Multi-User communications

MICAS 921

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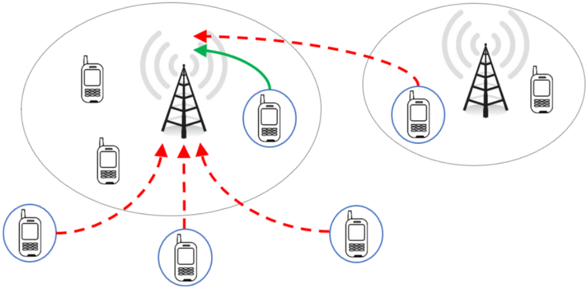
A Coordinated Uplink Scheduling and Power Control Algorithm for Multicell Networks

- A ditributed iterative algorithm that schedules and allocates power jointly
- Uplink existing procedure does not account for interference
- Some Attempt with Game-theory
- [1] (Pb.4 of the lecture) is the closest related problem we have studied together
- **Usefull Toolbox from the lecture :** Fix point fractionnal programming (Dinkelbach 1967 [2]) (Jongs 2012 [3])

Framework [6]



(a) Uplink multicell network

Problem Formulation

We consider J BSs each and denote U_i the set of users associated to BS i. We would like to maximize the utility of the network.

$$(P_0):\begin{cases}\max_{\mathbf{s},\mathbf{p}} f_0(\mathbf{s},\mathbf{p}) = \sum_{i=1}^{J} w_{s_i} \log\left(1 + \frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j\neq i} |h_{i,s_j}|^2 p_{s_j} + \sigma^2}\right) \\ \text{s.t. } s_i \in U_i \text{ and } 0 \leq P_{s_i} \leq P\end{cases}$$

HARD ! Non-Convex Mixed-Integer Problem

What makes this problem hard ?

- **s** is discrete
- The problem is not convex in **p**
- scheduling and power depends with neighboring cells through denominator (inteference)

Then what can we do ? Remove the interference !

Removing the SINR

We introduce the variable γ_i for the uplink SINR at BS i. The problem can now be refromulated as

$$\begin{cases} \max_{\sim,\shortmid,\gamma} \sum_{i=1}^{J} w_{s_i} \log(1+\gamma_i) \\ s.t. \gamma_i = \frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j \neq i} |h_{i,s_j}|^2 p_{s_j} + \sigma^2} \\ s_i \in U_i \\ 0 \leq P_{s_i} \leq P \end{cases}$$

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Lagrangian Reformulation

$$L(s, p, \gamma, \lambda) = \sum_{i=1}^{J} w_{s_i} \log(1 + \gamma_1) - \sum_{i=1}^{J} \lambda_i (\gamma_i - \frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j \neq i} |h_{i,s_j}|^2 p_{s_j} + \sigma^2})$$

At the optimum we obtain

$$\gamma_i = \frac{w_{s_i}}{\lambda_i} - 1$$

and

$$\lambda_i = w_{s_i} - \frac{w_{s_i} |h_{i,s_i}|^2 p_{s_i}}{\sum_{j} |h_{i,s_j}|^2 p_{s_j} + \sigma^2}.$$

The new formulation without SINR

$$f_r(\mathrm{s}, \mathrm{p}, \gamma) = \sum_{i=1}^{J} w_{s_i} \log(1+\gamma_1) - \sum_{i=1}^{J} w_{s_i}\gamma_i + \sum_{i=1}^{J} \frac{w_{s_i}(\gamma_i + 1)|h_{i,s_i}|^2 p_{s_i}}{\sum_j |h_{i,s_j}|^2 p_{s_j} + \sigma^2}$$

Easier ! s, p are outside the logarithm. But how to handle the ratio ? (see [5],[3])

A quadratic trick !

Lemma

Let A, B be two positive real valued function defined over the constraint set $\mathcal{X}$. Then

$$\begin{cases} \max_x \frac{A(x)}{B(x)} \\ x \in \mathcal{X} \end{cases} \iff \begin{cases} \max_{x,y} 2y\sqrt{A(x)} - y^2 B(x) \\ x \in \mathcal{X} \end{cases}.$$

Démonstration.

