Image Radiometry

Arthur Leclaire

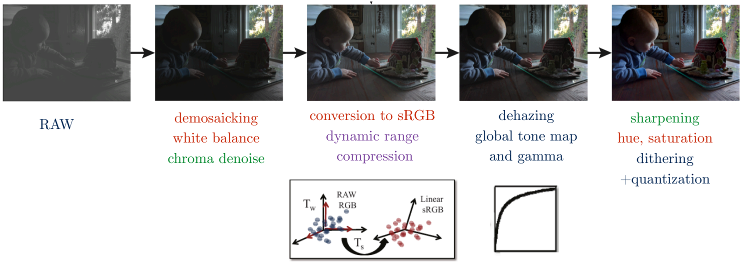
(based on the previous course given by Julie Delon)

MVA Introduction to Image Processing
October, 8th, 2025

Introduction

- A RAW image is acquired from the scene, and is affected by
  - blur (from defocus or motion)
  - sampling with the CCD sensor (which produces noise)
  - quantization (on discrete graylevel/color values)

- Going from the RAW image to the output RGB image involves several steps:



- In the two next sessions, we will discuss the steps related to graylevel/color transforms.

Plan

Histograms and Contrast Changes

Optimal Transport and Histograms

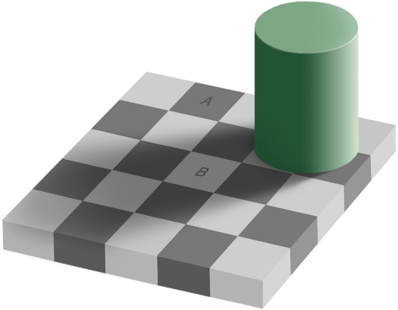
Related Topics

Contrast Change

- Our Human Visual System (HVS) is quite robust to increasing transformations of the gray levels.
- One can thus look for graylevel transformations that increase perceived quality while not changing the geometric content.

Local Perception and Optical Illusions

Our visual perception is strongly guided by our sensitivity to local contrasts. (Kanisza, 1980)



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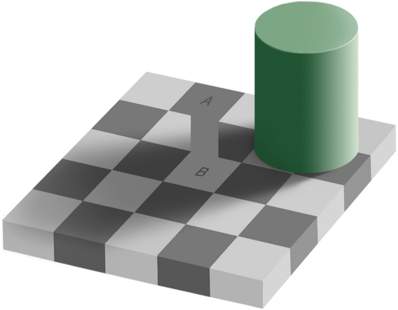

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Negative Image

However, our HVS does not compensate easily for decreasing contrast changes.

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Image Histogram

Let $u : \Omega \to \mathbf{R}$ be an image defined on a rectangle Ω with $M \times N$ pixels.

Definition

The histogram of u is the probability distribution on $\mathbf{R}$ defined by

$$h_u = \frac{1}{|\Omega|} \sum_{x \in \Omega} \delta_{u(x)}.$$

Remarks:

- h_u is the probability distribution of $u(X)$ where $X \sim \mathcal{U}(\Omega)$.
- If u has values in $A = \{a_1, \dots, a_K\}$ (e.g. $\{0, \dots, 255\}$), one can write

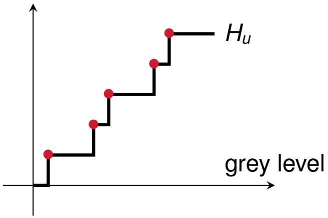
$$h_u = \sum_{k=0}^{K} p_k \delta_{a_k} \quad \text{where} \quad p_k = \frac{1}{|\Omega|} \left|\{ x \in \Omega \mid u(x) = a_k \}\right| = \mathbb{P}(u(X) = a_k).$$

Cumulative Histogram

Definition

The cumulative histogram of u is the fonction $H_u : \mathbf{R} \to [0, 1]$ defined by

$$\forall t \in \mathbf{R}, \quad H_u(t) = \int_{-\infty}^{t} h_u(s)ds = \frac{1}{|\Omega|} |\{ x \in \Omega \mid u(x) \leqslant t \}|.$$



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- By definition, H_u is the cumulative distribution function (c.d.f.) of the graylevel distribution h_u of u.
- We have $H_u' = h_u$ in the distribution sense (i.e. H_u is a primitive of h_u).

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- If u takes $|\Omega|$ distinct values, then for $x \in \Omega$, $|\Omega| H_u(u(x))$ is the rank of $u(x)$ among all u values.
- $H_u(u(x))$ can be understood as the “normalized rank” of $u(x)$.

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- $H_u(u(x))$ can be understood as the “normalized rank” of $u(x)$.
- We have

$$\begin{aligned} u(x) < u(y) &\implies H_u(u(x)) < H_u(u(y)) \\ u(x) \leq u(y) &\implies H_u(u(x)) \leq H_u(u(y)) \end{aligned}$$

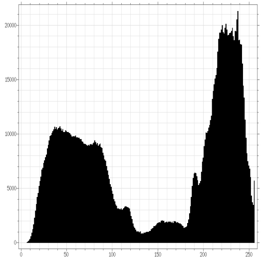
  and therefore

$$u(x) \leq u(y) \iff H_u(u(x)) \leq H_u(u(y)).$$

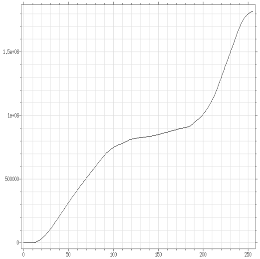
Example of Image Histogram

Here is an example of histogram and cumulative histogram for an 8-bit image of size 796 $\times$ 572.

u



h_u



H_u

Looking at the histogram allows you to see the proportions of dark or light areas in the image.

