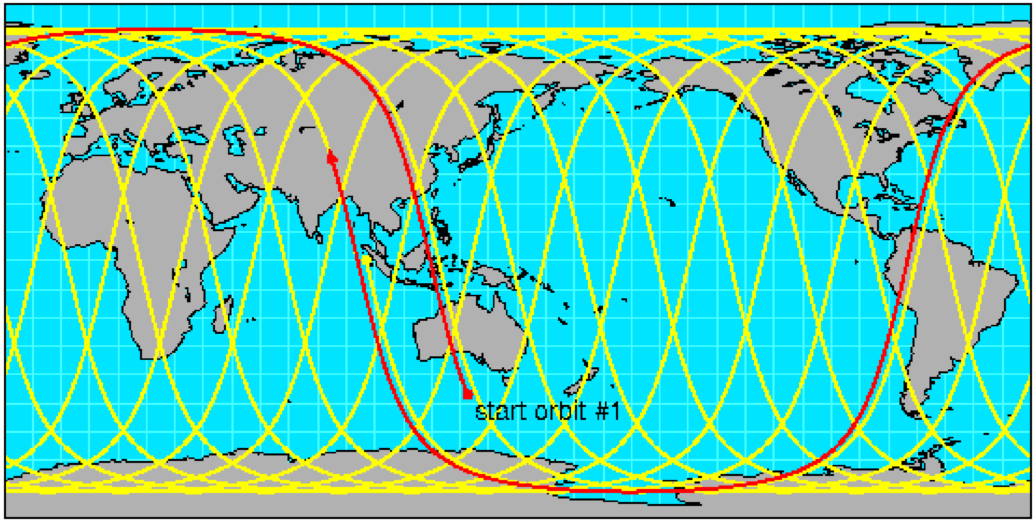
  - Applications : **meteo, survey of catastrophies**, telecoms, TV



Orbit choice

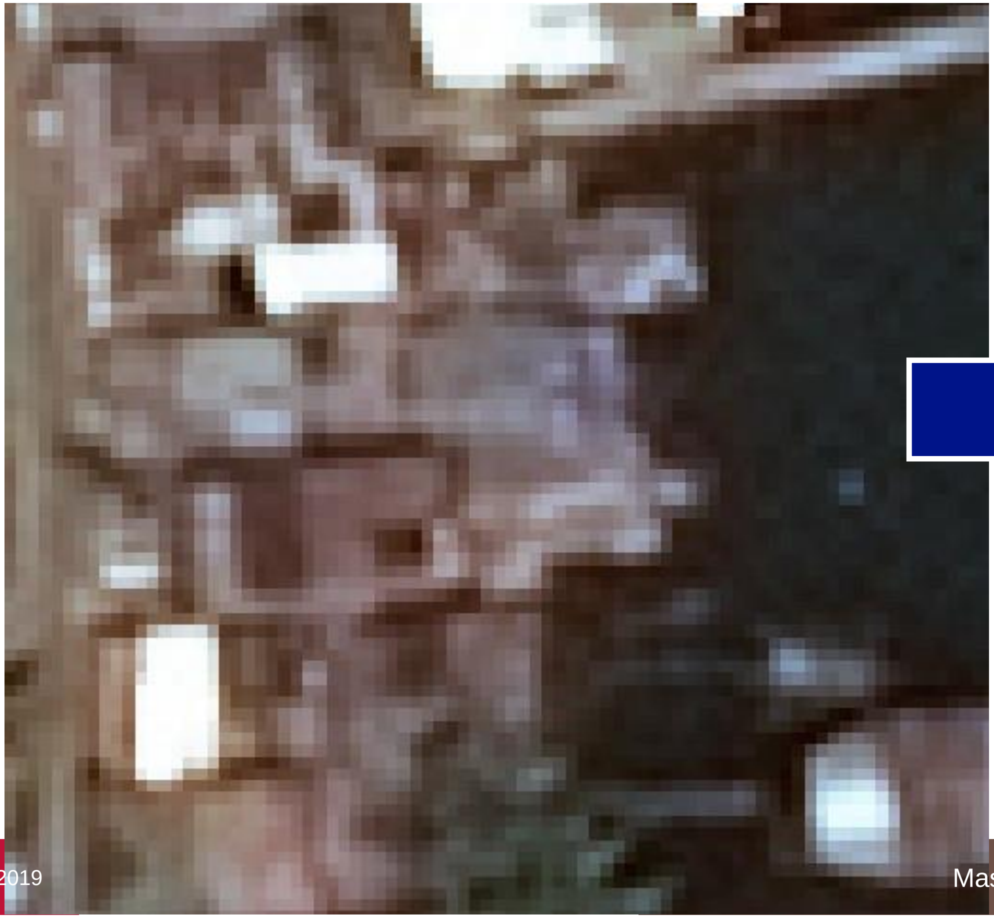
- **2) Processing satellite (low orbit)**
  - Altitude ~ 800 km (down to 250 km)
  - Circular ~ N/S
  - Trajectory : ± polar
  - ~ 15 revolutions / day
  - Helio-synchronous





Choice of resolution

- **Pixel size = smallest measured terrain on the ground**
  - from 30 cm to 10 km



SPOT 5

Δx = 2,5m

On Ground resolution

- **Depends on:**
  - Sensor :

    Photosites size: δx

    $G = \frac{f}{D}$ = enlargement

    $\Delta x = \frac{\delta x}{G}$ = smallest detail

  - The camera lens

    $\delta' x = \frac{\lambda f}{d}$ = diffraction limited resolution

$$\Delta x_{min} = \frac{\lambda f}{Gd} = \frac{\lambda D}{d}$$

➔ Smallest detail

D = satellite-Earth distance
~ 1 000 km = 10^6 m

λ = wave length
= 0,5 . 10^{-6} m

d = lens diameter
~ 0,5 m

$$\boldsymbol{\Delta x_{min} = 1\ m}$$

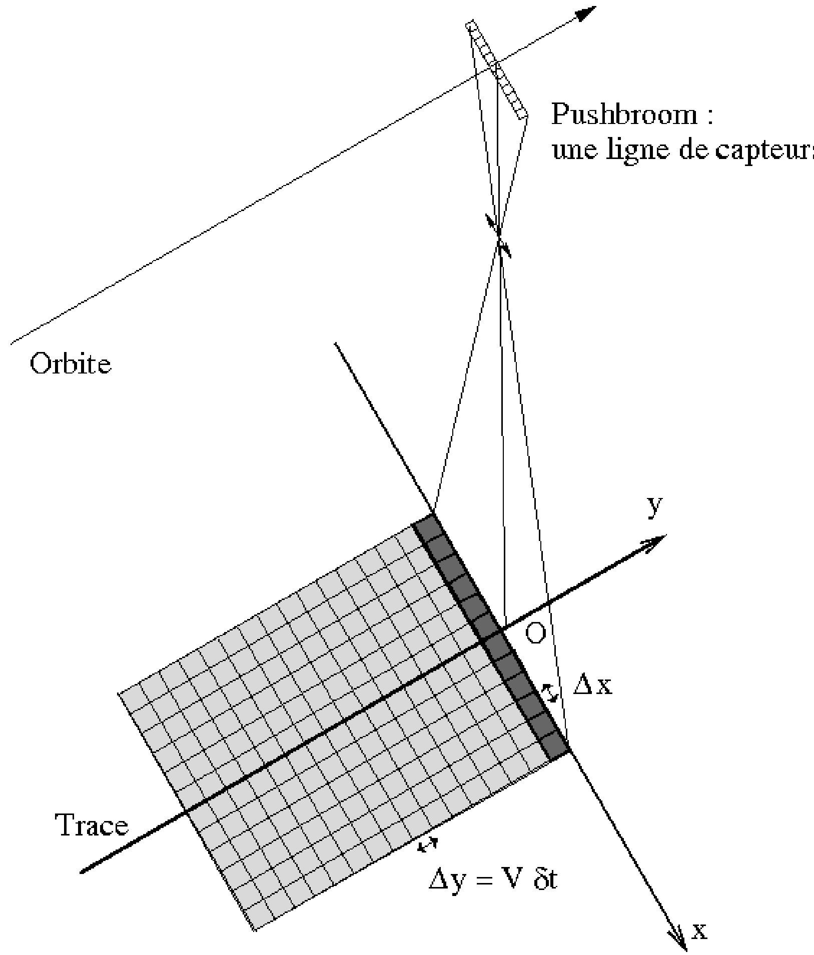
Possible with : f = 1 m

if $\delta' x = \frac{\lambda f}{d}$ = 1µm

the photosite measures 10^{-6} m

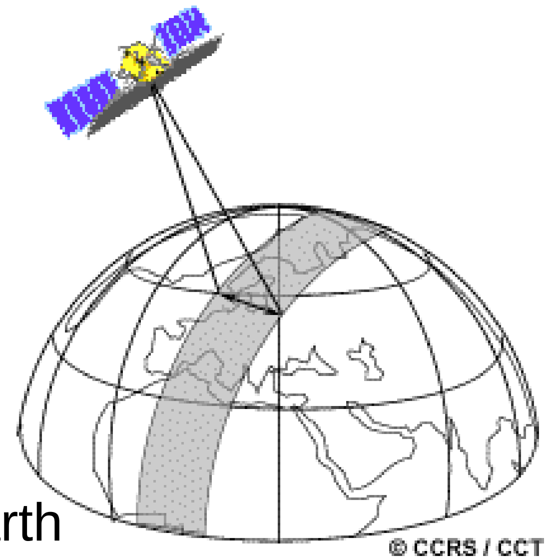
Often *push-broom* *sensor*

- **Sensor size along track:**
  - On line sensor
  - = speed x aperture time
- **In the other direction**
  - Number of sensors on a linene
  - from 6 000 to 40 000
- **Resolution :**
  - Depends on the lens

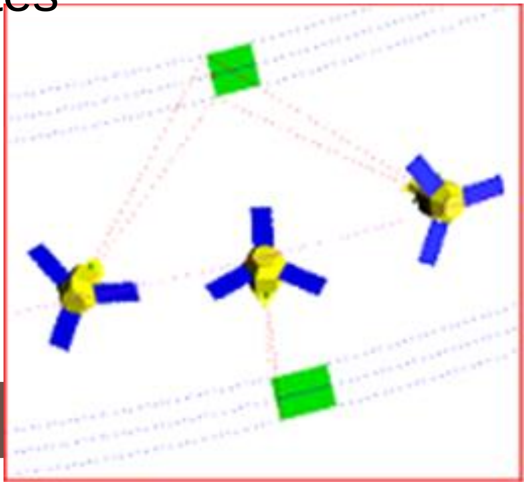


Swath choice

- **Swath = image width**
  - from 10 km to 10 000 km
  - = from 3 000 to 40 000 pixels / line
  - Given by the sensor size
  - Limited by the communication link with Earth



- **Revisit delay**
- **15 min for geostationnary sat. (to dump the memory)**
  - from 1h30 (min) to 1 month for processing satellites
  - But … sensor agility!

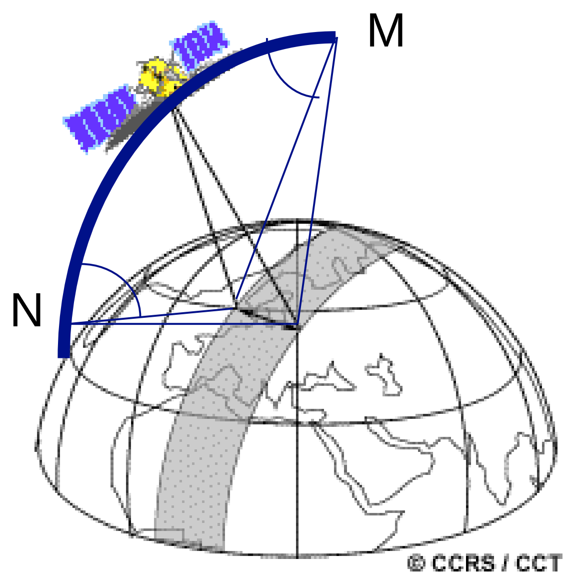


Video possibility

- **Angle of view ~ + or – 50 degrees:**
  - MN ~ 2000 km
  - 1 rotation around the Earth = 90 min

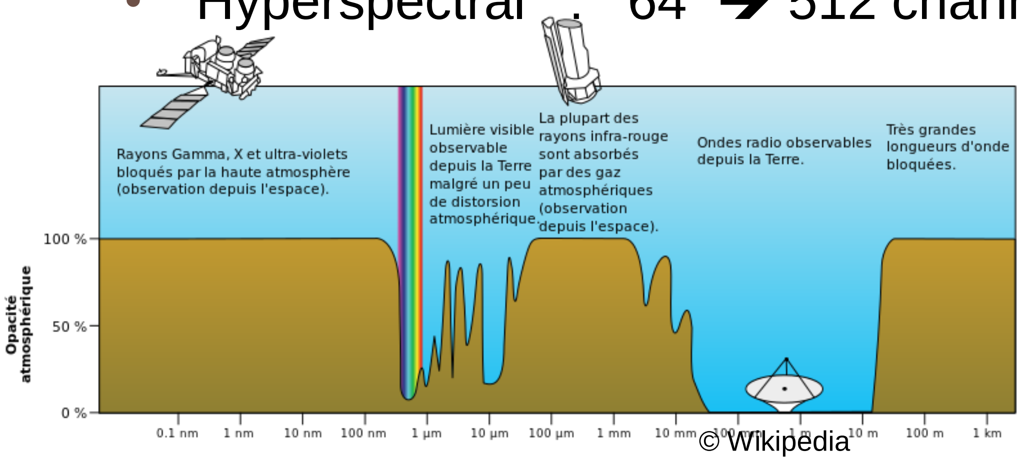
    ~ 40 000 km
  - Time to go from M to N

    = 90*2000/40000 = 4 min 30 s



Which wave length?

