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## Inferring additional knowledge from $QTC_N$ relations

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### ABSTRACT

It is widely held that people tend to use qualitative rather than quantitative phrases when raising or answering questions about moving objects. Queries about whether an object is moving towards or away from another object or whether objects are getting closer to each other or further away from each other, require qualitative responses. This characteristic should be reflected in a calculus to be used to describe and reason about continuously moving objects. In this paper, we present a qualitative trajectory calculus of relations between two disjoint moving objects, whose movement is constrained by a network. The proposed calculus ( $QTC_N$ ) is formally introduced and illustrated. Particular attention is placed on how to infer additional knowledge from  $QTC_N$  relations by means of composition tables and the transformation of  $QTC_N$  relations into relations defined by the Relative Trajectory Calculus on Networks ( $RTC_N$ ).

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## 1. Introduction

Continuously moving objects are prevalent in many domains such as human movement analysis (such as traffic planning or sports scene analysis) and animal behaviour science [38]. Most applications focus on the positional movement of the object, abstracted to a single point.<sup>1</sup> Recent advances in various positioning technologies (e.g. GPS, LBA, wireless communication) [61] allow the capture and storage of large quantities of such moving point data. Research has addressed the generation [5,43], indexing [1,18,39,47], modelling [29,33,34] and querying [16,25,37,50] of moving objects in spatiotemporal databases. However, only recently has work been conducted in reasoning about the relations between moving point objects and the transitions between these relations, especially in a qualitative framework [7,52]. A specific proposal for qualitative relations between disjoint moving point objects is the Qualitative Trajectory Calculus (QTC), which formally defines qualitative relations between disjoint moving point objects [52].

In this paper, building on [52], QTC is adapted to objects moving in networks, resulting in  $QTC_N$ , and its power for representing and reasoning with qualitative information for objects moving in networks is shown. The paper is structured as follows. Section 2 describes the difference between qualitative and quantitative information and explains why qualitative information can be useful. Section 3 briefly introduces the Qualitative Trajectory Calculus (QTC), which is the basis for the Qualitative Trajectory Calculus for Networks ( $QTC_N$ ) and which is formally outlined in Section 4. The next two sections focus on reasoning with  $QTC_N$  relations. Section 5 presents the composition of  $QTC_N$  relations, while Section 6 shows how

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<sup>1</sup> In the rest of this paper, when we refer to “moving point objects”, we mean such a moving object whose spatial extent has been abstracted to a single point, for example its centroid.

QTC<sub>N</sub> relations can be transformed into relations defined by the Relative Trajectory Calculus on Networks (RTC<sub>N</sub>). Section 7 discusses the usefulness of QTC<sub>N</sub> in possible applications, leading to conclusions and directions for further research in Section 8.

## 2. Qualitative versus quantitative questions

When raising or answering questions about moving objects, both qualitative and quantitative responses are possible. Typically, when responding to a question in a quantitative sense, a predefined unit of a quantity on a continuous measuring scale is used [27]. For example, when asked for the speed of a car, the most likely quantitative answer to that question would be that the car drives at, say, 30 km/h. As Galton says [24], quantitative information is ‘measured by quantity’. In the qualitative approach, the expected answer will be ‘the car is driving slowly’. Qualitative information is concerned with information which ‘depends on a quality’ [24]. A key aspect of qualitative information, is to find ways to represent continuous aspects of the world (space, time, quantity, etc.) by a small set of symbols [7,17]. In the qualitative approach, continuous information is qualitatively discretised by landmarks separating neighbouring open intervals, resulting in discrete quantity spaces [60]. For instance, one might say that a car driving more than 30 km/h is driving fast, and a car driving less than 30 km/h is driving slowly.

When describing the movement of objects, a qualitative description can sometimes give a more satisfactory answer than a quantitative one. For example, if one does not know the exact speed of a car and a bicycle, but one knows that the speed of the car is higher than the speed of the bicycle, one can say that the car is moving faster than the bicycle, labelling this with the qualitative value ‘+’. One could also say that the bicycle is moving slower than the car, by assigning the qualitative value ‘−’ to this relation. Finally, both objects can also move at the same speed, resulting in a qualitative value ‘0’. Note that a distinction is only introduced if it is relevant to the current context [6,7].

Of particular interest in describing qualitative information, are representations that form a finite set of jointly exhaustive and pairwise disjoint (JEPD) relations [46]. In a set of JEPD relations, any two entities are related by exactly one of these relations, and they can be used to represent definite knowledge with respect to the given level of granularity. Incomplete or partial knowledge can be specified by coarse relations representing unions (i.e. disjunctions) of possible JEPD relations.

There are a variety of other grounds why reasoning with qualitative information can be considered complementary to reasoning in a quantitative way, in areas such as artificial intelligence and geographic information science. A key motive is the fact that human beings are more likely to prefer to communicate in qualitative categories, supporting their intuition, rather than using quantitative measures [22]. Representing and reasoning with qualitative information can overcome information overload. Information overload occurs whenever more information has to be handled than can be processed [42]. For example, it is easier to communicate a certain slope characteristic of a region (e.g. flat, steep, hilly) than to provide over a thousand height points [12]. Also, spatial expressions in natural language are rarely precise (e.g. the library is located in the centre of the town; he is moving towards the cinema) [28]; in other words, they usually do not provide enough information to identify the exact geographical location of an object or event [36]. Abstract, non-coordinate-based methods are necessary to deal with these uncertainties [20]. Although reasoning with qualitative information may lead only to a partial answer, such an answer is often better than having no answer at all [21]. In addition, since the information is more granular, qualitative reasoning can be computationally easier than its quantitative counterpart [22]. Finally, qualitative data often provides an ideal way to deliver insights into a particular problem rapidly, in order to identify potential issues that warrant a more detailed quantitative analysis [35].

## 3. The qualitative trajectory calculus

Mereotopology is the most developed area of qualitative spatial reasoning [3,8,9]. However, when it comes to moving point objects, topological models such as the 9-intersection model merely distinguish two trivial topological relations between two point objects: equal and disjoint [15]. Since in the real world the mereotopological relationship between most moving objects is that of being disjoint, and topological models cannot further differentiate between disjoint objects, nor indeed can any purely topological representation, important questions remain unanswered. An obvious example is the case of two airplanes, where it is imperative to know whether both airplanes are likely to stay in a disjoint relation; if not, the consequences are catastrophic. In order to represent and reason about moving objects the Qualitative Trajectory Calculus (QTC) was introduced [52]. This calculus deals with qualitative relations between two disjoint moving point objects. QTC can distinguish a number of basic binary relationships between two moving objects. An object can be moving towards another object; it can be moving away from another object; or it can be stable with respect to the other object. In [52], two QTC calculi are defined. The Qualitative Trajectory Calculus – Double Cross (QTC<sub>C</sub>) [52,54] examines relations between moving point objects based on three reference lines forming a so-called double cross. The Qualitative Trajectory Calculus – Basic (QTC<sub>B</sub>) [55,56] defines these relations by comparing differences in distance over time. In order to elaborate a QTC calculus for network-based moving objects, we will build on QTC<sub>B</sub> since QTC<sub>C</sub> is not suitable to use in a network environment, as it utilises a direction-based spatial reference for defining relations. In the remainder of this section we will briefly introduce QTC<sub>B</sub> as defined in [52].

In QTC, time is assumed to be continuous and linear. This time line can be represented by the set of real numbers ( $\mathbb{R}$ ) and it has a total order associated with it. This implies that one cannot identify two time points next to each other. The density of

$\mathbb{R}$  allows no notion “nextness” [41]. In order to formally define the qualitative relations available in  $QTC_B$ , we introduce the following notations and definitions:

- $k|t$  denotes the position of an object  $k$  at time point  $t$ .
- $v_k|t$  denotes the speed of  $k$  at time point  $t$ .
- $d(x,y)$  denotes the distance between two positions  $x$  and  $y$ .

**Definition 3.1.** A relation in  $QTC_B$  at level 1 between a first object  $k$  and a second object  $l$  at a time point  $t$  is defined by a two character label. This label represents the following two relationships:

1. Movement of  $k$  with respect to  $l$  at  $t$ :

–:  $k$  is moving towards  $l$ :

$$\begin{aligned} & \exists t_1(t_1 < t \wedge \forall t^-(t_1 < t^- < t \rightarrow d(k|t^-, l|t) > d(k|t, l|t)) \wedge \\ & \exists t_2(t < t_2 \wedge \forall t^+(t < t^+ < t_2 \rightarrow d(k|t, l|t) > d(k|t^+, l|t))) \end{aligned} \quad (1)$$

+ :  $k$  is moving away from  $l$ :

$$\begin{aligned} & \exists t_1(t_1 < t \wedge \forall t^-(t_1 < t^- < t \rightarrow d(k|t^-, l|t) < d(k|t, l|t)) \wedge \\ & \exists t_2(t < t_2 \wedge \forall t^+(t < t^+ < t_2 \rightarrow d(k|t, l|t) < d(k|t^+, l|t))) \end{aligned} \quad (2)$$

0:  $k$  is stable with respect to  $l$ : all other cases

2. Movement of  $l$  with respect to  $k$  at  $t$ :

Can be described as in 1, with  $k$  and  $l$  interchanged, hence:

$$- : lis \text{ moving towards } k \quad (3)$$

$$+ : lis \text{ moving away from } k \quad (4)$$

$$0 : lis \text{ stable with respect to } k$$

**Definition 3.2.** A relation in  $QTC_B$  at level 2 between a first object  $k$  and a second object  $l$  at a time point  $t$  is defined by a three character label. The first two characters are defined as in Definition 3.1. The third character represents the relative speed and is defined as follows:

3. Relative speed of  $k$  with respect to  $l$  at  $t$ :

–:  $k$  is moving slower than  $l$ :

$$v_k|t < v_l|t \quad (5)$$

+ :  $k$  is moving faster than  $l$ :

$$v_k|t > v_l|t \quad (6)$$

0:  $k$  and  $l$  are moving equally fast:

$$v_k|t = v_l|t \quad (7)$$

#### 4. The qualitative trajectory calculus on networks

Having introduced QTC, we will now elaborate the definition of QTC on networks. Moreira et al. [40] differentiate between two kinds of moving objects: objects that have a completely free trajectory, only constrained by the dynamics of the object itself (e.g. a bird flying through the sky) and objects that have a constrained trajectory (e.g. a train on a railway track). Many trajectories involving humans are bounded to a network. Hence, there is a need to develop a calculus that defines qualitative relations between two disjoint moving objects on trajectories constrained by a network. An informal description and definition of  $QTC_N$  was presented in [4,52,53], while a conceptual neighbourhood diagram for  $QTC_N$  was presented in [4]. In this paper,  $QTC_N$  is defined formally. Also, we explore the power of this calculus to infer additional information from the basic  $QTC_N$ -relations.

