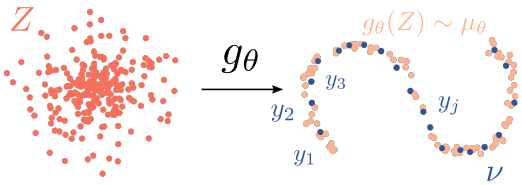
Generative Adversarial Networks

Arthur Leclaire

4AI07

Learning a Generative Network



GOAL: Estimate a generative model that fits a database $(y_j)_{1 \leq j \leq J}$ of images

Outline

In this session, we will study two approaches for learning generative models:

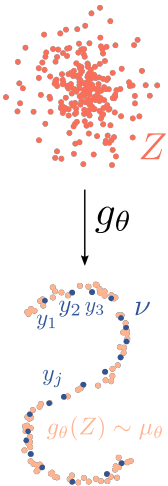
- **Generative Adversarial Networks (GANs)**
  [Goodfellow et al., 2014]
- **Wasserstein Generative Adversarial Networks (WGANs)**
  [Arjovsky et al., 2017]

Adversarial training: training simultaneously **a generator and a discriminator**.

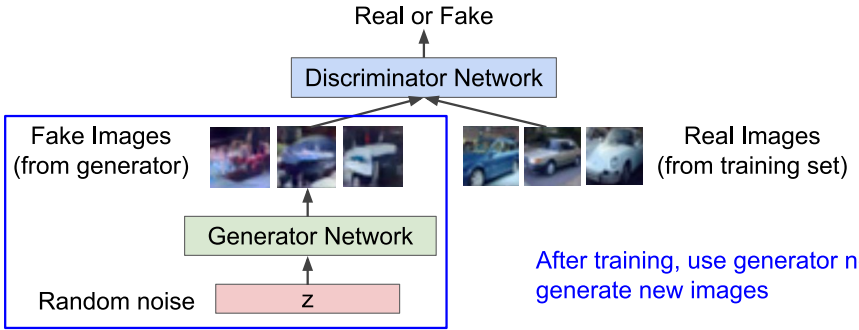
The discriminator D is trained to **discriminate real and fake samples**.

The generator g_θ is trained to fool the discriminator.

Adversarial training can be implemented with an alternate algorithm.



Generator v.s. Discriminator



After training, use generator network to generate new images

Plan

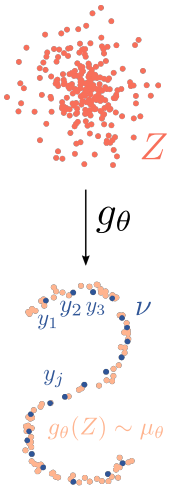
Loss function for Generative Modeling

Today, we will learn Generative Networks by solving

$$\inf_{\theta \in \Theta} \mathcal{L}(\mu_\theta, \nu)$$

where

- $\mathcal{L}$ is a loss function between probability distributions μ, ν on $\mathcal{X}, \mathcal{Y} \subset \mathbf{R}^d$.
- μ_θ is a probability distribution on a compact $\mathcal{X} \subset \mathbf{R}^d$:
  $g_\theta(Z) \sim \mu_\theta$ with g_θ neural network and $Z \sim \zeta$ input noise.
- The generator is parameterized by a θ in a open set $\Theta \subset \mathbf{R}^q$.
- ν is a probability on a compact $\mathcal{Y} \subset \mathbf{R}^d$:
  ν is the empirical distribution of the data.
  $Y \sim \nu$ will be a randomly chosen data point.



The Gist of Adversarial Training

- Train simultaneously a generator g_θ and a discriminator D with alternating updates:

$\rightarrow$ Push the discriminator $D : \mathbf{R}^d \rightarrow [0, 1]$ to discriminate between real and fake samples:

$D(g_\theta(z))$ should be close to 0 for any z
$D(y_j)$ should be close to 1 for any data point y_j

$\rightarrow$ Push the generator g_θ to fool the discriminator

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- The discriminator solves a binary classification problem between real and fake images:

$$\max_{D \in \mathcal{D}} \mathbb{E}[\log D(Y)] + \mathbb{E}[\log(1 - D(g_\theta(Z))]$$

where $\mathcal{D}$ is a (parametric) set of measurable functions $D : \mathbf{R}^d \rightarrow [0, 1]$. ($\log 0 = -\infty$.)

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- Finally, GAN training is a **min-max** two-player game:

$$\min_{\theta \in \Theta} \max_{D \in \mathcal{D}} \mathcal{L}_{\text{GAN}}(\theta, D) := \mathbb{E}[\log D(Y)] + \mathbb{E}[\log(1 - D(g_\theta(Z))]$$

Classification of fake points vs data points

In practice, the expectations are approximated using finite sums on samples.

With a finite sample $(x^{(i)})$ of real and fake points, it comes down to a logistic regression problem.

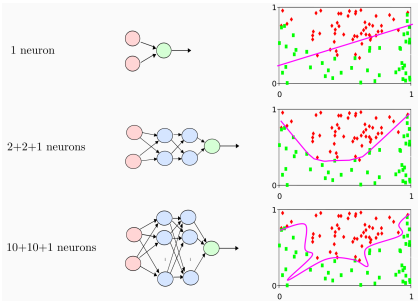
Let

- $\ell^{(i)} = 1$ if $x^{(i)}$ is one of the data points (y_j),
- $\ell^{(i)} = 0$ if $x^{(i)}$ is a generated point $g_\theta(Z)$.

We can then use the binary cross-entropy loss:

$$\frac{1}{N}\sum_{i=1}^{N}\left[\ell^{(i)}\log D(x^{(i)}) + (1-\ell^{(i)})\log\left(1 - D(x^{(i)})\right)\right]$$

$\longrightarrow$ `torch.nn.BCELoss`



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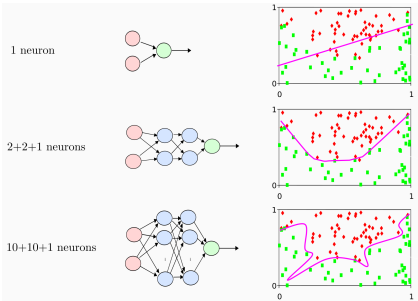

Training Algorithm

- In practice, g_θ and D are parameterized by neural networks.
  Here, D must have values in $[0, 1]$: take last layer as sigmoid activation $\sigma(x) = \frac{1}{1+e^{-x}}$.
  (Alternately, use `torch.nn.BCEWithLogitsLoss`)

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Illustration with a 2D example

Question: can you imagine a good discriminator for the following configuration?

