Large-Scale Generative Modeling

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MVA Generative Modeling
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Last Week

- We introduced GAN and WGAN training
- We studied a low-dimensional example explaining some instabilities...
- ... and gave some solutions to avoid them (Lipschitz penalties)
- We made some connections with optimal transport, in particular with semi-discrete WGAN

Further topics on Learning Stability

- Non-convergence of GAN/WGAN training dynamics (in simple cases) [Mescheder et al., 2018]
  The authors also propose two regularizations to get convergence (continuous dynamics)
- Statistical consistency when working on a sampled version of ν
  [Biau et al., 2018], [Biau et al., 2020]
- Learning multimodal distributions require generators with very large Lipschitz constants!
  [Salmona et al., 2022]

Today

- We will discuss quantitative evaluation of generative models
- We will go back to the particular case of texture generation
- We will focus on popular generative models for large-scale image generation

Plan

Quality Metrics for Generative Models

Generative Texture Models

Large-Scale GAN Training

Quality of a Generative Model

- **Question:** How to measure that the generator covers well the training data?
- **Main idea:** Comparing image distributions is hard...
  but comparing measurements from it is easier.
- Classification neural networks provide a set of deep non-linear features
  For example, VGG19 [Simonyan and Zisserman, 2015], or Inception Networks [Szegedy et al., 2016]
- **Measure quality of the generative model by looking at how deep statistics are preserved**
  Somehow, this ensures that the database is well-covered
- Keep in mind that
  - The network used to get the features must be relevant w.r.t. the generative task at play
  - Quantitative results highly depend on the network and implementation details

Inception Score ↑ [Salimans et al., 2016]

- The inception score measures if μ generates a diverse collection of meaningful pictures
- For an image x, `Inception-v3` gives a label distribution $p(y|x)$ (discrete on $N = 1000$ labels)
- Images containing meaningful objects have $p(y|x)$ with low entropy
- In order to generate various images, $p(y) = \int p(y|x)\mu(dx)$ should have high entropy

The Inception Score then writes as

$$\mathrm{IS}(\mu) = \exp\left(\int \mathrm{KL}\left(p(y|x)|p(y)\right)\mu(dx)\right) \in [1, N]$$

It is 1 iff for a.e. x, $p(\cdot|x) = p(\cdot)$ (label distribution does not depend on x)
It is N iff for a.e. x, $p(\cdot|x)$ is concentrated on one label, and $\forall y$, $\int p(y|x)\mu(dx) = \frac{1}{N}$

How to compute it in practice:

- Compute an estimate $\hat{p}(y)$ of $p(y) = \int p(y|x)\mu(dx)$ by drawing samples of μ
- Estimate $\int \mathrm{KL}(p(y|x)|\hat{p}(y))\mu(dx)$ by drawing samples of μ

Fréchet Inception Distance (FID) $\downarrow$ [Heusel et al., 2017]

The FID measures how close are two image distributions μ, ν in terms of features distributions.
It is based on the response of `Inception-v3` [Szegedy et al., 2016] before last pooling layer:

$$f : \mathbf{R}^d \to \mathbf{R}^m$$

that extracts $m = 2048$ features (as a generic image summary)
NB: Images may have to be resized/normalized to fit into this network.

Algorithm to compute the FID score:

1. Draw samples (x_i) and (y_j) of $X \sim \mu$ and $Y \sim \nu$ and compute the features $(f(x_i)), (f(y_j))$
2. Fit Gaussian distributions $\mathcal{N}(m_X, \Sigma_X)$ and $\mathcal{N}(m_Y, \Sigma_Y)$ to $(f(x_i)), (f(y_j))$ (in $\mathbf{R}^{2048}$)
3. Return the 2-Wasserstein distance between the Gaussian distributions, i.e. the Fréchet distance: [Dowson and Landau, 1982]

$$W_2^2\Big(\mathcal{N}(m_X, \Sigma_X), \mathcal{N}(m_Y, \Sigma_Y)\Big) = \|m_X - m_Y\|_2^2 + \mathrm{Tr}\Big(\Sigma_X + \Sigma_Y - (\Sigma_X \Sigma_Y)^{\frac{1}{2}}\Big)$$

NB: FID can be adapted to the “single-image” case: **SiFID** [Shaham et al., 2019]
SiFID compares distributions of features obtained after a convolution layer (spatially averaged)

Comments on Generative Quality

- Inception Score does not depend on the target distribution ν.
- Need to distinguish "precision/recall" for evaluating quality [Lucic et al., 2018].
  "Precision" is the probability that a fake image falls within the distribution of real images.
  "Recall" is the probability that a real image falls within the distribution of fake sample.
  IS mainly captures precision. FID captures both precision and recall.
- The IS and FID are not enough to measure the fact that samples are photo-realistic [Barratt and Sharma, 2018],
- Other measures have been proposed better correlated with Human prediction of quality [Kolchinski et al., 2019]