##### 4.1. Definitions and restrictions concerning networks and moving objects

A network, such as a road, rail or river network, is usually described as a set of interconnected linear spatial features; each such linear feature can be regarded as a curve, describing a linear path through the space it is embedded in. Thus, in essence, a network is a co-dimensional structure. The concept of co-dimensionality can be used to express the difference in dimension between spatial entities (point: zero-dimensional; line: one-dimensional, region: two-dimensional, etc.) and the space they are embedded in [24]. In the case of a network, one-dimensional structures (a set of interconnected lines) are embedded

in a two dimensional (co-dimension one) or three dimensional space (co-dimension two). Therefore, we assume an underlying spatial framework  $S$  for specifying locations. Typically this would be  $\mathbb{R}^2$ , but  $S$  could be any set with a metric distance function  $d(x,y)$  obeying the triangle inequality, and a notion of curve defined, such that  $\text{curves}(S)$  denotes the set of simple non-closed curves in  $S$ .

In order to formally define  $\text{QTC}_N$  relations for two moving point objects, using the network in which they are embedded as a reference frame, three functions are defined on curves. For any curve  $c$ :

- $\text{len}(c)$  denotes the length of  $c$ ;
- $\text{end}(c,x)$  is true if  $x$  is an endpoint of  $c$ ;
- if  $x$  and  $y$  are two points incident in  $c$ , then  $\text{subcurve}(c,x,y)$  denotes the subcurve of  $c$  between and including  $x$  and  $y$ .

The network in which objects move in  $\text{QTC}_N$  is characterised by a graph, whose edges represent a set of linear features and the nodes of the graph represent the endpoints which bound these linear features (Definition 4.1). A function  $\text{loc}(x)$  embeds these nodes and edges in the spatial framework  $S$  (Definitions 4.2 and 4.3). As stated above, the edges should represent simple non-closed curves. To formally define this property, we do not allow two nodes to lie at the same location (Restriction 4.1), the edges should be bounded by two different nodes (Definition 4.3) and two different edges can only intersect at their respective endpoints (Restriction 4.2). The number of edges representing curves which intersect at a node denotes the degree of that node (Definition 4.4).

**Definition 4.1.** If  $W$  is a network then  $\text{nodes}(W)$  is its set of nodes and  $\text{edges}(W)$  is its set of edges.

**Definition 4.2.** If  $n$  is a node then  $\text{loc}(n) \in S$  is the spatial location of  $n$  in  $S$ .

**Restriction 4.1.** If  $W$  is a network then  $\forall \{n_i, n_j\} \subseteq \text{nodes}(W) [\text{loc}(n_i) \neq \text{loc}(n_j)]$ .

**Definition 4.3.** If  $W$  is a network and  $e \in \text{edges}(W)$  then  $\text{loc}(e) \in \text{curves}(S)$  is the curve denoted by  $e$  in  $S$ , and  $[\exists \{n_i, n_j\} \subseteq \text{nodes}(W) [\text{end}(\text{loc}(e), \text{loc}(n_i)) \wedge \text{end}(\text{loc}(e), \text{loc}(n_j))]]$ .

**Restriction 4.2.** If  $W$  is a network then  $\forall e_i \forall e_j [ \{e_i, e_j\} \subseteq \text{edges}(W) \Rightarrow \text{loc}(e_i) \cap \text{loc}(e_j) \subseteq \{\text{loc}(n) : n \in \text{nodes}(W)\} ]$

**Definition 4.4.** If  $W$  is a network then the degree of a node  $n \in \text{nodes}(W)$ ,  $\text{deg}(n) = |\{e : e \in \text{edges}(W) \wedge \text{loc}(n) \in \text{loc}(e)\}|$

The movement of objects in  $\text{QTC}_N$  is restricted by the network, which implies that the location of an object should at all times be situated on an edge (Definition 4.5). As stated in Section 3, QTC only considers relations between disjoint objects, thus, two different objects cannot be at the same place at the same time (Restriction 4.3).

**Definition 4.5.** An object  $k$  at a time point  $t$  is located in a network  $W$  iff  $\exists e \in \text{edges}(W) [k|t \in \text{loc}(e)]$ .

**Restriction 4.3.** All two non identical objects  $k$  and  $l$  are not instantaneously coincident at a time point  $t$ :

$$\forall t \forall k \forall l [k \neq l \Rightarrow k|t \neq l|t].$$

To relate two objects in  $\text{QTC}_N$ , there needs to be at least one path between both objects (see Section 4.2). A path is composed of a connected sequence of edges. Since the objects do not necessarily lie at the endpoint of an edge, a notion of *edge segments* is required (Definition 4.6). The notation  $\text{seg}(e,x,y)$  denotes an edge segment which represents (i.e. whose location is) that part of an edge  $e$  between a point  $x$  and an endpoint  $y$  of the edge  $e$  (including  $x$  and  $y$ ). If  $x$  is the other endpoint of  $e$ , then the edge segment equals the edge  $e$  (as a special case). Thus, a path between two objects is composed of a sequence such that the first and last elements are edge segments on which the two objects are located (possibly the same segment), and any intermediate edges form a connected path, such that no edge occurs more than once (Definition 4.7). The length of a path is defined as the sum of the length of its edges and edge segments (Definition 4.8). A shortest path is defined as a path such that there is no path having a shorter length between the same two nodes (Definition 4.9). There can be more than one shortest path between two objects at the same time. If, in this special case, the first edge segment is different for all of these shortest paths, we refer to these shortest paths as bifurcating shortest paths (Definition 4.10 and Fig. 1).

**Definition 4.6.** If  $e$  is an edge then  $e' = \text{seg}(e,x,y)$  is an *edge segment* of  $e$  iff  $\text{end}(e,y) \wedge x \in \text{loc}(e) \wedge x \neq y \wedge \text{loc}(e') = \text{subcurve}(e,x,y)$

**Definition 4.7.** A path  $p$  between two different objects  $k$  and  $l$  in a network  $W$  at time point  $t$  is a sequence  $\langle e_1, \dots, e_m \rangle$ ,  $m \geq 1$  such that

$$\text{end}(e_1, k|t) \wedge \text{end}(e_m, l|t) \wedge \{e_2, \dots, e_{m-1}\} \subseteq \text{edges}(W) \wedge \exists (e'_1, e'_m, u, v) [\{e'_1, e'_m\} \subseteq \text{edges}(W) \wedge e_1 = \text{seg}(e'_1, k|t, u) \wedge e_m = \text{seg}(e'_m, l|t, v)] \wedge \forall 1 \leq i < m [\text{loc}(e_i) \cap \text{loc}(e_{i+1}) \neq \emptyset] \wedge \forall 1 \leq i < j \leq m [\text{loc}(e_i) \neq \text{loc}(e_j)].$$

**Definition 4.8.**  $|p| = \sum_{e \in \text{plen}} (\text{loc}(e))$  is the length of a path  $p$ .

**Definition 4.9.** A shortest path  $SP_{Wkl}^t$  in a network  $W$  from an object  $k$  to an object  $l$  at a time point  $t$  is a path  $p$  such that there is no path from  $k$  to  $l$  of length less than  $|p|$ . We may write  $SP_{Wkl}^t(p)$  when  $p$  is such a shortest path.

**Definition 4.10.** If there are at least two different shortest paths  $p_1, \dots, p_m$  from an object  $k$  to an object  $l$  at a time point  $t$ , then there is a bifurcating shortest path from  $k$  to  $l$  at  $t$  iff

$$\exists n \in \text{nodes}(W) [k|t = \text{loc}(n)] \wedge \forall (p_i = \langle e_1, \dots, e_i \rangle, p_j = \langle e'_1, \dots, e'_j \rangle) [1 \leq i < j \leq m] \Rightarrow e_i \neq e'_1$$

It is obvious that objects moving on a network do not always move along the same edge simultaneously. Objects can move from one edge to another. When doing so, they pass a node (Definition 4.11). If  $k$  passes a node lying at the intersection of two edges  $e^-$  and  $e^+$  at time point  $t$ , and neither of these edges is along a shortest path from  $k$  to  $l$  at  $t$ , this event is referred to as a shortest path omitting node pass event (Definition 4.12 and Fig. 2).