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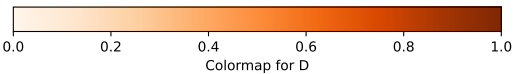

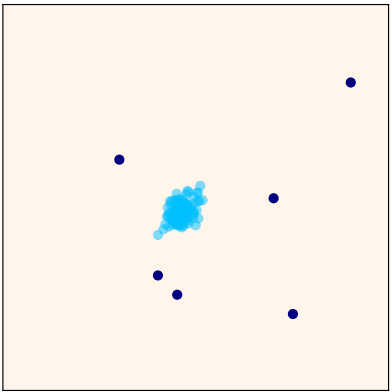

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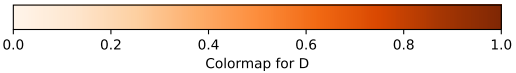

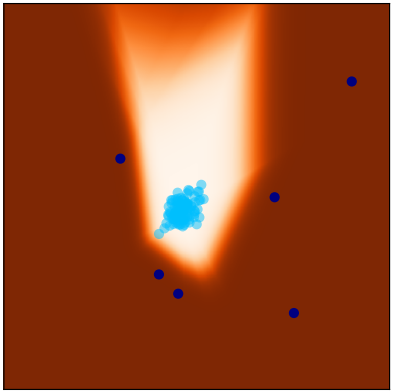

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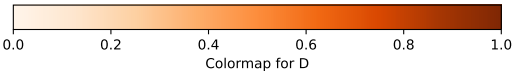

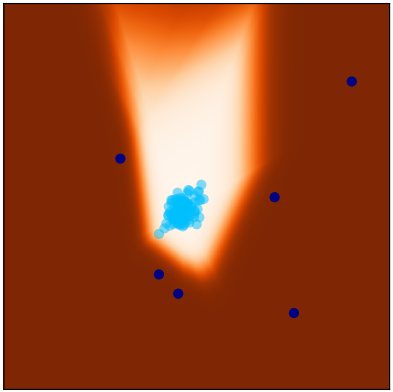

Problem: D is close to 0 on $\text{Supp}(\mu_\theta)$ $\rightarrow$ **"vanishing gradients" issue** (on ∇_θ)

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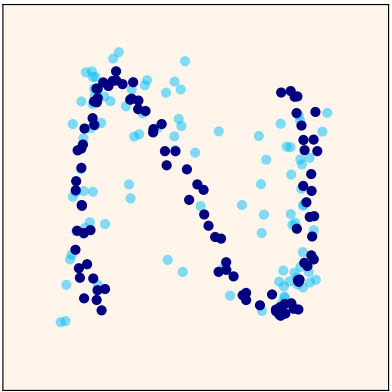

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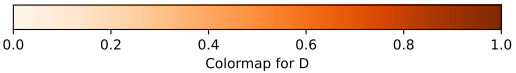

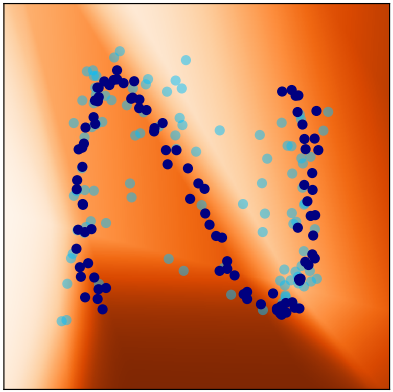

Optimal Discriminator

Let $\mathcal{D}_\infty$ the set of measurable functions from $\mathbf{R}^d$ to $[0,1]$, and let

$$L(\theta, D) = \mathcal{L}_{\text{GAN}}(\theta, D) = \int \log(D) d\nu + \int \log(1-D) d\mu_\theta.$$

Remark that

$$0 \geqslant \sup_{D \in \mathcal{D}_\infty} L(\theta, D) \geqslant L(\theta, \tfrac{1}{2}) = -\log 4.$$

Proposition ([Biau et al., 2018])

For fixed θ, assume that μ_θ and ν have densities p_θ, q with respect to a reference M. Then

$$\sup_{D \in \mathcal{D}_\infty} L(\theta, D) = L(\theta, D_\theta^*) \quad \textit{with} \quad D_\theta^* = \frac{q}{q + p_\theta}.$$

Relation with Jensen-Shannon divergence

Let us define the Kullback-Leibler divergence between probability measures μ, ν by

$$\mathrm{KL}(\mu|\nu) = \begin{cases} \int \log(\frac{d\mu}{d\nu}) d\mu & \text{if } \frac{d\mu}{d\nu} \text{ exists} \\ +\infty & \text{otherwise} \end{cases}$$

and the Jensen-Shannon divergence by $\mathrm{JS}(\mu, \nu) = \frac{1}{2}\,\mathrm{KL}(\mu, \frac{\mu+\nu}{2}) + \frac{1}{2}\,\mathrm{KL}(\nu, \frac{\mu+\nu}{2})$.

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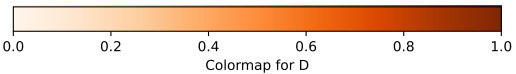
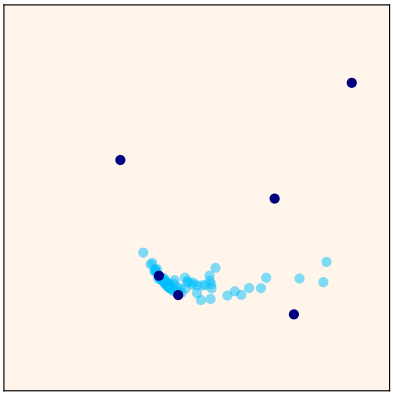
- If μ_θ and ν have disjoint supports, then $\mathrm{JS}(\mu_\theta, \nu) = \log 2$. $\rightarrow$ vanishing gradient issue!
- Why does it work then?
  $\rightarrow$ Because the parameterized discriminator is in practice smoother than D^*_θ.

What did you expect?

Final configuration. What is the final discriminator?