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Large-Scale GAN Training

Exemplar-based Texture Synthesis

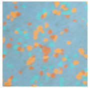
- Examplar texture :

$$u_0 : \Omega \to \mathbf{R}^d$$

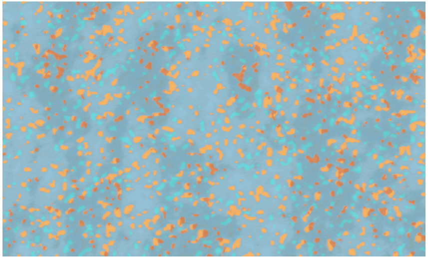
defined on a discrete rectangle $\Omega \subset \mathbb{Z}^2$ with d channels

- Texture model: stationary random field

$$V : \mathbb{Z}^2 \to \mathbf{R}^d$$



Exemplar u_0



Synthesis v

The problem can be split into

- Estimate a model V
- Draw one (or several) samples of V

What do we want to preserve ?

- Covariance, Fourier spectrum
  [Lewis, 1984], [Van Wijk, 1991], [Galerne et al., 2011], [Gilet et al., 2014]
- Wavelet statistics
  [Heeger & Bergen, 1995], [Zhu et al., 1998], [Portilla & Simoncelli, 2000], [Tartavel et al., 2014], [Zhang & Mallat, 2017], [Bruna & Mallat, 2019]
- Local Aspect, Patch statistics
  [Efros & Leung, 1999], [Kwatra et al., 2005], [Lefebvre & Hoppe, 2005]
- Neural Statistics
  [Gatys et al., 2015], [Lu et al., 2015], [Ulyanov et al., 2016]

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- **Neural Statistics**
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Synthèse de Textures par Transport Optimal de Patches

[Galerne et al., 2018]

QUESTION : **How to prescribe the patch distribution at several resolutions ?**

PRINCIPLE OF THE "TEXTO" MODEL:

- Initialize with a Gaussian field at coarse resolution
- At each resolution, apply a patch transport map to reimpose the exemplar patch distribution ν_s
- Upsample cleverly to go from one scale to the next

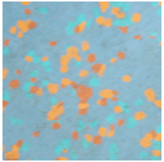


Image u^0
Patch distrib ν^0

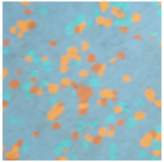


Image u^1
Patch distrib ν^1

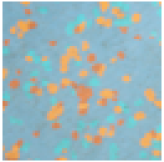


Image u^2
Patch distrib ν^2

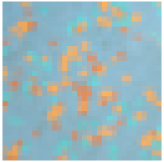


Image u^3
Patch distrib ν^3

The Texto Model

Compute exemplar $u^s : \Omega \cap (2^s\mathbb{Z}^2) \to \mathbf{R}^d$ at different scales $s = 0, \dots, S-1$
and corresponding patch distributions ν^s

Initialize synthesis with Gaussian field U_{S-1} at the coarse scale

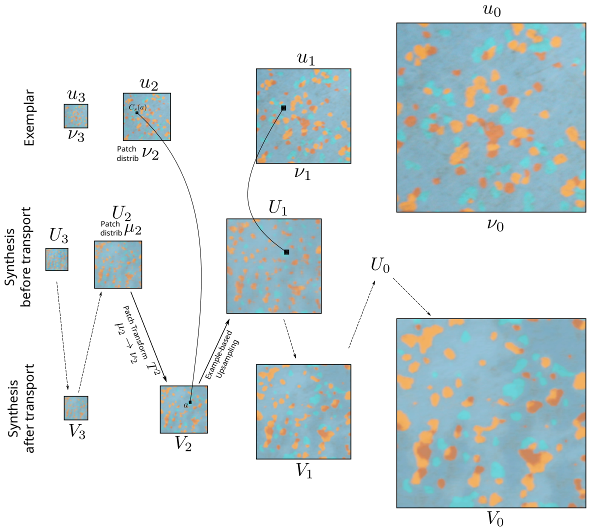
For $s = S-1, \dots, 0$,

- ■ Estimate the patch distribution μ_s of U_s
- ■ Learn a patch semi-discrete OT map T_s such that $T_s\sharp\mu_s \approx \nu_s$
  (Recall that T_s is a biased nearest neighbor assignment!)
- Apply T_s to all patches of U_s and recompose by averaging to an image V_s
- If $s > 0$, upsample V_s to initialize the next scale U_{s-1}
  (For that, use patches at the same positions, but twice larger.)