- **1 – Passive sensors: measure the energy sent back from Sun by Earth or the energy radiated by Earth**
  - Emitted from the Sun (Wien’s law) x Atmosphere transparency x Ground Reflexion
  - Black and White (Panchromatic)
  - Visible = Blue - Green - Red
  - Visible and Near Infra-Red : G - R - IR = false colors
  - Multispectral : 7 ➔ 20 channels
  - Hyperspectral : 64 ➔ 512 channels



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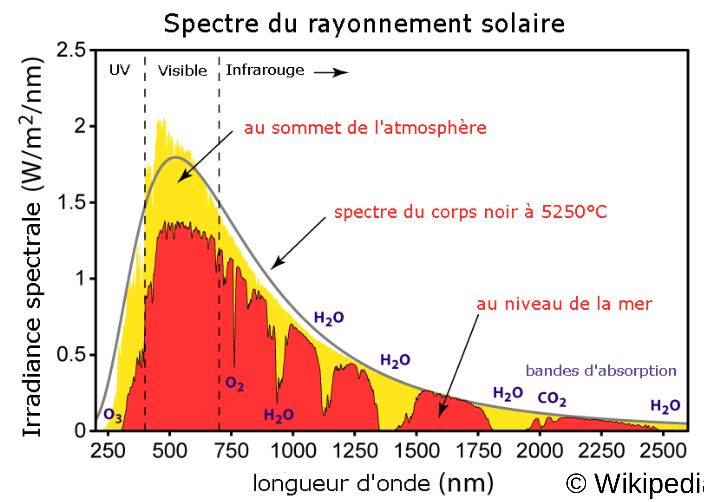
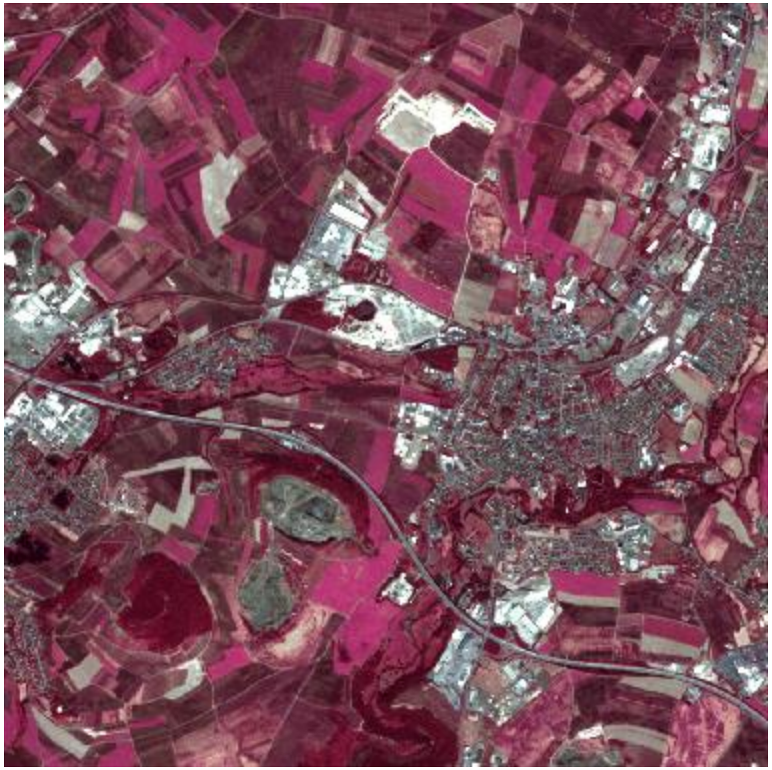


© Wikipedia

False colors : NIR-R-G → R-G-B

vegetation = red



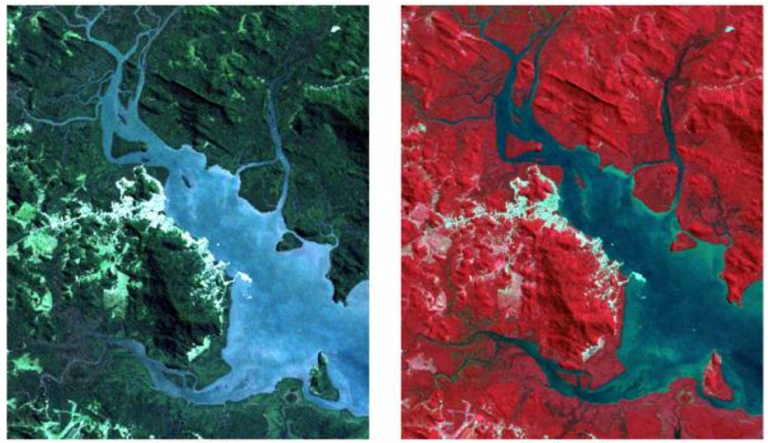
False colors

True colors

Multispectral image visualisation: pseudo colors

Landsat = 7 channels

321



(a) combinaison 321 (b) combinaison 432

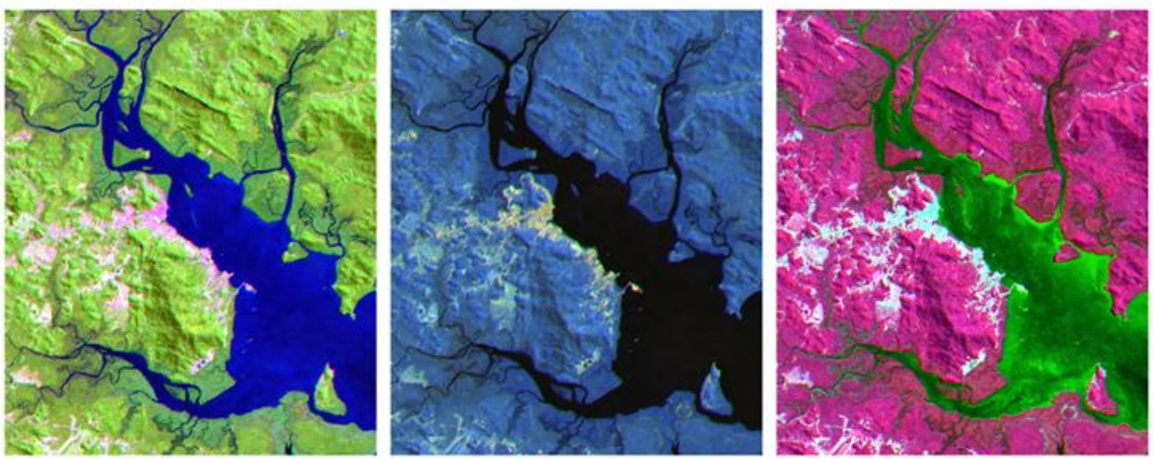
432

1
2
3
4
5
6
7

R
G
+ B

41(7+5)

542



(c) combinaison 542 (d) combinaison 754 (e) combinaison 435

435

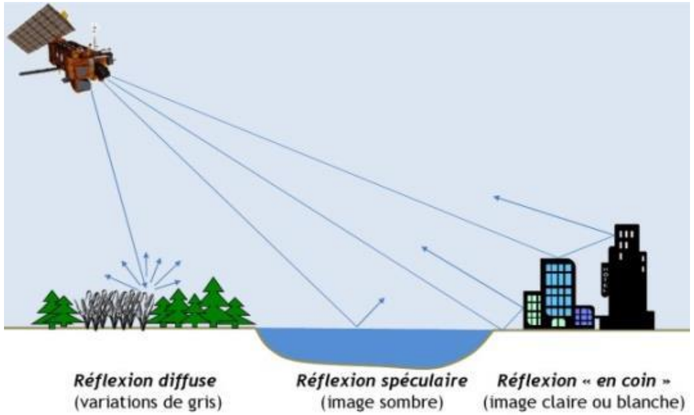
754

© UVED

Which wave length?

- **2 – Active sensors: EM emitter + receiver**

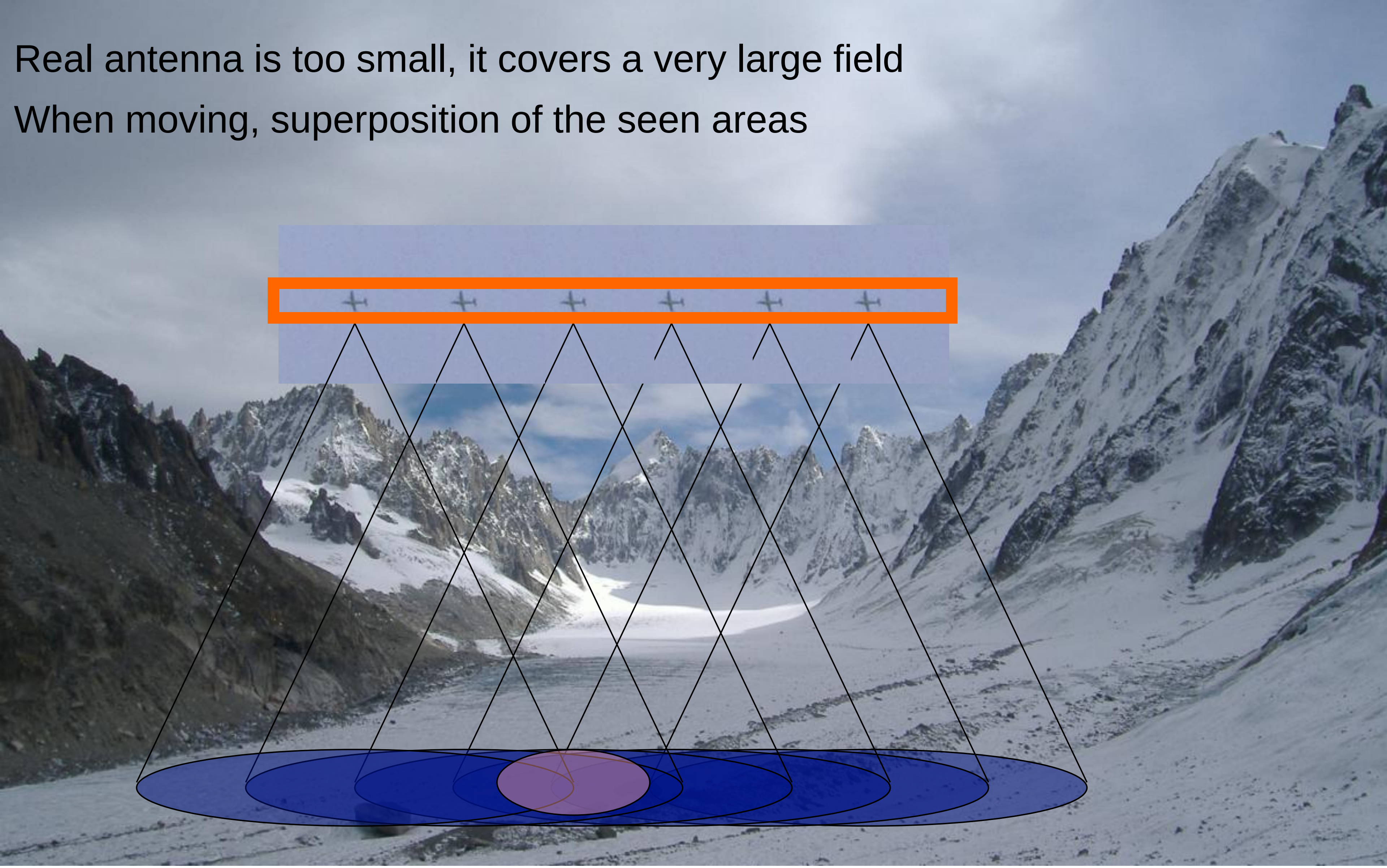
  radar = Micro waves: λ= 1 cm to 10 m



- But low resolution : $\Delta x = \frac{\lambda f}{Gd}$
- With complex processing: Synthetic Aperture Radar ➔ hi resolution

Real antenna is too small, it covers a very large field

When moving, superposition of the seen areas



One point is seen from several antenna positions

From computation we obtain an accurate information = synthetic antenna

Satellite images = big data !

