Evaluating the Posterior Sampling Ability of Plug&Play Diffusion Methods in Sparse-View CT

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Advances in learning-based image restoration

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Liam Moroy

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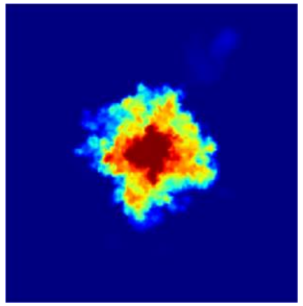
SUMMARY

- Context: tomographic imaging
- Goal : the *posterior gap*
- Plug & Play diffusion models
    - Approximations examples
- Evaluation of the *posterior gap*
    - In practice
    - Quantitative results

Context: tomographic imaging

Jet density image



$\mathbf{x}$

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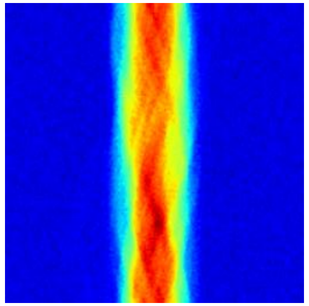
Sinogram



$\mathbf{y}_p$

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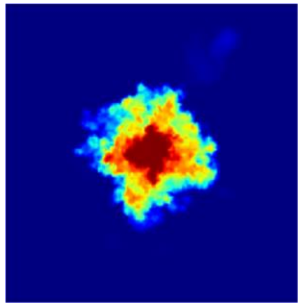

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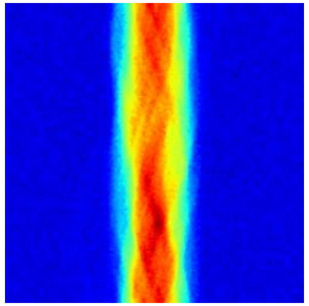

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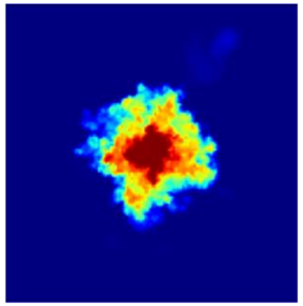

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Observation model:

$$\mathbf{y}_p = \mathbf{H}_p\mathbf{x} + \sigma_y\mathbf{n}$$

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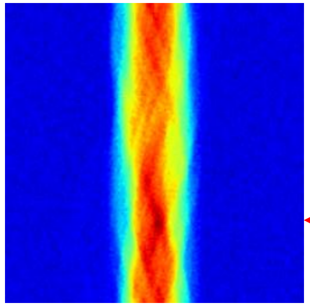

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$p(\mathbf{y}_p|\mathbf{x})$

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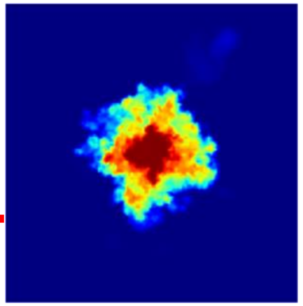

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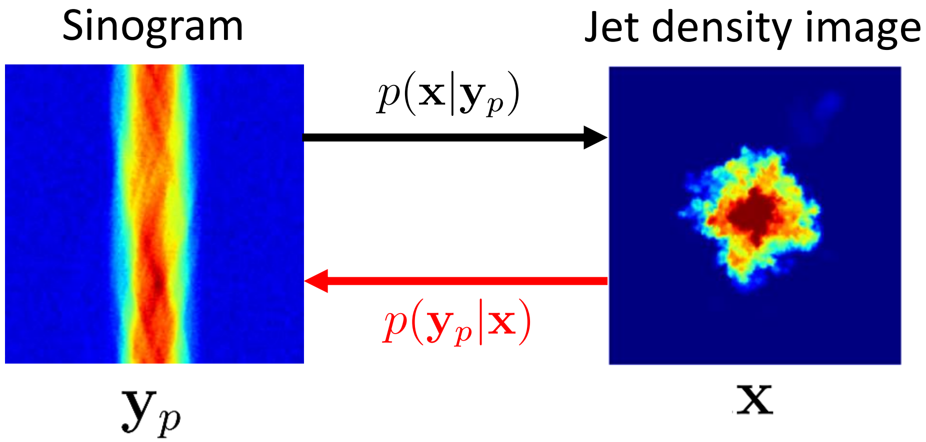

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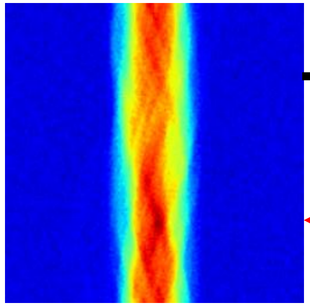

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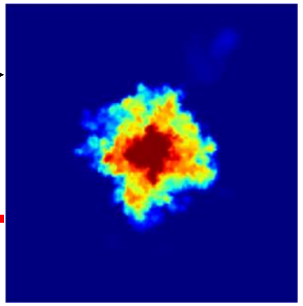

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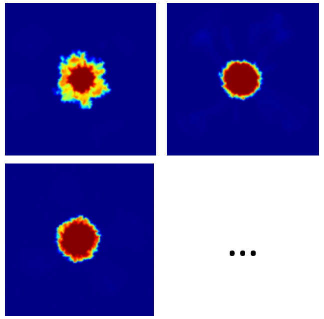

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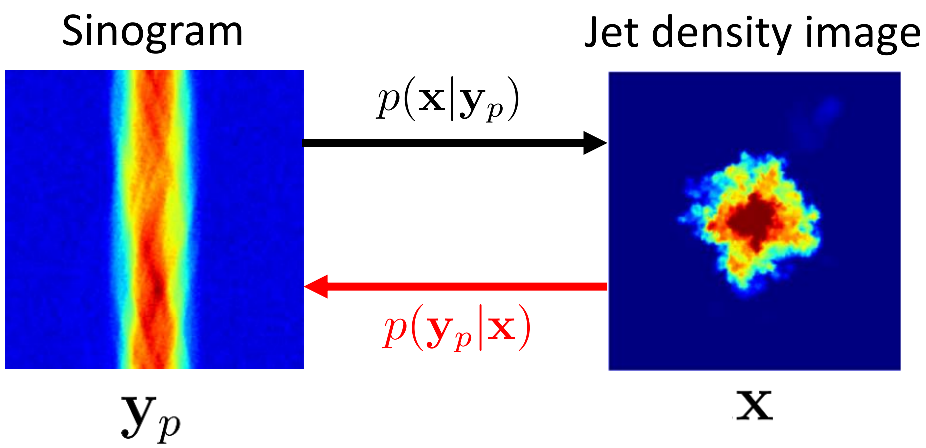

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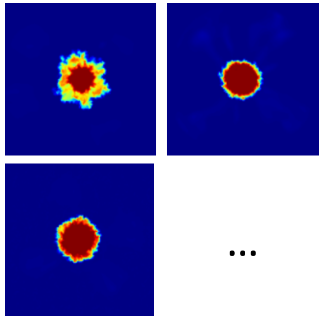

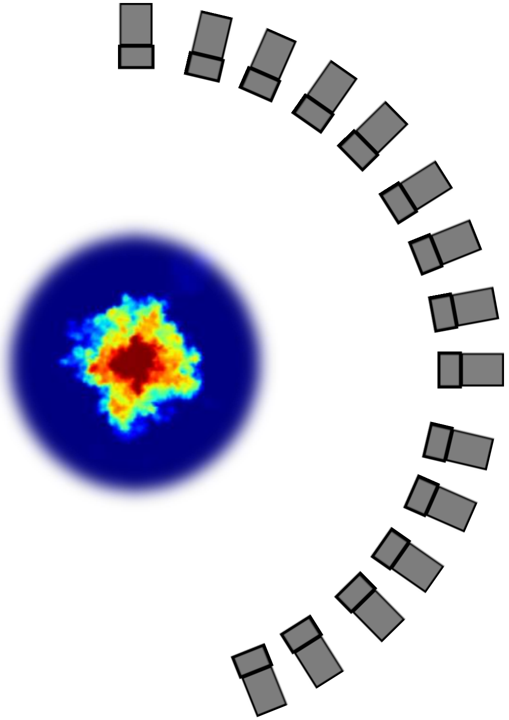

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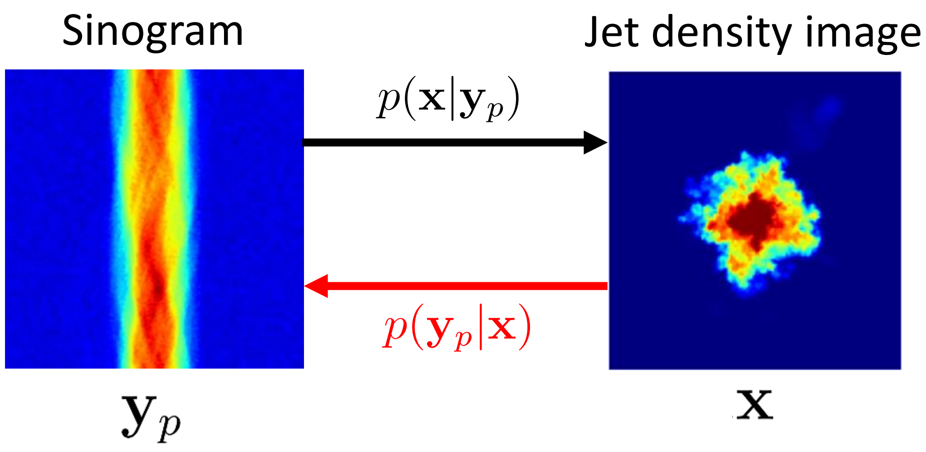

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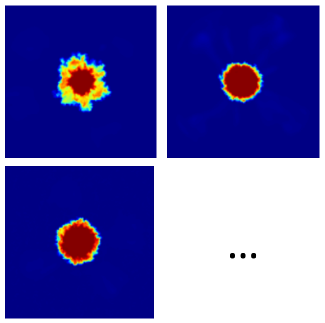