**Definition 4.11.** An object  $k$  on a network  $W$  is in a node pass event along edges  $e^-, e^+$  at a time point  $t$ ,  $\text{NPE}(k, t, e^-, e^+)$  iff

$$\{e^-, e^+\} \subseteq \text{edges}(W) \wedge e^- \neq e^+ \wedge \exists (t^-, t^+) [t^- < t \wedge t < t^+] \wedge \forall t_1 [t^- \leq t_1 \leq t] \Rightarrow k|t_1 \in \text{loc}(e^-) \wedge \forall t_2 [t \leq t_2 \leq t^+] \Rightarrow k|t_2 \in \text{loc}(e^+).$$

**Definition 4.12.** An object  $k$  on a network  $W$  is in a shortest path omitting node pass event with respect to another object  $l$  at a time point  $t$  iff  $\text{NPE}(k, t, e^-, e^+) \wedge \forall p [SP_{Wkl}^t(p) \Rightarrow [e^- \notin p \wedge e^+ \notin p]]$

4.2. Definition of  $QTC_N$  relations

The reference used to qualitatively assess the relation between two objects is the distance measured along the shortest path. If there is no path between two objects, then there is no  $QTC_N$  relation between these objects. Put differently, these objects are either not moving along a network, or they occupy disjoint parts of a disconnected network and will hence re-

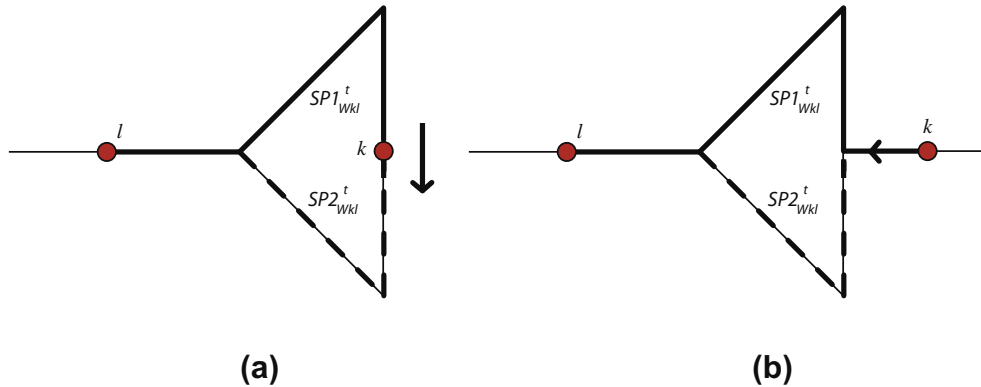


Fig. 1. Bifurcating (a) and non-bifurcating shortest paths (b).

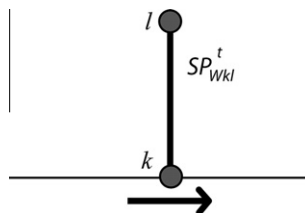


Fig. 2. A shortest path omitting node pass event.

main disjoint. The shortest path is chosen because it seems to encode what it means for one object to approach or recede from another object in a network. (In Euclidean space, one might naturally define approaching in terms of an angular measure, but this is not applicable in networks, and shortest path is the appropriate equivalent notion.) In a network, an object can only approach another object if and only if it moves along a shortest path between these two objects [4]. Using this property, we can state that an object  $k$  can only approach another object  $l$  at a time point  $t$  in a network  $W$  if it does not lie on  $SP_{Wkl}^t$  immediately before  $t$  and if it lies on  $SP_{Wkl}^t$  immediately after  $t$ .  $k$  moves away from  $l$  at  $t$  if it is on  $SP_{Wkl}^t$  immediately before  $t$  and if it does not lie on  $SP_{Wkl}^t$  immediately after  $t$ . If  $k$  lies on  $SP_{Wkl}^t$  only at  $t$ , but not immediately before and immediately after  $t$ , or if  $k$  is on  $SP_{Wkl}^t$  immediately before and immediately after  $t$ , then  $k$  is stable with respect to  $l$  (although this relation may only last for an instantaneous moment in time). This property allows reformulating conditions (1) and (2) of Definition 3.1 for the construction of the first level relation of  $QTC_B$  to a  $QTC_N$  setting.

**Definition 4.13.** A relation in  $QTC_N$  at level 1 between a first object  $k$  and a second object  $l$  on a network  $W$  at a time point  $t$  is defined by a two character label. This label represents the following two relationships:

1. Movement of  $k$  with respect to  $l$  at  $t$ :

–:  $k$  is moving towards  $l$ :

$$\begin{aligned} & \exists t_1 (t_1 < t \wedge \forall t^- (t_1 < t^- < t \rightarrow k|t^- \notin SP_{Wkl}^t)) \wedge \\ & \exists t_2 (t < t_2 \wedge \forall t^+ (t < t^+ < t_2 \rightarrow k|t^+ \in SP_{Wkl}^t)) \end{aligned} \quad (8)$$

+ :  $k$  is moving away from  $l$ :

$$\begin{aligned} & \exists t_1 (t_1 < t \wedge \forall t^- (t_1 < t^- < t \rightarrow k|t^- \in SP_{Wkl}^t)) \wedge \\ & \exists t_2 (t < t_2 \wedge \forall t^+ (t < t^+ < t_2 \rightarrow k|t^+ \notin SP_{Wkl}^t)) \end{aligned} \quad (9)$$

0:  $k$  is stable with respect to  $l$  (all other cases):

$$\exists t_1 (t_1 < t \wedge \forall t^- (t_1 < t^- < t \rightarrow k|t^- \in SP_{Wkl}^t)) \wedge \quad (10)$$

$$\exists t_2 (t < t_2 \wedge \forall t^+ (t < t^+ < t_2 \rightarrow k|t^+ \in SP_{Wkl}^t))$$

$$\exists t_1 (t_1 < t \wedge \forall t^- (t_1 < t^- < t \rightarrow k|t^- \notin SP_{Wkl}^t)) \wedge \quad (11)$$

$$\exists t_2 (t < t_2 \wedge \forall t^+ (t < t^+ < t_2 \rightarrow k|t^+ \notin SP_{Wkl}^t))$$

2. Movement of  $l$  with respect to  $k$  at  $t$ :

Can be described as in 1 with  $k$  and  $l$  interchanged, hence:

$$- : lis \text{ moving towards } k \quad (12)$$

$$+ : lis \text{ moving away from } k \quad (13)$$

$$0 : lis \text{ stable with respect to } k \quad (14, 15)$$

The second level relation of  $QTC_N$  is defined identically to the definition in  $QTC_B$  – cf. Definition 3.2 (Definition 4.14).

**Definition 4.14.** A relation in  $QTC_N$  at level 2 between a first object  $k$  and a second object  $l$  in a network  $W$  at a time point  $t$  is defined by a three character label. The first two characters are defined as in Definition 4.13. The third character represents the relative speed and is defined as follows:

3. Relative speed of  $k$  with respect to  $l$  at  $t$ :

–:  $k$  is moving slower than  $l$ :

$$v_k|t < v_l|t \quad (16)$$

+ :  $k$  is moving faster than  $l$ :

$$v_k|t > v_l|t \quad (17)$$

0:  $k$  and  $l$  are moving equally fast:

$$v_k|t = v_l|t \quad (18)$$

Based on Definition 4.14, we can construct all canonical cases for  $QTC_N$  relations at level 2. Let us analyse all possible movements of a first object  $k$  with respect to a second object  $l$  in a  $QTC_N$  relation at time point  $t$ .  $k$  can be stationary, i.e. not moving with respect to the network, or not. If  $k$  is stationary at  $t$ , it will be located on a shortest path to  $l$  at  $t$  (and immediately before and immediately after  $t$ ), and therefore the definition yields '0' for the first character in the label (i). If  $k$  is moving at  $t$ , then by definition there are four possibilities (ii – v).  $k$  can be on a shortest path to  $l$  immediately before  $t$  and not

immediately after  $t$ , which returns '+' for the first character in the label (ii),  $k$  can be on a shortest path to  $l$  immediately after  $t$  but not immediately before  $t$ , which returns '-' for the first character in the label (iii). When  $k$  is in a shortest path omitting

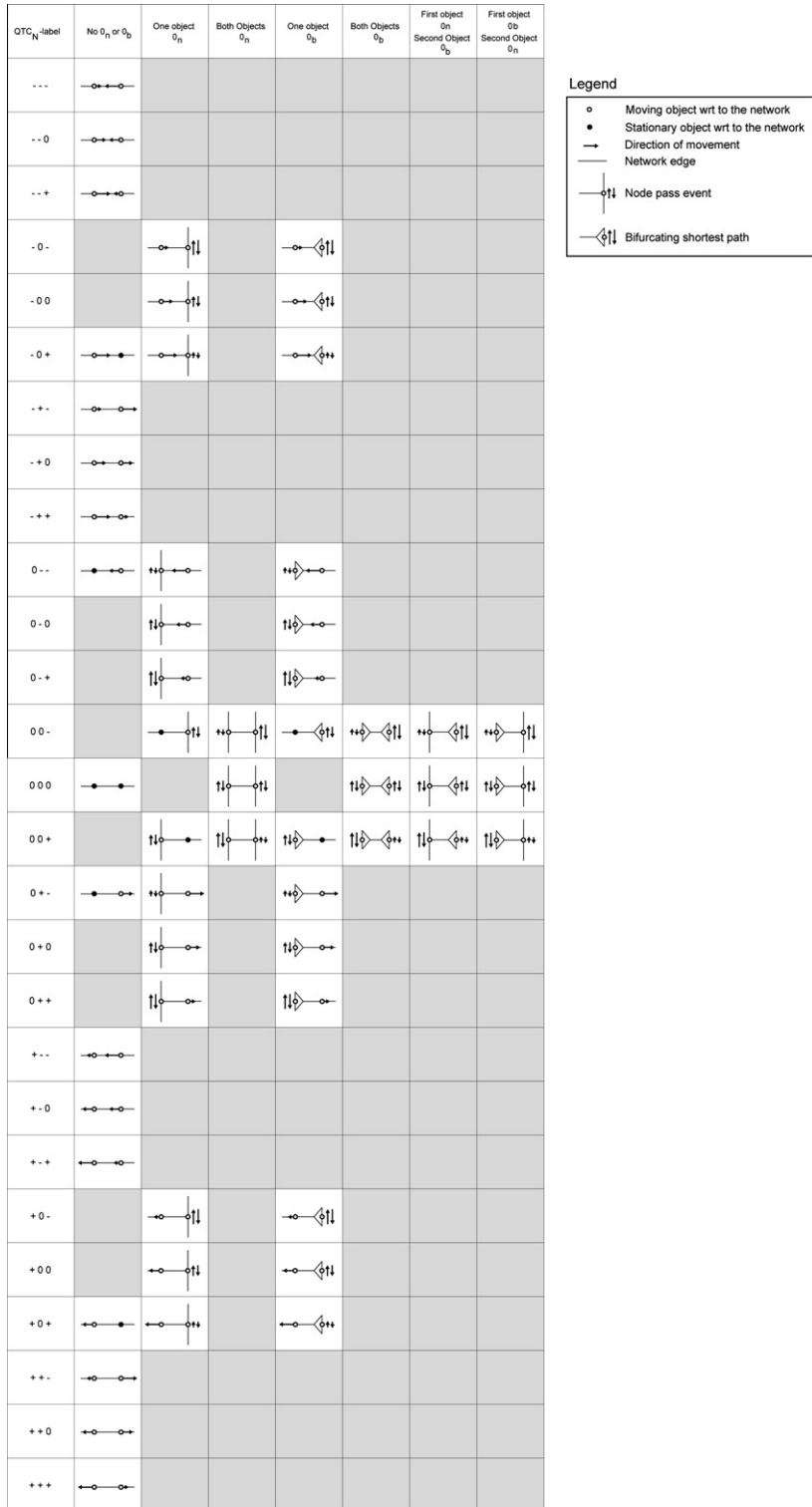


Fig. 3. 57 Canonical cases for QTC<sub>N</sub> at level 2.