Contrast Change

Definition

- A contrast change is a non-decreasing function $g : \mathbf{R} \rightarrow \mathbf{R}$.
- Applying a contrast change to the image u consists in computing the image

$$g \circ u(x) = g(u(x)).$$

This modified image is sometimes denoted (abusively) by $g(u)$.

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Proposition

- *If* $g : \mathbf{R} \to \mathbf{R}$ *is continous and increasing, then* $H_{g \circ u} = H_u \circ g^{-1}$.
- *If* $g : \mathbf{R} \to \mathbf{R}$ *is increasing, then*

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REMARK: If g is only non-decreasing, there may exists values a_k, a_ℓ of u such that $g(a_k) = g(a_\ell)$. In this case, after contrast change g, the areas of values a_k, a_ℓ are fused (information loss).

Simple Contrast Changes

There are several explicit contrast changes that are often helpful:

- Affine contrast change

$$m + \sigma \times \frac{u - \text{mean}(u)}{\text{std}(u)}$$

  allows to prescribe the mean and standard deviation to m and σ.

- Logarithmic change

$$\log(1 + u)$$

  helps to display images $u \geq 0$ with very large range (e.g. Fourier modulus).

- Piecewise affine functions may help to rescale more precisely some part of the image dynamics.

Generalized Inverse

Let μ be a probability distribution with cumulative distribution function (cdf) F_μ.

Definition

The generalized inverse of F_μ (also called “quantile function”) is defined by

$$F_\mu^-(\alpha) = \inf\{\, t \in \mathbf{R} \mid F_\mu(t) \geqslant \alpha \,\} \quad (\alpha \in]0,1[)\ .$$

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$$\forall \alpha \in]0,1[, \quad \{\, t \in \mathbf{R} \mid F_\mu(t) \geqslant \alpha \,\} = [F_\mu^-(\alpha), +\infty[\; ,$$

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REMARK: One can also show that for any $t \in \mathbf{R}$, $F_\mu \circ F_\mu^{-1}(t) \geq t$ and $F_\mu^{-1} \circ F_\mu(t) \leq t$.

Histogram Specification

Let $u : \Omega \to \mathbf{R}$ an image defined on a $M \times N$ rectangle Ω.
Let μ be a probability distribution on $\mathbf{R}$.

QUESTION: How to design a contrast change g such that the histogram $h_{g \circ u}$ is close to μ?

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QUESTION: How to design a contrast change g such that the histogram $h_{g \circ u}$ is close to μ?

There are several ways to measure how close two distributions are.

Proposition

Assume that μ has an increasing cdf F_μ. Let us consider $g \circ u$ with

$$g = F_\mu^- \circ H_u.$$

Then $H_{g \circ u}$ and F_μ coincide on the values of $g \circ u$.

EXERCISE: Suppose that u has histogram $h_u = \frac{1}{4}\delta_1 + \frac{1}{2}\delta_2 + \frac{1}{4}\delta_3$, and let $\mu = \mathcal{U}(0, 1)$.
Compute the histogram of the image $g \circ u$ introduced in the proposition.

Histogram Specification: other choices

There exist other choices for g such that $h_{g\circ u}$ and μ are “close”.

Assume that u has values $\{a_1, \dots, a_K\}$ and F_μ is increasing. Let g be a contrast change such that

$$g(a_k) = F_\mu^{-1} \circ \left(\frac{H_u(a_k) + H_u(a_{k-1})}{2} \right).$$

Then one can show (see L. Moisan’s course), that any such contrast change solves

$$\operatorname*{Argmin}_g \int_{\mathbf{R}} |H_{g\circ u}(t) - F_\mu(t)|^2 dt.$$

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Remark: Of course, we cannot ask $g \circ u$ to have exactly distribution μ.

Histogram Equalization

Let $u : \Omega \to \mathbf{R}$ defined on a discrete rectangle Ω.

Definition

Performing an histogram equalization of u consists in finding a contrast change g such that $h_{g\circ u}$ is "close" to the uniform distribution $\mathcal{U}(0,1)$.

- A standard way to perform histogram equalization is to use the contrast change $g = H_u$.
- This is justified by the fact that
$$\forall t \in \mathbf{R}, \quad H_{H_u \circ u}(t) \leq t$$
with equality at any t that can be written $t = H_u(u(x))$.
In this sense, the cumulative distribution function of $H_u \circ u$ is close to the one of $\mathcal{U}(0,1)$.