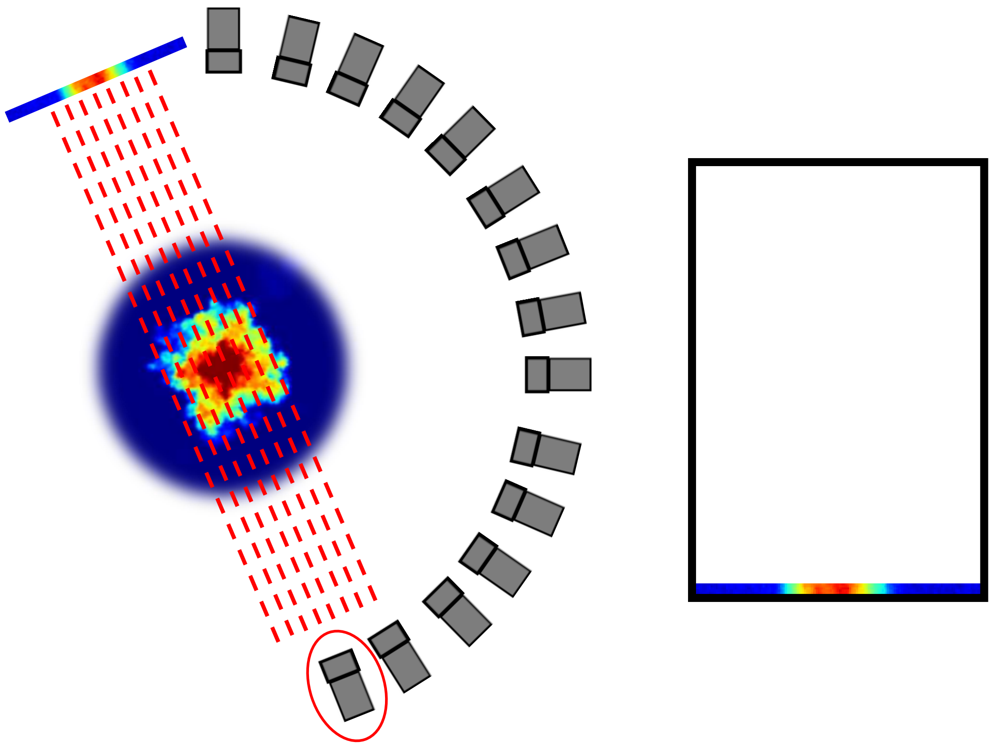

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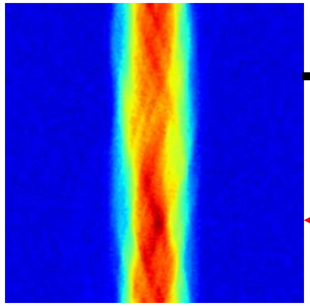

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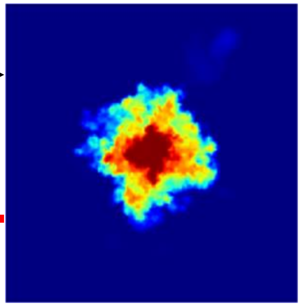

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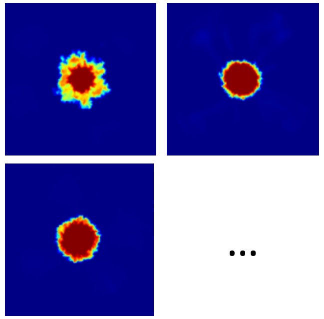

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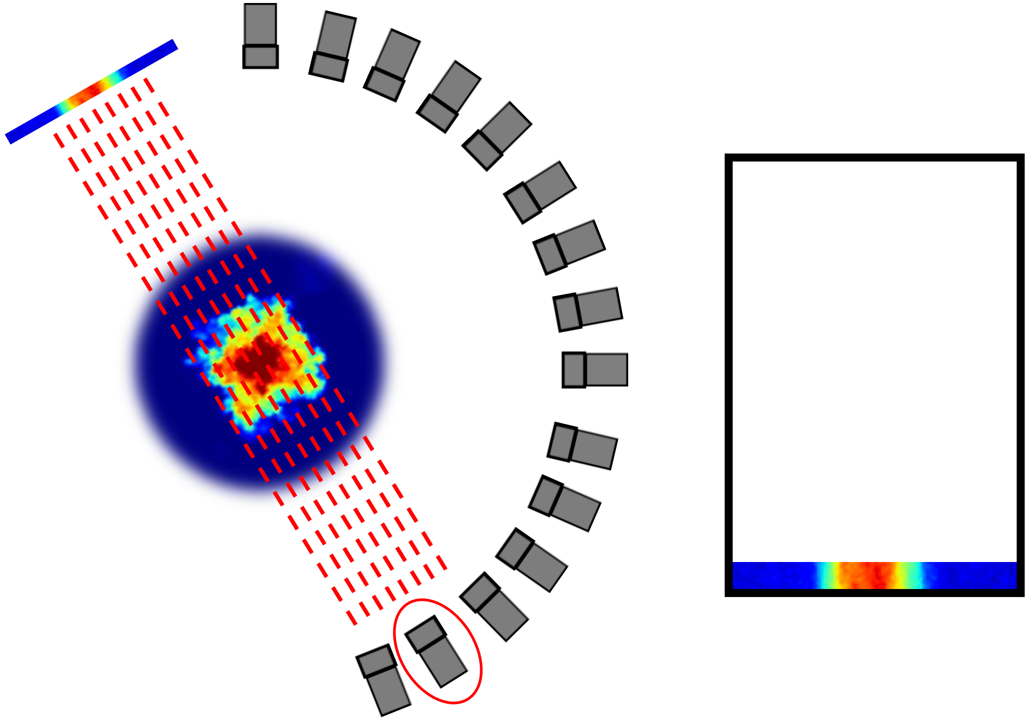

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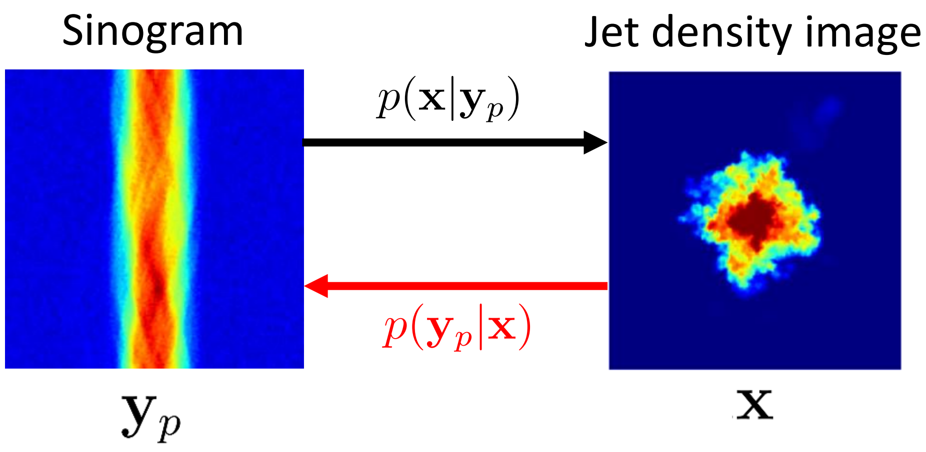

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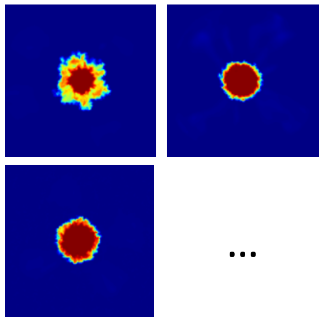

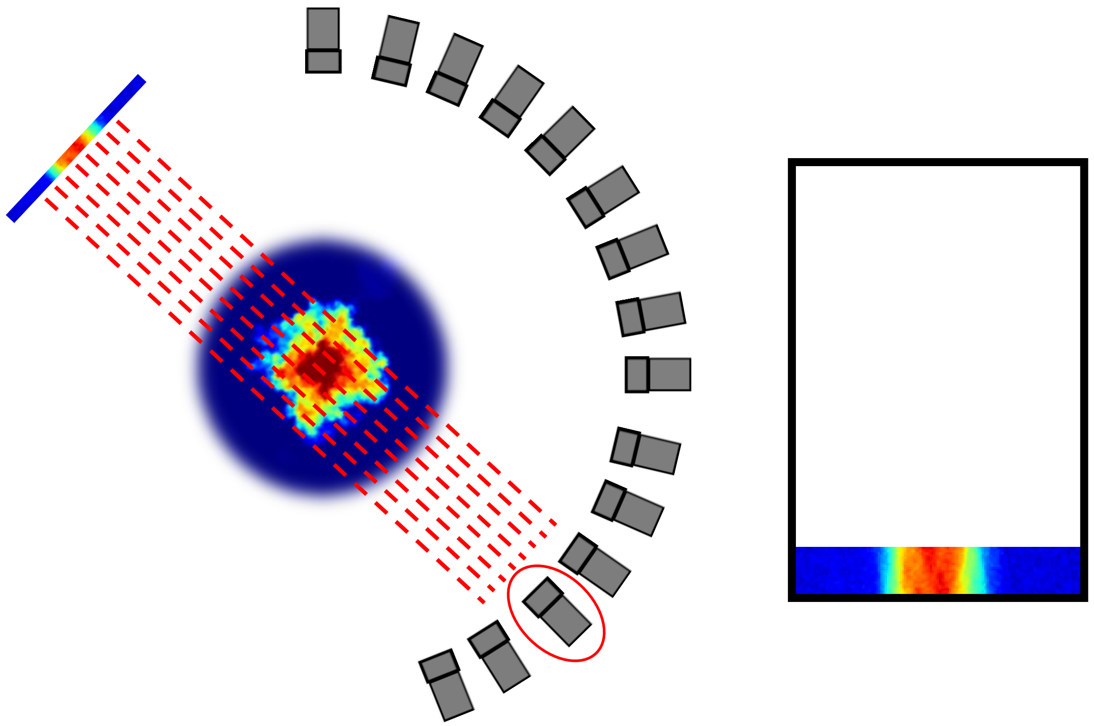

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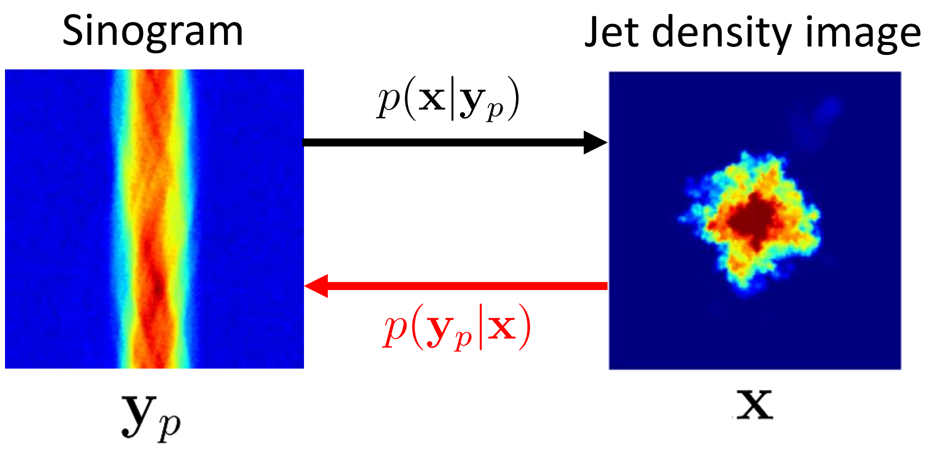