Output: synthesis at fine scale V_0

Remark: Once the model learnt, one can discard the learning steps ■ to do synthesis on-the-fly

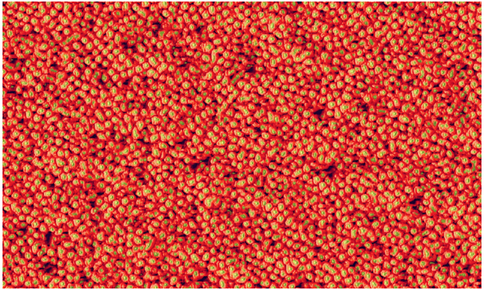
Texto in one diagram



Texto Results

- Long-range independence property
- Patches are transformed independently
  $\rightarrow$ allows for parallel computations
- Patch OT maps can be computed offline.
  $\rightarrow$ allows for very fast synthesis
- Synthesis slightly blurry
  due to patch averaging

Original
128 $\times$ 128



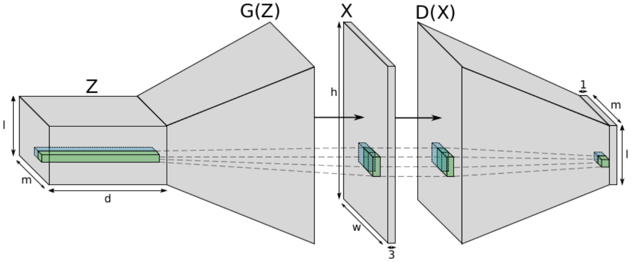
Synthesis
1280 $\times$ 768 (4 scales, 1s)

Spatial GANs [Jetchev et al., 2016], [Bergmann et al., 2017]

- Symmetric Convolutional Networks for G and D (as DCGAN, see later)
- From a $l \times m$ noise Z, $g_\theta(Z)$ generates a $h \times w$ image (in practice $l = m = 4$ and $h = w = 640$)
- Standard GAN loss (binary cross-entropy) but averaged over spatial positions (λ, μ):

$$\sum_{\lambda,\mu} \mathbb{E}[\log(1 - D_{\lambda,\mu}(g_\theta(Z)))] + \mathbb{E}[\log D_{\lambda,\mu}(Y')] \quad \text{where } Y' \text{ is a patch from } u_0$$

- PSGAN works on an augmented noise input Z, with local, global and periodic parts



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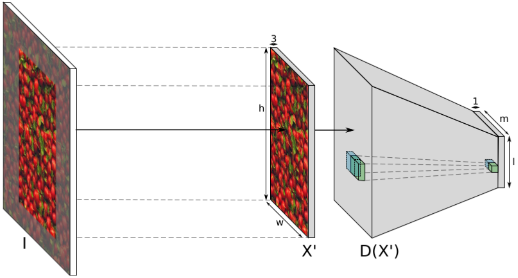
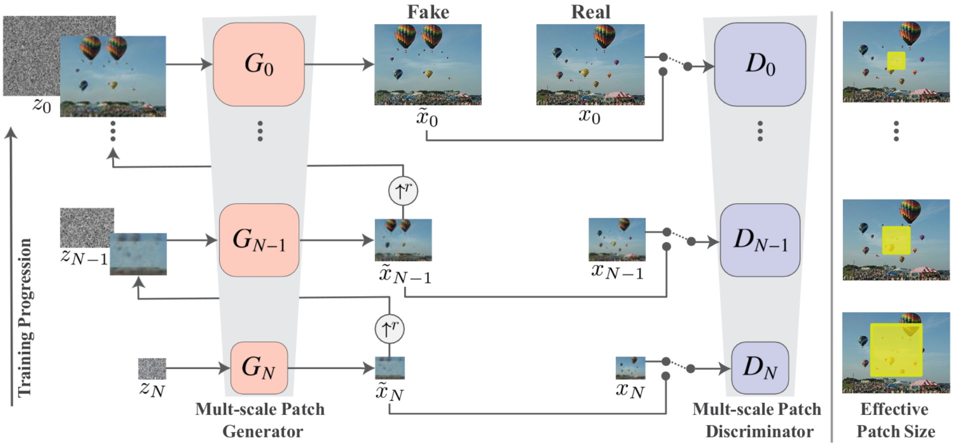


SinGAN: Learning from a Single Image [Shaham et al., 2019]

- Capture the multi-scale patch distributions of an image (possibly non-texture)
- Coarse-to-fine generator
- Patch-based dicriminator learned with WGAN-GP loss, at each scale
- Loss defined over all patches of the image, and not randomly selected patches
  $\rightarrow$ allows the network to learn boundary conditions

SinGAN: Learning from a Single Image [Shaham et al., 2019]