Television HD	1 280 x 720 pixels
Television 4k	4 000 x 2 000 pixels
PC display screen	1 600 x 1 200 pixels
Photo camera	5 000 x 4 000 pixels
Spot 1 … 4	**6 000 x 6 000 pixels**
SPOT 5	**24 000 x 24 000 pixels**
Quickbird	**40 000 x 40 000 pixels**

1 600 000 000 pixels = 1,6 Gpixels

= 800 PC display screens

1 SPOT 5 image = 10 s of satellite run

Diversity of Remote Sensing Images (slides are not presented in the lecture notes)

Part II – Remote Sensing Image Mining

Remote Sensing Imaging: Archiving Problems and Issues

Satellite Image archives

- How can we store millions of images?
- How can we ensure durability of storage?
- **How knowing that information exists?**
- **How retrieving information?**
- **How exploiting information?**

→ Data Mining directly on image files

When searching in a small set of images

→ Indexing images when received
→ data mining on index

When searching in large sets

RS Image mining IS NOT MultiMedia Image Mining

Mining in Multimedia Image databases

- **Multimedia information retrieval :**
  - Either from **semantic information**: name, description, caption, text (90 % of Google-like retrieval)
  - Or from **instance** (i.e. with a reference image) (Face or fingerprint recognition)

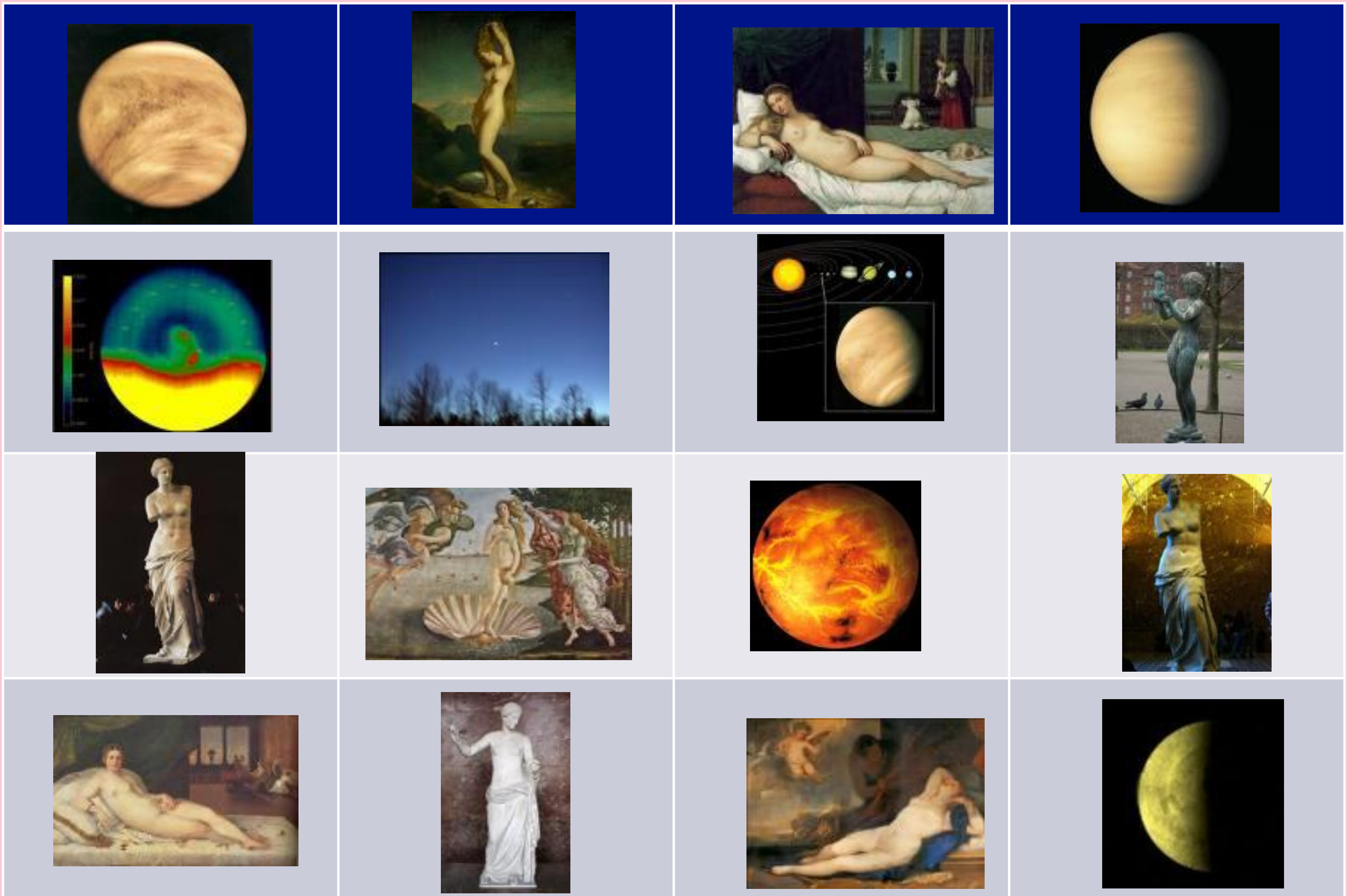
- **I – Classical Machine Learning techniques (2000-2012)**
  - Hand-crafted feature detection and/or salient point detection
  - Classification in p-dimensional space
    - ➔ few parameters
    - ➔ few learning images (groundtruth) ~ 1000

- **II – Deep neural networks (2012 - …)**
  - Directly with images as input and/or with extracted features
  - Several +/- linear classifiers in cascade
    - ➔ thousands of parameters
    - ➔ hundred of thousands of images as groundtruth

Multimedia image mining: handcrafted features + classification

- **Multimedia information retrieval from exemple:**
  - Choices: to be robust to possible differences
    - scale, lighting, orientation, color, … ➔ **invariance**
  - Strategy: detect invariant features
    - Histograms, color distribution, area-based segmentation, graph description, …
    - Textures
    - Salient point detection: Harris, SIFT, SURF, …
  - Represent the image as a vector in a *p* dimensional space $\mathbb{R}^p$
  - Classification : Bayès, k-NN, dynamic clustering, SVM (Support Vector Machine), Graph tree, …

Ambiguous semantics: Venus



Textual categorisation

invariance

Salient points: SIFT

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y),$$

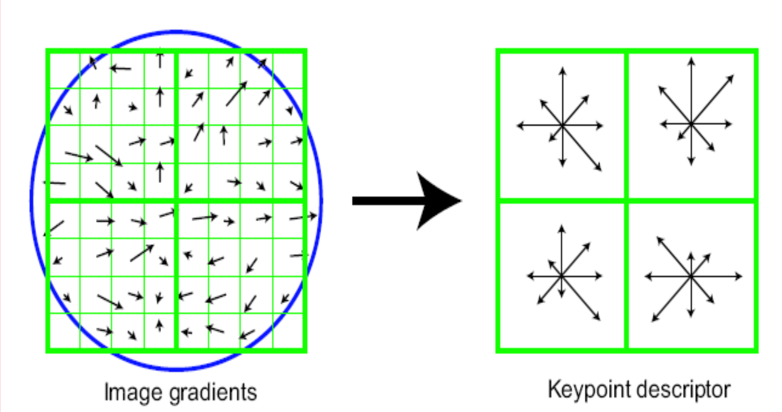
$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G.$$

$$\begin{aligned} D(x, y, \sigma) &= (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma). \end{aligned}$$

$$D(\mathbf{x}) = D + \frac{\partial D^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2} \mathbf{x}$$

$$\hat{\mathbf{x}} = -\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}.$$

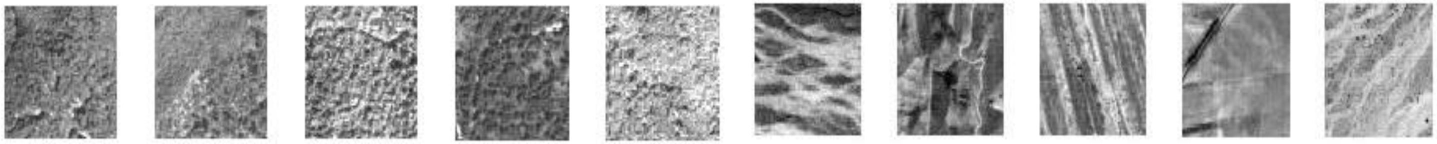
$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r}$$



Specificities of RS Image mining

Category-based retrieval in specific data-bases

- **Examples:**
  - Biomedical
  - Biology
  - Astronomy
  - Remote sensing and satellite images
- **Goal: to retrieve images « looking the same » as a given sample in very specialized data-bases**



- **Different from : retrieving the exact object in a very broad data-base**

Satellite images

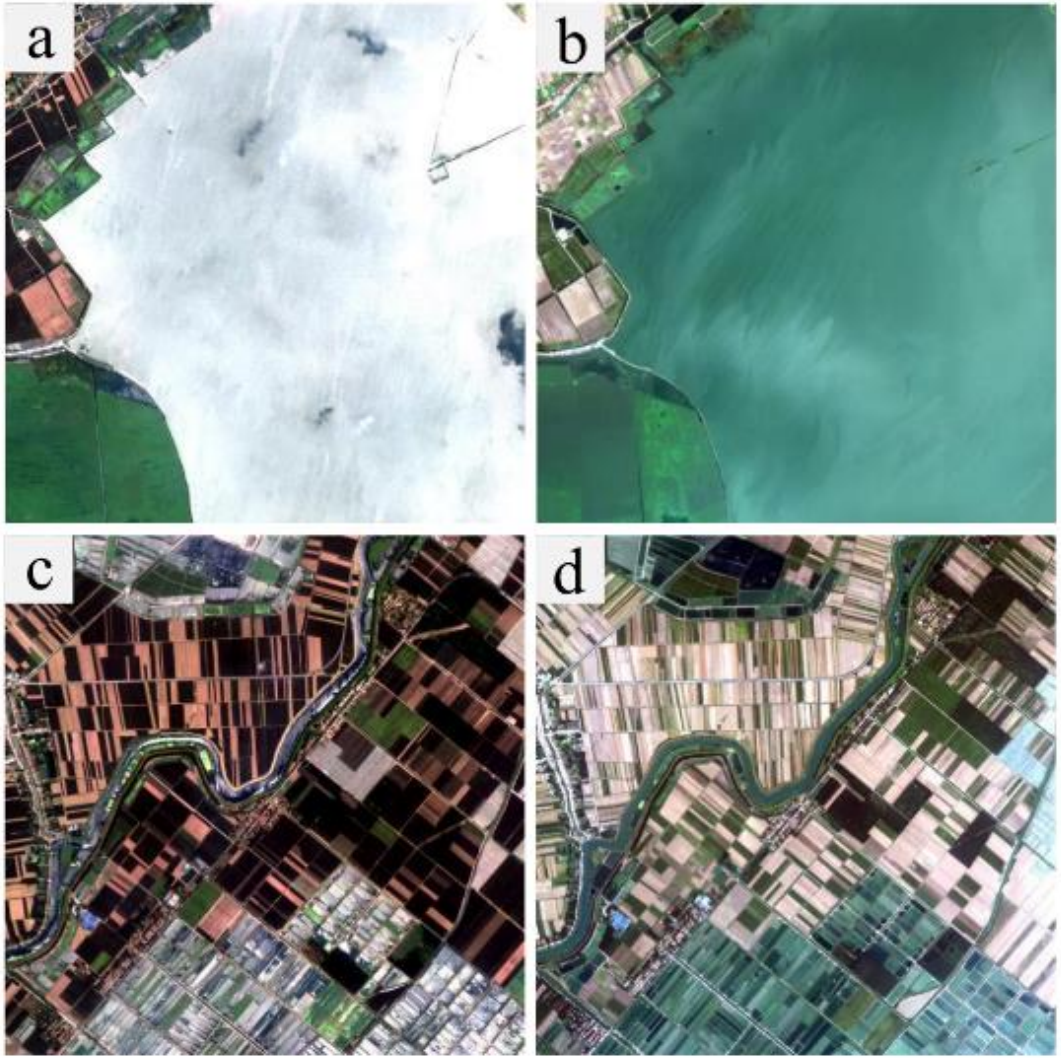
- **A very specific content**

Fields

City

Forest

A same region, different signals



From : Tong et al.
arXiv 1807.05713 - 2018

The role of scale

High-Badakchan, Tadjikistan - Ikonos

15 m

1 m

Main scales

- **<1 meter = Very high resolution :** fine details in urban context, cars, pedestrians, containers, fences, small boats, …

- **1 m < … < 5 m = High resolution :** urban fine structures, houses, streets, gardens, individual trees, railway & road networks, …

- **5 m < … < 30 m = Middle resolution:** fine landcover, coarse urban structure: dense urban, residential or commercial areas,

- **> 30 m = low resolution:** global landcover

Available information on satellite images (semantic information) = Ancillary data