- Dark blue: data points $(y_j)_{1 \leq j \leq 6}$
- Light blue: 100 samples $(g_\theta(z_k))_{1 \leq k \leq 100}$ of μ_θ





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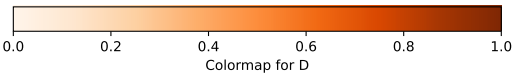

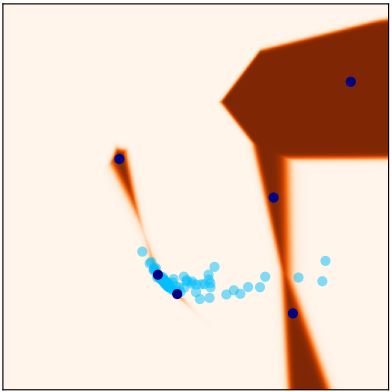

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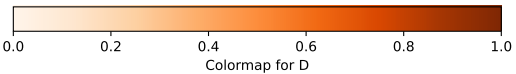

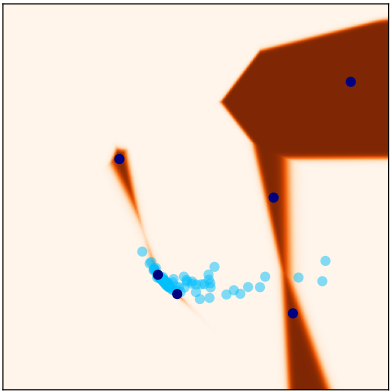

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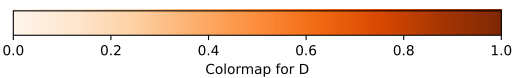

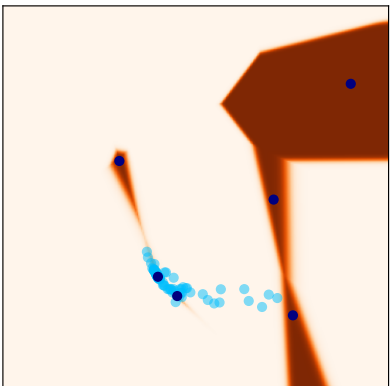

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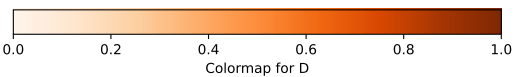

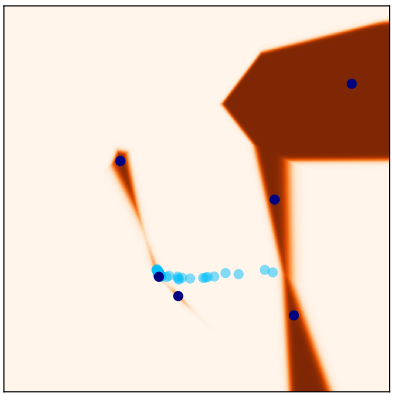

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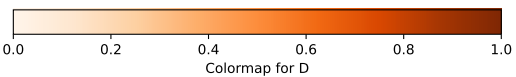

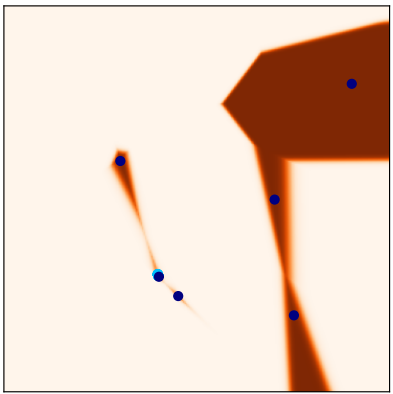

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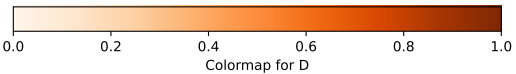

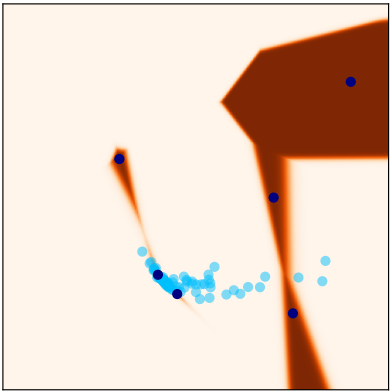

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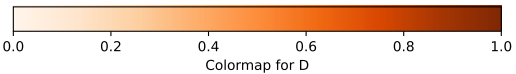
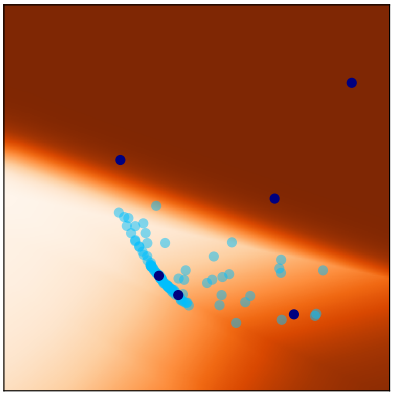




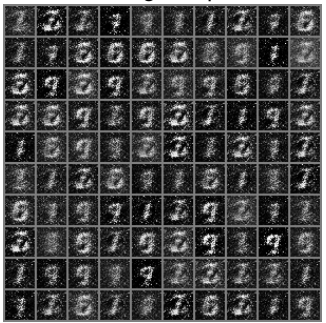
GAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 1



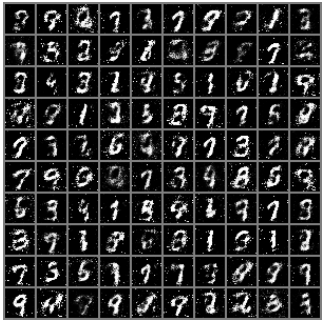
GAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 10



GAN Training for MNIST digits

Training with MNIST (60 000 images)

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Real images

Fake images, epoch 100

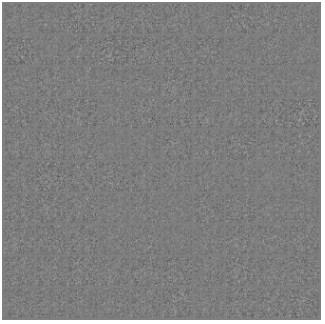
GAN Training for MNIST digits

Training GANs is quite unstable!