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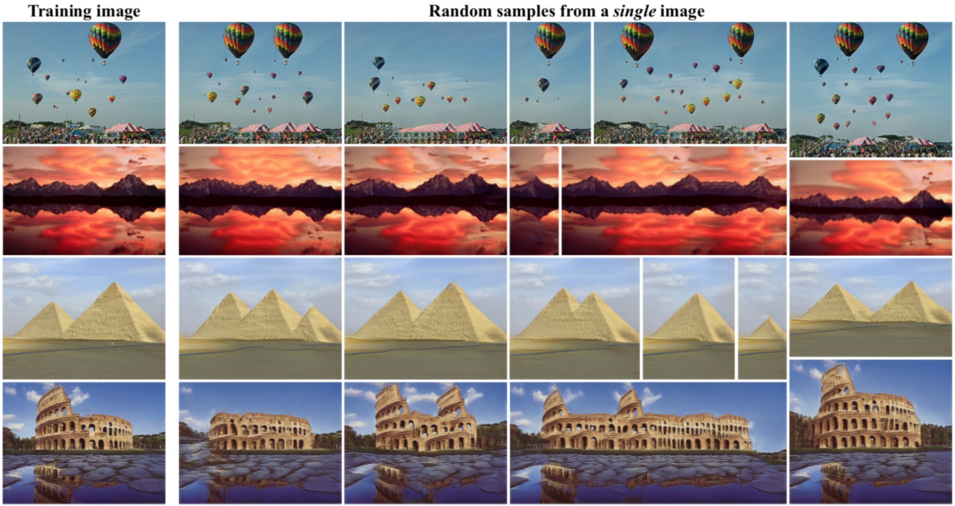


Generative Networks for Texture Synthesis [Houdard et al., 2023]

IDEA : Build a generative network g_θ that directly constrains features distributions where

$$\mathcal{F}_p(u) : \Omega \to \mathbf{R}^{d_p} \text{ extracts features fo type } p.$$

For each feature type p, let

- $\mu_{\theta p}$: distribution of features $\mathcal{F}_p(g_\theta(Z))$
- ν_p : empirical distribution of features $\mathcal{F}_p(u_0)$

Examples:

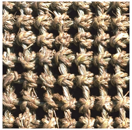
- $\mathcal{F}_p(u) : \Omega \to \mathbf{R}^{s_p \times s_p}$ extracts the $s_p \times s_p$ patches of u
- $\mathcal{F}_p(u) : \Omega_p \to \mathbf{R}^{d_p}$ extracts the response to layer p of a neural network (e.g. VGG)

Learning of GOTEX model

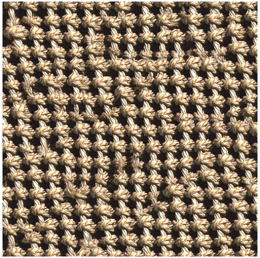
$$\inf_\theta \sum_p W(\mu_{\theta p}, \nu_p)$$

$\longrightarrow$ Alternate optimization with one dual variable ψ_p for each p

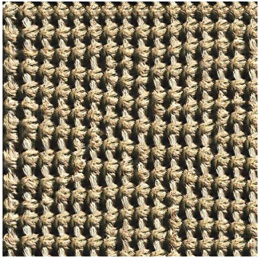
Samples of Texture Networks



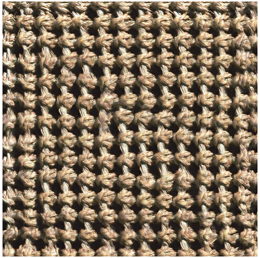
Original



GOTEX
[Houdard et al., 2023]

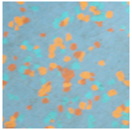


PSGAN
[Bergmann et al., 2017]

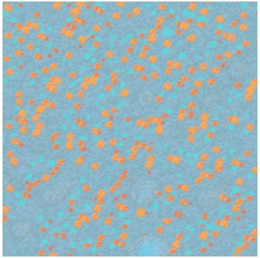


SinGAN
[Shaham et al., 2019]

Samples of Texture Networks



Original



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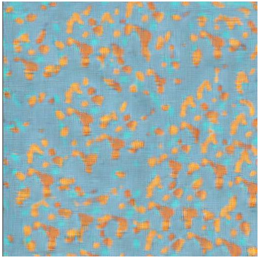
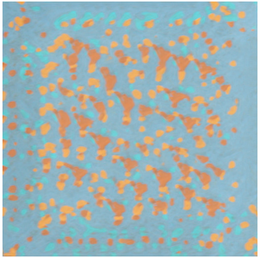


PSGAN
[Bergmann et al., 2017]



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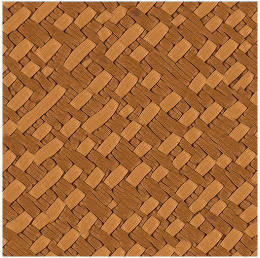
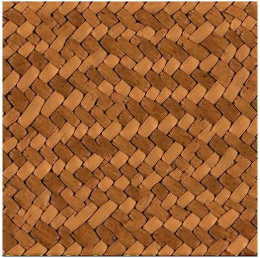


GOTEX
[Houdard et al., 2023]

PSGAN
[Bergmann et al., 2017]