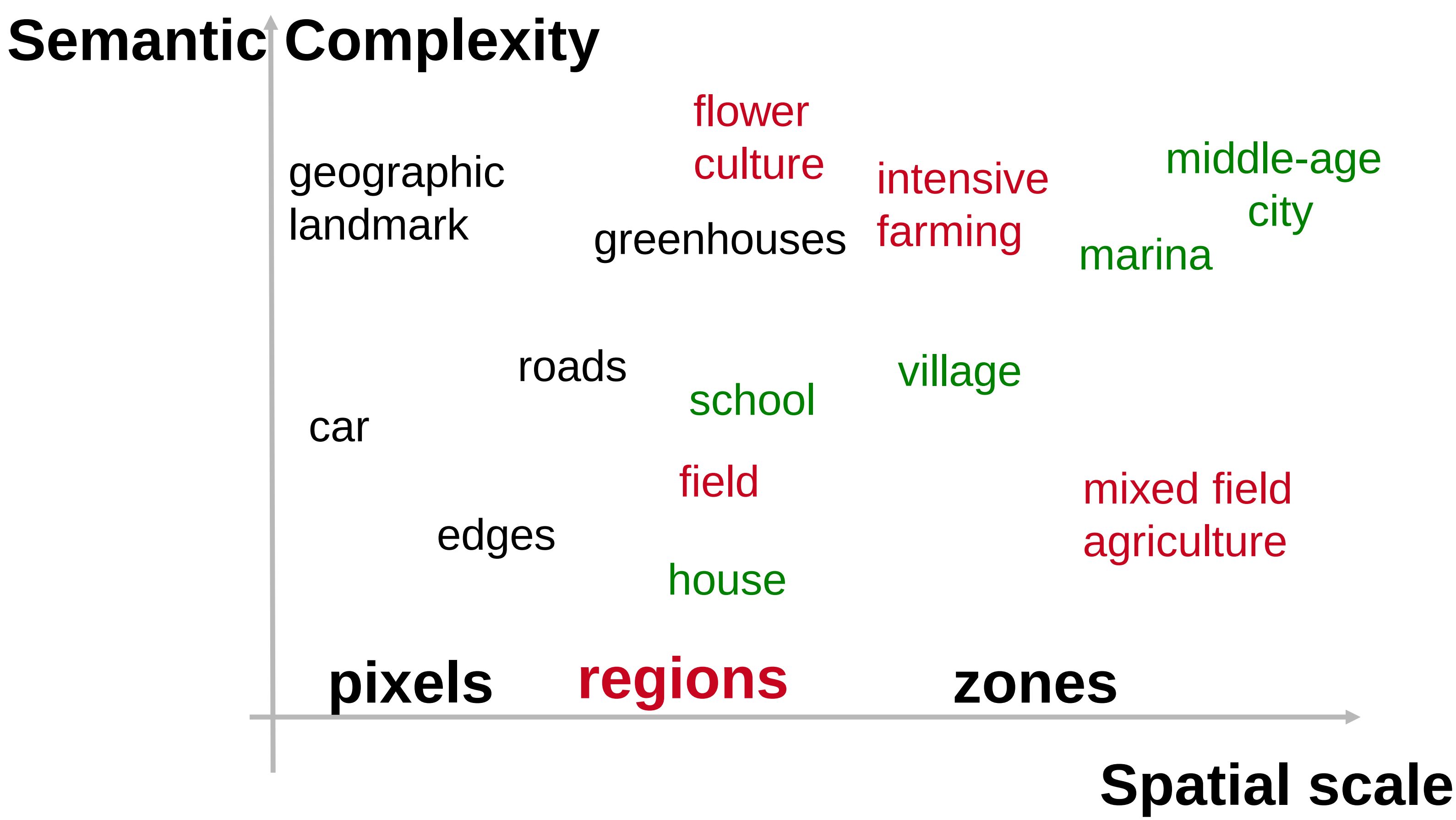
- **Accurate positionning in universal geographical references:** UTM, Mercator, Lambert, etc…
- **Precise time referencing:** seasonal variations (vegetation, insolation, agricultural production, …), sun positionning (shadows), tide effects (precise coast-line, harbours and fishering activities), meteorological conditions (snow, floods, …)
- **Satellite parameters:** resolution, spectral sensitivity, noise
- **Often:** Image quality, cloud cover

Satellite images

- **What are we looking for?** **It is not clear!**

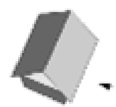
  - Precise objects:
    - A boat — a road-crossing
    - A building — an airplane landing area
  - Generic objects:
    - A marina — a forest fire
    - Greenhouse cultures — refugee camps
    - Oil pipeline — typhoon hazards
    - A geological synclinal
  - Specific terrain configurations:
    - Conducive to: … floods, … desertification, … urban pollution, …
    - Conducive to: … build a factory, … plan a bombing, … cultivate marijuana

Spatial scale vs. Semantic complexity

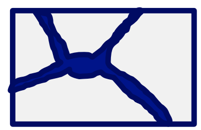


Hierarchical representation

- **Pixel**
  - spectral properties (R,G,B,IR)
  - contrast / texture
  - edges, contours


house

network

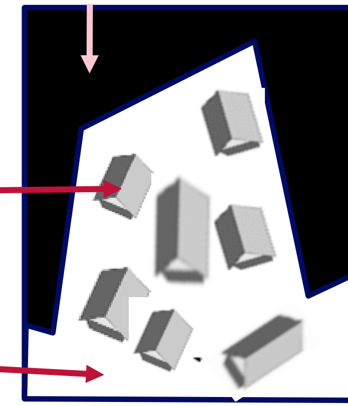


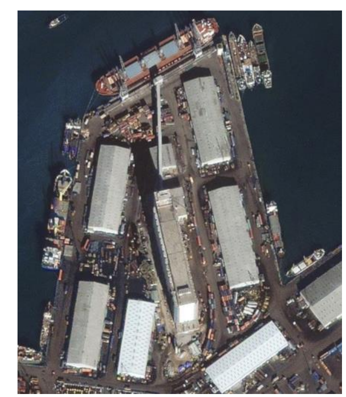
➔ **Objects** ➔ **Scene**

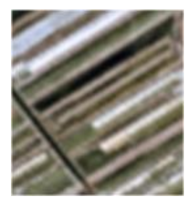
sea

warehouse

wharf






fields

- form / shape
- **Region**
  - content (spectral : textural)

Increasing semantics

RS image processing & hand-crafted feature detection

Mining in RS Image databases

- **Semantic information retrieval :**
  - From ancillary data

- **I – Classical Machine Learning techniques (2000-2012)**
  - Image Processing
  - Hand-crafted feature detection and/or salient point detection
  - Classification in p-dimensional space
    - → few parameters
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- **II – Deep neural networks (2012 - …)**
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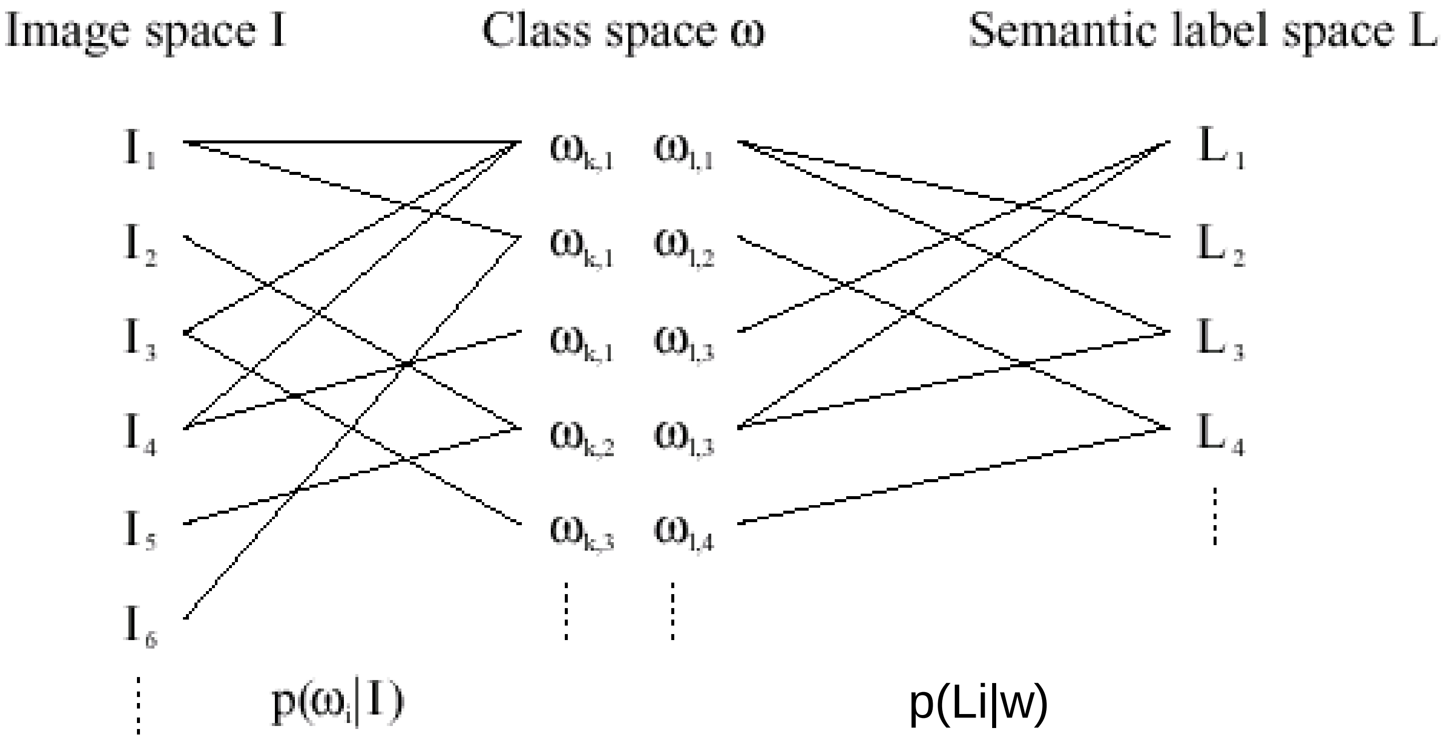
  - Several +/- linear classifiers in cascade
    - → thousands of parameters
    - → hundred of thousands of images as groundtruth

Probabilistic evaluation



Hand crafted features

- **Radiometry**
  - Multispectral : channels
  - Specific combinations for remote sensing : NDVI + IB + ISU
- **Textures**
  - Gabor Filters
  - Haralick cooccurrence matrices and their descriptors
  - Quadratic Mirror Filters (wavelets)
  - Contourlet decomposition
  - Steerable wavelets
  - Markov random fields parameters (Gaussian, Laplacian, Log-laplacian …)
- **Structures**
  - Contours, regions, lakes, forests, deserts
  - Objects : roads, buildings, rivers, lakes
  - Roads, Train or River networks

Some efficient choices

- **Indexing:** small subimages: (~ 64 x 64 pixels) = 320 m x 320 m on the ground for SPOT 5 images
- **Mixed features:**
  - Radiometry (Panchro only)
  - Structure (contours)
  - wavelets : 2 directions, 4 scales
- **Automatic feature selection (supervised:** ReliefF, Fisher FS, SVM-RFE or Unsupervised: MIC (*Max Information Compression*), k-means FS**)**

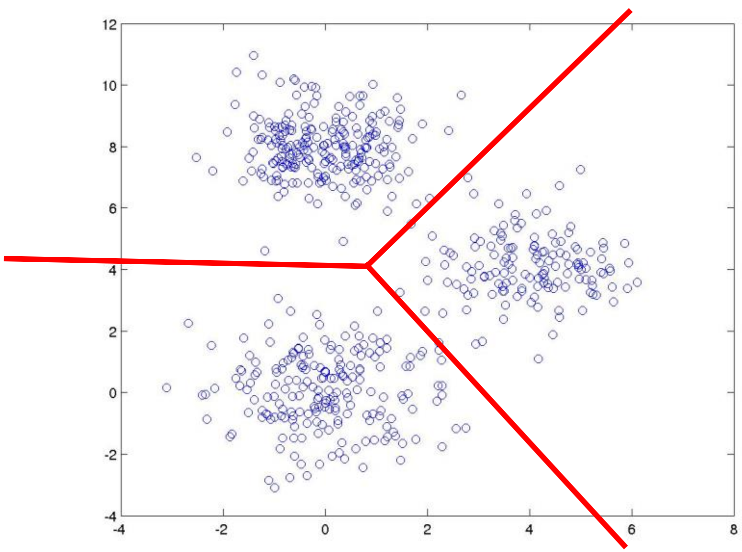
~ 100 features with redundancy **or** ➔ **10 to 20 features without redundancy**

- **Give names to classes (***from label to name***)**
  - Waste fields
  - Cultures
  - Housing
  - Road and river networks

Classification

Many different classifiers:

- MAP & Bayes decision
- K-nearest neigbours
- Graph tree
- Kernel methods (SVM = Support Vector Machine)
- Hierarchical clustering



label = 24

or

Semantic labelling

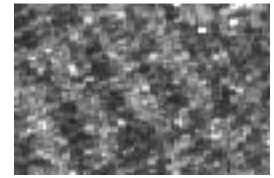
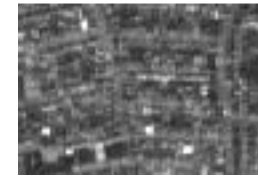
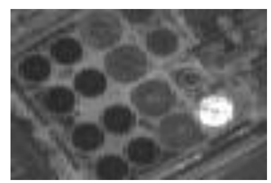
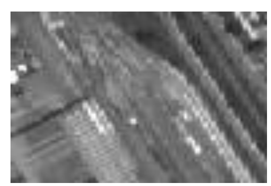
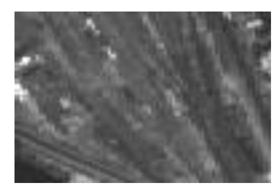
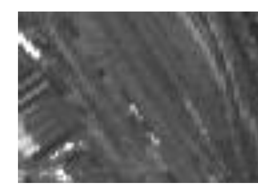
name = « Corn field »

Supervised

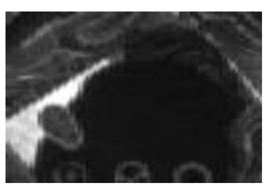
or

Unsupervised

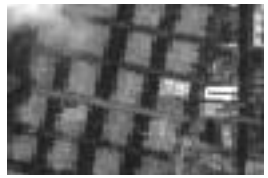
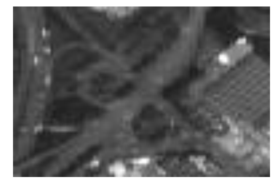
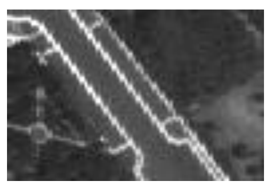
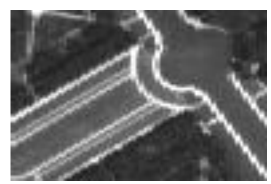
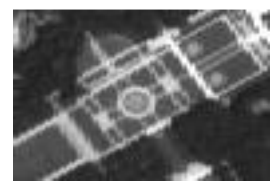
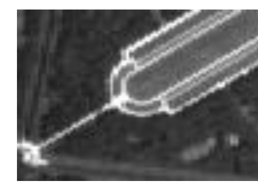
Supervised classes

Residential areas				
Planes				
Industrial tanks & cisterns				
Railway marshalling yard				

Supervised classes

factories				
Dense urban area				
villages				
Urban parks				

Supervised classes

Graveyards				
Road interchange				
Castle parks				
Parking lots				

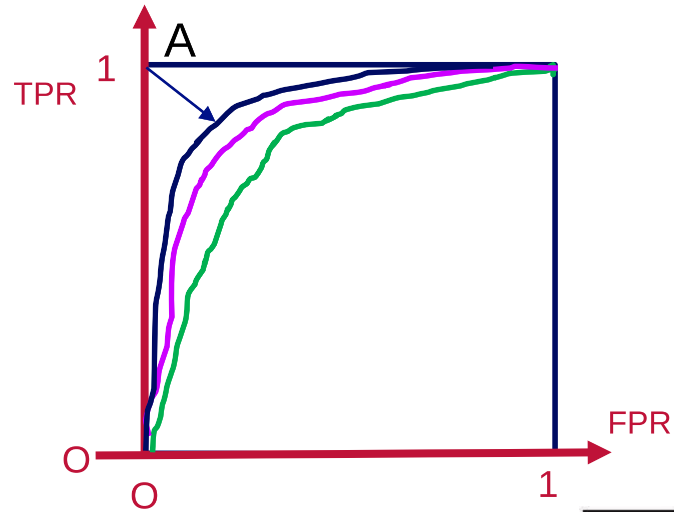
How to express results?