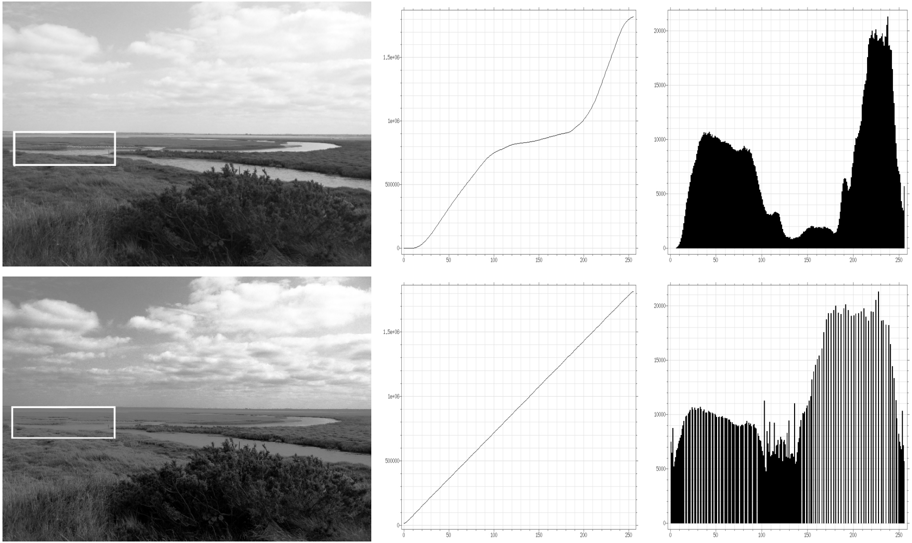
Algorithm for Histogram Equalization

To perform histogram equalization in practice:

- Order the pixel values $u(x)$ keeping the ranks $r(x) \in \{0, \ldots, |\Omega| - 1\}$.
- At each pixel x, replace the value $u(x)$ by the normalized rank $\frac{r(x)}{|\Omega|} \in [0, 1]$.
- You may apply a normalization to go back to a desired range. (for example, $\times 255$)

NB: If two pixels x, y are ex-aequo, you can either put the same rank or disambiguate arbitrarily.

Example of Histogram Equalization



Information Loss

Before equalization:

After equalization:

Increasing quantization noise

Histogram equalization sometimes amplifies noise in the image.

Original image

Equalized image

Plan

Motivation: Image Comparison

Images taken by Lionel Moisan.

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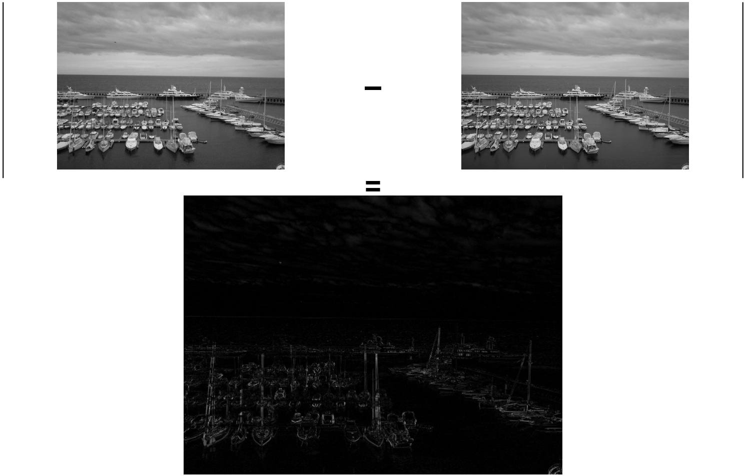


Motivations

Comparing two images is easier after color/histogram equalization.

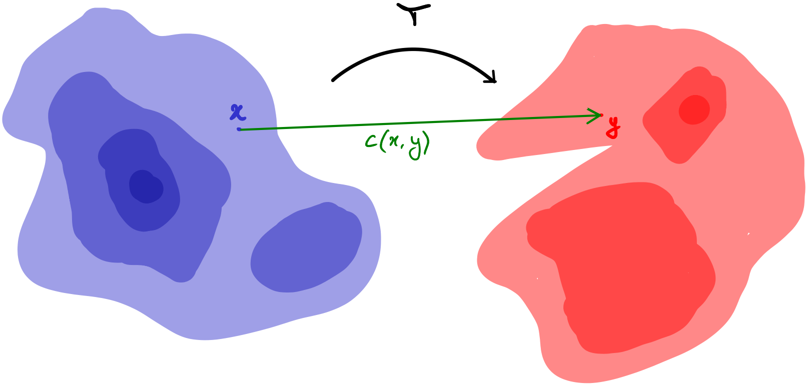
Correcting or transforming the radiometry or colors of an image or a set of images

- compute a color palette that is common to N images
- correcting flicker effects in films
- color palette specification (for example: day to night effect)

This requires a kind of distance or interpolation between graylevel/color distributions.

"Les déblais et les remblais"...

Optimal transport has been introduced by Gaspard Monge in his
Mémoire sur la Théorie des Déblais et des Remblais (1784)

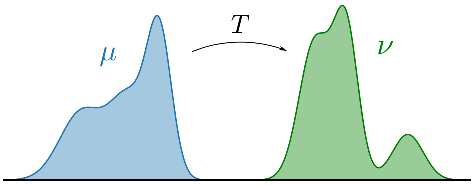


Optimal Transport (see G. Peyré's or Villani's books)

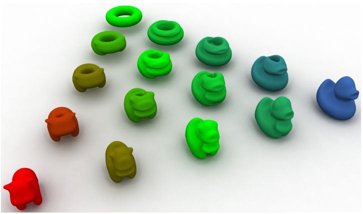
For μ, ν probability measures on $\mathbf{R}^d$, let

$$\mathrm{OT}(\mu, \nu) = \min_T \int_{\mathbf{R}^d} c(x, T(x)) d\mu(x)$$

where T should send μ onto ν.