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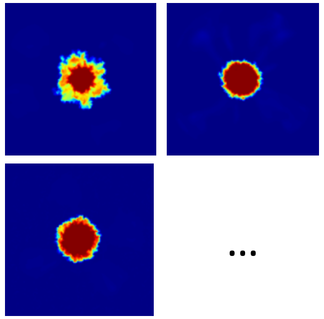

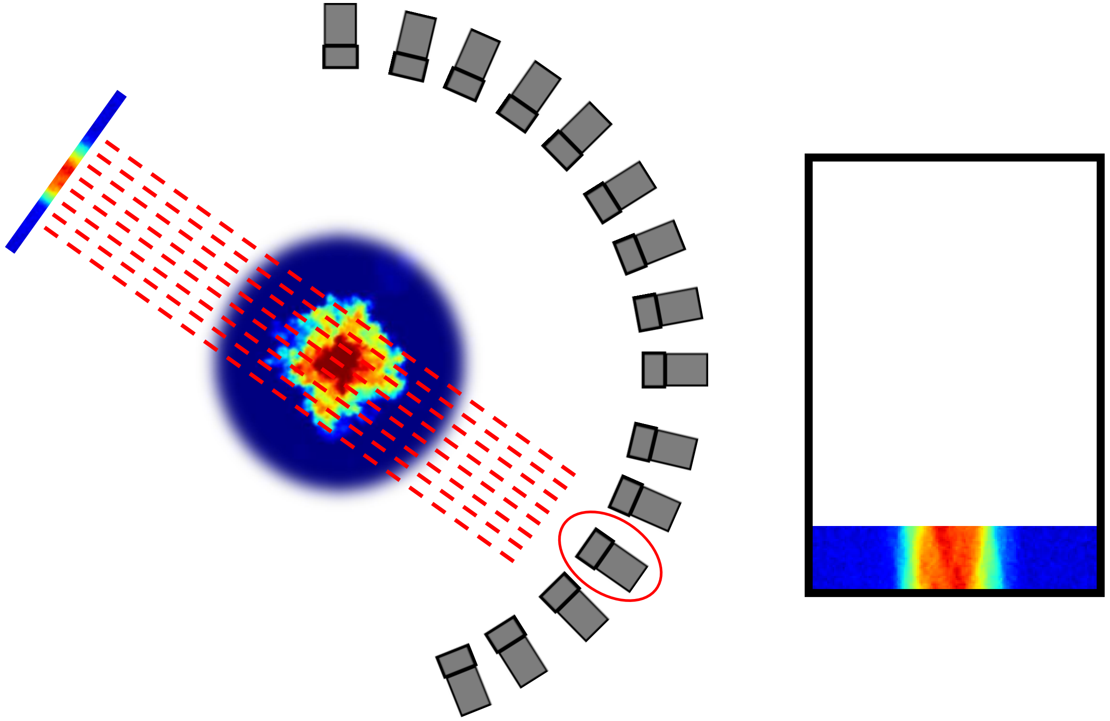

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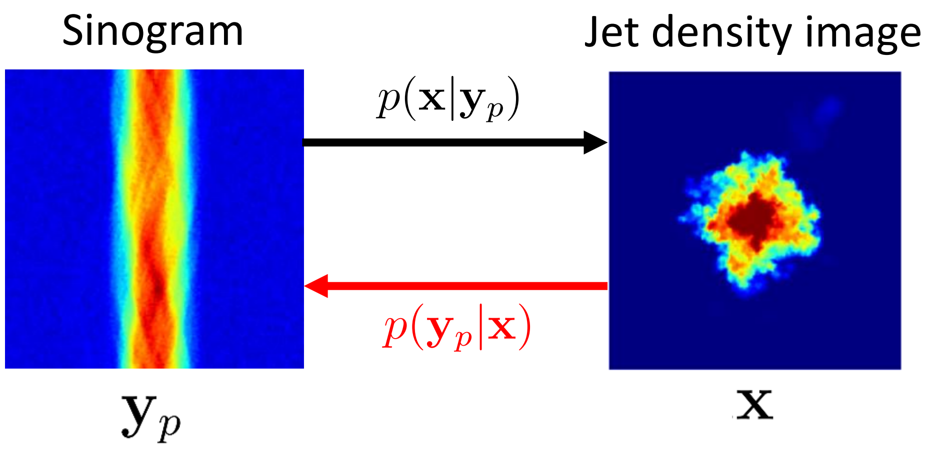

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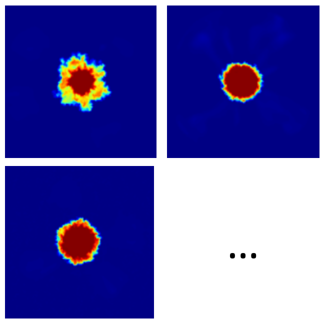

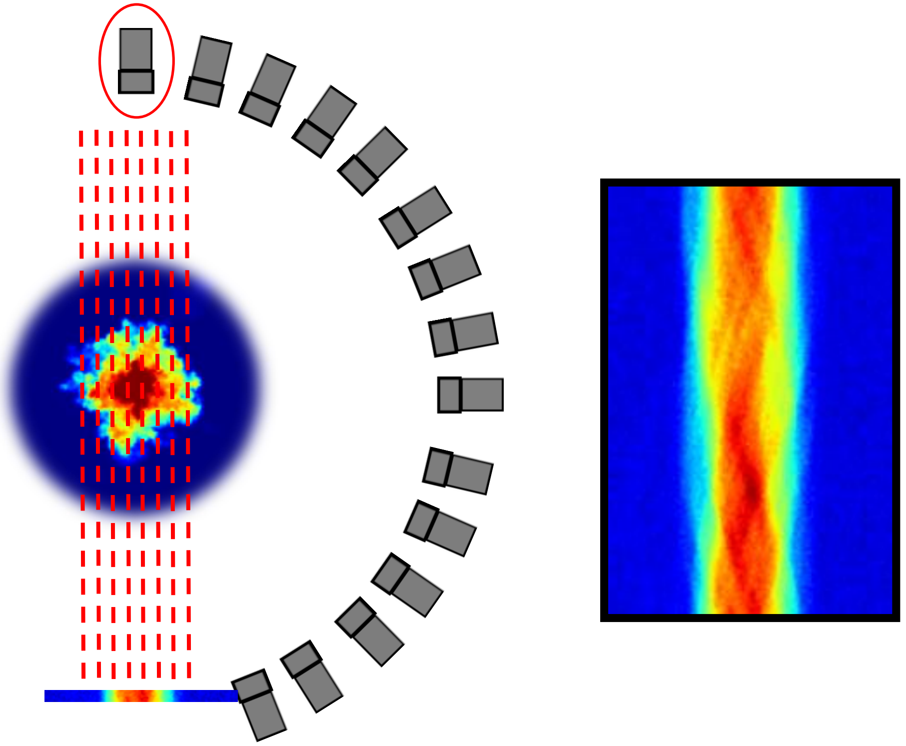

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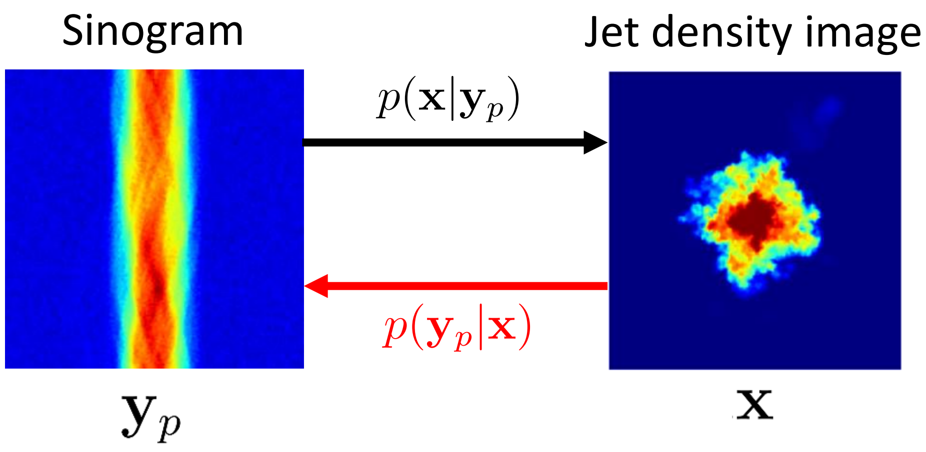

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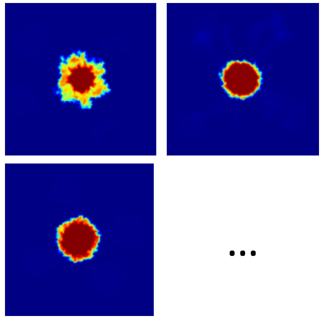

