A Mathematical Description of the Speed/Accuracy Trade-off of Aimed Movement

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ABSTRACT
Target clicking having proved an indispensable building block of interface design, it is little surprise that the speed/accuracy trade-off of aimed movement has always been a keen concern of HCI research. The trade-off is described by the Fitts law. In HCI and psychology likewise, the traditional approach has focused on the time-minimisation paradigm of Fitts [5], ignoring other relevant paradigms in which the Fitts law fails, such as the spread-minimisation paradigm of Schmidt et al. [18]. This paper aims at unearthing and consolidating the foundations of the speed/accuracy trade-off problem. Taking mean movement time as our speed measure and relative spread as our accuracy measure, we show that a small set of obvious mathematical axioms predict not only the data from the Fitts and the Schmidt paradigms but also the data from the more recent dual-minimisation paradigm of Guiard et al. [7]. The new mathematical framework encourages a more complete understanding: not only is it possible to estimate an amount of resource, a quantity equivalent to the classic throughput, it is also possible to characterize the resource-allocation strategy — the other, no less important facet of the trade-off problem which has been left aside so far. The proposed approach may help HCI practitioners obtain from their experimental data more reliable and more complete information on the comparative merits of design options.

Categories and Subject Descriptors
CCS → Human-centered computing → Human computer interaction (HCI) → Interaction techniques → Pointing

Keywords
Fitts law, aimed movement, pointing, speed/accuracy trade-off, resource, resource allocation.

1. THE FITTS LAW
The more exacting the accuracy demand on a movement, the slower. In his famous 1954 paper Fitts [5] was able to give this general observation the form of a simple mathematical equation, known as the Fitts law. In its *task* version, the Fitts law reads:

$$\mu_T = a + b \log_2 (d/w + 1), \quad (1)$$

where μ_T is the average time it takes people to reach a target of width w whose centre is located at distance d, and where a and $b > 0$ stand for empirically adjustable constants. The logarithmic term is called the index of difficulty (i_d). Most researchers actually use the *behavioural* version of the Fitts law [26], which reads:

$$\mu_T = a + b \log_2 (\mu_A/\sigma_A + 1), \quad (2)$$

where μ_A and σ_A denote the mean and the standard deviation of movement amplitude, the logarithmic term being called the index of effective difficulty I_{de}.

1.1 Terminology and Mathematical Notation
To tackle the subject of this paper, the trade-off of speed and accuracy in the execution of aimed movement, we first need to fix our terminology and our mathematical notation (see Appendix 1). In this research we care about a number of distinctions that have been treated somewhat casually in the literature. We use lowercase letters to denote deterministic quantities and uppercase letters to denote randomly-varying quantities. Thus we note target distance and target width, under full control of experimenters, d and w, whereas we note the time duration and the amplitude of the movement, subject to random variability, T and A. We let μ_X and σ_X denote the mean and the standard deviation of a random variable X. Notice that the error $E = A - d$, being the difference between the random quantity A and the deterministic quantity d, is itself a random quantity; and the variability of E being entirely due to the variability of A, we have $\sigma_E = \sigma_A$.

1.2 Shortcomings of the Fitts Law
The Fitts law has been justly praised as an empirically robust rule of thumb whose mathematical formulation has received formal justifications in light of information theory [5][19][9]. Nevertheless the law has some shortcomings that must be discussed seriously.

1.2.1. *A Loosely-Constrained Independent Variable*

We have problems with the right-hand side of Fitts-law equations. All known variants of i_d lack a true zero, meaning that the y-intercept of the Fitts law is uninterpretable [7]. Second, it has been a tradition in the description of the Fitts law to omit to specify the range of i_d or I_{de} values over which the law is supposed to hold — in Meehl's [13] terminology the *spielraum* or range of interest. Most corroborations of the Fitts law, with r squares computed over arbitrarily narrow ranges of difficulty, look rather like confirmations that the Earth is locally flat.

1.2.2 *A Paradigm-Dependent Rule of Thumb*
The Fitts law being of the form $\mu_T = f(\mu_A/\sigma_A)$, where the function f is nonlinear, the relation should remain nonlinear when recast as $\sigma_A = f(\mu_A/\mu_T)$. This is precisely the relation investigated in the so-called Schmidt paradigm [18], in which participants are to minimize endpoint spread σ_A, treated as the dependent measure, while covering pre-specified amplitudes in pre-specified amounts of time (thus average speed μ_A/μ_T is the independent variable). Even though the Schmidt paradigm involves strictly the same three quantities — namely μ_A, μ_T, and σ_A — it has been reported to deliver a linear trade-off [18]. The linearity of the Schmidt law,

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as it is called, does not seem to have troubled Fitts-law theorists too much. Yet, the mere fact that the Fitts law is apparently jeopardised by a simple rearrangement of the terms of its equation — or, at the practical level, by a change in the experimental paradigm — raises the concern that the Fitts law, valid within just one particular paradigm, lacks generality.

1.2.3. *Half of the Question Ignored*

A trade-off between two quantities x and y is a dual phenomenon with both a *conservation* facet and a *change* facet: a certain combination of x and y is conserved despite a change in the respective contributions of x and y to the conserved quantity. Being a conservation across a certain transformation, an invariance — e.g., the invariance of shape under such transformations as rotations, translations, or rescalings [11] — is something far more interesting than a trivial constancy [21].