COLOR TRANSFER



SHAPE INTERPOLATION

Two OT formulations (see (Santambrogio, 2015))

Let μ, ν two probability distributions supported in $\mathcal{X}, \mathcal{Y} \subset \mathbf{R}^d$.
Let $c : \mathcal{X} \times \mathcal{Y} \to \mathbf{R}$ a function (called the *ground cost*).

OPTIMAL TRANSPORT COST WITH MONGE FORMULATION:

$$\mathrm{OT}(\mu, \nu) = \min_{T\sharp\mu=\nu} \int_{\mathbf{R}^d} c(x, T(x))d\mu(x) \qquad \text{(OT-Monge)}$$

where $T\sharp\mu(A) = \mu(T^{-1}(A))$ for all A.
A solution of (OT-Monge) is called an **optimal transport map** from μ to ν.

NB: There exist singular cases of this problem. For some (μ, ν) there is no T such that $T\sharp\mu = \nu$.

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OPTIMAL TRANSPORT COST WITH KANTOROVICH FORMULATION:

$$W(\mu, \nu) = \inf_{\pi \in \Pi(\mu,\nu)} \int_{\mathcal{X} \times \mathcal{Y}} c(x, y) \, d\pi(x, y) \qquad \text{(OT-Kanto)}$$

where $\Pi(\mu, \nu)$ is the set of distributions π on $\mathcal{X} \times \mathcal{Y}$ with marginals μ, ν.
A solution of (OT-Kanto) is called an **optimal transport plan** for (μ, ν).

NB: If T solves (OT-Monge), then the law of $(X, T(X))$ (with $X \sim \mu$) solves (OT-Kanto).
Also, under weak regularity assumptions on μ, $\mathrm{OT}(\mu, \nu) = W(\mu, \nu)$ (Santambrogio, 2015).

Metric Properties

For $c(x,y) = \|x-y\|^p$, $p \in [1,\infty)$, the p-Wasserstein cost is defined by

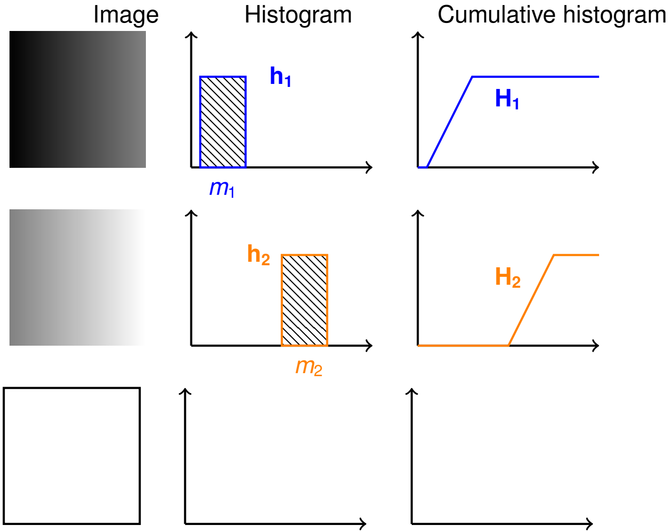
$$W_p(\mu,\nu) = \inf_{\pi \in \Pi(\mu,\nu)} \int_{\mathcal{X}\times\mathcal{Y}} \|x-y\|^p \, d\pi(x,y).$$

Theorem (See e.g. Chap 6 of (Villani, 2009))

Let $\mathcal{P}_p$ the set of probability measures μ on $\mathbf{R}^d$ such that $\int \|x\|^p d\mu(x) < \infty$.

- $W_p^{\frac{1}{p}}$ *is a distance on* $\mathcal{P}_p$.
- $\mu_n \xrightarrow[n\to\infty]{W_p} \mu$ *if and only if* $\begin{cases} \forall \varphi \in \mathscr{C}_b(\mathbf{R}^d), \quad \int \varphi d\mu_n \to \int \varphi d\mu \\ \int \|x\|^p d\mu_n(x) \to \int \|x\|^p d\mu(x) \end{cases}$.

How to interpolate between histograms?



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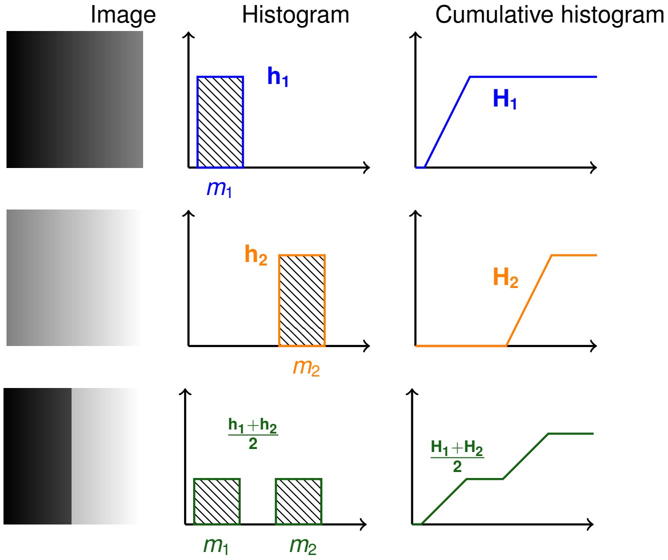