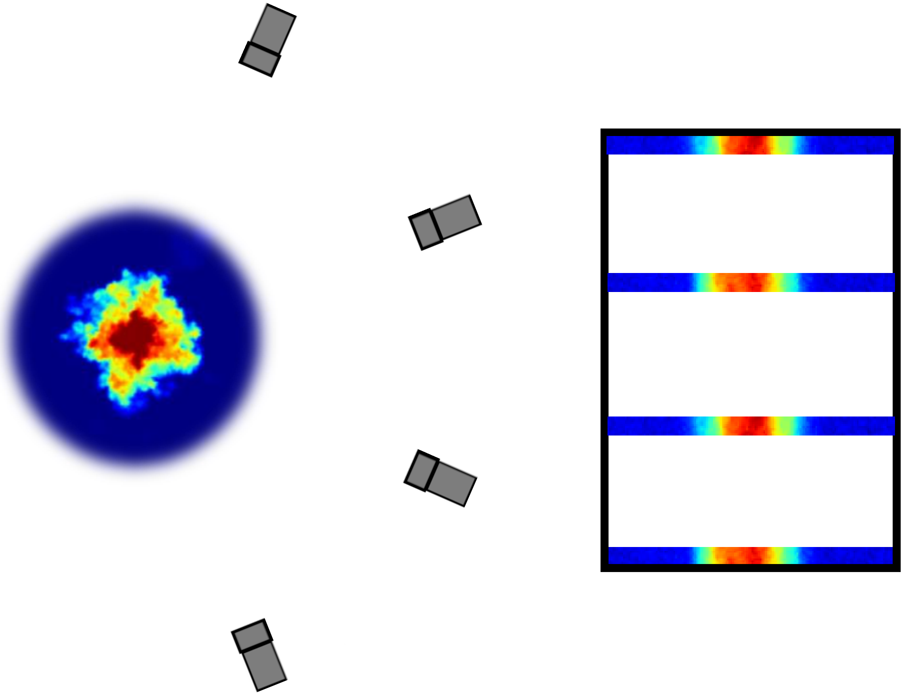

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Posterior gap : $\mathrm{dist}(\widetilde{p}(\mathbf{x}|\mathbf{y}_p),\ p(\mathbf{x}|\mathbf{y}_p))$ (ex: Kullback-Leibler divergence*)

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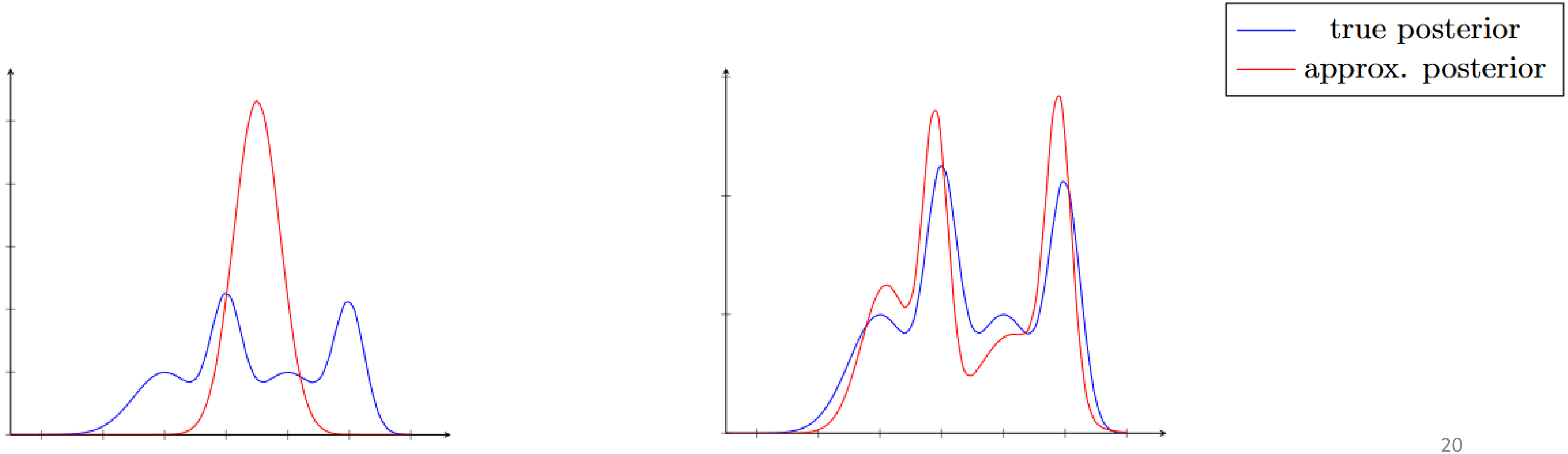
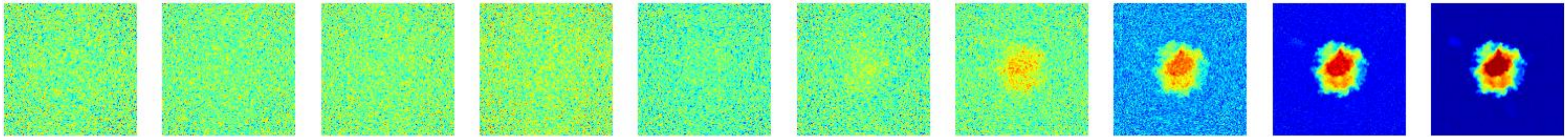


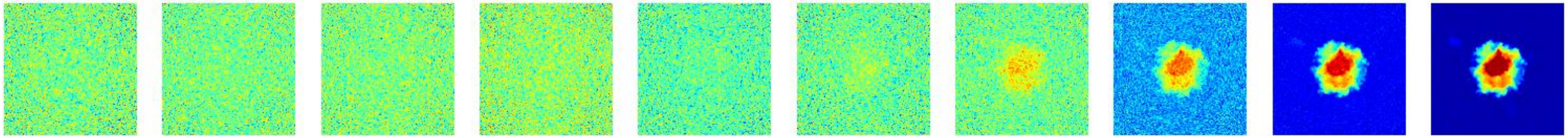
Posterior sampling



$$\mathrm{d}\mathbf{x} = -g^2(t)\nabla_{\mathbf{x}} \log p_t(\mathbf{x}|\mathbf{y}_p)\mathrm{d}t + g(t)\mathrm{d}\bar{\mathbf{w}}$$

Plug & Play diffusion models

Posterior sampling



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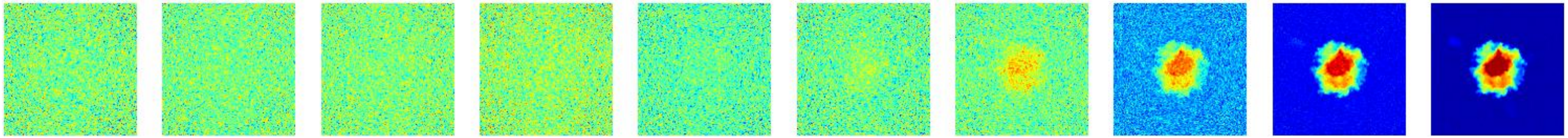

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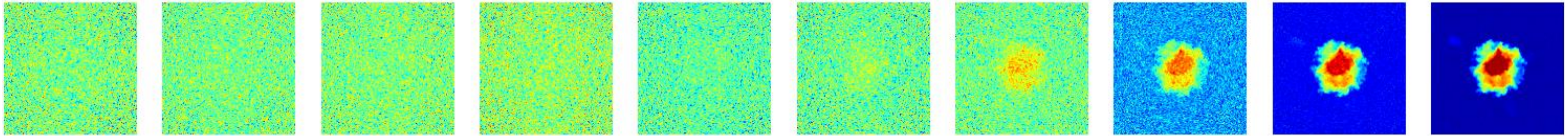
$$\mathrm{d}\mathbf{x} = -g^2(t)\left(\underbrace{\nabla_{\mathbf{x}} \log p_t(\mathbf{x})}_{\text{Prior guidance}} + \underbrace{\nabla_{\mathbf{x}} \log p_t(\mathbf{y}_p|\mathbf{x})}\right)\mathrm{d}t + g(t)\mathrm{d}\bar{\mathbf{w}}$$

$\approx \mathbf{s}_{\boldsymbol{\theta}}(\mathbf{x}, t)$

denoiser

Posterior sampling



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Prior guidance

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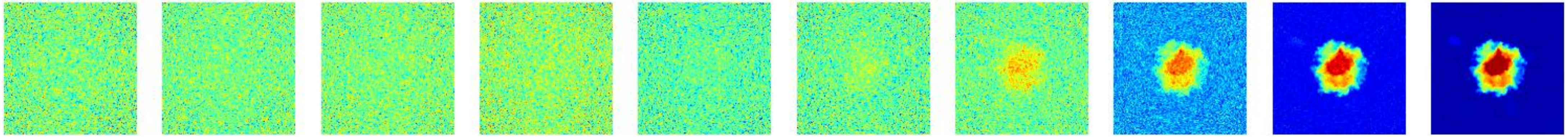
denoiser

Measurement consistency guidance

Plug & Play diffusion models

Posterior sampling



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Prior guidance $\approx \mathbf{s}_{\boldsymbol{\theta}}(\mathbf{x}, t)$

denoiser

Intractability of $\nabla_{\mathbf{x}} \log p_t(\mathbf{y}_p|\mathbf{x})$ → $\boxed{\text{Analytical approximation of } p_t(\mathbf{y}_p|\mathbf{x})}$

Plug & Play diffusion models : approximations examples

Tweedie' estimator :

$$\hat{\mathbf{x}}_0(\mathbf{x}_t) = \mathbf{x}_t + \sigma^2(t)\,\mathbf{s}_\theta(\mathbf{x}_t, t) \approx \mathbb{E}\left[\mathbf{x}_0|\mathbf{x}_t\right]$$