- **Classification rate** 97.3 % (or **error rate**: 2.7 %)
- **Confusion matrix**

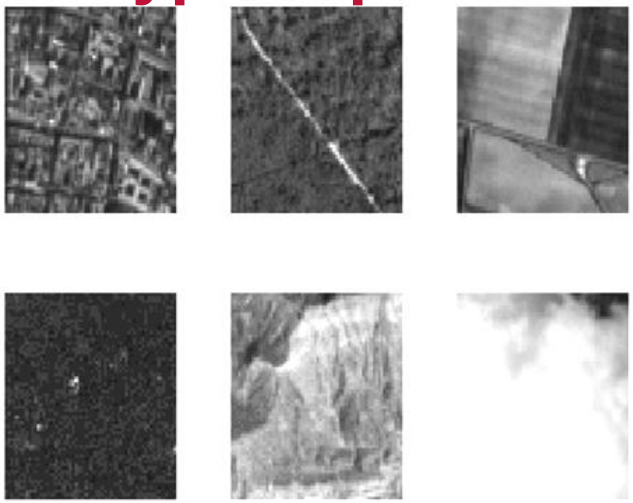
	Present object	Absent object
Positive detection	True positive (TP)	False positive (FP) (type I error)
Negative detection	False negative (type II error)	True Negative

- **Receiver Operating Characteristic (ROC Curve)**

Convert TP and FP into FPR and TPR ϵ [0,1]
Plot TPR = f(FPR) for many different parameters
Without specific instruction, take the closest
point from A = (0,1) as working condition



Typical performances of algorithms



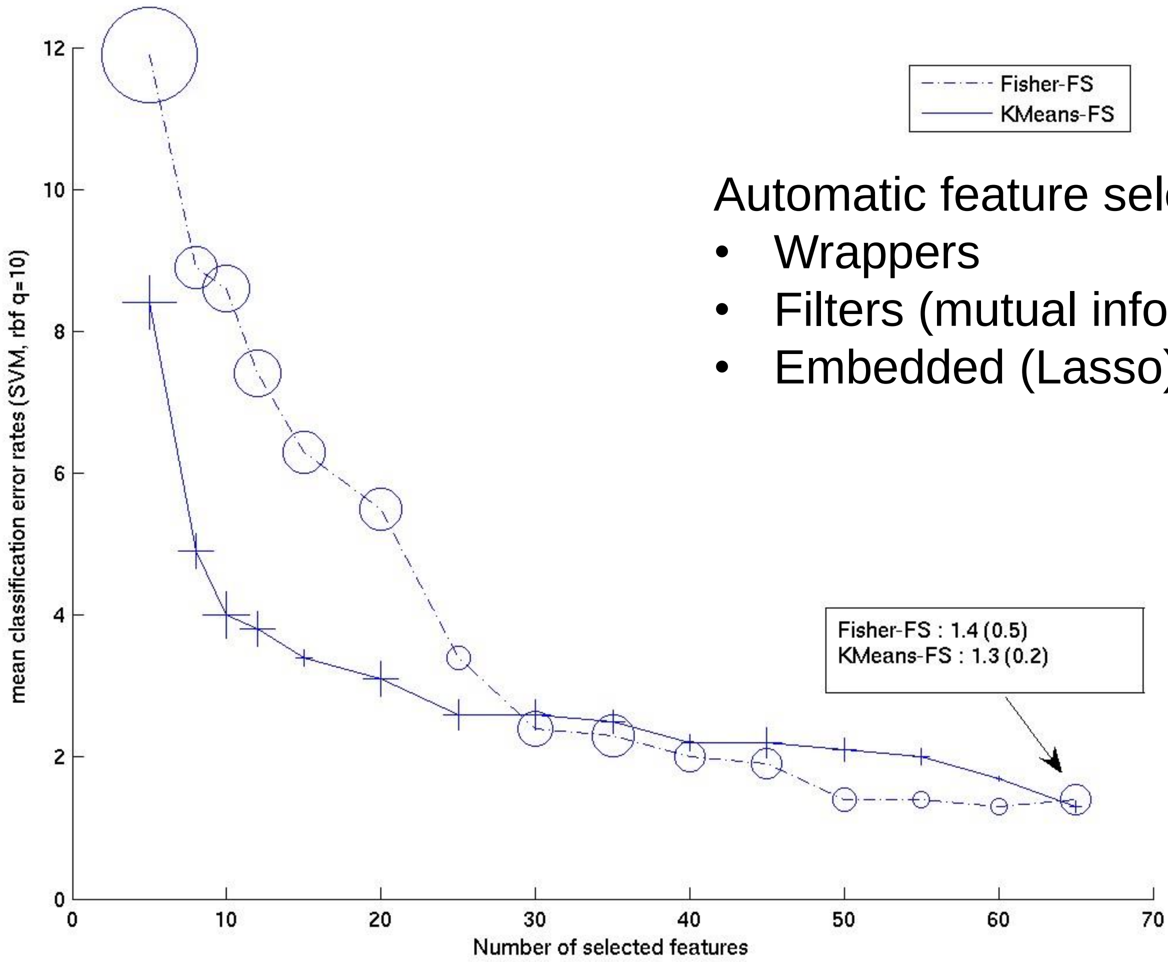
Sub image classification (128 x 128) :

city, wood, fields, sea, desert & clouds

600 images for each class
Results: Gaussian SVM,
Mean error 1.4% ± 0.4%
(147 features, cross validated)

True\Found (%)	city	clouds	desert	fields	woods	sea
city	98.8	0	0	0.5	0	0
cloud	0	99.3	0.2	0	0	0
desert	0	0	99.0	0.3	0	0
fields	0.5	0.2	0.8	98.1	0.3	0.4
woods	0	0.2	0	0	98.0	1.4
sea	0.7	0.3	0	1.0	1.7	98.2

How many features?



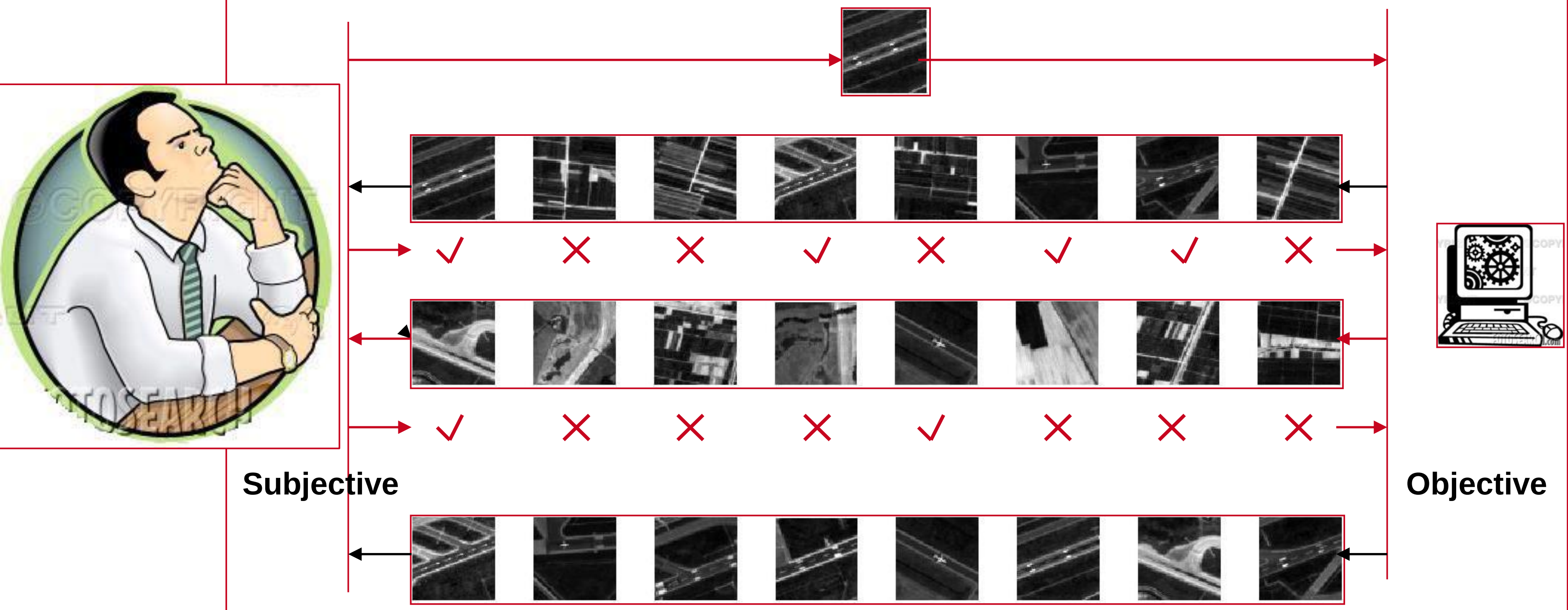
Automatic feature selection

- Wrappers
- Filters (mutual information)
- Embedded (Lasso)

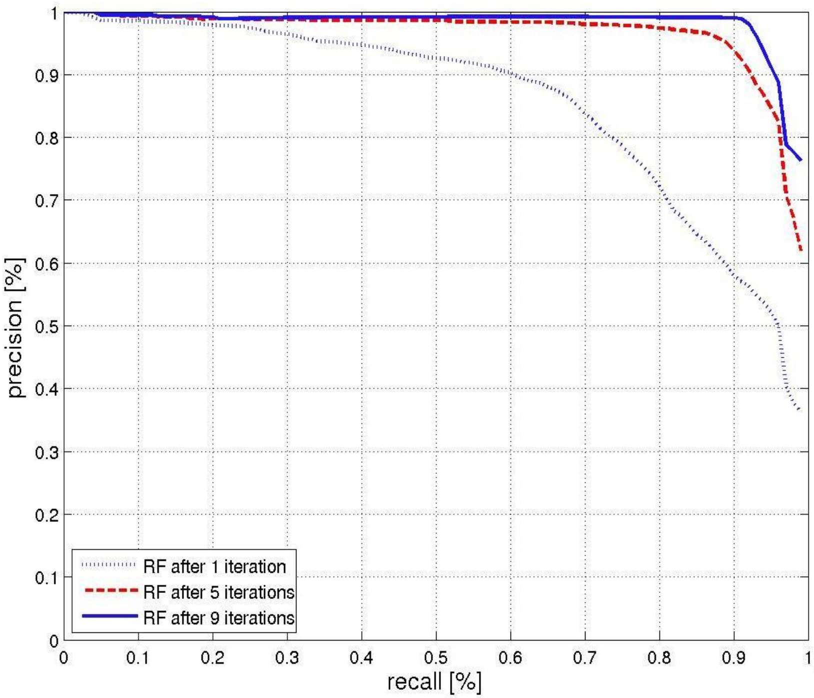
Using a human expert to improve learning