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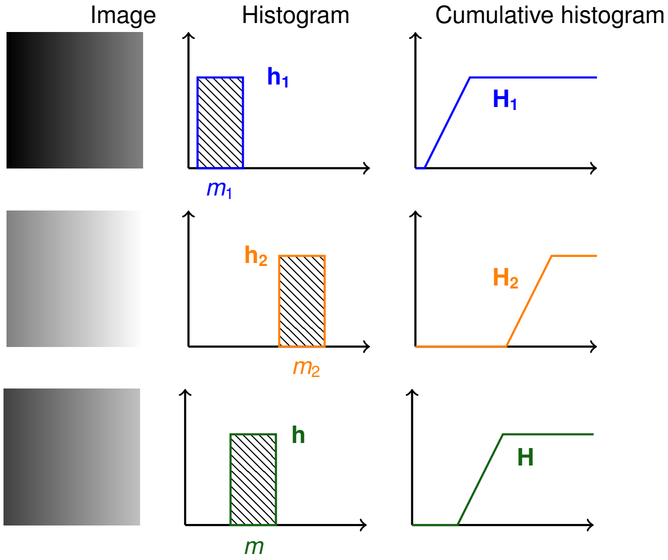

Optimal Transport - 1D case

Let μ, ν two probability distributions on $\mathbf{R}$ and $c : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ lower bounded.
Recall that F_μ is the cumulative distribution of μ.

Proposition

Suppose that μ has no atom. Then $T = F_\nu^- \circ F_\mu$ satisfies $T_\sharp \mu = \nu$.

Theorem (See e.g. in (Santambrogio, 2015))

Suppose that $c(x, y) = h(x - y)$ with $h : \mathbf{R} \to \mathbf{R}$ strictly convex. Then

- *$(F_\mu^-, F_\nu^-)_\sharp \mathcal{U}(0, 1)$ is an optimal transport plan for (μ, ν).*
- *If μ is atomless, then $T = F_\nu^- \circ F_\mu$ is an optimal transport map from μ to ν.*

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- *If μ is atomless, then $T = F_\nu^- \circ F_\mu$ is an optimal transport map from μ to ν.*

Consequence: The theorem gives an expression of the OT cost

$$W(\mu, \nu) = \int_0^1 h(F_\mu^-(t) - F_\nu^-(t))dt.$$

In particular, the $W_2^{1/2}$ distance is the L^2 distance between the inverse cdf of μ and ν.

Remark: One main practical issue is that we usually work with discrete histograms.
$\longrightarrow$ Thus we have to deal with *ex-aequo* values.

Kantorovich Barycenter

The Kantorovich barycenter of L probability distributions $(\nu^1, \dots, \nu^L)$ on $\mathbf{R}^d$ with weights $(\rho_1, \dots, \rho_L)$ such that $\sum_\ell \rho_\ell = 1$ is defined by

$$\underset{\mu}{\text{Argmin}} \sum_{l=1}^{L} \rho_l W(\mu, \nu^l)^2$$

where the minimum goes over all probability distributions μ on $\mathbf{R}^d$.

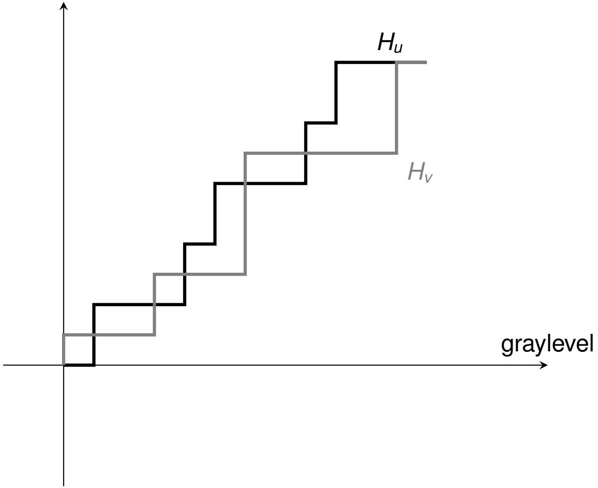
Definition

Let u, v two images defined on a discrete rectangle Ω.
The midway histogram associated to u, v is the distribution h_{midway} whose inverse cdf is

$$H^-_{midway} = \frac{H^-_u + H^-_v}{2}.$$

- This definition makes sense because H^-_{midway} is a piecewise constant left-continuous function.
- The midway histogram solves the W_2 isobarycenter problem.

Example of Midway Histogram



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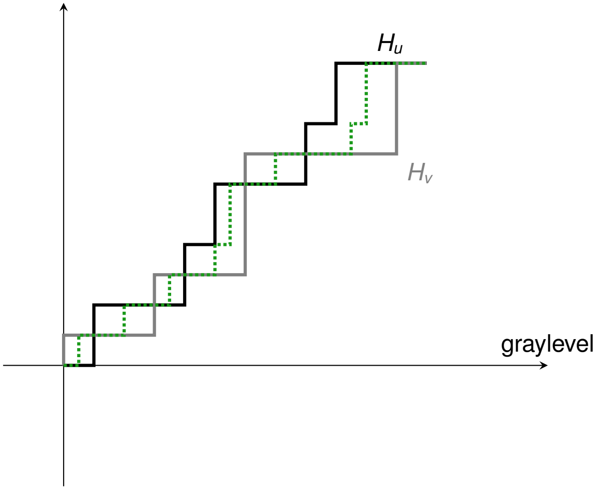


Midway Equalization

(Delon, 2004b)

The midway equalization consists in applying respectively to images u and v the contrast changes

$$H_{midway}^{-} \circ H_u \qquad \text{and} \qquad H_{midway}^{-} \circ H_v$$

Since $H_u^{-} \circ H_u \circ u = u$, this is equivalent to applying the contrast changes

$$g_u = \frac{1}{2}(\mathrm{Id} + H_v^{-} \circ H_u) \qquad \text{and} \qquad g_v = \frac{1}{2}(\mathrm{Id} + H_u^{-} \circ H_v).$$

- Then images $g_u(u), g_v(v)$ have quasi-identical distributions, close to h_{midway}.
- In practice, we can use an approximate formula based on ranking (faster to implement).