The generator can suffer from *mode collapse*:
i.e. it always produces the same image (one mode only).
Example: same as before **but with SGD instead of Adam**.

Real images

Fake images, epoch 1



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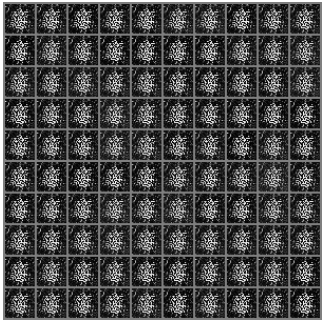

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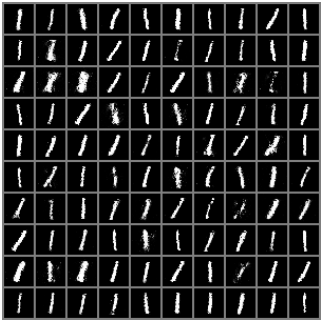
Fake images, epoch 100



Plan

Wasserstein GANs and Optimal Transport

WGAN are based on **Optimal Transport**.

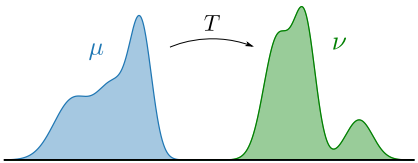
The Optimal Transport Cost measures the cost of moving some mass μ onto some mass ν.

It is sensitive to a "groundcost"

$$c : \mathbf{R}^d \times \mathbf{R}^d \longrightarrow \mathbf{R}.$$

For example: $c(x, y) = \|x - y\|$.

OT has many applications in image processing.



COLOR TRANSFER

Two formulations of the OT cost

Let μ, ν two probability distributions supported in $\mathcal{X}, \mathcal{Y} \subset \mathbf{R}^d$. Let $c : \mathcal{X} \times \mathcal{Y} \longrightarrow \mathbf{R}$ be continuous.

The optimal transport cost is defined by

$$W(\mu, \nu) = \inf_{\pi \in \Pi(\mu, \nu)} \int_{\mathcal{X} \times \mathcal{Y}} c(x, y)\, d\pi(x, y)$$

where $\Pi(\mu, \nu)$ is the set of distributions π on $\mathcal{X} \times \mathcal{Y}$ with marginals μ, ν.

For $c(x, y) = \|x - y\|^p, p \in [1, \infty[$, this defines the p-Wasserstein cost W_p.

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For $c(x, y) = \|x - y\|^p, p \in [1, \infty[$, this defines the p-Wasserstein cost W_p.

The "dual formulation" of the 1-Wasserstein cost is given by

$$W_1(\mu, \nu) = \sup_{\psi \in \text{Lip}_1} -\int \psi(x) d\mu(x) + \int \psi(y) d\nu(y)$$

where Lip_1 is the set of 1 Lipschitz functions $\psi : \mathbf{R}^d \to \mathbf{R}$.

Wasserstein Generative Networks (WGAN)

[Arjovsky et al., 2017]

Learning a Wasserstein WGAN consists in solving

$$\operatorname*{Argmin}_{\theta \in \Theta} W_1(\mu_\theta, \nu),$$

For $c(x, y) = \|x - y\|$, we get the usual WGAN formulation:

$$\mathcal{L}_{\text{WGAN}}(\theta, D) = \mathbb{E}[D(Y)] - \mathbb{E}[D(g_\theta(Z))],$$

where the discriminator D should be 1-Lipschitz.

✓ Benefit of the Wasserstein cost over the Jensen-Shannon divergence:
It is sensitive to the distance between the supports of μ_θ and ν.

Compare Loss functions

- Loss function for **GAN**:

$$\sup_{D \in \mathcal{D}_\infty} \mathbb{E}[\log D(Y)] + \mathbb{E}[\log(1 - D(g_\theta(Z)))]$$

- Loss function for **WGAN**:

$$\sup_{D \in \text{Lip}_1} \mathbb{E}[D(Y)] - \mathbb{E}[D(g_\theta(Z))].$$

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- The WGAN training algorithm alternates between
  · Ascent step(s) on $D \mapsto \mathbb{E}[D(Y)] - \mathbb{E}[D(g_\theta(Z)]$
  · Descent step(s) on $\theta \mapsto \mathbb{E}[-D(g_\theta(Z))]$

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- But, we have to constrain $D \in \mathrm{Lip}_1$ along the way...

Learning Lipschitz discriminators

- The original WGAN paper [Arjovsky et al., 2017] uses weight clipping to decrease the Lipschitz constant:

```
for p in D.parameters():
    p.data.clamp_(-c, c)
```

- Alternately, [Gulrajani et al., 2017] proposed to control more precisely the Lipschitz constant of D.
- We can do that by constraining the norms of ∇D at many points a_i.

The Gradient Penalty

[Gulrajani et al., 2017]

- Actually, Gulrajani et al. propose to use a finer property of W_1:
  the optimal dual potential φ satisfies $\|\nabla\phi\| = 1$ on segments joining samples from μ_θ and ν.
  (see e.g. [Santambrogio, 2015], and also a remark later in these slides)
- Therefore, they proposed to include a "gradient penalty" in the loss:

$$\mathrm{GP}(D) = \mathbb{E}[(\|\nabla D(A)\| - 1)^2] \quad \text{where} \quad A \sim \mathcal{U}([g_\theta(Z), Y]).$$

  Warning: the gradient is with respect to the variable x and not the parameters θ.
- This leads to the **WGAN-GP** discriminator loss (with penalty weight $\lambda > 0$):

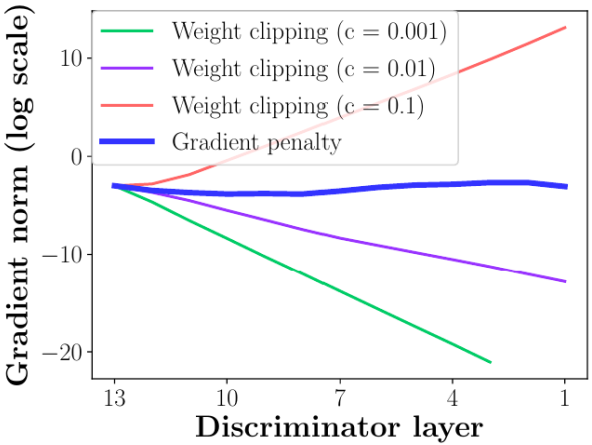
$$\sup_D \mathbb{E}[D(Y)] - \mathbb{E}[D(g_\theta(Z))] - \lambda \mathrm{GP}(D).$$

- In practice, the GP is estimated by drawing batches of A and computing $\nabla D(A)$ by automatic differentiation.