SinGAN
[Shaham et al., 2019]

Plan

Popular Image Databases

- MNIST (digits): 60000 images with 28^2 px
- Fashion-MNIST (clothes): 70000 images with 28^2 px
- CIFAR-10: 60000 images with 32^2 px
- CelebA: $\approx$ 200000 images with 178×278 px
- CelebA-HQ: $\approx$ 30000 images with 1024^2 px
- LSUN (Bedroom/Cat/Churches/...): $\approx 10^5, 10^6$ images with 256^2 px
- FFHQ (or FFHQ-U): 70000 images with 1024^2 px

Neural Network architecture

Generator and discriminator networks can have various layers:

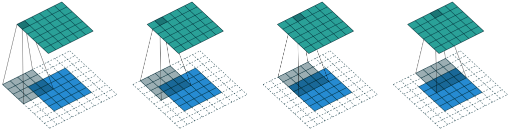
- Fully connected (FC) layers
- Upsampling (interpolation) or Subsampling (max/average pooling) layers
- Convolution/Transposed convolution (with stride), see next slide
- Activation functions: RELU, leakyRELU, sigmoid, tanh, etc
- BatchNorm
- ...

Convolution and Transposed convolution

The convolution of u, v is defined by

$$w * u(i) = \sum_j w(j)u(i-j),$$

where $u(j) \in \mathbf{R}^d$, $w(j) \in \mathbf{R}^{d' \times d}$.



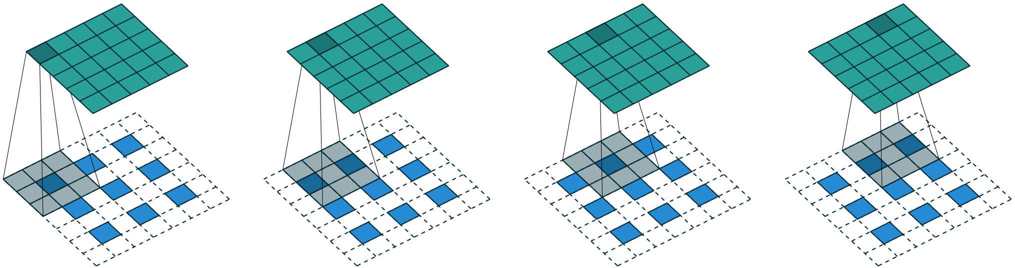
Notice that

- The transpose of a convolution with a $k \times k$ kernel is a convolution with a $k \times k$ kernel
- The transpose of a border crop is zero-padding the borders.
- The transpose of a crude subsampling is zero-inserting.

Fractionally strided convolutions:

- This is the transpose operator of convolution+subsampling (convolution with stride).
- Called `ConvTranspose2d` in PyTorch

One Example from [Dumoulin and Visin, 2016]



"The transpose of convolving a 3×3 kernel over a 5×5 input padded with a 1×1 border of zeros using 2×2 strides (i.e., $i = 5$, $k = 3$, $s = 2$ and $p = 1$). It is equivalent to convolving a 3×3 kernel over a 3×3 input (with 1 zero inserted between inputs) padded with a 1×1 border of zeros using unit strides (i.e., $i' = 3$, $\tilde{i}' = 5$, $k' = k$, $s' = 1$ and $p' = 1$)."

BatchNorm layer

Principle of BatchNormalization: for any batch $(x_i)_{i\in B}$ of a K-dimensional feature, transform

$$y_i^{(k)} = \gamma^{(k)}\hat{x}_i^{(k)} + \beta^{(k)} \quad \text{with} \quad \hat{x}_i^{(k)} = \frac{x_i^{(k)} - m_i^{(k)}}{\sqrt{(\sigma_i^{(k)})^2 + \varepsilon}}$$

where $m_i^{(k)}, \sigma_i^{(k)}$ are mean and std of the k-feature over this batch,
and where $\gamma^{(k)}, \beta^{(k)}$ are trainable parameters.

At inference: normalization is done with $m_i^{(k)}, \sigma_i^{(k)}, \gamma^{(k)}, \beta^{(k)}$ learned during training.
Switch to inference mode with `model.eval()`

Convolutional GAN

[Radford et al., 2016]

Important principles of the construction:

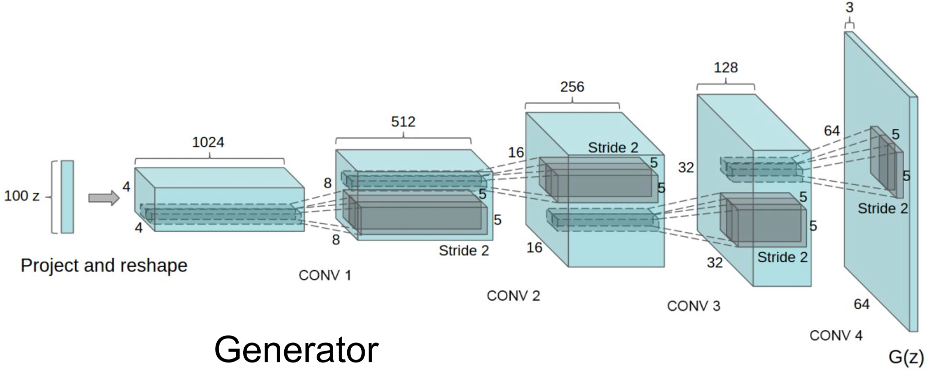
- “All convolutional”: remove max pooling layers, and learn downsampling instead
- Eliminate Fully-Connected Layers
- Batch Normalization to stabilize learning (except on generator output, and discriminator input)
- ReLU activations for the generator
- LeakyReLU activations for the discriminator

Generator: upsampling network with **fractionally strided convolutions** (i.e. the transpose operator of convolution+subsampling , called `ConvTranspose2d` in PyTorch)

Discriminator: convolutional network with strided convolutions

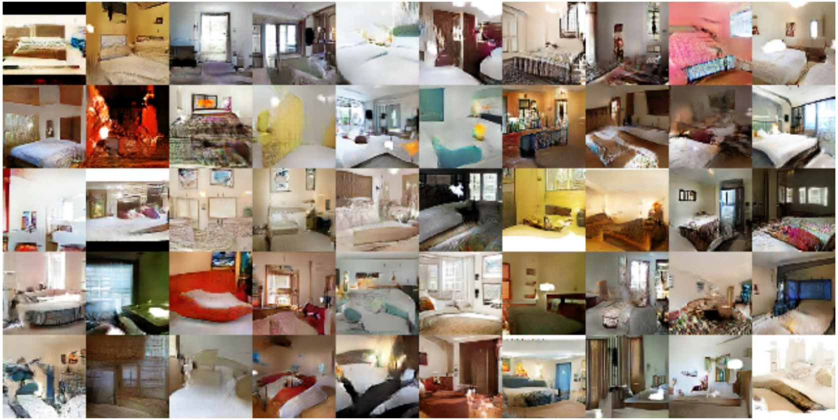
DCGAN Architecture

[Radford et al., 2016]



Generator

Image Generation with DCGAN [Radford et al., 2016]

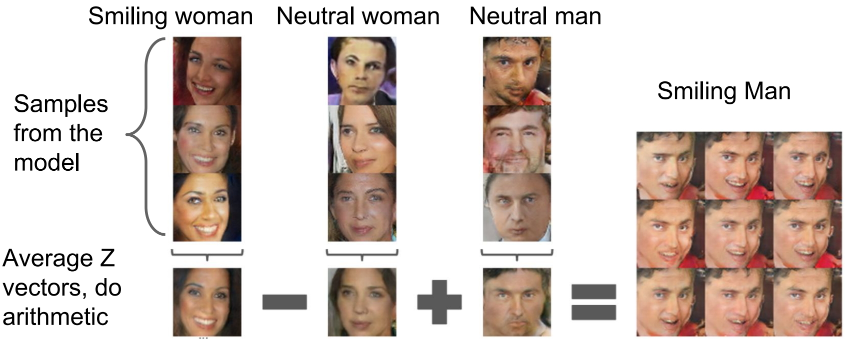


Generations of realistic bedrooms pictures, from randomly generated latent variables.

Image Interpolation with DCGAN [Radford et al., 2016]

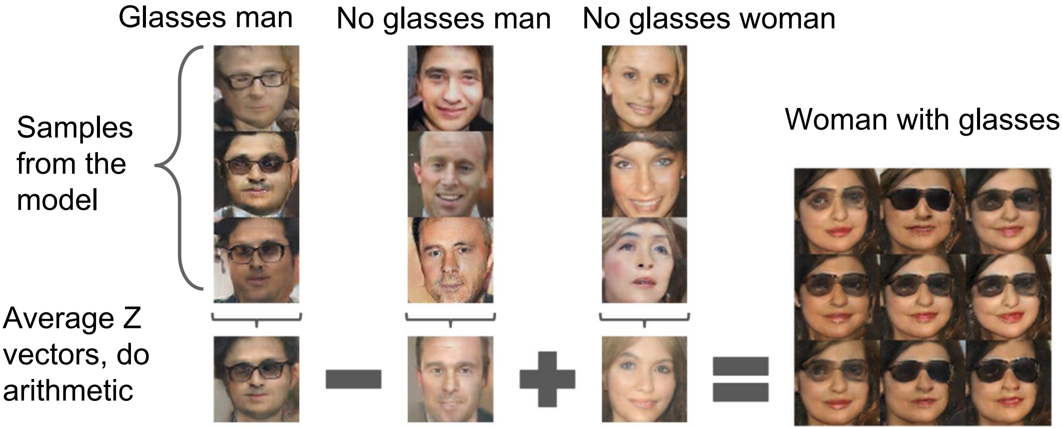
Interpolation in between points in latent space.