Methods	Approx. of $p(\mathbf{y}|\mathbf{x}_t)$	Approx. of $\nabla_{\mathbf{x}_t} \log p_t(\mathbf{y}_p|\mathbf{x}_t)$
DPS [1]	$\mathcal{N}(\mathbf{y}|\ \mathbf{H}\hat{\mathbf{x}}_0(\mathbf{x}_t),\ \sigma_y^2\mathbf{I})$	$\alpha_{\mathrm{DPS}}(\mathbf{x}_t, \mathbf{y})\dfrac{\partial \hat{\mathbf{x}}_0(\mathbf{x}_t)}{\partial \mathbf{x}_t}\mathbf{H}^{\mathrm{t}}(\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}_0(\mathbf{x}_t))$
MCG [2]	$\mathcal{N}(\mathbf{y}|\ \mathbf{H}\hat{\mathbf{x}}_0(\mathbf{x}_t),\ \mathbf{H}\mathbf{H}^{\mathrm{t}})$	$\alpha_{\mathrm{MCG}}(\mathbf{x}_t, \mathbf{y})\dfrac{\partial \hat{\mathbf{x}}_0(\mathbf{x}_t)}{\partial \mathbf{x}_t}\mathbf{H}^{\dagger}(\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}_0(\mathbf{x}_t))$
ΠG [3]	$\mathcal{N}(\mathbf{y}|\ \mathbf{H}\hat{\mathbf{x}}_0(\mathbf{x}_t),\ \alpha_{\Pi\mathrm{G}}(\sigma_t^2)\mathbf{H}\mathbf{H}^{\mathrm{t}} + \sigma_y^2\mathbf{I})$	$\dfrac{\partial \hat{\mathbf{x}}_0(\mathbf{x}_t)}{\partial \mathbf{x}_t}\mathbf{H}^{\mathrm{t}}(\alpha_{\Pi\mathrm{G}}(\sigma_t^2)\mathbf{H}\mathbf{H}^{\mathrm{t}} + \sigma_y^2\mathbf{I})^{-1}(\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}_0(\mathbf{x}_t))$

References:
[1] - Diffusion posterior sampling for general noisy inverse problems, H. Chung *et al.*, ICLR 2023
[2] - Improving diffusion models for inverse problems using manifold constraints, H. Chung *et al.*, NIPS 2022
[3] - Pseudoinverse-guided diffusion models for inverse problems, J. Song *et al.*, ICLR 2023

Evaluation of the *posterior gap*

Posterior gap: $\mathrm{dist}(\widetilde{p}(\mathbf{x}|\mathbf{y}_p),\ p(\mathbf{x}|\mathbf{y}_p))$

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- Posterior-Prior Similarity (PPS):

$$\mathbb{E}_{p(\mathbf{y}_p)}\left[p(\mathbf{x}|\mathbf{y}_p)\right] = p(\mathbf{x})$$

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- Normalized Measurement Consistency (NMC):

$$\frac{1}{m_p\sigma_y^2}\mathbb{E}_{p(\mathbf{y}_p)}\mathbb{E}_{p(\mathbf{x}|\mathbf{y}_p)}\left[\|\mathbf{y}_p - \mathbf{H}_p\mathbf{x}\|_2^2\right] = 1$$

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$$\frac{1}{m_p\sigma_y^2}\mathbb{E}_{p(\mathbf{y}_p)}\mathbb{E}_{\tilde{p}(\mathbf{x}|\mathbf{y}_p)}\left[\|\mathbf{y}_p - \mathbf{H}_p\mathbf{x}\|_2^2\right] = 1 \quad ?$$

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Independence in p

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$$\mathbb{E}_{p(\mathbf{y}_p)}\left[\widetilde{p}(\mathbf{x}|\mathbf{y}_p)\right] = \widetilde{p}(\mathbf{x}) = p(\mathbf{x}) \quad ?$$

- Normalized Measurement Consistency (NMC):

$$\frac{1}{m_p\sigma_y^2}\mathbb{E}_{p(\mathbf{y}_p)}\mathbb{E}_{p(\mathbf{x}|\mathbf{y}_p)}\left[\|\mathbf{y}_p - \mathbf{H}_p\mathbf{x}\|_2^2\right] = 1$$

$$\frac{1}{m_p\sigma_y^2}\mathbb{E}_{p(\mathbf{y}_p)}\mathbb{E}_{\widetilde{p}(\mathbf{x}|\mathbf{y}_p)}\left[\|\mathbf{y}_p - \mathbf{H}_p\mathbf{x}\|_2^2\right] = 1 \quad ?$$

Evaluation of the *posterior gap* – In practice

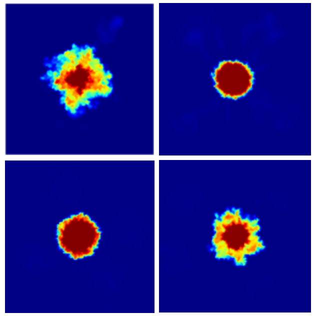
- Posterior-Prior Similarity (PPS):

$$\mathbb{E}_{p(\mathbf{y}_p)}\left[\widetilde{p}(\mathbf{x}|\mathbf{y}_p)\right] = p(\mathbf{x}) \quad ?$$

Evaluation of the *posterior gap* – In practice

- Posterior-Prior Similarity (PPS):

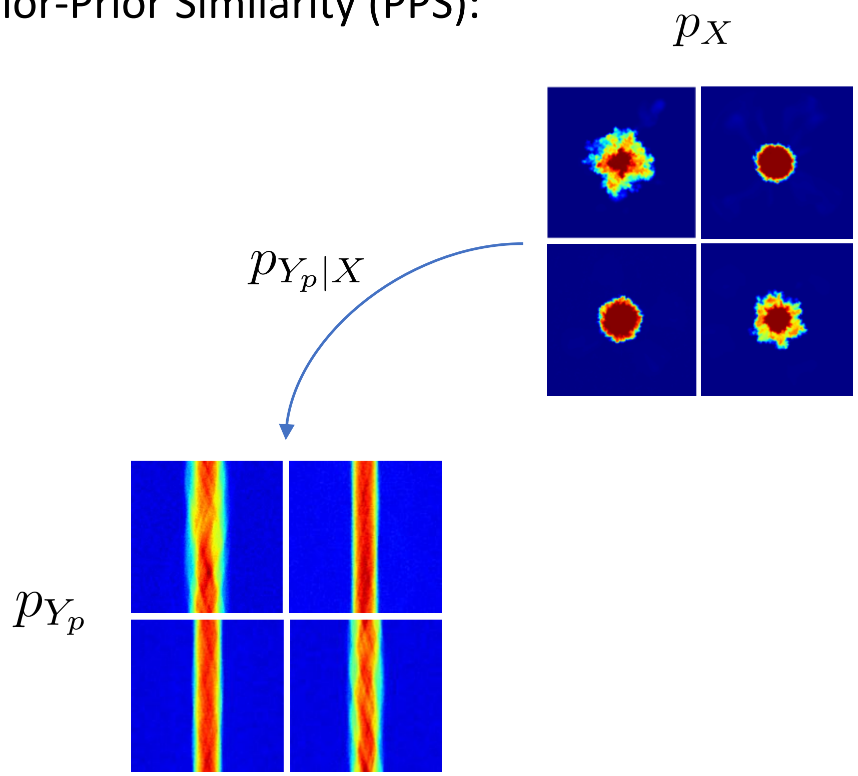
p_X



$$\mathbb{E}_{p(\mathbf{y}_p)}\left[\widetilde{p}(\mathbf{x}|\mathbf{y}_p)\right] = p(\mathbf{x}) \quad ?$$

Evaluation of the *posterior gap* – In practice

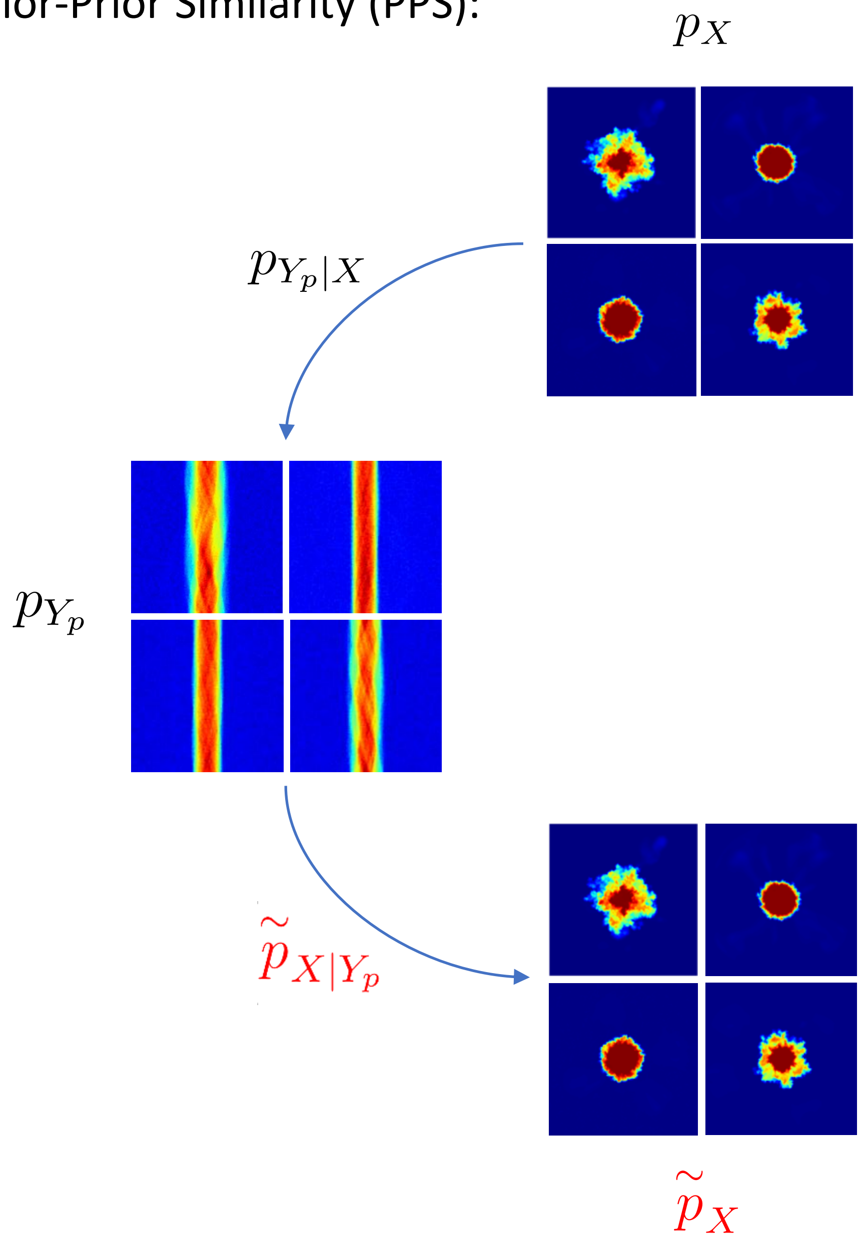
- Posterior-Prior Similarity (PPS):



$$\mathbb{E}_{p(\mathbf{y}_p)}\left[\tilde{p}(\mathbf{x}|\mathbf{y}_p)\right] = p(\mathbf{x}) \ ?$$