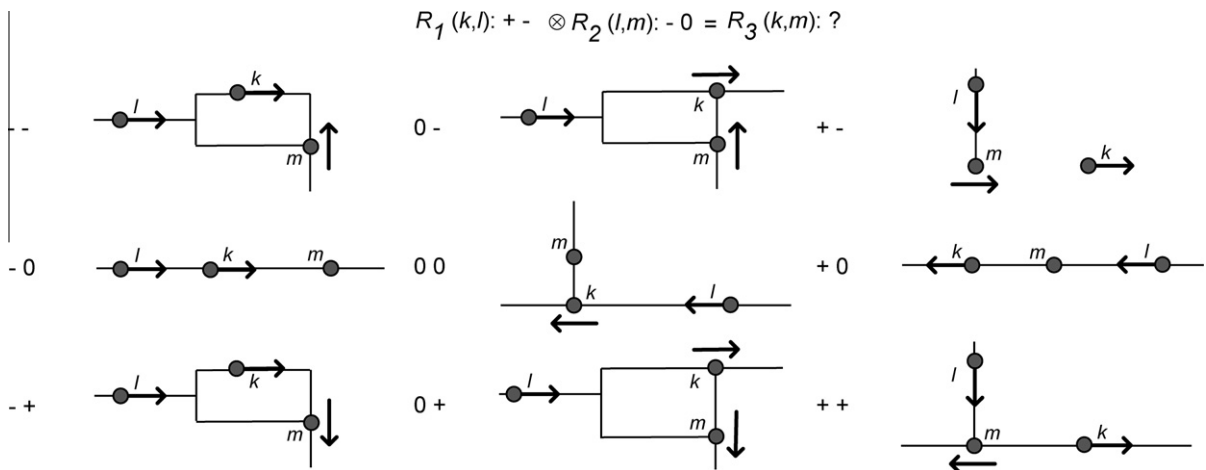


Fig. 4. Animations for the composition of (+ -) and (- 0); a movement arrow next to an object indicates that the object is passing a node.

node pass event with respect to  $l$ , it will not be on a shortest path to  $l$  just before and after  $t$ , resulting in a '0' for the first character in the label (iv). If there is a bifurcating shortest path from  $k$  to  $l$ , then  $k$  will be on a shortest path to  $l$  just before and after  $t$ , which also yields '0' for the first character in the label (v). The same five cases exist for the movement of the second object in the relation. Hence, there exist 25 ( $5^2$ ) canonical cases looking at the first level of  $QTC_N$ . When considering the second level, the additional three possibilities for the third label character might be expected to yield 75 ( $25 \times 3$ ) canonical cases. However, due to the impossibility for a stationary object to move faster than or equally as fast as a non-stationary object, 18 of these relations cannot physically occur. The remaining 57 canonical cases are presented in Fig. 3. The first column in the figure presents the  $QTC_N$  relation. In the other columns, an icon is sketched for all canonical cases. A '0n' denotes a '0' due to a shortest path omitting node pass event. A '0b' denotes a '0' due to the existence of a bifurcating shortest path between the objects. The left and right dots represent the positions of  $k$  (the first object) and  $l$  (the second object), respectively. A dot is filled if the object can be stationary. The arrow symbols represent the potential movement directions of the objects. The arrows can have different lengths indicating the difference in relative speed.

### 5. Composition

People often make inferences of and from qualitative relations in daily life [6]. For example, if we know that Nico is taller than Philippe and Frank is taller than Nico, we infer that Frank is taller than Philippe. A specific type of inference mechanism, which is a fundamental part of a relational calculus, is the composition of its relations [51]. The idea behind composition is to compose a finite set of new facts and rules from existing ones, i.e. if two existing relations  $R_1(k,l)$  and  $R_2(l,m)$  share a common object ( $l$ ), they can be composed into a new relation  $R_3(k,m)$ , denoted by:  $R_1(k,l) \otimes R_2(l,m) = R_3(k,m)$  – note that  $R_3$  may be a disjunction of base relations. If, for a set of relations, the compositions of all combinations of base relations can be computed, they are usually stored in a *composition table*. Composition tables make sense from a computational point of view, since a compositional inference can simply be looked up, instead of needing complex computations [3,58]. Ever since their introduction, composition tables have been precomputed for many different temporal (e.g. the interval calculus [2] and the semi interval calculus [21]), spatial (e.g. topological calculi [14,44], directional calculi [19,22], distance calculi [32], and spatiotemporal calculi (e.g. QTC [57])).

#### 5.1. Composition of $QTC_N$ relations

Since the composition of relative speed (represented by the third character of a level 2  $QTC_N$  relation) is straightforward, this section will focus on the composition of  $QTC_N$  at level 1. Nine ( $3^2$ )  $QTC_N$  base relations can be distinguished at level 1. As a consequence, the composition table at level 1 has 81 ( $9^2$ ) entries, each of which potentially contains a subset of these nine relations. Thus, 729 ( $9^3$ ) possible combinations of three relations need to be examined for their existence or non-existence. For each possibility that actually exists, a simple 'animation' can be drawn to demonstrate its existence. Examples of such animations for the composition of (+ -) and (- 0) are shown in Fig. 4.

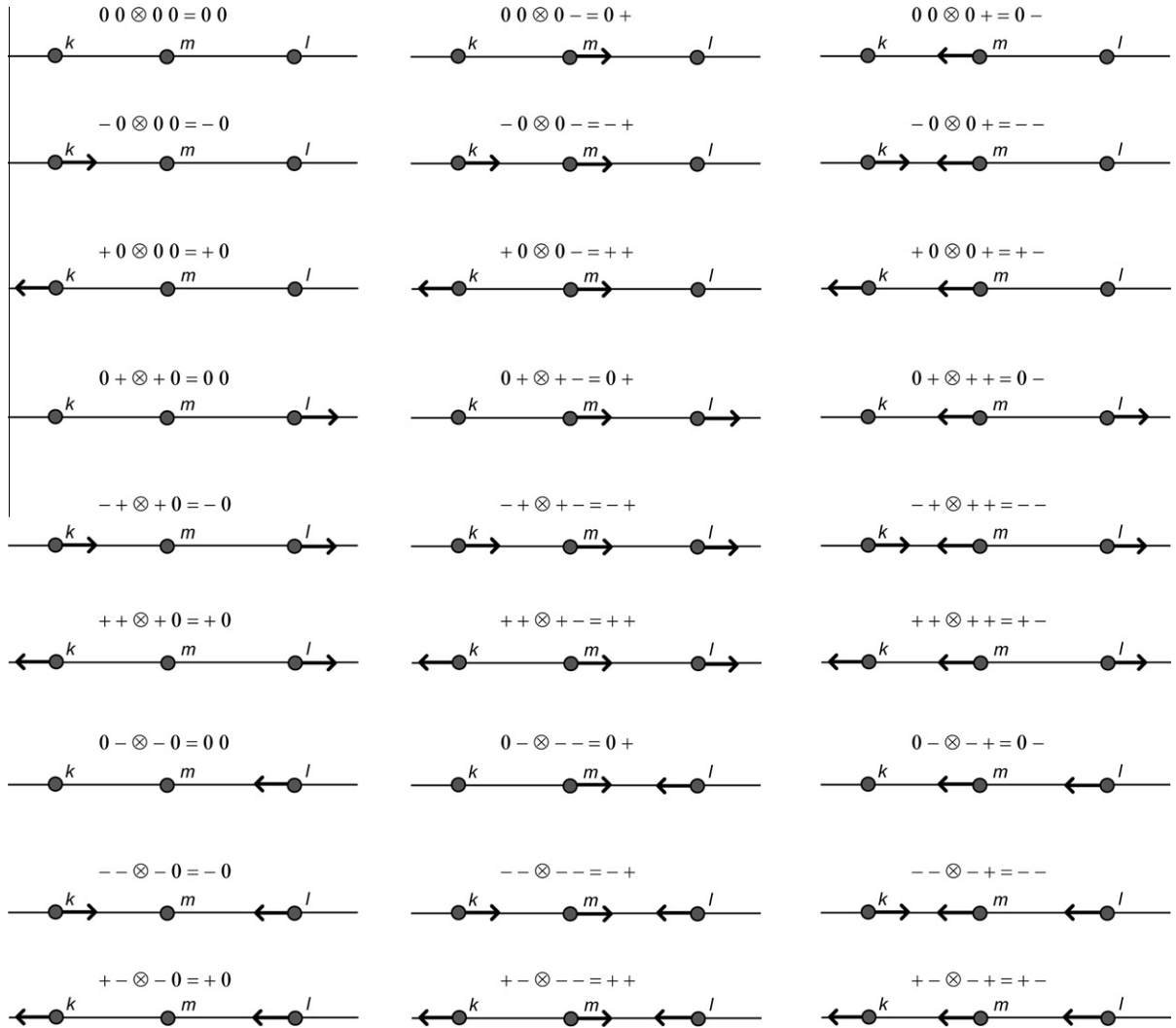
Since each composition yields the entire set of base relations, the construction of a composition table is trivial. This triviality results from the fact that  $QTC_N$  relations do not provide sufficient information about the spatiotemporal configuration of the network. Therefore, in order to obtain sparser composition tables, additional knowledge of the relation between the network and the moving objects is required. This can be acquired by imposing temporal as well as spatial constraints.