Gradient Penalty - Implementation

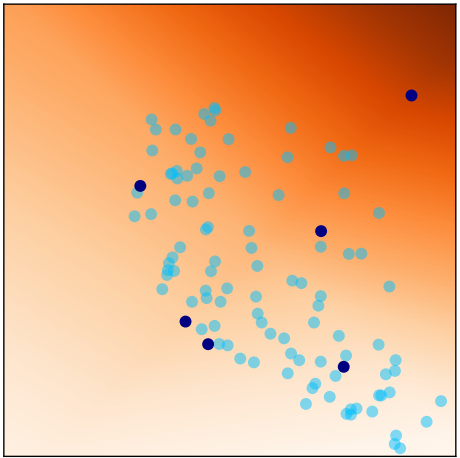
```
def gradient_penalty(D,x,y):
    # Calculate interpolation
    b = x.shape[0]
    if y.shape[0]!=b:
        print('wrong size')
    alpha = torch.rand((b,1,1,1),device=device)
    interp = (alpha * y + (1 - alpha) * x)
    interp.requires_grad_()

    # Calculate discriminator gradients on interpolated points
    Di = D(interp).view(-1)
    gradout = torch.ones(Di.size()).to(device)
    gradients = torch.autograd.grad(outputs=Di, inputs=interp, grad_outputs=gradout,
       create_graph=True, retain_graph=True)[0]

    # Derivatives of the gradient close to 0 can cause problems because of
    # the square root, so manually calculate norm and add epsilon
    gradients_norm = torch.sqrt(torch.sum(gradients ** 2, dim=[1,2,3]) + 1e-12)

    # Return gradient penalty
    return ((gradients_norm - 1) ** 2).mean()
```

WGAN: Gradient Penalty v.s. Weight clipping

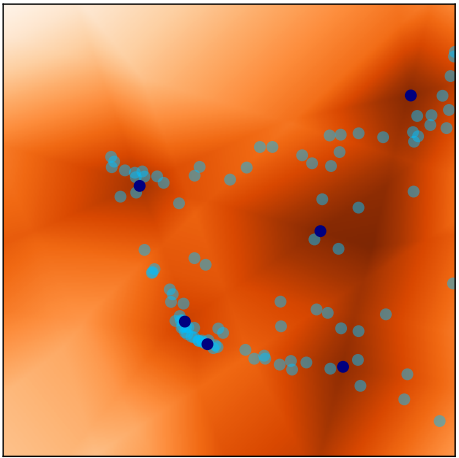


(source: [Gulrajani et al., 2017])

Example of WGAN training



WGAN-WC



WGAN-GP

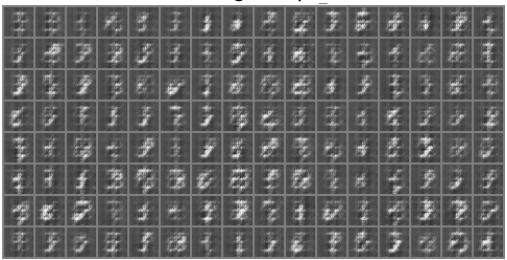
WGAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 1



WGAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 10

WGAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 20

WGAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 30

WGAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 40

WGAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 50

WGAN Training for MNIST digits

Training with MNIST (60 000 images)

- Adam optimizer
- Learning rate 0.0002 for both the discriminator and the generator

Real images

Fake images, epoch 100

WGAN Stability

WGAN-GP is a more stable way to train deep convolutional generators/discriminators.
But the results still depend highly on the optimization strategy and on the networks architectures.

Figure 2: Different GAN architectures trained with different methods. We only succeeded in training every architecture with a shared set of hyperparameters using WGAN-GP.

(source: [Gulrajani et al., 2017])

Plan

Popular Image Databases

- MNIST (digits): $60k$ images with 28^2 px (10 classes)
- Fashion-MNIST (clothes): $70k$ images with 28^2 px (10 classes)
- CIFAR-10: $60k$ images with 32^2 px (10 classes)
- ImageNet: $\approx 1430k$ images of various size (1000 classes)
- CelebA: $\approx 200k$ images with 178×278 px
- CelebA-HQ: $\approx 30k$ images with 1024^2 px
- LSUN (Bedroom/Cat/Churches/...): $\approx 100k$ or $1M$ images with 256^2 px
- FFHQ (or FFHQ-U): $70k$ images with 1024^2 px
- LAION-5B: $5.85B$ images of various size

Neural Network architecture

Generator and discriminator networks can have various layers:

- Fully connected (FC) layers
- Upsampling (interpolation) or Subsampling (max/average pooling) layers
- Convolution/Transposed convolution (with stride)
- Activation functions: RELU, leakyRELU, sigmoid, tanh, etc
- BatchNorm
- ...

Input noise Z has often uniform distribution $\mathcal{U}([0,1]^p)$ or Gaussian distribution $\mathcal{N}(0, \mathrm{Id})$.

Convolution

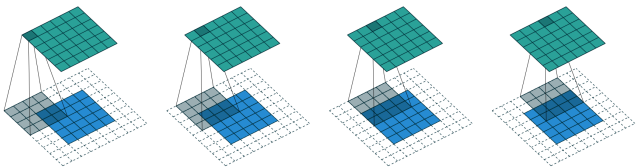
Let $u : \Omega \to \mathbf{R}^C$ be defined on $\Omega = [0 : M-1] \times [0 : N-1]$.

Let $w : \omega \to \mathbf{R}^{C' \times C}$ be defined on a small $\omega \subset \mathbb{Z}^2$. (Often, $\omega = [-k, k]^2$)

Definition

The convolution $w * u$ of the image u with kernel w is defined by

$$w * u(x) = \sum_{y \in \omega} w(y) u(x - y) = \sum_{z \in -\omega} \tilde{w}(z) u(x + z) \quad \text{where} \quad \tilde{w}(z) = w(-z).$$



NB: There are several possible border conditions (restriction, constant padding, periodic, ...)