Evaluation of the *posterior gap* – In practice

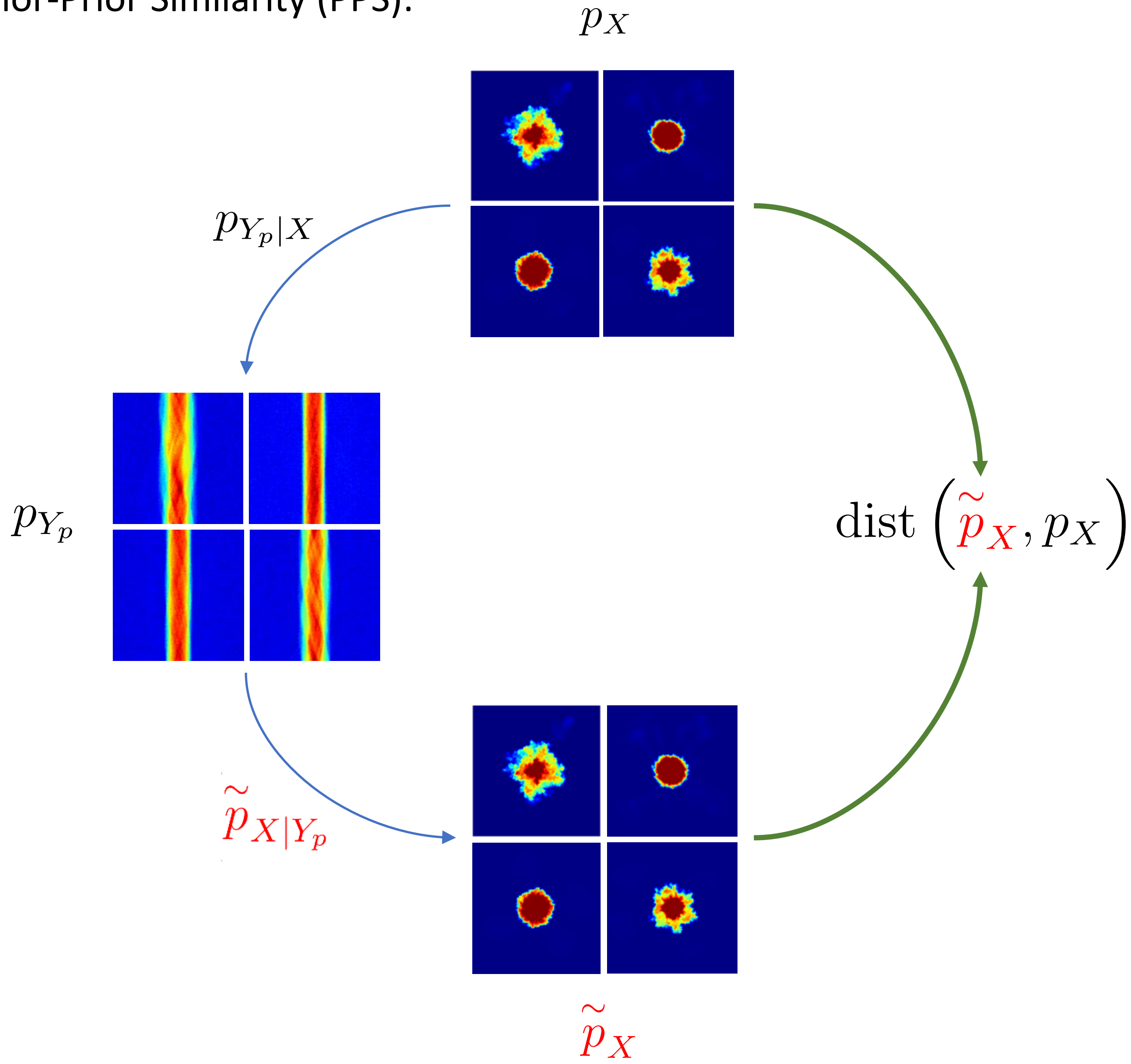
- Posterior-Prior Similarity (PPS):



$$\mathbb{E}_{p(\mathbf{y}_p)}\left[\widetilde{p}(\mathbf{x}|\mathbf{y}_p)\right] = p(\mathbf{x}) \quad ?$$

Evaluation of the *posterior gap* – In practice

- Posterior-Prior Similarity (PPS):

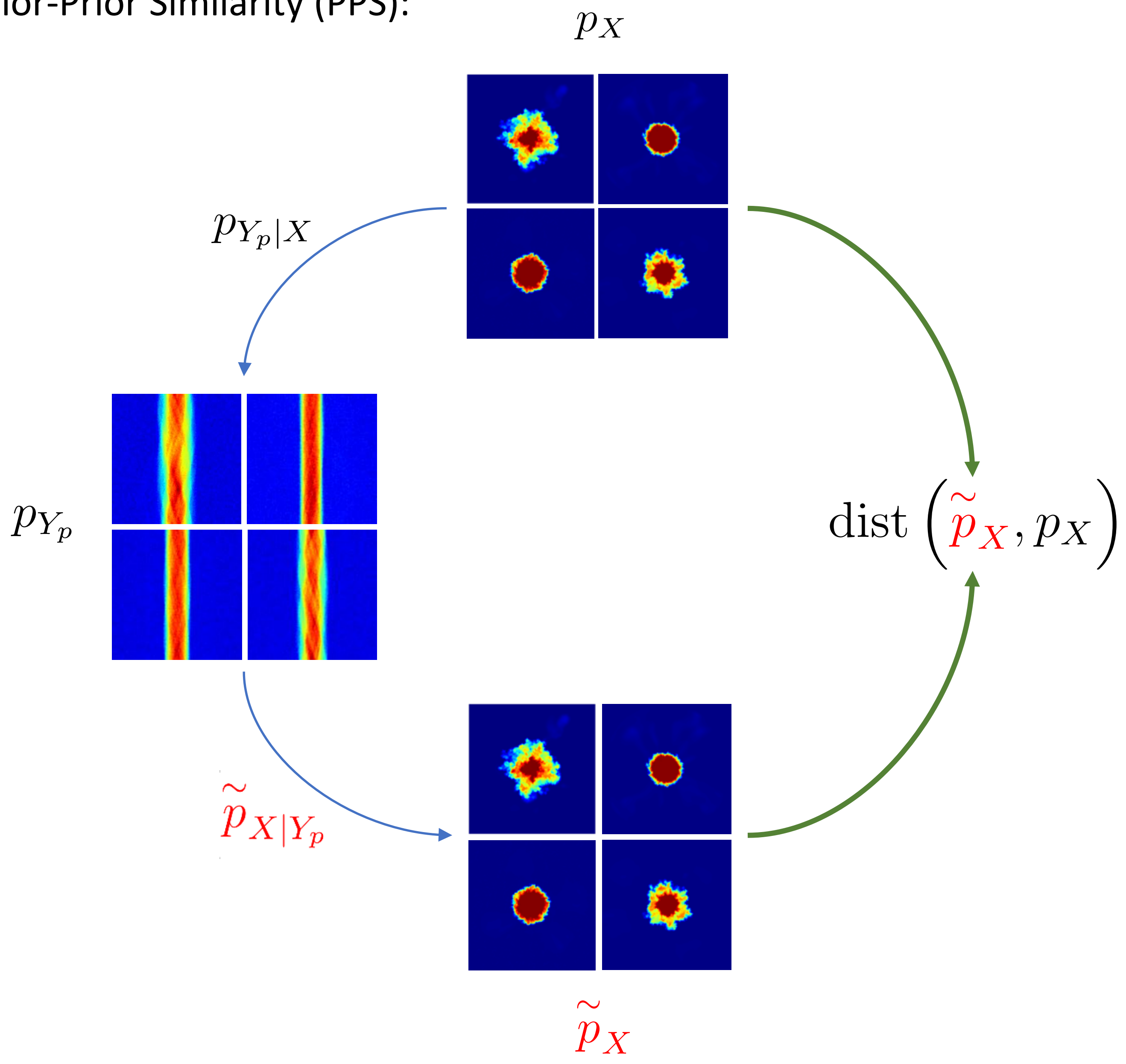


$$\mathbb{E}_{p(\mathbf{y}_p)}\left[\tilde{p}(\mathbf{x}|\mathbf{y}_p)\right] = p(\mathbf{x}) \ ?$$

$$\text{PPS} = \text{dist}(\tilde{p}_X, p_X)$$

Evaluation of the *posterior gap* – In practice

- Posterior-Prior Similarity (PPS):



$$\mathbb{E}_{p(\mathbf{y}_p)}\left[\widetilde{p}(\mathbf{x}|\mathbf{y}_p)\right] = p(\mathbf{x}) \quad ?$$