**Table 1**

Composition table for  $QTC_N$  at level 1 restricted to relations lasting over time intervals;  $A_0$  and  $B_0$  stand for the set  $\{-, +\}$ .

$R_1 \otimes R_2$	--	- 0	- +	0 -	0 0	0 +	+ -	+ 0	++
--	$A_0B_0$	$A_00$	$A_0B_0$	$\emptyset$	$\emptyset$	$\emptyset$	$A_0B_0$	$A_00$	$A_0B_0$
- 0	$\emptyset$	$\emptyset$	$\emptyset$	$A_0B_0$	$A_00$	$A_0B_0$	$\emptyset$	$\emptyset$	$\emptyset$
- +	$A_0B_0$	$A_00$	$A_0B_0$	$\emptyset$	$\emptyset$	$\emptyset$	$A_0B_0$	$A_00$	$A_0B_0$
0 -	$0 B_0$	$0 0$	$0 B_0$	$\emptyset$	$\emptyset$	$\emptyset$	$0 B_0$	$0 0$	$0 B_0$
0 0	$\emptyset$	$\emptyset$	$\emptyset$	$0 B_0$	$0 0$	$0 B_0$	$\emptyset$	$\emptyset$	$\emptyset$
0 +	$0 B_0$	$0 0$	$0 B_0$	$\emptyset$	$\emptyset$	$\emptyset$	$0 B_0$	$0 0$	$0 B_0$
+ -	$A_0B_0$	$A_00$	$A_0B_0$	$\emptyset$	$\emptyset$	$\emptyset$	$A_0B_0$	$A_00$	$A_0B_0$
+ 0	$\emptyset$	$\emptyset$	$\emptyset$	$A_0B_0$	$A_00$	$A_0B_0$	$\emptyset$	$\emptyset$	$\emptyset$
++	$A_0B_0$	$A_00$	$A_0B_0$	$\emptyset$	$\emptyset$	$\emptyset$	$A_0B_0$	$A_00$	$A_0B_0$



**Fig. 5.** Possible relative movement configurations in  $QTC_N$  for  $R_1(k,l) \otimes R_2(l,m)$  where  $m$  lies on the simple shortest path between  $k$  and  $l$  and none of the objects is located at a node 9.

5.2. Temporal constraints

As a first approach to achieve sparser composition tables, temporal constraints can be considered. One valuable temporal constraint, perhaps the only general one, is to consider which relations lasting over a time interval (rather than holding only instantaneously). A '0' in a level 1  $QTC_N$  label can only hold over a time interval when an object is stationary with respect to

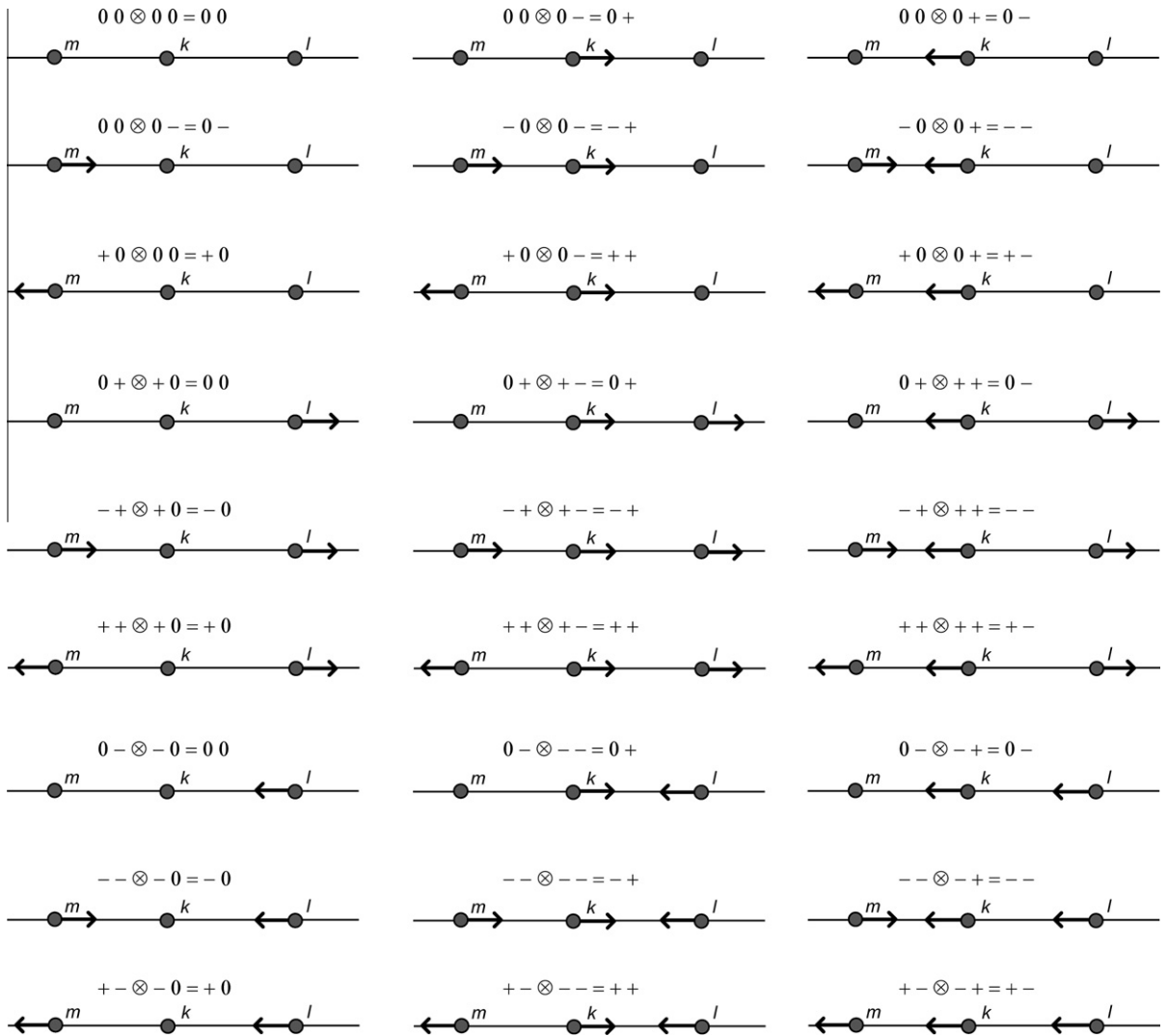


Fig. 6. Possible relative movement configurations in  $QTC_N$  for  $R_1(k,l) \otimes R_2(l,m)$  where  $k$  lies on the simple shortest path between  $m$  and  $l$  and none of the objects is located at a node 9.

Table 2

Composition table for relative movement in  $QTC_N$ , for  $R_1(k,l) \otimes R_2(l,m)$  where  $m$  lies on the simple shortest path between  $k$  and  $l$  and none of the objects is located at a node.

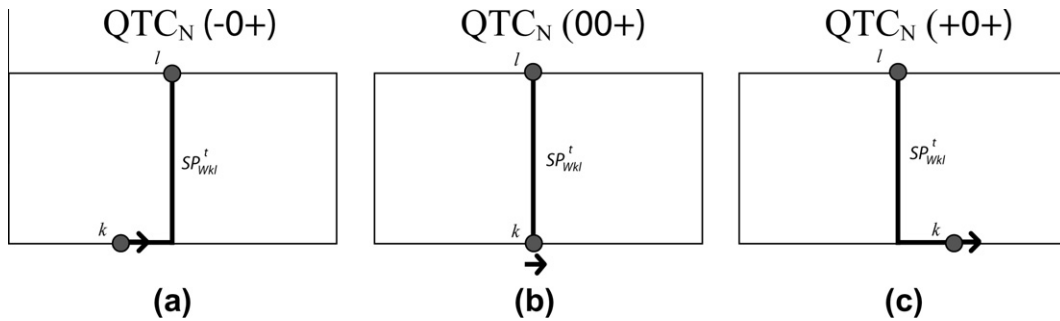
$R_1 \otimes R_2$	--	-0	-+	0-	00	0+	+-	+0	++
--	-+	-0	--	∅	∅	∅	∅	∅	∅
-0	∅	∅	∅	-+	-0	--	∅	∅	∅
-+	∅	∅	∅	∅	∅	∅	-+	-0	--
0-	0+	00	0-	∅	∅	∅	∅	∅	∅
00	∅	∅	∅	0+	00	0-	∅	∅	∅
0+	∅	∅	∅	∅	∅	∅	0+	00	0-
+-	++	+0	+-	∅	∅	∅	∅	∅	∅
+0	∅	∅	∅	++	+0	+-	∅	∅	∅
++	∅	∅	∅	∅	∅	∅	++	+0	+-

the network, as can be proven using the constraints of continuity [4]. As a consequence, an object which is stationary with respect to one object will also be stationary with respect to any other object. The composition table according to this restriction is provided in Table 1. The composition table consists of five fine results (i.e. singleton base relations), all being (0 0), 18

**Table 3**

Composition table for relative movement in  $QTC_N$ , for  $R_1(k,l) \otimes R_2(l,m)$  where  $k$  lies on the simple shortest path between  $m$  and  $l$  and none of the objects is located at a node.

$R_1 \otimes R_2$	--	-0	-+	0-	00	0+	+ -	+0	++
--	+ -	+0	++	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
-0	$\emptyset$	$\emptyset$	$\emptyset$	+ -	+0	++	$\emptyset$	$\emptyset$	$\emptyset$
-+	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	+ -	+0	++
0-	0-	00	0+	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
00	$\emptyset$	$\emptyset$	$\emptyset$	0-	00	0+	$\emptyset$	$\emptyset$	$\emptyset$
0+	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	0-	00	0+
+ -	--	-0	-+	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
+0	$\emptyset$	$\emptyset$	$\emptyset$	--	-0	-+	$\emptyset$	$\emptyset$	$\emptyset$
++	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	--	-0	-+



**Fig. 7.** A transition in  $QTC_N$  from  $(-0+)$  via  $(00+)$  to  $(+0+)$ .

disjunctions of two relations, 22 disjunctions of four relations and 36 inconsistent compositions (denoted by the empty set). Thus, the total number of possibilities is reduced from 729 to 129.