Example of Midway Equalization

u

v

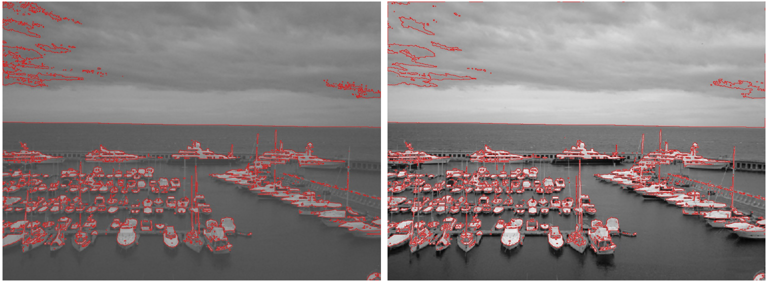
Example of Midway Equalization

$g_u(u)$

$g_v(v)$

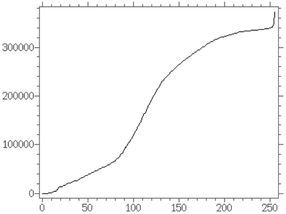
Action on the Level Sets

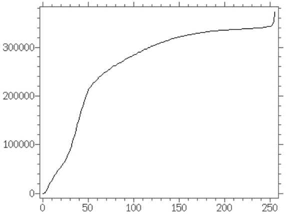
During midway equalization, the gray levels with same rank in the two images are averaged.



Median level sets between the two images (71 in first image, and 155 in second image).

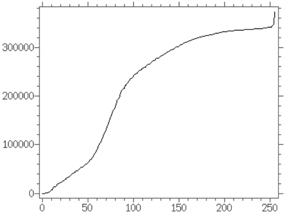
Examples of Midway Equalization

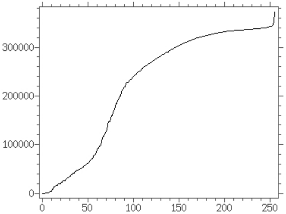




Before midway equalization

Examples of Midway Equalization





After midway equalization

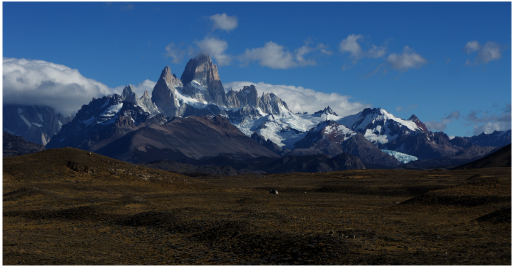
Examples of Midway Equalization

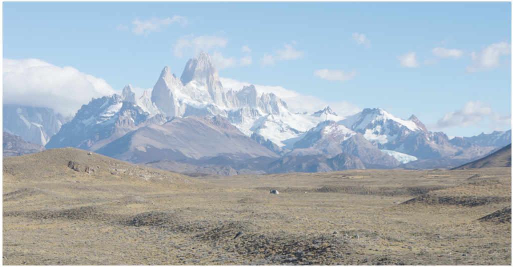
Before midway equalization

Examples of Midway Equalization

After midway equalization

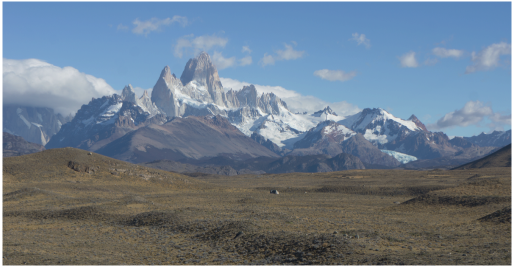
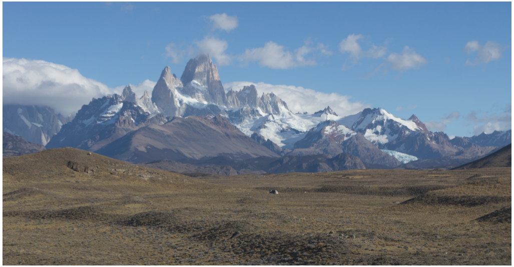
Examples of Midway Equalization





Before midway equalization

Examples of Midway Equalization

After midway equalization

Application to satellite images

(Delon, 2004a)