Convolution and Transposed convolution

Notice that

- The transpose of a convolution with a $k \times k$ kernel is a convolution with a $k \times k$ kernel
- The transpose of a border crop is zero-padding the borders.
- The transpose of a crude subsampling is zero-inserting.

Strided convolutions:

- A "convolution with stride" is a convolution followed by subsampling.
- Called `Conv2d` in PyTorch

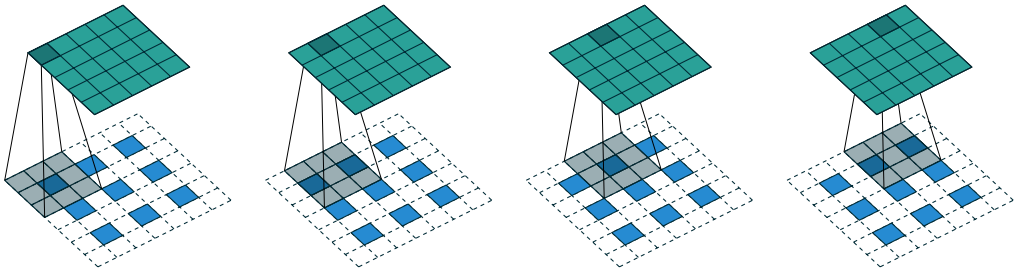
Fractionally strided convolutions:

- This is the transpose operator of convolution with stride.
- Called `ConvTranspose2d` in PyTorch

Useful webpage to compute the size of output after a Conv2D layer:

`https://madebyollin.github.io/convnet-calculator/`

One Example from [Dumoulin and Visin, 2016]



"The transpose of convolving a 3×3 kernel over a 5×5 input padded with a 1×1 border of zeros using 2×2 strides (i.e., $i = 5$, $k = 3$, $s = 2$ and $p = 1$). It is equivalent to convolving a 3×3 kernel over a 3×3 input (with 1 zero inserted between inputs) padded with a 1×1 border of zeros using unit strides (i.e., $i' = 3$, $\tilde{i}' = 5$, $k' = k$, $s' = 1$ and $p' = 1$)."

BatchNorm layer

Principle of BatchNormalization:

- Consider a batch $(x_n)_{1 \leq n \leq N}$ of N responses to a neural layer with C features.
- For each n, $x_{n,i} \in \mathbf{R}^{W \times H}$ is the i-th feature map of the n-th image.
- Batch normalization consists in computing for any n, i

$$y_{n,i} = \gamma_i z_{n,i} + \beta_i \quad \text{with} \quad z_{n,i} = \frac{x_{n,i} - m_i}{\sqrt{\sigma_i^2 + \varepsilon}}$$

  where m_i, σ_i are the mean and std of the gathered feature maps $(x_{n,i})_{1 \leq n \leq N}$.
  (In other words, m_i, σ_i contains averages over N and spatial dimensions H, W.)
- γ_i, β_i are trainable parameters.
- Implemented in `BatchNorm2d` in PyTorch.

At inference: normalization is done with $m_i, \sigma_i, \gamma_i, \beta_i$ learned during training.
Switch to inference mode with `model.eval()`.

Different Kinds of Normalization

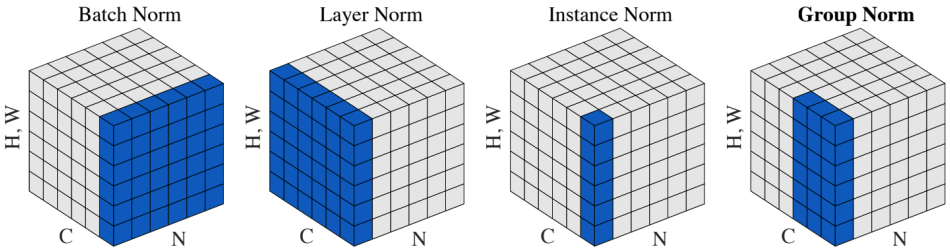


Diagram from [Wu and He, 2018]

- H, W: spatial dimensions
- C: channel dimension
- N: batch dimension

(See the formula for InstanceNorm in [Ulyanov et al., 2017])

Convolutional GAN

[Radford et al., 2016]

Important principles of the construction:

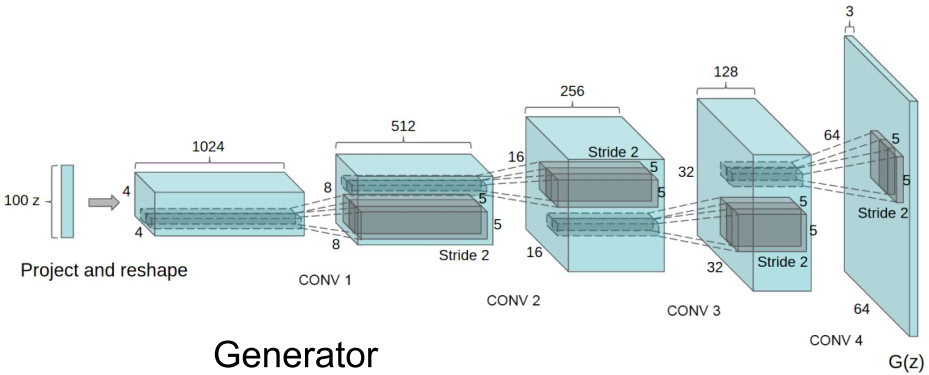
- "All convolutional": remove max pooling layers, and learn downsampling instead
- Eliminate Fully-Connected Layers
- Batch Normalization to stabilize learning (except on generator output, and discriminator input)
- ReLU activations for the generator
- LeakyReLU activations for the discriminator