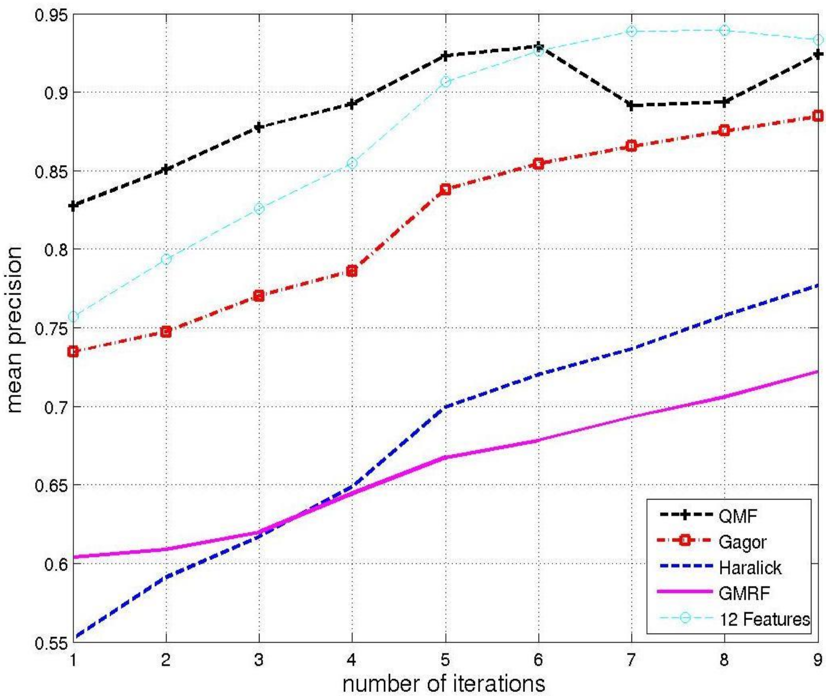
Learning with Relevance feedback

- **Man Machine dialog**



- **Database composed of 600 SPOT5 images divided in 6 classes**
- **Used features: Gabor, Haralick, QMF and GMRF**
- **Gaussian Kernel**
- **System evaluation: Precision-Recall graphs**





Deep Neural Networks

Mining in RS Image databases

- **Semantic information retrieval :**
  - From ancillary data

- **I – Classical Machine Learning techniques (2000-2012)**
  - Image processing
  - Hand-crafted feature detection and/or salient point detection
  - Classification in p-dimensional space
    - ➔ few parameters
    - ➔ few learning images (groundtruth) ~ 1000

- **II – Deep neural networks (2012 - …)**
  - Directly with images as input and/or with extracted features
  - Several +/- linear classifiers in cascade
    - ➔ thousands of parameters
    - ➔ hundred of thousands of images as groundtruth

Some references (dated 01/01/2019)

- Zhang, L., Zhang, L., & Du, B. (2016). Deep learning for remote sensing data: A technical tutorial on the state of the art. *IEEE Geoscience and Remote Sensing Magazine*, *4*(2), 22-40.
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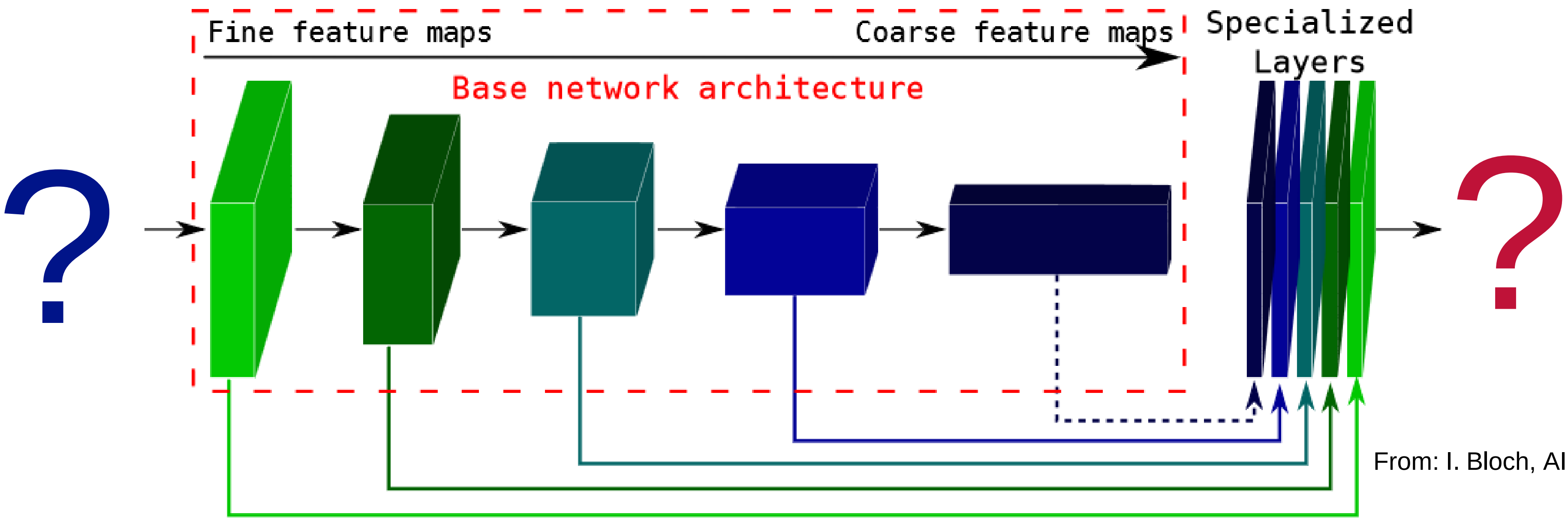
- Marmanis, D., Datcu, M., Esch, T., & Stilla, U. (2016). Deep learning earth observation classification using ImageNet pretrained networks. *IEEE Geoscience and Remote Sensing Letters*, *13*(1), 105-109.
- Kussul, N., Lavreniuk, M., Skakun, S., & Shelestov, A. (2017). Deep learning classification of land cover and crop types using remote sensing data. *IEEE Geoscience and Remote Sensing Letters*, *14*(5), 778-782.
- Penatti, O. A., Nogueira, K., & dos Santos, J. A. (2015). Do deep features generalize from everyday objects to remote sensing and aerial scenes domains?. In *Proceedings of the IEEE conference on computer vision and pattern recognition workshops* (pp. 44-51).

Deep Neural Network



From: I. Bloch, AIC

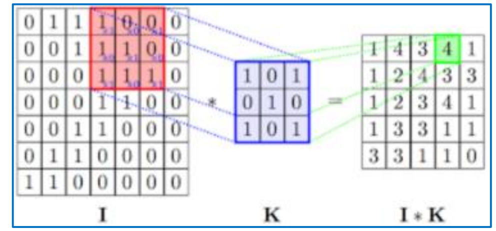
- **Which input?**
- Raw image
- Processed image (filtered, segmented …)
- Feature detected image (classified, edge detected, …)
- Features

- **Which architecture?**
  - # layers,
  - type of layers
- **Which protocole?**
  - Feature learning
  - Fine tuning

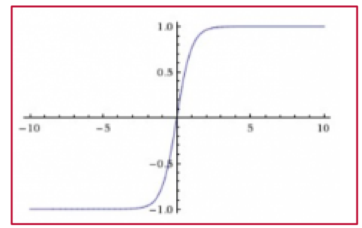
- **Which output?**
- Densely classified image
- Detected targets
- List of targets
- List of Feature

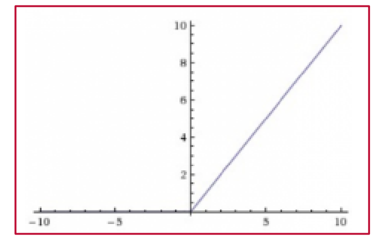
CNN basic components

- **Convolutional layer**: with *rxr* kernel – down scaling

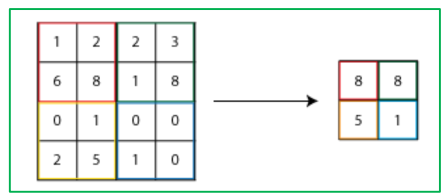


- **Nonlinearity**: sigmoïd or RELU (rectified linear unit)

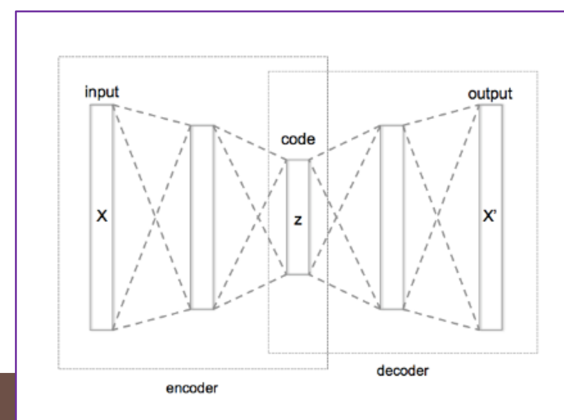




- **Pooling layer**: single value taken from a set of values - ex: *max* on a *rxr* patch

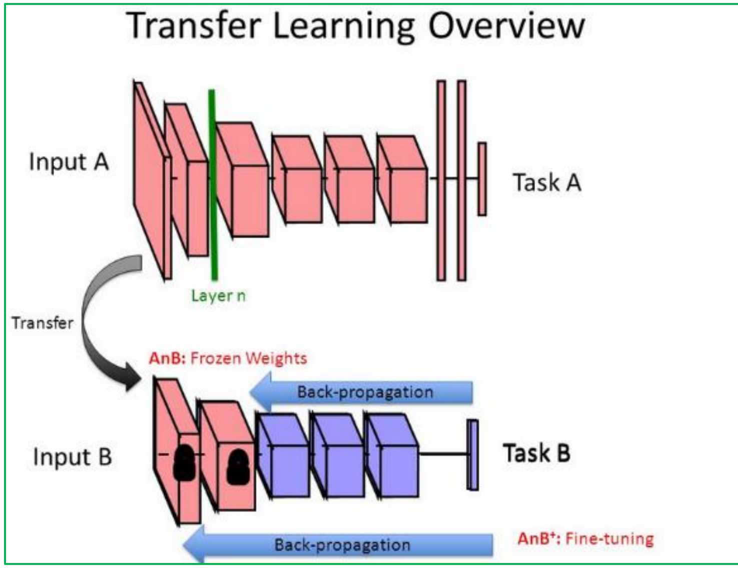


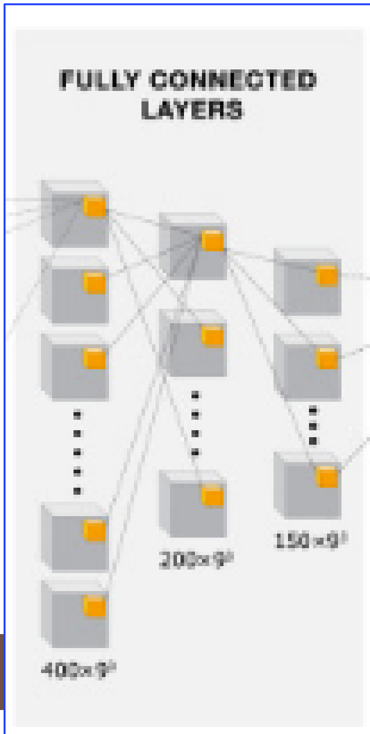
- **Autoencoder**: symetrical NN to reduce the model dimensionality

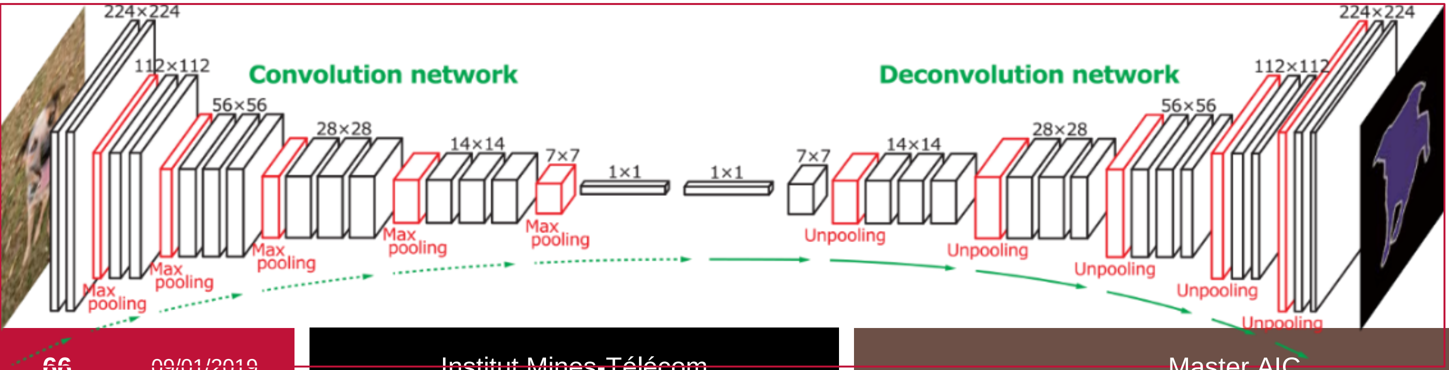


CNN basic components

- **Fully convolutional layer**: to perform a large distance context dependance
- **Transfer coding:** to learn from a database and use for another one
- **Fine Tuning**: to specify a network to a given task after training on a general purpose data base
- **Yoyo architecture** : downsampling for feature extraction then upsampling for fine positioning of targets







Most used components for RS-CNN (2019)

- **CNN from the Pattern Recognition community**
  - AlexNet
  - GoogleNet
  - VGGNet
  - ResNet
  - Inception
- **Training sets**
  - ImageNet (General purpose image library for pattern recognition)
  - UC Merced DataSet (Aerial images / 21 classes)
  - OSM - OpenStreetMap (Aerial Image Database)
  - Google Street Map (hi level semantic)
  - NLCD - USGS data Base (Geological survey)
  - Corinne Landcover (Agriculture & vegetation)
  - Gaofen Image Dataset (GID) (Hi Resolution Satellite)
  - …

Instance # 1 : Basic CNN (DLR)