Arithmetic with DCGAN [Radford et al., 2016]



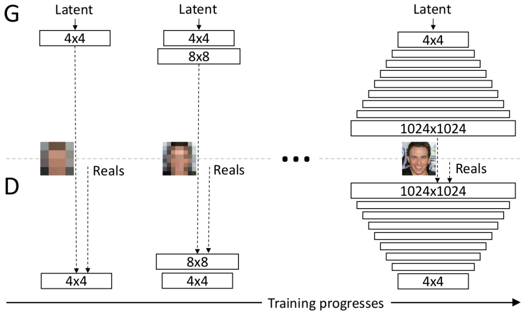
- Average latent vector of several samples
- After arithmetic, add a small random perturbation to get similar samples

Arithmetic with DCGAN [Radford et al., 2016]



Progressive Growing of GANs [Karras et al., 2018]

- Progressive Multiresolution Training
- Mirror architectures for *G* and *D*
- Simple upsampling/downsampling
  nearest neighbor upsampling;
  average pooling downsampling
- Minibatch statistics layer at the end of the discriminator
- Pixelwise feature normalization
- Training with WGAN-GP

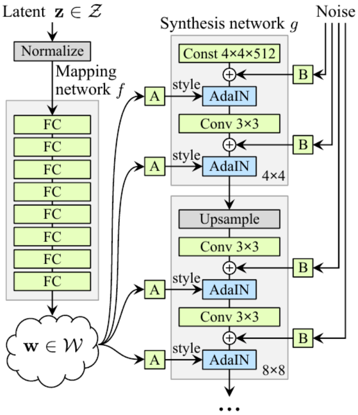


StyleGAN [Karras et al., 2019]

- “separation of high-level features (pose, identity) from stochastic variation (freckles, hair)”
- Embed input latent code z into an intermediate latent space w with a multilayer perceptron (8 FC layers)
- Spatially invariant style vector $y = (y_s, y_b)$ for each feature map, obtained from w
- AdaIN: Adaptive Instance Normalization

$$\mathrm{AdaIN}(x_i, y) = y_{s,i} \frac{x_i - \mu(x_i)}{\sigma(x_i)} + y_{b,i}$$

  where the feature map x_i is normalized separately
- Style mixing (playing with two latent codes w_1, w_2)



(b) Style-based generator

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- Style mixing (playing with two latent codes w_1, w_2)

StyleGAN [Karras et al., 2019]

StyleGAN allows for style mixing at different scales (by using the corresponding subpart of w).

StyleGAN2 [Karras et al., 2020]

- AdaIN causes droplet artifacts in StyleGAN
  $\rightarrow$ Weight modulation/demodulation instead of AdaIN
- Path length regularization:
  fixed step-size in w results in fixed magnitude change in imag
- Residual connections with downsampling in D
- Skip connections in G
- No progressive growing (which leads to *phase artifacts*)

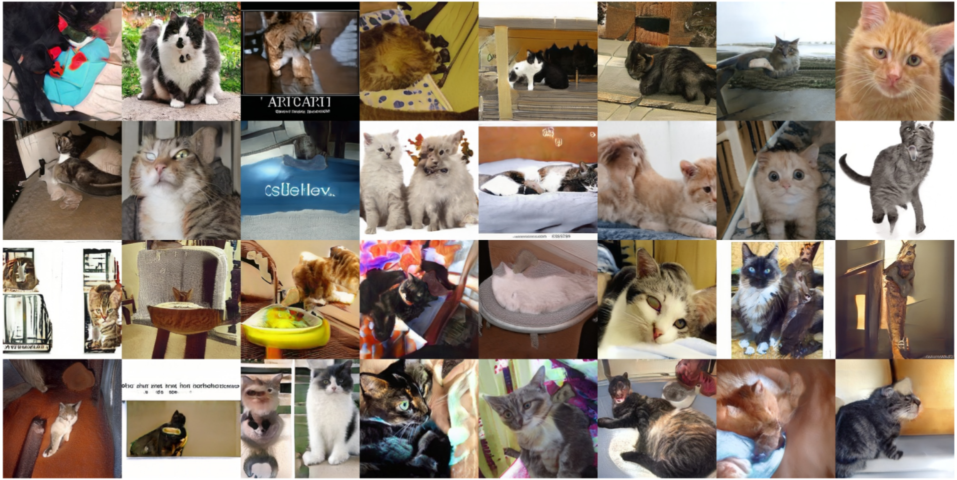
StyleGAN vs StyleGAN2

First row: real images
Second row: samples of StyleGAN after projection on the latent code