$$\text{PPS} = \text{dist}(\widetilde{p}_X, p_X)$$

$$\text{PPS}_{\text{FID}} = \text{FID}(\widetilde{p}_X, p_X)$$

$$\text{PPS}_{\text{CMMD}} = \text{CMMD}(\widetilde{p}_X, p_X)$$

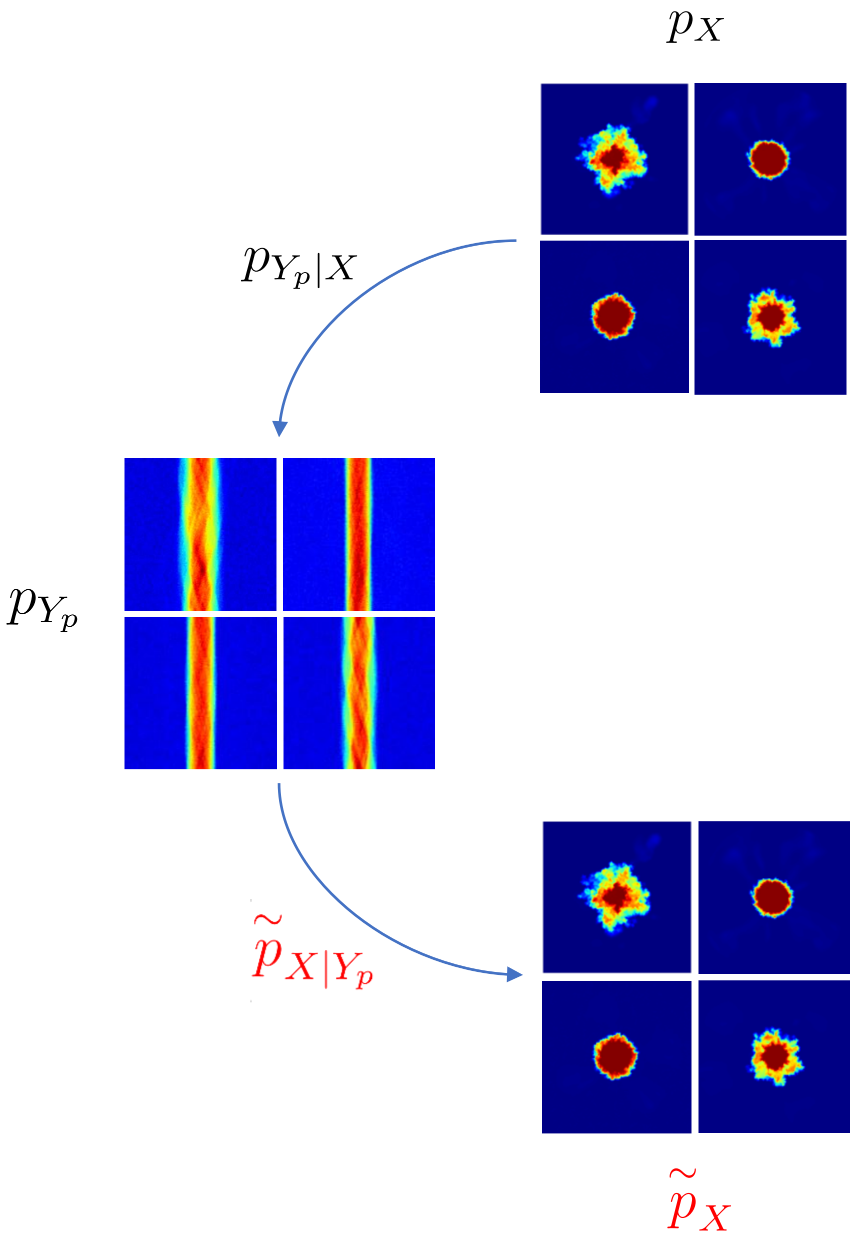
Evaluation of the *posterior gap* – In practice

- Normalized Measurement Consistency (NMC):

$$\frac{1}{m_p \sigma_y^2} \mathbb{E}_{p(\mathbf{y}_p)} \mathbb{E}_{\tilde{p}(\mathbf{x}|\mathbf{y}_p)} \left[\|\mathbf{y}_p - \mathbf{H}_p \mathbf{x}\|_2^2 \right] = 1 \quad ?$$

Evaluation of the *posterior gap* – In practice

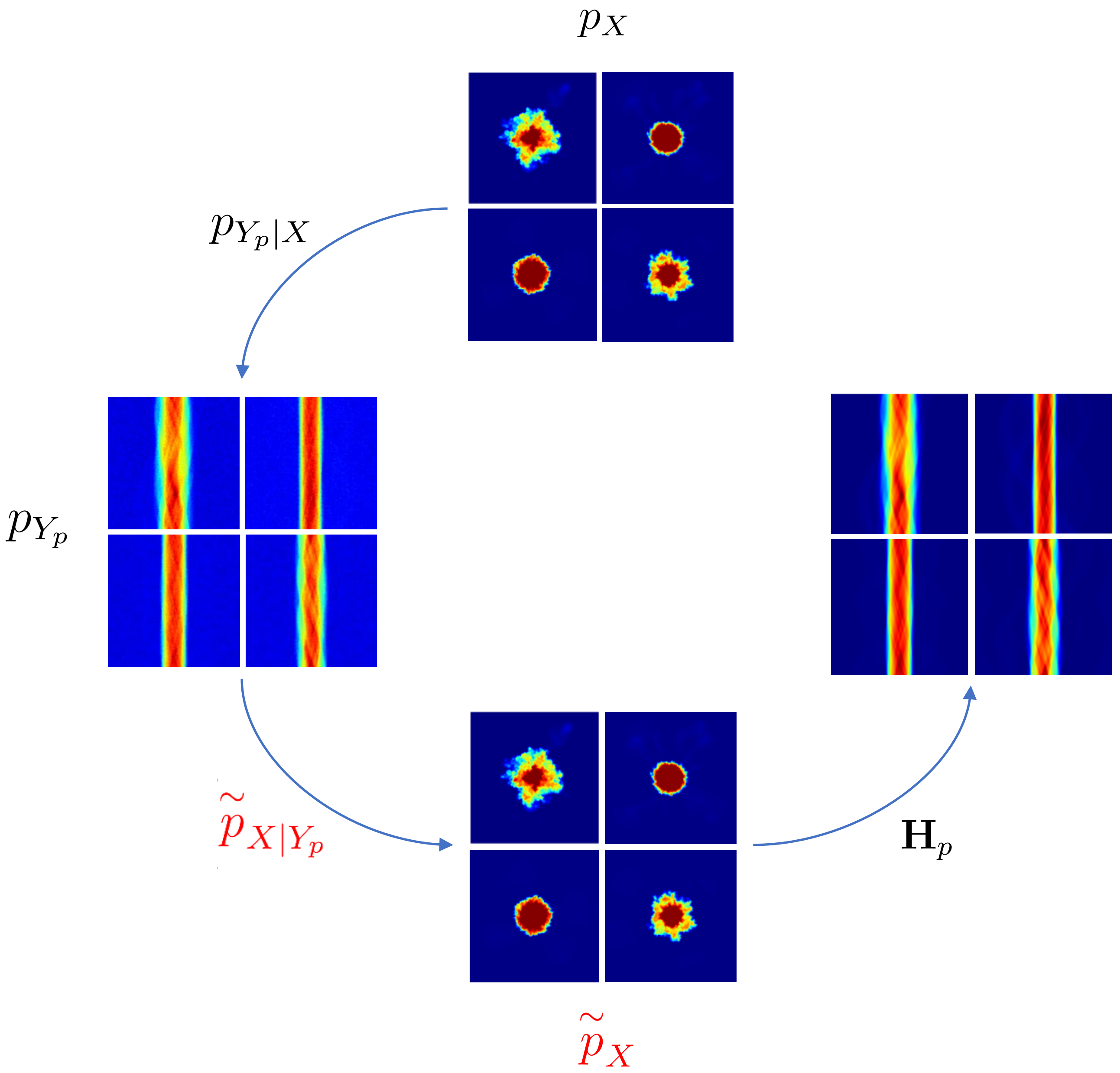
- Normalized Measurement Consistency (NMC):



$$\frac{1}{m_p \sigma_y^2} \mathbb{E}_{p(\mathbf{y}_p)} \mathbb{E}_{\tilde{p}(\mathbf{x}|\mathbf{y}_p)} \left[\|\mathbf{y}_p - \mathbf{H}_p \mathbf{x}\|_2^2 \right] = 1 \quad ?$$

Evaluation of the *posterior gap* – In practice

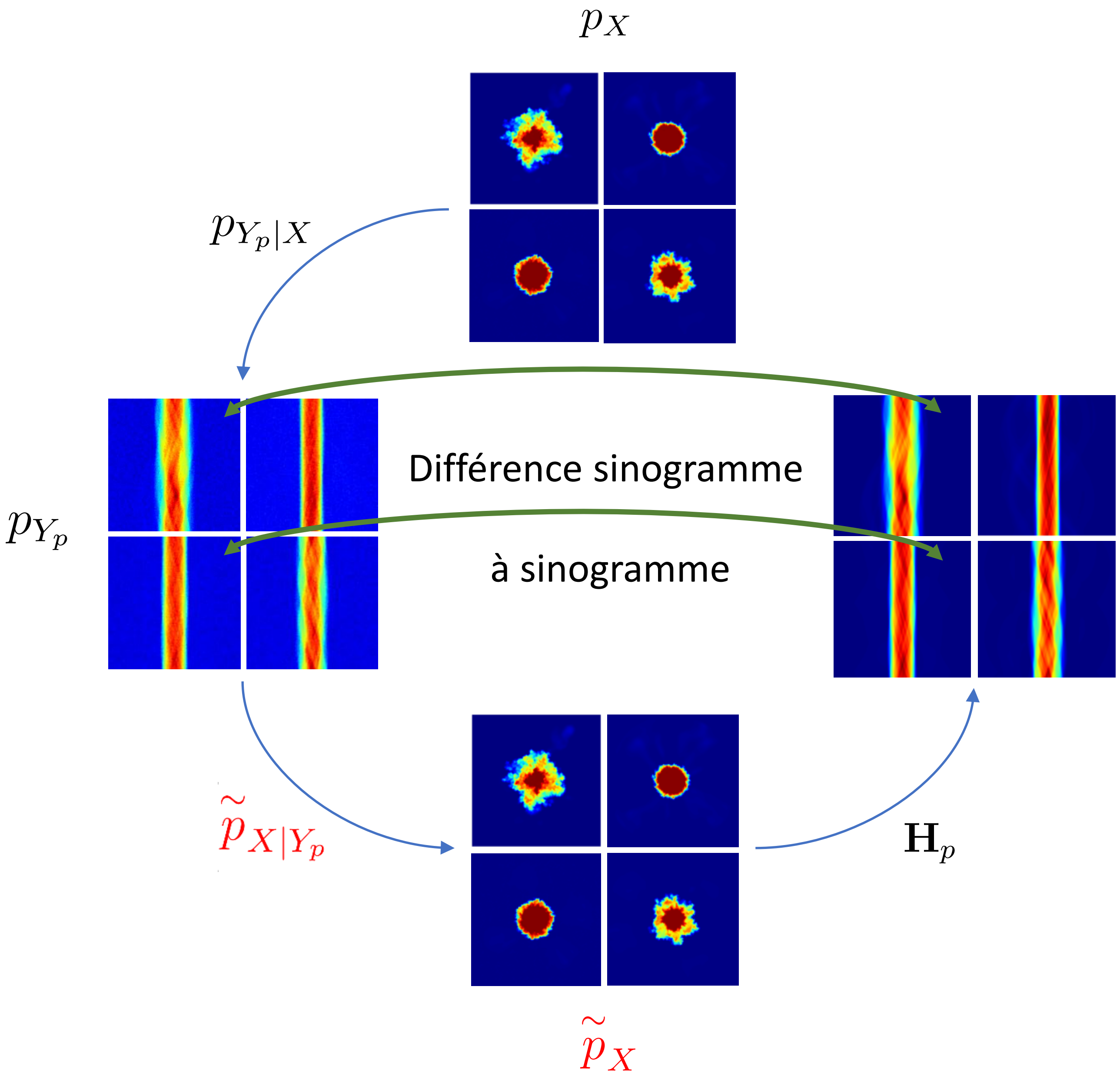
- Normalized Measurement Consistency (NMC):



$$\frac{1}{m_p\sigma_y^2}\mathbb{E}_{p(\mathbf{y}_p)}\mathbb{E}_{\tilde{p}(\mathbf{x}|\mathbf{y}_p)}\left[\|\mathbf{y}_p-\mathbf{H}_p\mathbf{x}\|_2^2\right]=1\ ?$$