Marmanis et al. IEEE TGRS, Jan 2016

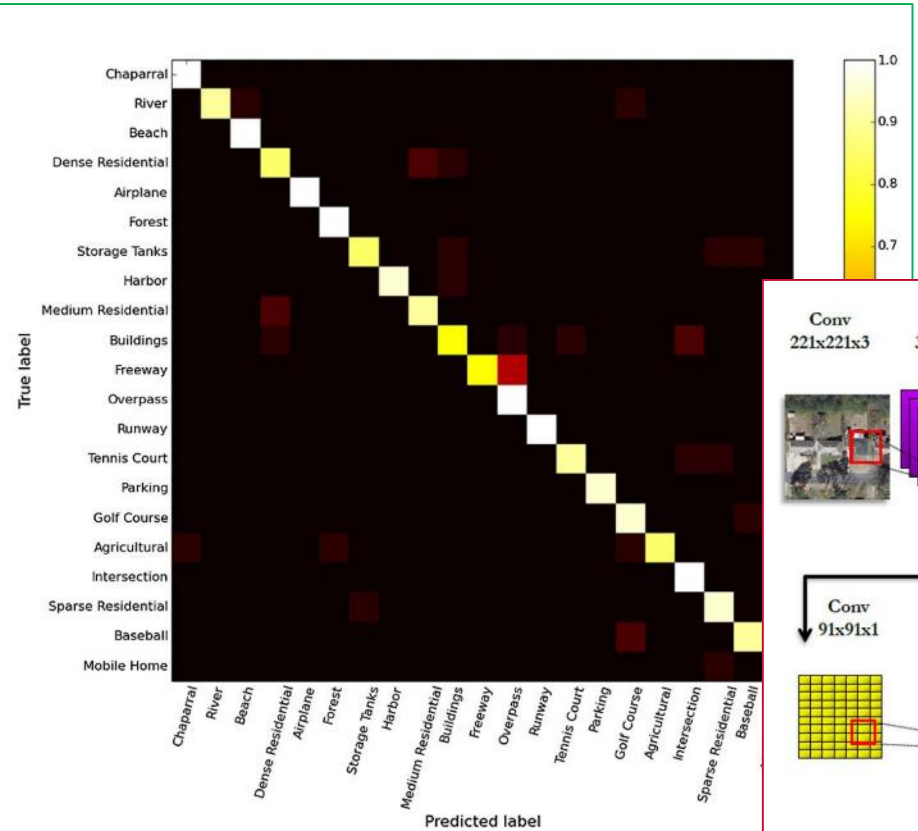
- With UC Merced Land database (aerial / 21 classes)
- With pre-trained CNN (Imagenet)
- Fine-tuned full convolutional layers with enhanced data

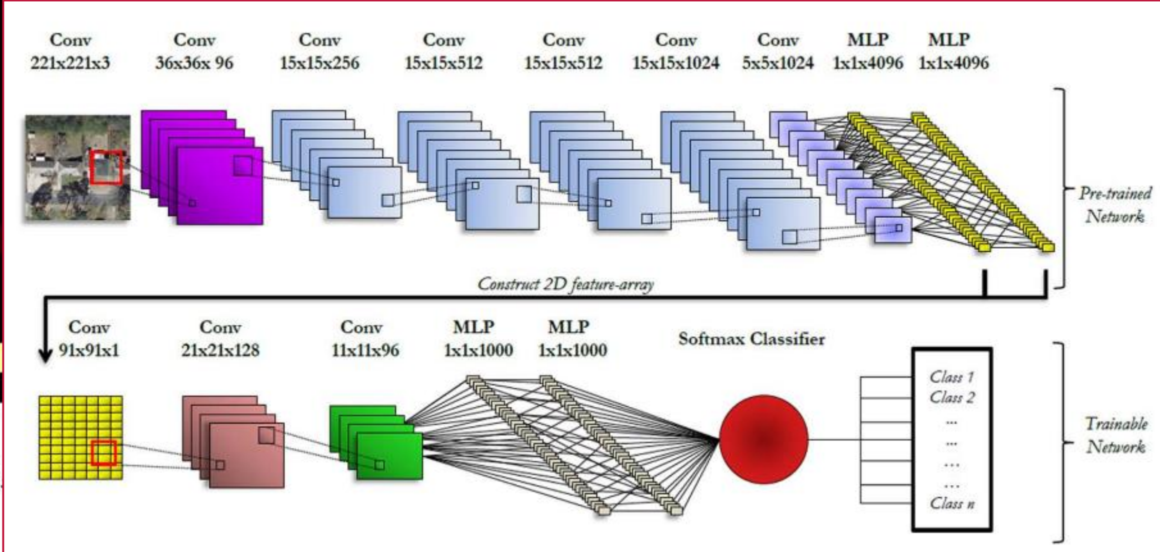
TABLE II
CLASSIFICATION COMPONENTS AND ALGORITHM COMPARISON

Method & Algorithm	Test-set Accuracy
Random Forest with RGB feature	44%
CNN with RGB feature	44.5%
Random Forest with Overfeat features	86.9%
CNN with Overfeat feature	**92.4 %**

TABLE III
METHOD COMPARISON OVER THE UCML BENCHMARK

Method & Algorithm	Test-set Accuracy
BOVW [2]	71.8%
SPMK [1]	74%
SPCK++ [2]	76%
Sparce Coding [4]	81.7%
Salient Unsupervised Learning [6]	82.8% ± 1.18%
MinTree + KD-Tree [3]	83.1% ± 1.2%
CNN with Overfeat feature	**92.4 %**

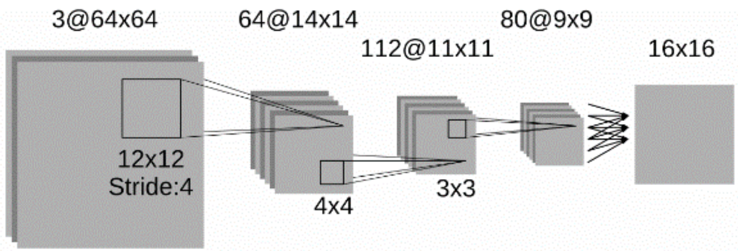




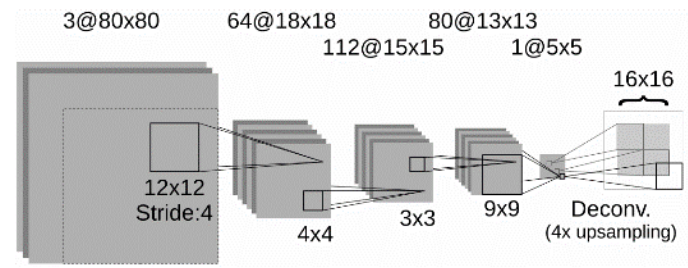
Instance # 2 : fully CNN (Inria)

Maggiori et al. IEEE TGRS, feb 2017

Patch-based CNN



Fully convolutional Patch -based CNN



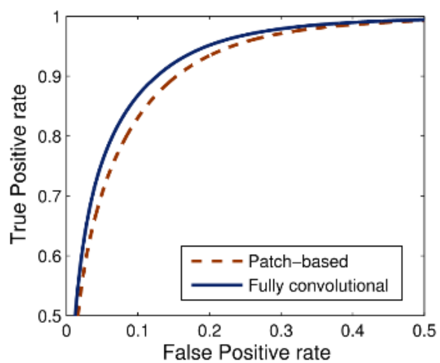


Image | ground truth | patch based | fully convolutional | SVM

(a) (b) (c) (d) (e)

From : Tong et al.
arXiv 1807.05713 - 2018

Instance # 3 : RS CNN (Liemars/Wuhan)

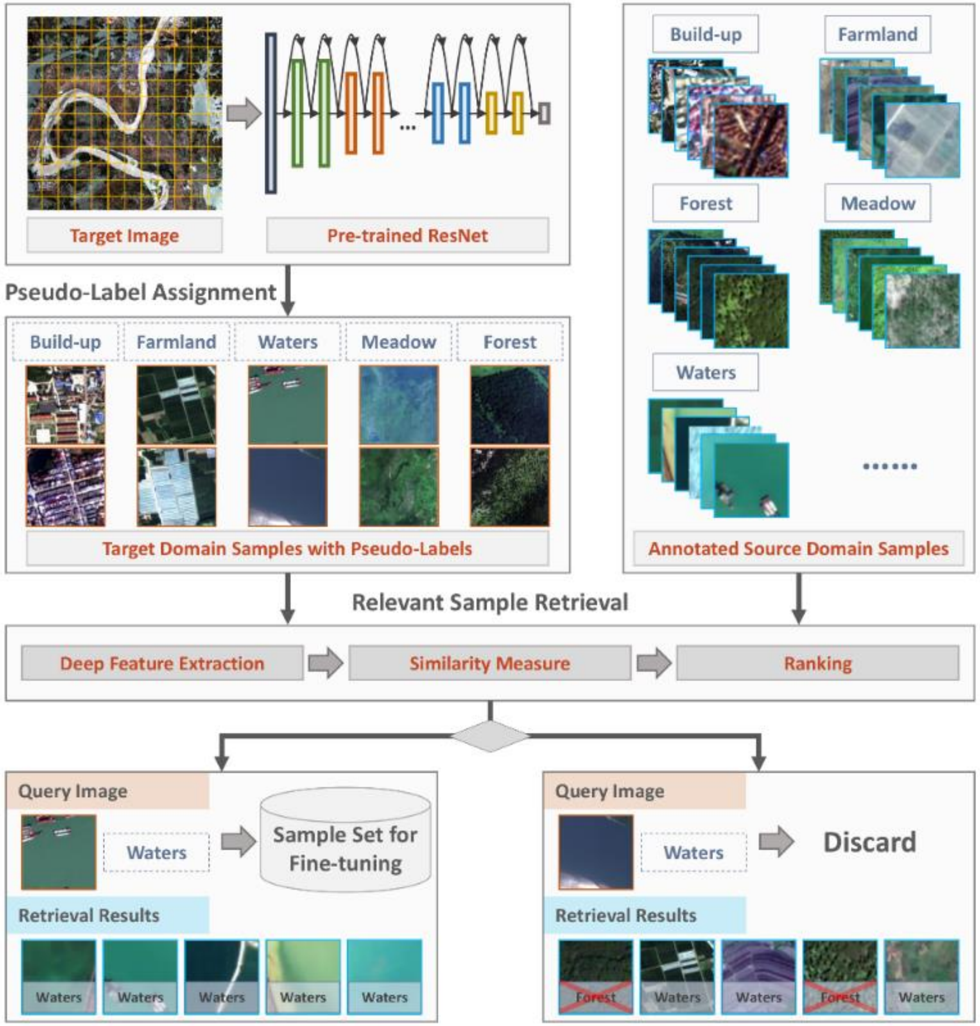
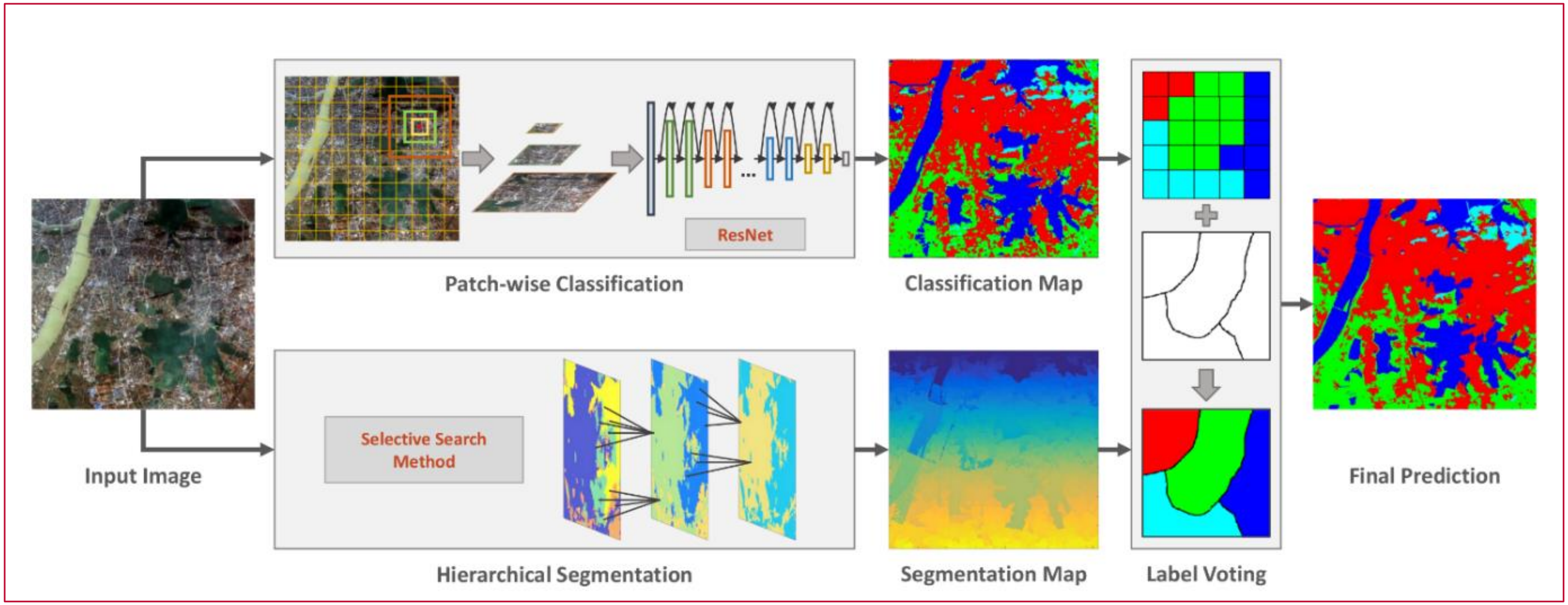


Figure 1: Sample selection for model fine-tuning.