**5.3. Spatial constraints**

While the composition results in Table 1 are already much sparser than those obtained without constraints, they merely provide five fine results. Therefore, as a second approach, spatial constraints can be imposed on top of the temporal restriction. As shown in Section 4, the determination of a level 1  $QTC_N$  relation merely involves knowledge about the relative movement with respect to the shortest path(s) between the objects concerned. In composition, this relative movement is known for the first two object pairs, while nothing is known about the shortest path(s) of the third pair, leaving all relations possible to occur. For three objects  $k, l$  and  $m$ , assume that the relations  $R_1(k,l)$  and  $R_2(l,m)$  are given and  $R_3(k,m)$  is unknown, implying that  $SP_{kl}^t$  and  $SP_{lm}^t$  are known and that  $SP_{km}^t$  is unknown. If it is known that  $k|t$  is on  $SP_{lm}^t$  or that  $m|t$  is on  $SP_{kl}^t$ , a simple non-closed curve can be drawn containing the positions of all three objects at  $t$ . On this curve, each object has three movement possibilities: it can be stable or move in one of two opposite directions. Hence, there are 27 ( $3^3$ ) movement configurations of these three objects. An illustration of each specific configuration is shown in Figs. 5 and 6, respectively illustrating the cases of  $m|t$  lying on  $SP_{kl}^t$  and  $k|t$  lying on  $SP_{lm}^t$ . The associated composition tables are presented in Tables 2 and 3. This kind of composition is very useful, since it always leads to exact knowledge: both tables contain 27 fine composition results, whereas 54 compositions are inconsistent.

**6. Transforming  $QTC_N$  into the relative trajectory calculus on networks**

Having defined the  $QTC_N$  relations between a pair of moving objects, a set of trivial qualitative questions can be answered. For example, by looking at the third character of the label, one can identify which object is moving the fastest. Looking at the first two characters of the  $QTC_N$  label, queries such as whether an object is moving towards or away from another object can be resolved. In addition to these trivial questions,  $QTC_N$  at level 2 has the power to answer additional questions using the information contained by all three characters in the label. This information can be obtained by transforming QTC relations into relations defined by the Relative Trajectory Calculus (RTC) [52].

In contrast to QTC, where distances between objects at different times are compared (e.g. in Definitions 3.1 and 4.13), RTC defines relations based on the relative motion of an object against another object at the same moment in time [52] (Definition 6.1).

**Table 4**  
Transformations from all QTC<sub>N</sub> canonical cases to RTC<sub>N</sub> relations.

QTC <sub>N</sub> -label	RTC <sub>N</sub> -label	QTC <sub>N</sub> -label	RTC <sub>N</sub> -label	QTC <sub>N</sub> -label	RTC <sub>N</sub> -label
--	⇒ -	0s 0s 0	⇒ 0	0n 0n +	⇒ 0
-- 0	⇒ -	0s 0s +	⇒ [redacted]	0s + -	⇒ +
-- +	⇒ -	0b 0s -	⇒ [redacted]	0s + 0	⇒ [redacted]
- 0s -	⇒ [redacted]	0b 0s 0	⇒ [redacted]	0s ++	⇒ [redacted]
- 0s 0	⇒ [redacted]	0b 0s +	⇒ 0	0b + -	⇒ +
- 0s +	⇒ -	0n 0s -	⇒ [redacted]	0b + 0	⇒ 0
- 0b -	⇒ 0	0n 0s 0	⇒ [redacted]	0b ++	⇒ 0
- 0b 0	⇒ 0	0n 0s +	⇒ 0	0n + -	⇒ +
- 0b +	⇒ -	0s 0b -	⇒ 0	0n + 0	⇒ 0
- 0n -	⇒ 0	0s 0b 0	⇒ [redacted]	0n ++	⇒ 0
- 0n 0	⇒ 0	0s 0b +	⇒ [redacted]	+ - -	⇒ -
- 0n +	⇒ -	0b 0b -	⇒ 0	+ - 0	⇒ 0
- + -	⇒ +	0b 0b 0	⇒ 0	+ - +	⇒ +
- + 0	⇒ 0	0b 0b +	⇒ 0	+ 0s -	⇒ [redacted]
- + +	⇒ -	0n 0b -	⇒ 0	+ 0s 0	⇒ [redacted]
0s - -	⇒ -	0n 0b 0	⇒ 0	+ 0s +	⇒ +
0s - 0	⇒ [redacted]	0n 0b +	⇒ 0	+ 0b -	⇒ 0
0s - +	⇒ [redacted]	0s 0n -	⇒ 0	+ 0b 0	⇒ 0
0b - -	⇒ -	0s 0n 0	⇒ [redacted]	+ 0b +	⇒ +
0b - 0	⇒ 0	0s 0n +	⇒ [redacted]	+ 0n -	⇒ 0
0b - +	⇒ 0	0b 0n -	⇒ 0	+ 0n 0	⇒ 0
0n - -	⇒ -	0b 0n 0	⇒ 0	+ 0n +	⇒ +
0n - 0	⇒ 0	0b 0n +	⇒ 0	+ + -	⇒ +
0n - +	⇒ 0	0n 0n -	⇒ 0	+ + 0	⇒ +
0s 0s -	⇒ [redacted]	0n 0n 0	⇒ 0	+ + +	⇒ +

**Definition 6.1.** A relation in RTC between a first object *k* and a second object *l* at a time point *t* is defined by a single character label. This label represents the comparison of the distance between *k* and *l* immediately before *t* with the distance between *k* and *l* immediately after *t*. This results in three possibilities:

-: the distance between *k* and *l* decreases:

$$\exists t_1, t_2 (t_1 < t < t_2 \wedge \forall t^-, t^+ (t_1 < t^- < t < t^+ < t_2 \rightarrow d(k|t^-, l|t^-) > d(k|t^+, l|t^+))) \tag{19}$$

0: the distance between *k* and *l* remains the same:

$$\exists t_1, t_2 (t_1 < t < t_2 \wedge \forall t^-, t^+ (t_1 < t^- < t < t^+ < t_2 \rightarrow d(k|t^-, l|t^-) = d(k|t^+, l|t^+))) \tag{20}$$

+: the distance between *k* and *l* increases:

$$\exists t_1, t_2 (t_1 < t < t_2 \wedge \forall t^-, t^+ (t_1 < t^- < t < t^+ < t_2 \rightarrow d(k|t^-, l|t^-) < d(k|t^+, l|t^+))) \tag{21}$$

RTC<sub>N</sub> describes the RTC relations on networks. In what follows, we will show that every QTC<sub>N</sub> relation can be mapped onto a single RTC<sub>N</sub> relation. This allows QTC<sub>N</sub> at level 2 to answer questions such as whether two objects are getting closer to each other or whether they are getting further away from each other. To this end, we will first consider the cases where the union of all shortest paths over the entire time span can be described as a simple curve without junctions. Note that this excludes, among others, the case of bifurcating shortest paths (Fig. 1) and shortest path omitting node pass events (Fig. 2). Hence, the following equalities apply for QTC<sub>N</sub> relation between the objects *k* and *l* at time point *t*:

A '-' in the first character of the relation label implies:

$$\begin{aligned} d(k|t^+, l) + d(k|t, k|t^+) &= d(k|t, l) \\ d(k|t^-, l) - d(k|t, k|t^-) &= d(k|t, l) \end{aligned} \tag{22}$$

A '+' in the first character of the relation label implies:

$$\begin{aligned} d(k|t^+, l) - d(k|t, k|t^+) &= d(k|t, l) \\ d(k|t^-, l) + d(k|t, k|t^-) &= d(k|t, l) \end{aligned} \tag{23}$$

Analogous reasoning applies for '-' and '+' in the second label character, yielding (24, 25). Regardless of the QTC<sub>N</sub> relation it follows from (23–25) that:

$$\begin{aligned} d(k|t^+, k|t^-) &= d(k|t, k|t^+) + d(k|t, k|t^-) \\ d(l|t^+, l|t^-) &= d(l|t, l|t^+) + d(l|t, l|t^-) \end{aligned} \tag{26}$$

**Theorem 1.** A  $QTC_N$  relation  $(- - -)$  between the objects  $k$  and  $l$  at a time point  $t$  can be transformed into an  $RTC_N$  relation  $(-)$ , such that the  $RTC_N$  relation is true whenever the  $QTC_N$  relation is true.

**Proof.** By definition, the first two characters of  $(- - -)$  imply:

$$d(k|t^-, l|t) > d(k|t, l|t) > d(k|t^+, l|t) \tag{27}$$

$$d(k|t, l|t^-) > d(k|t, l|t) > d(k|t, l|t^+) \tag{28}$$

From (27) and (28) it follows that:

$$d(k|t^-, l|t) > d(k|t, l|t^+) \tag{29}$$

$$\iff d(k|t^-, l|t) + d(l|t, l|t^-) > d(k|t, l|t^+) = -d(k|t, k|t^+) \tag{30}$$

$$\Rightarrow d(k|t^-, l|t^-) > d(k|t^+, l|t^+) \tag{31}$$

which is by definition equal to the  $RTC_N$  relation  $(-)$ .  $\square$

Analogously, it can be proven that the  $QTC_N$  relations  $\{(- - 0), (- - +), (- 0 +), (0 - -)\}$  can be converted into the  $RTC_N$  relation  $(-)$ .

**Theorem 2.** A  $QTC_N$  relation  $(+ + +)$  between the objects  $k$  and  $l$  at time point  $t$  can be transformed into an  $RTC_N$  relation  $(+)$ , such that the  $RTC_N$  relation is true whenever the  $QTC_N$  relation is true.