Before midway equalization

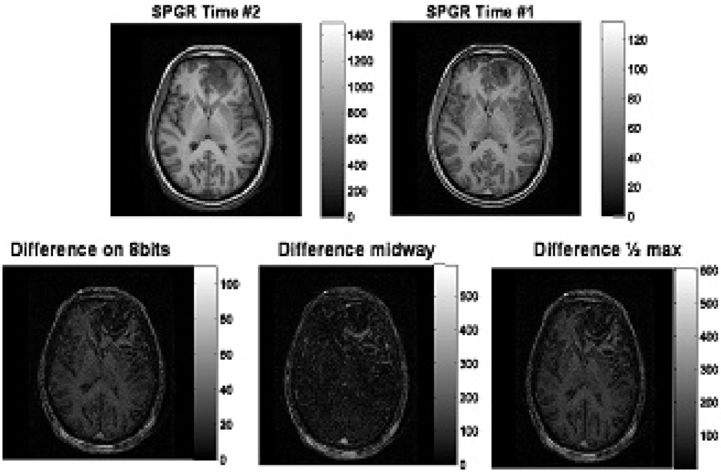
Application to satellite images

(Delon, 2004a)

After midway equalization

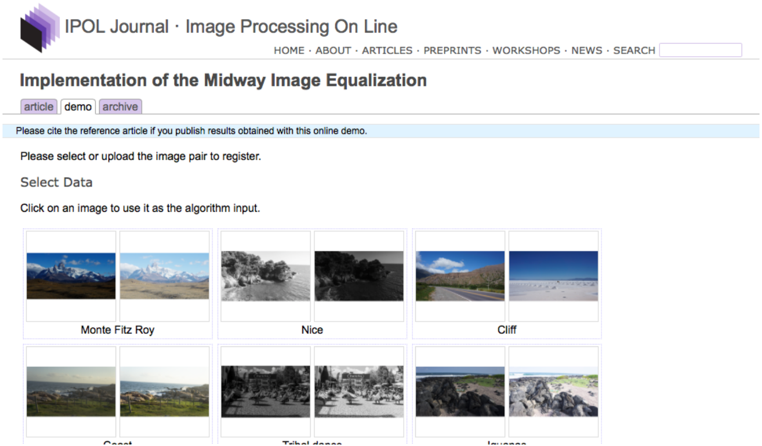
Application en imagerie médicale

(Angelini et al., 2007)



Mid-way Online Demo

(Guillemot and Delon, 2016)

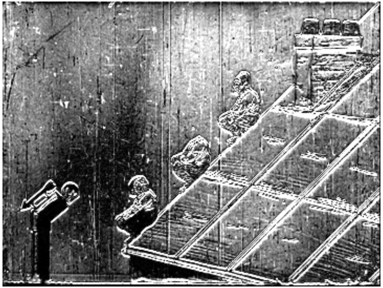


http://www.ipol.im/pub/art/2016/140/

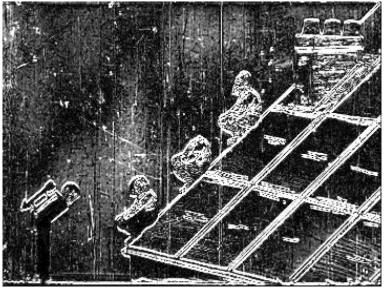
Midway Video Equalization

(Delon, 2005; Sánchez, 2017)

- Midway equalization can be generalized to K frames of a film.
- It is helpful to correct “flicker” artifacts (for example in old movies).



Before Correction



After Correction

Differences between consecutive frames of a film

Plan

Gamma Correction

Old screens called CRT (for cathode-ray tube) had a non-linear behavior (for electronic reasons).

If v is the input signal (with values in $[0,1]$), it can be approximately modeled by a power-law

$$u = v^{\gamma}\,, \quad \text{with} \quad \gamma \approx 2.2.$$

This make images appear darker than they are:

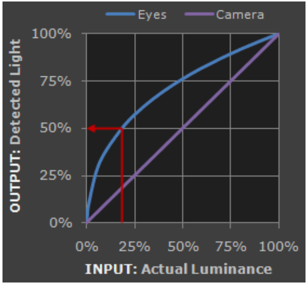
Gamma Correction

Some acquisition devices (cameras, scanner) apply the so-called **Gamma correction**:

$$v = v_{real}^{\frac{1}{\gamma}} \quad \text{with} \quad 1/2.2 \approx 0.45.$$

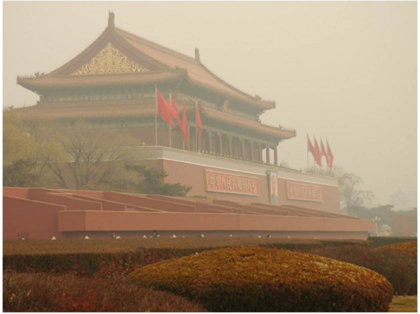
Most CCD (charged-couple device) sensors use this, and then use a uniform quantization.

In addition to compensate the non-linearity, it helps to better respect the Human perception:



Dehazing

Graylevel or Color Images may be affected by haze (or fog effects):



Dehazing

(He et al., 2010)

An image (with C channels) containing haze can be modeled as

$$u(x) = r(x)t(x) + a(1 - t(x))$$

where

- $u(x) \in \mathbf{R}^C$ is the observed intensity
- $r(x) \in \mathbf{R}^C$ is the scene radiance (same shape as u)
- $t(x) \in [0, 1]$ is the medium transmission : proportion of light that reaches the camera
  It depends on the depth of the observed point.
- $a \in \mathbf{R}^C$ is the global atmospheric light

Dehazing

Dehazing would consist in estimating r, t, a from u.