Instance # 3 : RS CNN (Liemars/Wuhan)

Cooperation between classifying (sparse) and segmenting (dense)



From : Tong et al.
arXiv 1807.05713 - 2018

Instance # 3 : RS CNN (Liemars/Wuhan)

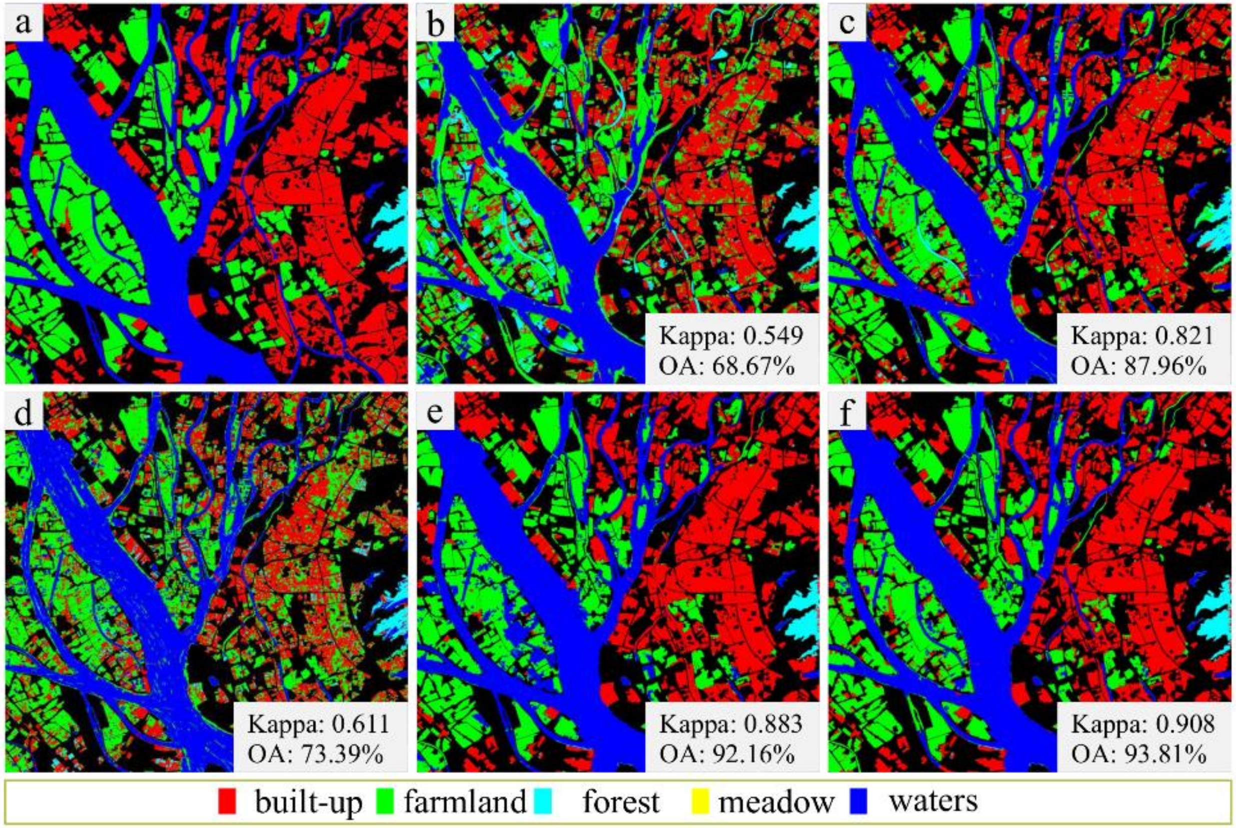


Figure 8: Land-use classification maps of the GF-2 image obtained in Dongguan, Guangdong Province on January 23, 2015. (a) Ground truth. (b)-(f) Results of eCognition, RF+Fusion, SVM+Fusion, PT-GID, and FT-$\mathbf{U}_{tg}$.

From : Tong et al.
arXiv 1807.05713 - 2018

From Low to High Level - Changing the scale

Complexity of images

Analysis window : real size
128 x 128 pixels

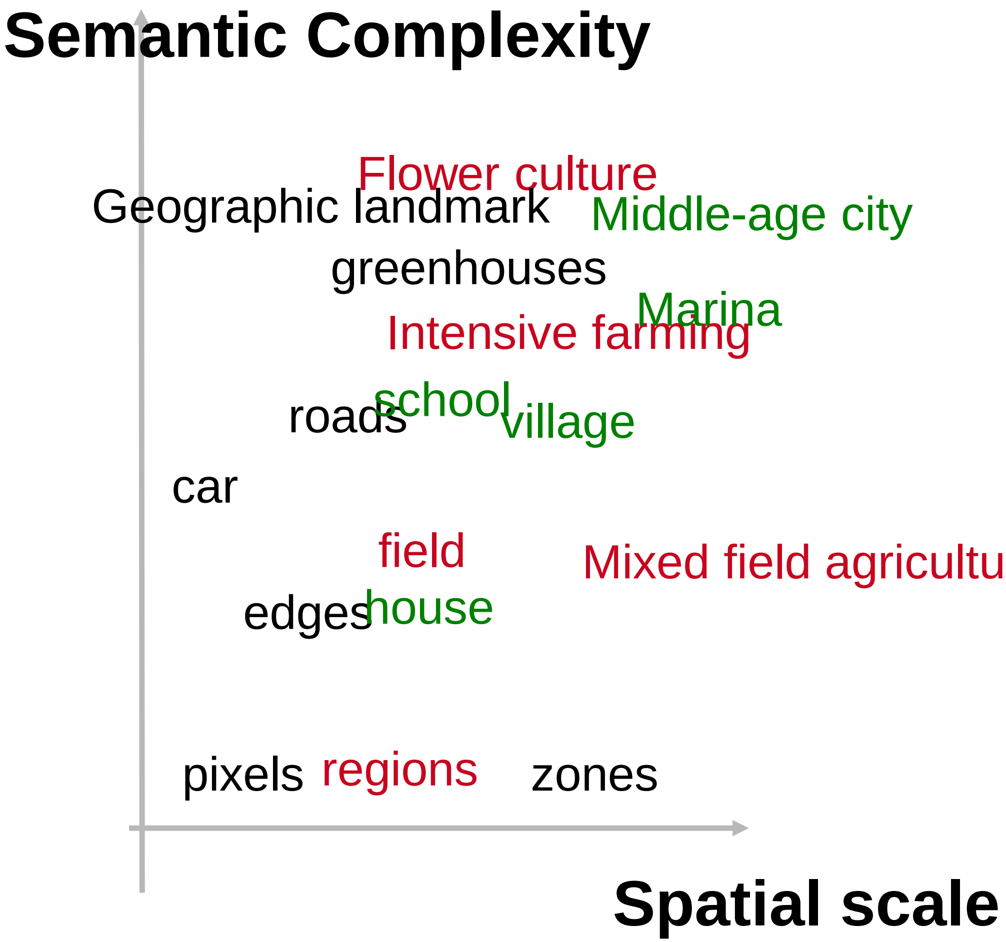
Analysis window : enlarged

Hierarchical representation

Two goals:
- Enlarge the field of view
- Increase the semantic level

Grouping strategy:
- Sliding window
- Pyramid
- Growing and Merging

Decision strategy:
- Bag of Visual Words (BOVW)



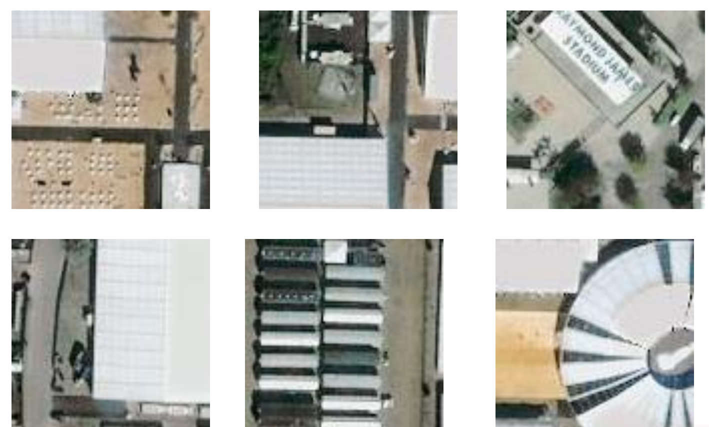
Increasing the semantics

Park = {trees+fields+tracks}

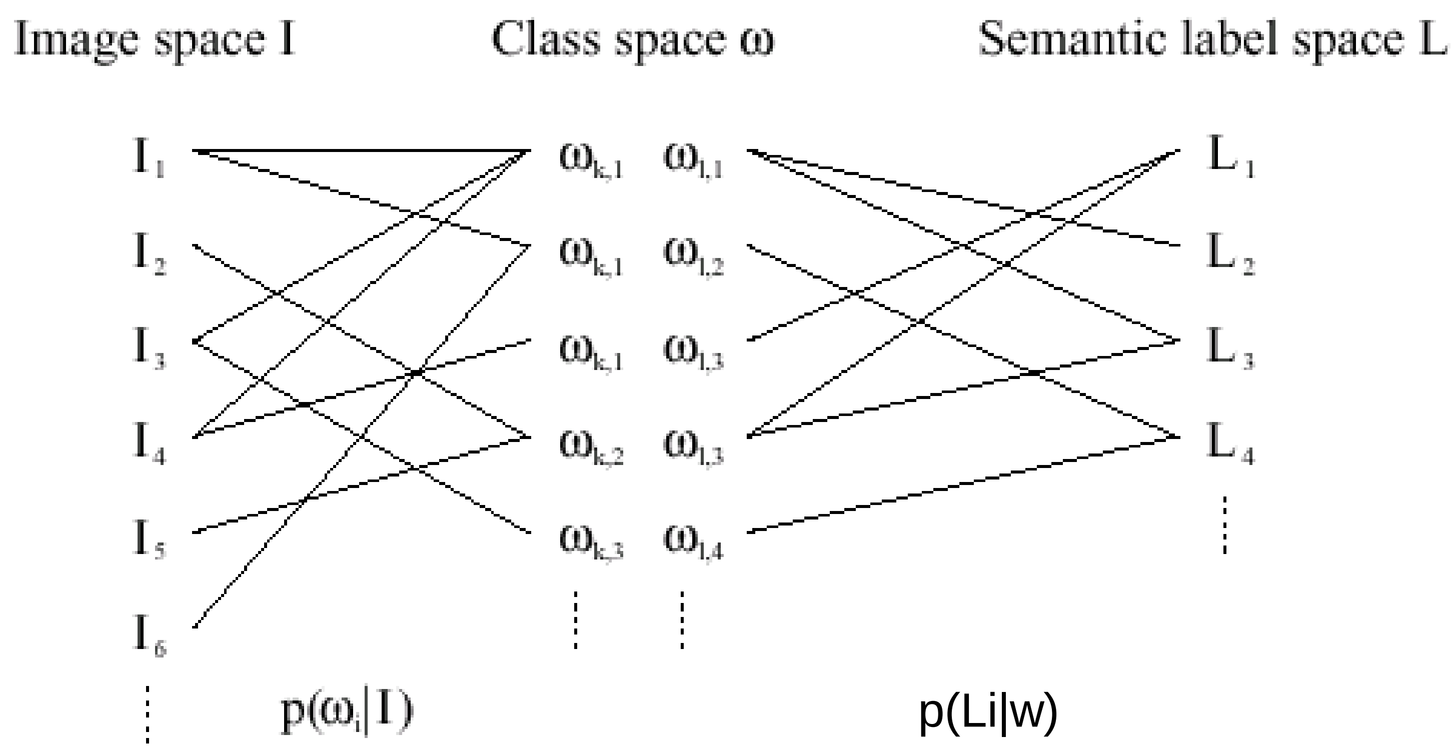
Residential area = {houses + lawns + pools + roads}

Waste area ={waste+lawns+trees+roads}

Commercial area = {buildings+houses+parking lots+ waste



Probabilistic evaluation



Decision making: Bag of Words

- **2 levels** ➔ **H=high** (unknown) **L = low** (known)
- **List of *N* classes at H** = $\{c_1, c_2, \ldots c_N\}$
- **At H : 1 super-region with *n* objects, each** ∈ **1 class = *n* labels** described by the ordered list of the probability (or the occurrence) of each class**:**

$$R_k=\{p_1,p_2, \ldots p_n\}$$

- **Classify H according to the** R_k
  - Naïve Bayes : $\boldsymbol{c}^* = \operatorname{argmax} \boldsymbol{p}(\boldsymbol{c}|\boldsymbol{x}) = \operatorname{argmax} \boldsymbol{p}(\boldsymbol{c}) \prod_{\boldsymbol{k}=1}^{\boldsymbol{n}} \boldsymbol{p}(\boldsymbol{x}_{\boldsymbol{k}}|\boldsymbol{c})$
  - Improving Naïve Bayes:
    - pLSA = Probabilistic Latent Semantic Analysis
    - LDA = Latent Dirichlet Analysis