Let $h_x : y \in \mathbb{R} \mapsto 2y\sqrt{A(x)} - y^2 B(x) \in \mathbb{R}$. Then $\max_y h_x(y) = \frac{A(x)}{B(x)}$. This prooves the equivalence.

Decoupling of (s_i, p_{s_i}) over i.

$$f_r(\mathrm{s}, \mathrm{p}, \gamma) = \sum_{i=1}^{J} w_{s_i} \log(1+\gamma_1) - \sum_{i=1}^{J} w_{s_i}\gamma_i + \sum_{i=1}^{J} \frac{w_{s_i}(\gamma_i+1)|h_{i,s_i}|^2 p_{s_i}}{\sum_j |h_{i,s_j}|^2 p_{s_j} + \sigma^2}$$

We apply the lemma J times for each ratio in the sum and switch the two sums. The function to maximise is now

$$f_q(\mathrm{s}, \mathrm{p}, \gamma, \mathrm{y}) = \sum_{i=1}^{J} \Bigg(w_{s_i} \log(1+\gamma_i) - w_{s_i}\gamma_i - y_i^2\sigma^2 + 2y_i\sqrt{w_{s_i}(\gamma_i+1)|h_{i,s_i}|^2 p_{s_i}} - \sum_{j=1}^{J} y_j^2 |h_{j,s_i}|^2 p_{s_i} \Bigg)$$

Decoupled (s_i, p_{s_i}) in each cell. Easier and can be distributed !

With fixed γ, s, p, the optimal y^* is obtained by setting the gradient to zero

$$y_i^* = \frac{\sqrt{w_{s_i}(1+\gamma_i)|h_{i,s_i}|^2 p_{s_i}}}{\sum_j |h_{i,s_j}|^2 p_{s_j} + \sigma^2}.$$

We then substitute y in the expression and find the optimal γ by setting the gradient to zero

$$\gamma_i^* = \frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j \neq i} |h_{i,s_j}|^2 p_{s_j} + \sigma^2}.$$

Finally we compute the optimal power allocation if user s is selected at BS i,

$$p_s^* = \min\Big(\frac{w_s(1+\gamma_i)|h_{i,s}|^2 y_i^2}{(\sum_j |h_{j,s}|^2 y_j^2)^2}, P\Big).$$

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Scheduling

$$s_i^* = \arg\max_{s \in \mathcal{U}_i} \left(G_i(s) - \sum_{j \neq i} D_j(s) \right)$$

where

$$G_i(s) = w_s \log(1+\gamma_i) - w_{s_i}\gamma_i + 2y_i\sqrt{w_s(1+\gamma_i)|h_{i,s_i}|^2 p_s^*} - p_s^* y_i^2 |h_{i,s}|^2$$

$$D_j(s) = y_j^2 |h_{j,s}|^2 p_s^*$$

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The algorithm in a slide !

- **Input :** Channel Matrix H and convergence threshold ϵ
- **Initialization :** Initialize **s** and **p**
- **While not converged in each base station** i **do :**
  - $\gamma_i \leftarrow \frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j \neq i} |h_{i,s_j}|^2 p_{s_j} + \sigma^2}$
  - $y_i \leftarrow \frac{\sqrt{w_{s_i}(1+\gamma_i)|h_{i,s_i}|^2 p_{s_i}}}{\sum_{j} |h_{i,s_j}|^2 p_{s_j} + \sigma^2}$
  - Update (s, p) with previous equations
  - Communicate (p_i, s_i) to the other base station

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Convergence Property

Theorem (The algorithm converges monotonically.)