**Proof.** By definition, the first two characters of  $(+ + +)$  imply:

$$d(k|t^+, l|t) > d(k|t, l|t) > d(k|t^-, l|t) \tag{32}$$

$$d(k|t, l|t^+) > d(k|t, l|t) > d(k|t, l|t^-) \tag{33}$$

From (32) and (33) it follows that:

$$d(k|t^+, l|t) > d(k|t, l|t^-) \tag{34}$$

$$\iff d(k|t^+, l|t) + d(l|t, l|t^+) > d(k|t, l|t^-) = d(k|t, k|t^-) \tag{35}$$

$$\Rightarrow d(k|t^+, l|t^+) > d(k|t^-, l|t^-) \tag{36}$$

which is by definition equal to the  $RTC_N$  relation  $(+)$ .  $\square$

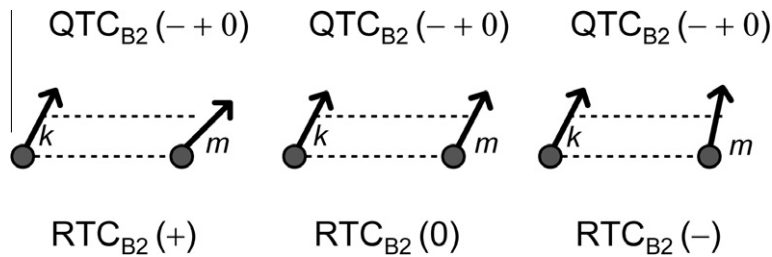


Fig. 8. Examples of transformations from  $QTC_B$  to  $RTC$ .

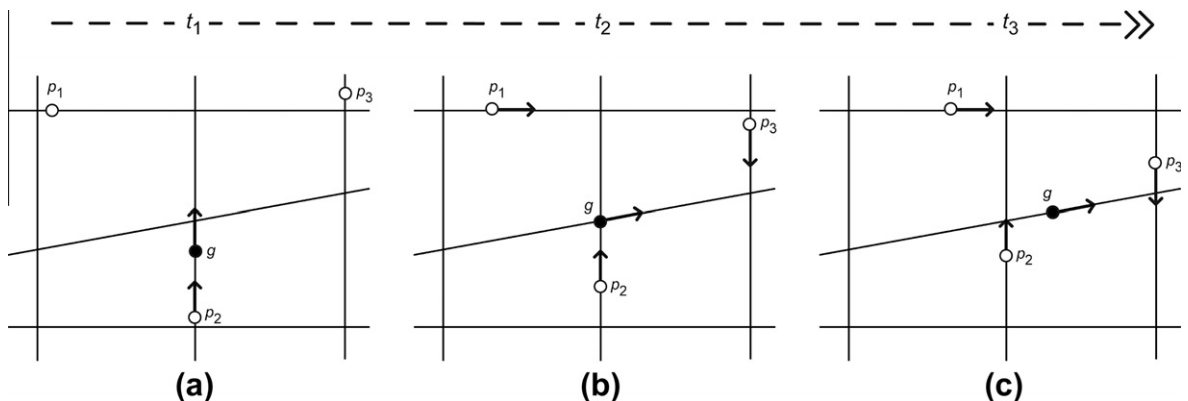


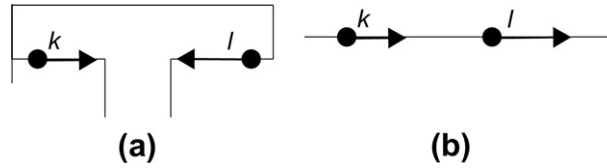
Fig. 9. Simplified animation of three policemen chasing a gangster.

**Table 5**  
Transformations from QTC<sub>N</sub> into RTC<sub>N</sub> relations.

QTC <sub>N</sub> -label	RTC <sub>N</sub> -label	QTC <sub>N</sub> -label	RTC <sub>N</sub> -label	QTC <sub>N</sub> -label	RTC <sub>N</sub> -label
---	⇒ -	0--	⇒ -	+--	⇒ -
--0	⇒ -	0--0	⇒ 0	+ - 0	⇒ 0
--+	⇒ -	0 - +	⇒ 0	+ - +	⇒ +
-0-	⇒ 0	00-	⇒ 0	+ 0 -	⇒ 0
-00	⇒ 0	000	⇒ 0	+ 0 0	⇒ 0
-0+	⇒ -	00+	⇒ 0	+ 0 +	⇒ +
-+-	⇒ +	0+-	⇒ +	++-	⇒ +
-+0	⇒ 0	0+0	⇒ 0	+ + 0	⇒ +
-++	⇒ -	0++	⇒ 0	+ + +	⇒ +

**Table 6**  
Composition results inferred over [t<sub>1</sub>, t<sub>3</sub>] due to spatial and temporal constraints.

Time	Known relations	Results inferred from temporal constraints	Results inferred spatial constraints
t <sub>1</sub>	R(p <sub>1</sub> , p <sub>2</sub> ) = (0 -), R(p <sub>1</sub> , p <sub>3</sub> ) = (0 0), R(p <sub>2</sub> , p <sub>3</sub> ) = (- 0), R(p <sub>2</sub> , g) = (- +)	R(p <sub>1</sub> , g) = (0 -) ∨ (0 +), R(p <sub>3</sub> , g) = (0 -) ∨ (0 +)	R(p <sub>1</sub> , g) = (0 -), R(p <sub>3</sub> , g) = (- 0)
]t <sub>1</sub> , t <sub>2</sub> [	R(p <sub>1</sub> , p <sub>2</sub> ) = (- -), R(p <sub>1</sub> , p <sub>3</sub> ) = (- +), R(p <sub>2</sub> , p <sub>3</sub> ) = (- -), R(p <sub>2</sub> , g) = (- +)	R(p <sub>1</sub> , g) = (- -) ∨ (- +) ∨ (+ -) ∨ (+ +), R(p <sub>3</sub> , g) = (- -) ∨ (- +) ∨ (+ -) ∨ (+ +)	R(p <sub>1</sub> , g) = z (- -), R(p <sub>3</sub> , g) = (- -)
t <sub>2</sub>	R(p <sub>1</sub> , p <sub>2</sub> ) = (0 -), R(p <sub>1</sub> , p <sub>3</sub> ) = (0 0), R(p <sub>2</sub> , p <sub>3</sub> ) = (- 0), R(p <sub>2</sub> , g) = (- +)	None possible	None possible
]t <sub>2</sub> , t <sub>3</sub> [	R(p <sub>1</sub> , p <sub>2</sub> ) = (0 -), R(p <sub>1</sub> , p <sub>3</sub> ) = (0 0), R(p <sub>2</sub> , p <sub>3</sub> ) = (- 0), R(p <sub>2</sub> , g) = (- +)	R(p <sub>1</sub> , g) = (- -) ∨ (- +) ∨ (+ -) ∨ (+ +), R(p <sub>3</sub> , g) = (- -) ∨ (- +) ∨ (+ -) ∨ (+ +)	R(p <sub>3</sub> , g) = (- -)
t <sub>3</sub>	R(p <sub>1</sub> , p <sub>2</sub> ) = (0 -), R(p <sub>1</sub> , p <sub>3</sub> ) = (0 0), R(p <sub>2</sub> , p <sub>3</sub> ) = (- 0), R(p <sub>2</sub> , g) = (- +)	R(p <sub>1</sub> , g) = (- -) ∨ (- +) ∨ (+ -) ∨ (+ +), 3 R(p <sub>3</sub> , g) = (- -) ∨ (- +) ∨ (+ -) ∨ (+ +)	R(p <sub>3</sub> , g) = (- -)



**Fig. 10.** Two scenes without collision danger for two moving objects.

Analogously, it can be proven that the QTC<sub>N</sub> relations {(+ + 0), (+ + -), (0 + +), (0 + -)} can be converted into the RTC<sub>N</sub> relation (+).

**Theorem 3.** A QTC<sub>N</sub> relation (- + -) between the objects k and l at time point t can be transformed into an RTC<sub>N</sub> relation (+), such that the RTC<sub>N</sub> relation is true whenever the QTC<sub>N</sub> relation is true.

**Proof.** By definition, the third character of (- + -) implies:

$$v_k < v_l \tag{37}$$

$$\iff \frac{\partial x_k}{\partial t} < \frac{\partial x_l}{\partial t} \tag{38}$$

$$\iff \frac{d(k|t^-, k|t^+)}{\partial t} < \frac{d(l|t^-, l|t^+)}{\partial t} \tag{39}$$

$$\Rightarrow d(k|t^-, k|t) + d(k|t, k|t^+) < d(l|t^-, l|t) + d(l|t, l|t^+) \tag{40}$$

$$\iff d(k|t^-, k|t) + d(k|t, l|t) + d(k|t, k|t^+) < d(l|t^-, l|t) + d(k|t, l|t) + d(l|t, l|t^+) \tag{41}$$

$$\iff d(k|t^-, k|t) + d(k|t, l|t) - d(l|t^-, l|t) < d(l|t^+, l|t) + d(k|t, l|t) - d(k|t, k|t^+) \tag{42}$$

$$\Rightarrow d(k|t^-, l|t^-) < d(k|t^+, l|t^+) \tag{43}$$

which is by definition equal to the RTC<sub>N</sub> relation (+). □

Analogously, it can be proven that the QTC<sub>N</sub> relation (- + +) can be converted into the RTC<sub>N</sub> relation (+), that the QTC<sub>N</sub> relations {(+ - -), (+ - +)} can be converted into the RTC<sub>N</sub> relation (-), and that the QTC<sub>N</sub> relations {(+ + 0), (+ - 0), (0 0 0)} can be converted into the RTC<sub>N</sub> relation (0).