He et al. introduced the **dark channel prior**

$$d(x) = \min_{y \in \mathcal{N}_x,\ c \in \{r,g,b\}} u_c(y)$$

where $\mathcal{N}_x$ is a 15×15 neighborhood of x.

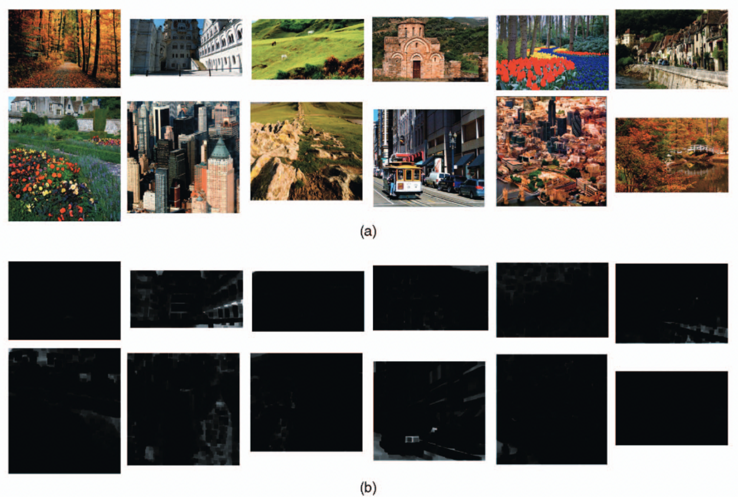
Gist: “on haze-free outdoor images: in most of the non-sky patches, at least one color channel has very low intensity at some pixels.”

Dehazing method:

- estimate atmospheric light a from the values of u in the 0.1% brightest pixels of d.
- the transmission is estimated by $\hat{t}(x) = 1 - \min_{y \in \mathcal{N}_x,\ c \in \{r,g,b\}} \frac{u_c(x)}{a_c}$.
- the radiance is recovered by

$$\hat{r}(x) = \frac{u(x) - a}{\max(\hat{t}(x), t_0)} + a.$$

Dehazing



Examples of dark channels (b) computed from images (a).

Dehazing

Input Images

Dehazing

Dehazed Images

Dehazing

By-product: Estimated Depth $-\log \hat{t}(x)$

Dynamic range and Human adaptation

The Human visual system (HVS) can adapt to a large dynamic range.

condition	illumination (cd/m^2)
star	10^{-3}
Moon	10^{-1}
artificial light	10^2
white sheet under sun light	10^4
sun light at noon	10^{10}

- **HVS** is sensitive to 10 orders of magnitude (5 in a fixed scene).
- **Numerical sensors** : 4 orders of magnitude for best sensors (otherwise, 3)
- **Screens** : 2 or 3 orders of magnitude

Example of a Saturated Image

The sensor capicity is limited. See the course of Y. Gousseau on HDR imaging.

Quantization

Storage/transmission require that images be encoded into normalized finite value ranges.

Exemples :

- **Numerical camera**: 12 bits images (or 8 bits after Gamma correction)
- **Satellite image**: 12 bits (4096 levels)
- **Color images**: 24 bits (8 for each channel).
- **HVS**: adapts to the ambiant light (but does not exceed 6 or 7 bits)

Quantization operator Let us consider values $(q_k)_{k=0,\dots K-1}$ and bins $(t_k)_{k=0,\dots K}$ with

$$t_0 \leq q_0 \leq t_1 \leq q_1 \leq \dots q_{K-1} \leq t_K.$$

The quantization operator Q is defined by $Q(\lambda) = q_i$ if $t_i \leq \lambda < t_{i+1}$.
The quantized image is then obtained as $Q \circ u : \Omega \rightarrow \{q_0, \dots q_{p-1}\}$.

Simple Quantization Methods

Uniform quantization:

Take $Y = \{0, \dots 255\}$ and some integer K that divides 256, and set

$$t_k = i\frac{256}{K}, \quad i = 0, \dots, K \quad \text{and} \quad q_k = \frac{t_k + t_{k+1}}{2} = (k + \frac{1}{2})\frac{256}{K}.$$

Histogram-based quantization:

Set $t_k = H_u^-(\frac{k}{p})$ and q_k (for example) the mean of the u values between $[t_k, t_{k+1}]$.

$\rightarrow$ This is equivalent to perform uniform quantization after histogram equalization.

Lloyd-Max quantization:

Set (t_k, q_k) with an iterative algorithm that minimizes

$$MSQE = \sum_{k=0}^{K-1} \sum_{x | u(x) \in [t_k, t_{k+1}[} h_k(u(x) - q_k)^2.$$

Quantization Examples

Uniform quantization

Quantization Examples

Histogram-based quantization

Quantization Examples

Lloyd-Max quantization

Dithering

- Dithering is a stochastic technique to improve the rendering of the image after quantization.
- Principle is to add independent noise values (depending on the graylevel) before quantization.
- It has been used (for a very long time!) for printing pictures in newspapers!

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Take-home Messages

SUMMARY AND COMMENTS:

- Histograms and Cumulative histograms represent the radiometry of an image.
- There are many solutions to perform contrast changes over images.
- Some of them optimize a criterion on cdf or inverse cdf.
- Optimal transport in 1D can be expressed with explicit formulae based on cdf.
- 1D optimal transport helps to design contrast changes or midway equalization.
- Various applications are related to contrast changes (e.g. flicker reduction, quantization, ...)

THANK YOU FOR YOUR ATTENTION!

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