$$\begin{aligned}
f_0(\mathsf{s}^{t+1}, \mathsf{p}^{t+1}) &= f_r(\mathsf{s}^{t+1}, \mathsf{p}^{t+1}, \gamma(\mathsf{s}^{t+1}, \mathsf{p}^{t+1})) \\
&\geq f_r(\mathsf{s}^{t+1}, \mathsf{p}^{t+1}, \gamma(\mathsf{s}^{t}, \mathsf{p}^{t})) \\
&= f_q(\mathsf{s}^{t+1}, \mathsf{p}^{t+1}, \gamma(\mathsf{s}^{t}, \mathsf{p}^{t}), \mathsf{y}(\mathsf{s}^{t+1}, \mathsf{p}^{t+1}, \gamma(\mathsf{s}^{t}, \mathsf{p}^{t}))) \\
&\geq f_q(\mathsf{s}^{t+1}, \mathsf{p}^{t+1}, \gamma(\mathsf{s}^{t}, \mathsf{p}^{t}), \mathsf{y}(\mathsf{s}^{t}, \mathsf{p}^{t}, \gamma(\mathsf{s}^{t}, \mathsf{p}^{t}))) \\
&\geq f_q(\mathsf{s}^{t}, \mathsf{p}^{t}, \gamma(\mathsf{s}^{t}, \mathsf{p}^{t}), \mathsf{y}(\mathsf{s}^{t}, \mathsf{p}^{t}, \gamma(\mathsf{s}^{t}, \mathsf{p}^{t}))) \\
&= f_r(\mathsf{s}^{t}, \mathsf{p}^{t}, \gamma(\mathsf{s}^{t}, \mathsf{p}^{t})) \\
&= f_0(\mathsf{s}^{t}, \mathsf{p}^{t})
\end{aligned}$$

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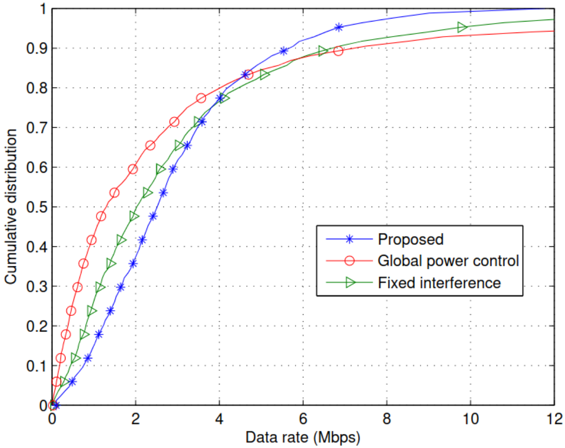
Experimental Validation



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Jongs Algorithm [4]

Algorithm 3.3: Jong's algorithm to optimally solve Problem 3.6.

Set $\epsilon > 0$, $t = 0$, initialize $\mathbf{u}^{(0)}$ and $\boldsymbol{\beta}^{(0)}$

Set $C_D := \epsilon + 1$.

while $C_D > \epsilon$ **do**

- Find $\mathbf{x}_t^*$ by optimally solving the problem defined by (3.27)-(3.28) with $\mathbf{u}^{(t)}$ and $\boldsymbol{\beta}^{(t)}$.
- Set $C_D := \|\psi(\boldsymbol{\beta}^{(t)}, \mathbf{u}^{(t)}, \mathbf{x}_t^*)\|$.
- For $k = 1, \ldots, j$, compute $u_k^{(t+1)}$ and $\beta_k^{(t+1)}$ using (3.31) and (3.32), respectively.
- Set $t = t + 1$.

end

Dinkelbach Algorithm [4]

Algorithm 3.1: Dinkelbach's algorithm to optimally solve Problem 3.4.

Set $\epsilon > 0$, $t = 0$ and $\lambda^{(0)} = 0$

Set $C_D = \epsilon + 1$.

while $C_D > \epsilon$ **do**

- Find $\mathbf{x}_t^*$ by optimally solving the problem defined by (3.17)-(3.18) with $\lambda^{(t)}$.
- Set $C_D = f_0(\mathbf{x}^*) - \lambda^{(t)} h_0(\mathbf{x}^*)$.
- Compute $\lambda^{(t+1)}$ using (3.19).
- Set $t = t + 1$.

end