Generator: upsampling network with **fractionally strided convolutions**

Discriminator: convolutional network with **strided convolutions**

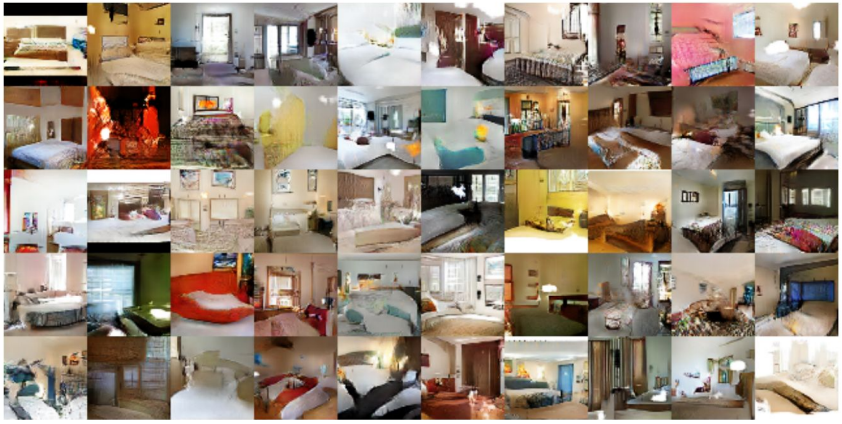
DCGAN Architecture

[Radford et al., 2016]



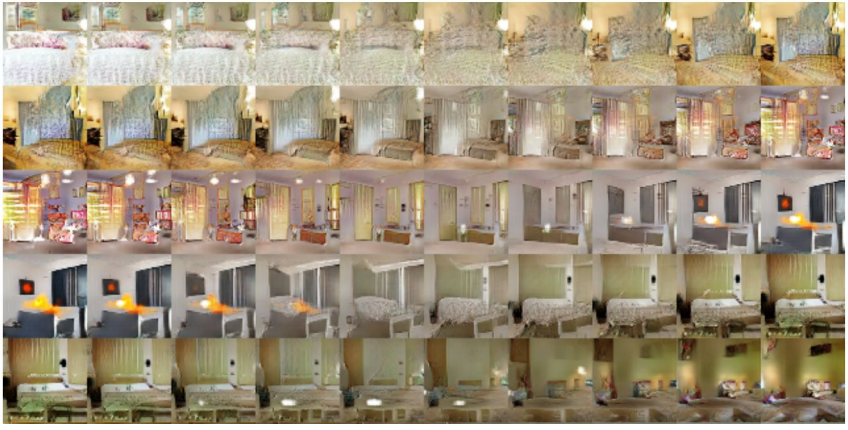
Generator

Image Generation with DCGAN [Radford et al., 2016]



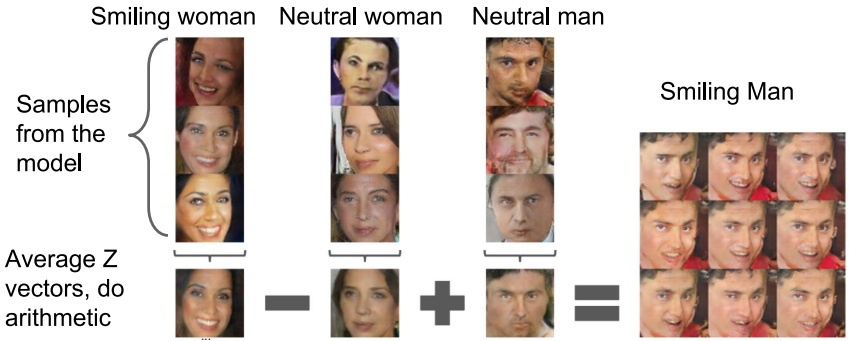
Generations of realistic bedrooms pictures, from randomly generated latent variables.

Image Interpolation with DCGAN [Radford et al., 2016]



Interpolation in between points in latent space.

Arithmetic with DCGAN [Radford et al., 2016]

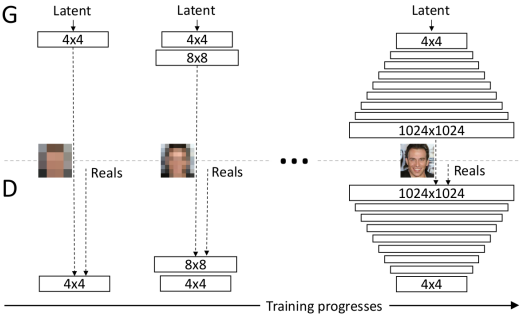


- Average latent vector of several samples
- After arithmetic, add a small random perturbation to get similar samples

Arithmetic with DCGAN [Radford et al., 2016]

Progressive Growing of GANs [Karras et al., 2018]

- Progressive Multiresolution Training
- Mirror architectures for G and D
- Simple upsampling/downsampling
  nearest neighbor upsampling;
  average pooling downsampling
  - Minibatch statistics layer at the end of D
  - Pixelwise feature normalization
- Training with WGAN-GP



StyleGAN [Karras et al., 2019]

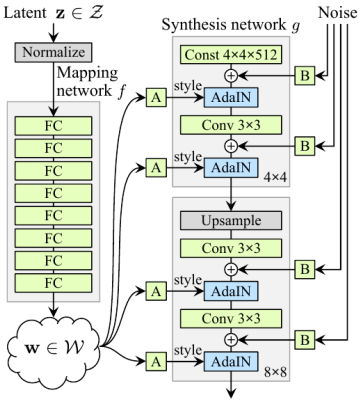
- "Separation of high-level features (pose, identity) from stochastic variation (freckles, hair)"
- Embed latent code z into an intermediate latent space w with a multilayer perceptron (8 FC layers)
- Spatially invariant style vector $y = (y_s, y_b)$ for each feature map, obtained from w
- AdaIN: Adaptive Instance Normalization

$$\text{AdaIN}(x_i, y) = y_{s,i} \frac{x_i - \mu(x_i)}{\sigma(x_i)} + y_{b,i}$$

where the feature map x_i is normalized separately. (No learned parameters γ, β here.)

$$\text{AdaIN}(x_{n,i}, y) = y_{s,i} \frac{x_{n,i} - \mu(x_{n,i})}{\sigma(x_{n,i})} + y_{b,i}$$

- Style mixing (playing with two latent codes w_1, w_2)



(b) Style-based generator

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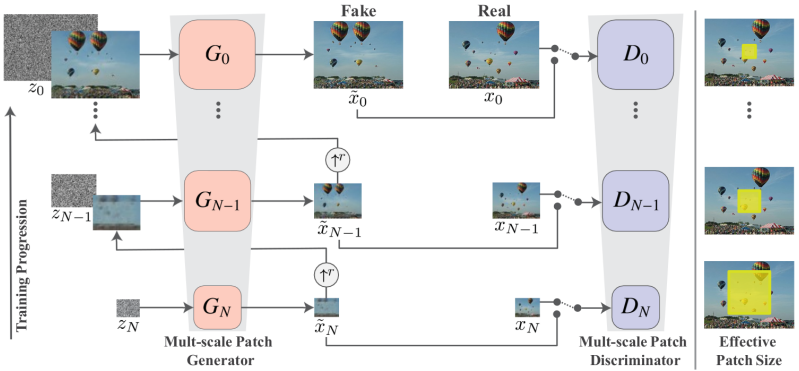
StyleGAN [Karras et al., 2019]

StyleGAN allows for style mixing at different scales (by using the corresponding subpart of w).