StyleGAN vs StyleGAN2

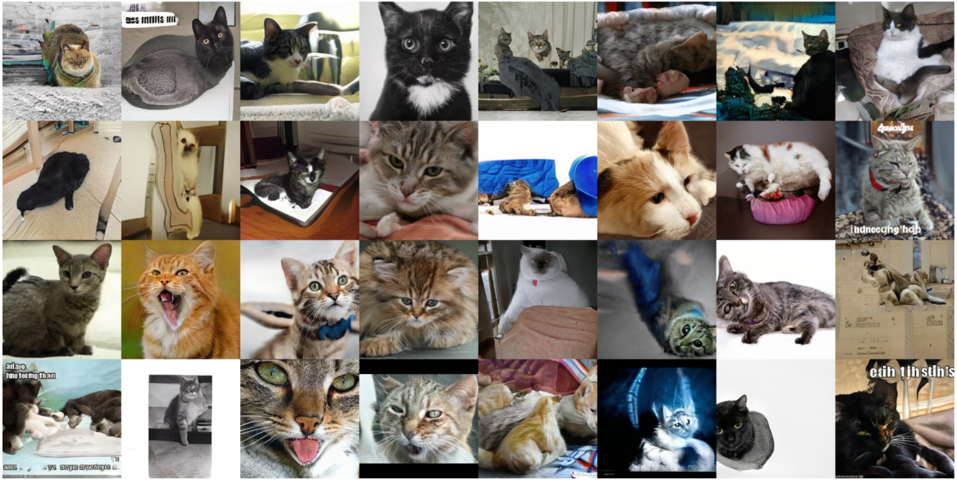
First row: real images
Second row: samples of StyleGAN2 after projection on the latent code

The Cat Challenge...



Samples of StyleGAN2-Model1 trained on LSUN Cat

The Cat Challenge...

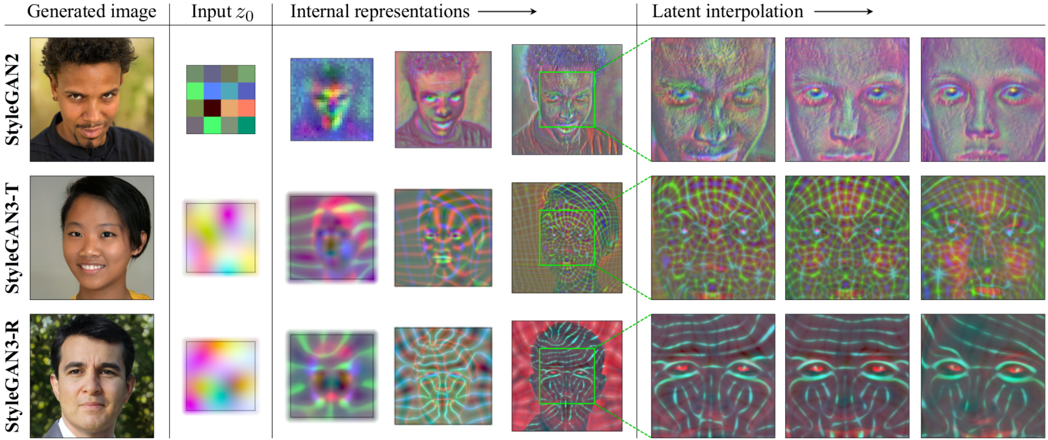


Samples of StyleGAN2-Model2 trained on LSUN Cat

StyleGAN3 aka Alias-free GANs

- Aliasing artifacts present in some GANs results due to:
  · non-ideal upsampling
  · pointwise activations
- Enforce continuous equivariance to sub-pixel translation (Shannon is back...)
- Also, ensure that no aliasing appears through the network:
  · use band-limited filters
  · use low-pass filters when needed

StyleGAN3 aka Alias-free GANs



Are GANs created equal?

[Lucic et al., 2018]

Many variants of GAN training exist, with various architectures and more or less stable training.

- Regarding quality of generated images, may GAN variants perform similarly.
  Lucic et al. proposed a large comparison framework, with a budget for hyperparameter tuning, and by averaging over several random seeds.
- "WGANs work because they fail" [Stanczuk et al., 2021], [Mallasto et al., 2019]
  The dual training in WGAN-GP does not approximate the Wasserstein distance correctly.
  But estimating it more precisely (e.g. semi-discrete WGAN) often leads to blurrier samples.
  $\rightarrow$ The quality of a generative network relies on good features learned by the discriminator.

Take-home Messages

- FID score gives a reasonble/simple way to measure the quality of a generative model... but it does not suffice to judge photo-realism of the samples
- We discussed several architectures for texture/image generation
- Large-scale synthesis benefits from architectures adapted for multi-resolution synthesis.
- Recent generative models crucially rely on
  · several tricks for training or designing the architecture
  · very long training of models with an extremely large number of parameters

THANK YOU FOR YOUR ATTENTION!

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