Note that the above mentioned theorems are not valid when the union of shortest paths does not constitute a simple curve over the considered time span, since Eqs. (23–25) are not valid. Based on restrictions imposed by continuity, it can be shown that, in these cases, there is also a unique transformation from a  $QTC_N$  relation into a single  $RTC_N$  relation. Consider a qualitative variable capable of taking any of the three qualitative values ‘–’, ‘0’ and ‘+’. Due to continuity, this variable cannot make a direct change from ‘–’ to ‘+’ and vice versa, since such a change must always pass the intermediate value ‘0’ [23]. Let us consider the shortest path omitting node pass event in Fig. 7. In Fig. 7a there is a  $QTC_N$  relation (– 0 +), which can be transformed into the  $RTC_N$  relation (–), according to Theorem 1. Analogously, (+ 0 +) can be transformed into (+) in Fig. 7c. Then, due to the above restriction imposed by continuity, the  $QTC_N$  relation (0 0 +) in Fig. 7b must be an  $RTC_N$  relation (0).

Similar transformations apply for all  $QTC_N$  relations occurring at shortest path omitting node pass events or when there are bifurcating shortest paths. Table 4 provides an overview of the transformations from each canonical case in  $QTC_N$  to the respective  $RTC_N$  relation. A ‘On’ denotes that a ‘0’ is due to a shortest path omitting node pass event. A ‘Ob’ denotes that a ‘0’ is due to the existence of a bifurcating shortest path between the objects. A ‘Os’ denotes a ‘0’ is due to a stationary object. The black cells indicate that no corresponding  $RTC_N$  relations physically exist.

Thus, there is a one-to-one mapping from  $QTC_N$  to  $RTC_N$  relations. This is notable since for QTC relations of objects having a free trajectory in  $\mathbb{R}^2$ , this is not the case [52]. The latter is illustrated in Fig. 8. Since the dotted line has a fixed length, the figure shows that a  $QTC_B$  relation (– + 0) can be transformed into all possible RTC relations.

## 7. Discussion

On the one hand, defining and examining the properties of a distance based calculus for moving objects constrained by networks is a worthwhile theoretical investigation into further aspects of QTC theory. On the other hand, we argue that this calculus is also convenient for the use in applications. In this section, we will illustrate this usefulness by means of two examples.

### 7.1. A police/Gangster example

In order to show the applicability of the composition of  $QTC_N$  relations at level 1 and the usefulness of the temporal and spatial constraints stated in Section 5, let us consider the following example where three policemen  $p_1$ ,  $p_2$  and  $p_3$  are at different locations in a city and wish to catch a gangster  $g$  along a road network (Fig. 9). It is assumed that the policemen know their mutual positions and therefore their mutual shortest paths at any time, but they can only see the gangster if they are in line of sight. At time  $t_1$ , while  $p_1$  and  $p_3$  are still awaiting instructions, policeman  $p_2$  has noticed and started chasing the gangster who started to escape, thus yielding  $R(p_2, g) = (- +)$  (Fig. 9a). Since all shortest paths are simple and  $g|_{t_1}$  is on both  $SP_{p_1 p_2}^{t_1}$  and  $SP_{p_3 p_2}^{t_1}$ , composition can be applied using Table 2, such that  $p_2$  can give the right orders to  $p_1$  and  $p_3$  concerning the direction in which they should move, i.e.  $p_2$  directs  $p_1$  and  $p_3$  to start moving towards  $p_2$  (since  $p_1$  and  $p_3$  know where  $p_2$  is), which causes  $p_1$  and  $p_3$  to move towards  $g$  just after  $t_1$ . At  $t_2$ ,  $g$  is at a junction. Hence, composition cannot be applied, since one cannot know which turn  $g$  will take (Fig. 9b). Immediately after  $t_2$ ,  $p_2$  will have seen  $g$  turning right, and so still knows at which edge  $g$  is, thus enabling composition with respect to  $p_3$  and  $g$ . This situation lasts until  $t_3$  (Fig. 9c) and will continue after  $t_3$ , probably until the gangster gets caught. Tables 5 and 6 lists the respective composition results inferred over  $[t_1, t_3]$ . As can be noted, results are only lacking at  $t_2$ , whereas during the rest of the period there is complete information due to the existing spatiotemporal constraints.

### 7.2. A collision avoidance application

An application where  $QTC_N$  at level 2 can be useful is in collision avoidance systems. If one wants to know if two objects are going to collide, then it is useful, as a first step, to restrict attention to the objects that might meet. In other words, only the objects which are getting closer to each other, i.e. objects in an  $RTC_N$  relation (–), are relevant, because otherwise they cannot collide. Thus,  $QTC_N$  relations at level 2 eliminate many movements from further examination, greatly reducing calculation times. Further examination of the remaining relations gives information on the type of collision. The relations (– + +) and (+ – –) indicate possible rear-end collisions, whereas (– – –), (– – 0), and (– – +) indicate possible head-on collisions. The relations (– 0 +) and (0 – –) may indicate collisions with a stationary object. Note that these  $QTC_N$  relations indicate potential collisions that do not necessarily result in real collisions. Related work on collision avoidance has, on the one hand, focussed on detecting possible collisions between objects which have a completely free trajectory in a two dimensional space [13,26,48]. These approaches mainly focus on the direction of movement. Although they have all shown their usefulness when the movement of objects is unconstrained, directional methods can not directly be transformed to networks, since they do not take into account the spatial structure of a network. The movement in Fig. 10a, for example, would announce a possible collision in all the above mentioned directional approaches, while from  $QTC_N$  analysis it follows that the objects move away from each other and therefore cannot collide. Furthermore, none of the methods above incorporates the relative speed between two moving objects. However, the notion of relative speed is crucial for collision detection in cases where the objects move in the same direction, while in the other cases, it may offer appealing insights into a finer subdivision of collision types. Consider the example in Fig. 10b. When using only directional information, this movement would trigger a



collision detection, but since  $l$  is moving faster than  $k$ , the distance between them increases, and hence, there is no true collision danger. For both these reasons, directional approaches over-predict possible collisions, while  $QTC_N$  does not.

Other techniques for collision avoidance considering network-constrained objects mainly focus on railway networks. Collisions in these systems are avoided by disallowing two trains to occupy the same track segment [30,31]. First of all, these methods also over-predict possible collisions, since two trains may travel on the same track without colliding (e.g. as in Fig. 10b with  $k$  moving slower than  $l$ ). Secondly, this sole constraint does not capture every possible collision situation. If two trains are on different segments, they can still be close and moving towards each other. Hence, not all collisions can be predicted in collision avoidance systems relying on this constraint (especially for objects colliding at network junctions).

## 8. Conclusions and future work

In this paper, we have formally presented the Qualitative Trajectory Calculus on Networks ( $QTC_N$ ) as a qualitative calculus to represent and reason about moving point objects which are constrained in their movement by networks.

We have shown two techniques to infer additional knowledge from the basic  $QTC_N$  relations. On the one hand we have presented the composition of  $QTC_N$  relations (Section 5). It was found that, at level 1, each  $QTC_N$  base relation is a possible result for each composition of two relations. While this result, at first, can be considered of limited use, it was shown how sparser and more powerful composition tables may be obtained by imposing realistic additional spatial and temporal constraints. By excluding instantaneous relations, we were able to reduce the total of 729 possibilities to 129 (18%). In addition, by restricting to the case where the union of shortest paths involved in the composition forms a non-closed curve, a further reduction was made to 27 (4%) fine results (i.e. singleton base relations). These sparser composition tables are more powerful and useful with respect to potential applications, as has been illustrated in Section 7.1.

On the other hand, we have demonstrated that  $QTC_N$  is able to answer qualitative questions such as whether objects on a network are moving towards or away from each other. These queries are not limited to trivial questions which merely relate to the relationship represented by a single  $QTC_N$  relation character. Hence, a  $QTC_N$  relation conveys more information than each of its individual label characters separately. As pointed out in Section 6, each canonical relation in  $QTC_N$  at level 2 can be uniquely transformed into a  $RTC_N$  relation (this is not the case for  $QTC_B$  in  $\mathbb{R}^2$  [52]). Therefore,  $QTC_N$  is capable of answering questions such as whether two objects are getting closer to each other or whether they are getting further away from each other. In Section 7.2, we have illustrated that the definition of  $QTC_N$  and the unique transformation of its relations into single  $RTC_N$  relations can be useful, for example in collision avoidance systems.

The theoretical contributions in this paper complement the earlier contributions vis-à-vis other calculi of the  $QTC$  family (see [10] for an overview) in general, and regarding  $QTC_N$  in particular. While  $QTC_N$  relations have been introduced in a brief and informal manner in earlier work [4,52], this paper offers a formal axiomatisation of  $QTC_N$ . In addition to the conceptual neighbourhood diagrams presented in [4], we have presented the composition tables for  $QTC_N$  as well as the transformation of  $QTC_N$  into  $RTC_N$  relations. Furthermore, we have explored and illustrated the reasoning power of  $QTC_N$  by means of its ability to answer qualitative queries. As has been recently shown for  $QTC_B$  and  $QTC_C$  [11], these contributions will allow  $QTC_N$  to be implemented in an information system in order to represent and reason about moving objects constrained by networks.

Among the qualitative calculi that deal with relations between moving objects,  $QTC_N$  is unique in its consideration of network-based objects. An exception is the work of Wang et al. [59] who extend the Directed Interval Algebra [45] to the Road Network Directed Interval Algebra (RNDIA). Although their algebra is also based on the notion of shortest paths, RNDIA differs from  $QTC_N$  as it defines relations among directed network tracks rather than relations among moving point objects. RNDIA is therefore less appropriate to represent and reason about instantaneous events occurring among objects along their trajectories. Collisions, for example, are not unambiguously represented in RNDIA as they may occur in the case of different RNDIA base relations (e.g. the equal, overlay, and cross relations [54]). Given that practically all traffic movements are bounded by networks,  $QTC_N$ -based applications are promising in the field of Intelligent Transportation Systems and Geographic Information Systems for Transportation (GIS-T) [49].

Ongoing research involving  $QTC_N$  is being conducted on cognitive aspects of the calculus. Major questions to be investigated in this respect include what specific terms such as motion verbs and prepositions do people attach to each of the canonical cases of the calculus. Future findings on these issues may provide insights on the power of  $QTC_N$  in natural language processing and human computer interaction.

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