SinGAN: Learning from a Single Image [Shaham et al., 2019]

- Capture the multi-scale patch distributions of an image (possibly non-texture)
- Coarse-to-fine generator
- Patch-based dicriminator learned with WGAN-GP loss, at each scale



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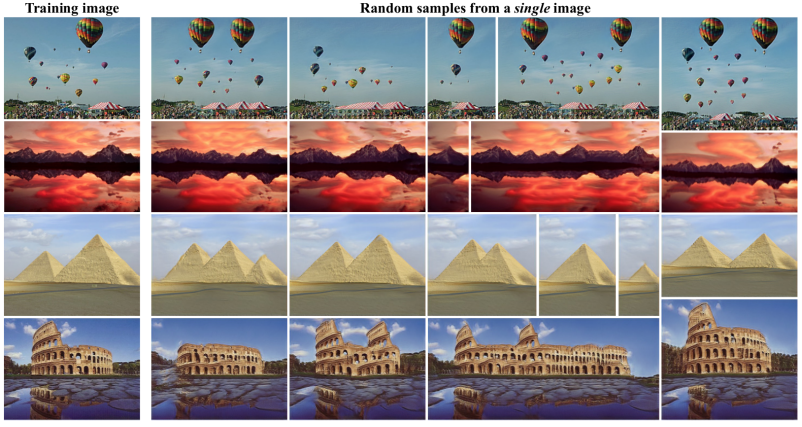

Take-home Messages

SUMMARY AND COMMENTS:

- We introduced GANs and Wasserstein GANs.
- Some constraints (Lipschitz) help to make training more stable.
- Results still depend on the generator/discriminator architectures and the optimization strategy.
- GANs can be used for image synthesis, with convolutional architectures.
- Assessing the quality of a trained generative network requires well-designed quantitative measures (see Appendix).
- Very large scale image synthesis requires tailored network architectures.

THANK YOU FOR YOUR ATTENTION!

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BatchNorm layer

Principle of BatchNormalization: for any batch $(x_i)_{i\in B}$ of a K-dimensional feature, transform

$$\forall k = 1, \dots, K, \quad y_i^{(k)} = \gamma^{(k)} \hat{x}_i^{(k)} + \beta^{(k)} \quad \text{with} \quad \hat{x}_i^{(k)} = \frac{x_i^{(k)} - m_i^{(k)}}{\sqrt{(\sigma_i^{(k)})^2 + \varepsilon}}$$

where $m_i^{(k)}, \sigma_i^{(k)}$ are mean and std of the k-feature over this batch,
and where $\gamma^{(k)}, \beta^{(k)}$ are trainable parameters.

At inference: normalization is done with $m^{(k)}, \sigma^{(k)}, \gamma^{(k)}, \beta^{(k)}$ learned during training.
Switch to inference mode with `model.eval()`.

Inception Score ↑ [Salimans et al., 2016]

- The inception score measures if μ generates a diverse collection of meaningful pictures
- For an image x, `Inception-v3` gives a label distribution $p(y|x)$ (discrete on $N = 1000$ labels)
- Images containing meaningful objects have $p(y|x)$ with low entropy
- In order to generate various images, $p(y) = \int p(y|x)\mu(dx)$ should have high entropy

The Inception Score then writes as

$$\mathrm{IS}(\mu) = \exp\left(\int \mathrm{KL}\left(p(y|x)|p(y)\right)\mu(dx)\right) \in [1, N]$$

It is 1 iff for a.e. x, $p(\cdot|x) = p(\cdot)$ (label distribution does not depend on x)
It is N iff for a.e. x, $p(\cdot|x)$ is concentrated on one label, and $\forall y$, $\int p(y|x)\mu(dx) = \frac{1}{N}$

How to compute it in practice:

- Compute an estimate $\hat{p}(y)$ of $p(y) = \int p(y|x)\mu(dx)$ by drawing samples of μ
- Estimate $\int \mathrm{KL}(p(y|x)|\hat{p}(y))\mu(dx)$ by drawing samples of μ

Fréchet Inception Distance (FID) ↓ [Heusel et al., 2017]

The FID measures how close are two image distributions μ, ν in terms of features distributions.
It is based on the response of `Inception-v3` [Szegedy et al., 2016] before last pooling layer:

$$f : \mathbf{R}^d \to \mathbf{R}^m$$

that extracts $m = 2048$ features (as a generic image summary)
NB: Images may have to be resized/normalized to fit into this network.

Algorithm to compute the FID score:

1. Draw samples (x_i) and (y_j) of $X \sim \mu$ and $Y \sim \nu$ and compute the features $(f(x_i))$, $(f(y_j))$
2. Fit Gaussian distributions $\mathcal{N}(m_X, \Sigma_X)$ and $\mathcal{N}(m_Y, \Sigma_Y)$ to $(f(x_i)), (f(y_j))$ (in $\mathbf{R}^{2048}$)
3. Return the 2-Wasserstein distance between the Gaussian distributions, i.e. the Fréchet distance: [Dowson and Landau, 1982]

$$W_2^2\Big(\mathcal{N}(m_X, \Sigma_X), \mathcal{N}(m_Y, \Sigma_Y)\Big) = \|m_X - m_Y\|_2^2 + \mathrm{Tr}\Big(\Sigma_X + \Sigma_Y - 2(\Sigma_X\Sigma_Y)^{\frac{1}{2}}\Big)$$

NB: FID can be adapted to the "single-image" case: **SiFID** [Shaham et al., 2019]
SiFID compares distributions of features obtained after a convolution layer (spatially averaged)