Evaluation of the *posterior gap* – In practice

- Normalized Measurement Consistency (NMC):



$$\frac{1}{m_p\sigma_y^2}\mathbb{E}_{p(\mathbf{y}_p)}\mathbb{E}_{\tilde{p}(\mathbf{x}|\mathbf{y}_p)}\left[\|\mathbf{y}_p-\mathbf{H}_p\mathbf{x}\|_2^2\right] = 1 \quad ?$$

$$\mathrm{NMC} = \frac{1}{m_p\sigma_y^2 N}\sum_{i=1}^{N}\|\mathbf{y}_p^{(i)}-\mathbf{H}_p\tilde{\mathbf{x}}^{(i)}\|_2^2$$

$$\text{où} \quad \tilde{\mathbf{x}}^{(i)} \sim \tilde{p}(\mathbf{x}|\mathbf{y}_p^{(i)})$$

Evaluation of the *posterior gap* - Quantitative results

		DPS			MCG			ΠG		
		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation	
	p	NMC $\rightarrow 1$	$\text{PPS}_{\text{FID}} \downarrow$	$\text{PPS}_{\text{CMMD}} \downarrow$	NMC $\rightarrow 1$	$\text{PPS}_{\text{FID}} \downarrow$	$\text{PPS}_{\text{CMMD}} \downarrow$	NMC $\rightarrow 1$	$\text{PPS}_{\text{FID}} \downarrow$	$\text{PPS}_{\text{CMMD}} \downarrow$
JET	180									
	90									
	30									
	18									
	12									
	6									
	3									
	1									

Evaluation of the *posterior gap* - Quantitative results

		DPS			MCG			ΠG		
		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation	
	p	NMC $\to 1$	PPS_{FID} $\downarrow$	PPS_{CMMD} $\downarrow$	NMC $\to 1$	PPS_{FID} $\downarrow$	PPS_{CMMD} $\downarrow$	NMC $\to 1$	PPS_{FID} $\downarrow$	PPS_{CMMD} $\downarrow$
JET	180	1.90	**1.03**	**0.040**						
	90	1.90	**1.01**	**0.040**						
	30	1.90	**0.96**	**0.042**						
	18	1.84	1.02	0.044						
	12	1.77	1.28	0.060						
	6	**1.64**	2.23	0.141						
	3	**1.59**	3.80	0.235						
	1	**1.84**	12.89	1.061						

Evaluation of the *posterior gap* - Quantitative results

	p	DPS			MCG			ΠG		
		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation	
		NMC $\to 1$	$\mathrm{PPS}_{\mathrm{FID}} \downarrow$	$\mathrm{PPS}_{\mathrm{CMMD}} \downarrow$	NMC $\to 1$	$\mathrm{PPS}_{\mathrm{FID}} \downarrow$	$\mathrm{PPS}_{\mathrm{CMMD}} \downarrow$	NMC $\to 1$	$\mathrm{PPS}_{\mathrm{FID}} \downarrow$	$\mathrm{PPS}_{\mathrm{CMMD}} \downarrow$
JET	180	1.90	**1.03**	**0.040**						
	90	1.90	**1.01**	**0.040**						
	30	1.90	**0.96**	**0.042**						
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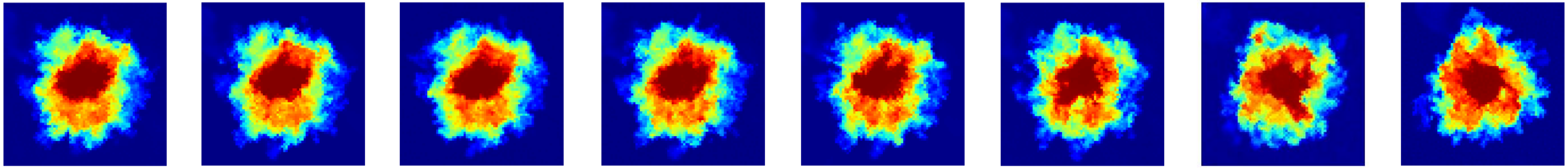
	12	1.77	1.28	0.060						
	6	**1.64**	2.23	0.141						
	3	**1.59**	3.80	0.235						
	1	**1.84**	12.89	1.061						

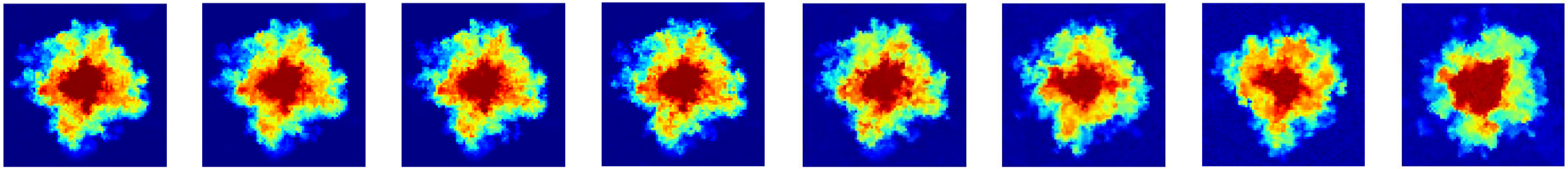


180 90 30 18 12 6 3 1

p

Evaluation of the *posterior gap* - Quantitative results

		DPS			MCG			ΠG		
		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation	
	p	NMC $\to 1$	PPS_{FID} $\downarrow$	PPS_{CMMD} $\downarrow$	NMC $\to 1$	PPS_{FID} $\downarrow$	PPS_{CMMD} $\downarrow$	NMC $\to 1$	PPS_{FID} $\downarrow$	PPS_{CMMD} $\downarrow$
JET	180	1.90	**1.03**	**0.040**	**1.11**	4.87	0.151			
	90	1.90	**1.01**	**0.040**	**1.15**	4.86	0.086			
	30	1.90	**0.96**	**0.042**	**1.21**	2.29	0.090			
	18	1.84	1.02	0.044	**1.27**	5.06	0.542			
	12	1.77	1.28	0.060	**1.40**	10.23	1.096			
	6	**1.64**	2.23	0.141	1.95	14.20	1.733			
	3	**1.59**	3.80	0.235	2.42	14.21	2.298			
	1	**1.84**	12.89	1.061	2.44	13.96	1.941			

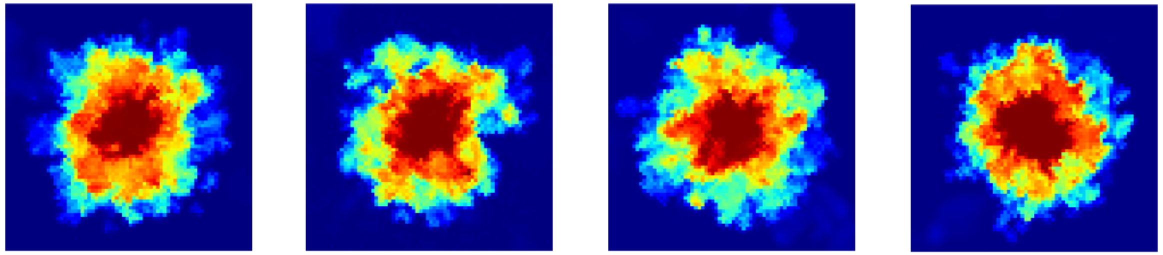


180 90 30 18 12 6 3 1

p

Evaluation of the *posterior gap* - Quantitative results

		DPS			MCG			ΠG		
		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation		Measurement adequation	Prior adequation	
	p	NMC $\rightarrow 1$	$\text{PPS}_{\text{FID}} \downarrow$	$\text{PPS}_{\text{CMMD}} \downarrow$	NMC $\rightarrow 1$	$\text{PPS}_{\text{FID}} \downarrow$	$\text{PPS}_{\text{CMMD}} \downarrow$	NMC $\rightarrow 1$	$\text{PPS}_{\text{FID}} \downarrow$	$\text{PPS}_{\text{CMMD}} \downarrow$
JET	180	1.90	**1.03**	**0.040**	**1.11**	4.87	0.151	–	–	–
	90	1.90	**1.01**	**0.040**	**1.15**	4.86	0.086	–	–	–
	30	1.90	**0.96**	**0.042**	**1.21**	2.29	0.090	–	–	–
	18	1.84	1.02	0.044	**1.27**	5.06	0.542	2.11	**0.62**	**0.008**
	12	1.77	1.28	0.060	**1.40**	10.23	1.096	5.25	**0.64**	**0.009**
	6	**1.64**	2.23	0.141	1.95	14.20	1.733	6.57	**0.63**	**0.009**
	3	**1.59**	3.80	0.235	2.42	14.21	2.298	7.99	**0.58**	**0.007**
	1	**1.84**	12.89	1.061	2.44	13.96	1.941	9.72	**0.56**	**0.005**



12 6 3 1

p

To sum it up

Evaluation of the posterior gap …
… through PPS and NMC properties …
… of 3 SoA P&P diffusion methods (DPS, MCG, πG).

As p decreases, the posterior gap inscreases !

Perpective

A more restrictive version of the NMC property.

Better implementation tools for the PPS property.

Find a more consistent approximation of $p(\mathbf{y}|\mathbf{x}_t)$