In the case of the speed/accuracy trade-off of aimed movement, the Fitts law [10][19] amounts to the statement of the invariance of the throughput across the variations of the index of difficulty, which controls the speed/accuracy balance. Unfortunately the Fitts-law literature has paid little or no attention to the variation of the speed/accuracy balance, the transformation facet of the speed/accuracy trade-off. A sign of this conceptual hemineglect is visible in the recent ISO standard [10], a set of guidelines for Fitts-law experimentation, which recommends to practitioners to retain only throughput estimates from their data. This implies the problematic assumption that a single number can fully characterize the performance achieved with a device or an interaction technique. In fact, quite independently of the throughput, devices and interaction techniques may differ to considerable extents in terms of the speed/accuracy strategy they elicit, and in some contexts these differences may be of much practical consequence.

2. INVENTORYING AND SIFTING POSSIBLE SPEED AND ACCURACY MEASURES

Shouldn't there exist a more general, paradigm-independent law? Notice that the Schmidt law and the Fitts law do not use the same measures of speed and accuracy. To measure speed the Schmidt law uses the ratio μ_A/μ_T, of dimension $[LT^{-1}]$, whereas the Fitts law uses μ_T, of dimension [T]; and to measure accuracy the Schmidt law uses the spread σ_A, of dimension [L], whereas the Fitts law uses the dimensionless ratio μ_A/σ_A. To be in a position to see whether or not different paradigms reveal different laws, we must make sure we use the same measures, and obviously valid ones. But several definitions of movement speed and accuracy are possible, and so we need to inventory the possibilities and examine the validity of each.

As first identified by Fitts [5], our speed/accuracy trade-off problem involves three crucial quantities, two central–trend statistics, mean movement time μ_T and mean amplitude μ_A, and one dispersion statistic, endpoint spread σ_E or σ_A. In Figure 1 these statistics outline a *speed* axis vertically and an *accuracy axis* horizontally. At the intersection of the two axes stands the scale parameter $\mu_A \approx d$, which specifies the absolute magnitude of the movement. By definition an *aimed*-movement task demands the specification of a certain target amplitude d that μ_A is supposed to approach, but μ_A is not involved in the trade-off of interest and does not constitute a utility. The aiming bias $\mu_E = \mu_A - d$ is indeed a negative utility (the less the better), but it is not involved in the trade-off we are talking about. The two negative utilities crucial here are mean movement time and endpoint spread.

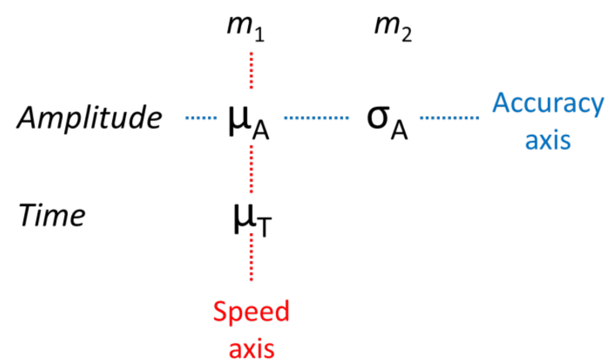

Figure 1. The three statistics that can be combined in the definition of speed and accuracy. m_1 and m_2 denote the first and the second moment of distributions.

There are three possible basic measures of movement speed: (1) *absolute slowness* μ_T (in s) as in the Fitts paradigm (2) *relative speed* μ_A/μ_T (in m/s) as in the Schmidt paradigm, and (3) *relative slowness* μ_T/μ_A (in s/m), an unused option.

Likewise there are three possible basic measures of movement accuracy: (1) *absolute spread* σ_A (mm) as in the Schmidt paradigm, (2) *relative amplitude* μ_A/σ_A (-) as in the Fitts paradigm, and *relative spread* σ_A/μ_A (-) as in the paradigm we recently introduced [8]. We now compare the merits of these different candidate measures in light of three independent criteria.

2.1 Scale Independence

More often than not in Fitts-law studies the scale parameter d is manipulated thoughtlessly in designs simply because it is hard for experimenters to obtain a reasonably large range of difficulties at a single level of d. Most designs suffer from a more or less severe factor confound, a positive correlation, between the index of difficulty and d [6]. For example the correlation was no less than +.68 in Fitts's [5] famous tapping experiment, as is easy to check from his Table 1 (p. 254). But the confound was essentially harmless in his case because to measure movement speed the author had chosen μ_T and Fitts's data show that d had essentially no effect on this dependent variable, in keeping with the isochrony principle reported in the handwriting literature — there is little or no change in the time it takes people to complete a given graphical form as they rescale the form up and down, within limits [20]. Had Fitts instead used relative speed μ_A/μ_T as his speed measure, his experiment would have produced confusing data because this ratio happens to be massively scale dependent. Thus, of the three candidate measures of speed only μ_T passes the scale-independence test.

Turning to accuracy, a simple Weber law argument predicts that absolute spread σ_A must be about proportional to μ_A, and the presence of this scale-dependency effect is verified in virtually any data set. In contrast, the other two candidate measures of accuracy, relative amplitude μ_A/σ_A and relative spread σ_A/μ_A, which consist of dimensionless ratios, are scale independent by construction.

2.2 Ratio-Scale Metric

Facing an empirical relation of the form $y = f(x)$ it is desirable to have a true physical zero not only on the y axis (as is nearly always the case) but also on the x axis, otherwise y-intercepts are uninterpretable [7].

All three candidate measures of speed have a true zero. Having passed the first test, mean movement time μ_T also passes the second. As for accuracy, at first sight the ratio μ_A/σ_A and its inverse σ_A/μ_A look like equally promising accuracy measures because it is mathematically trivial to transform one into the other. One should realise, however, that only the latter ratio enjoys a ratio-scale metric [7]. Relative distance μ_A/σ_A, which underlies the calculation of all indices of difficulty in the Fitts paradigm, has an arbitrary zero. No one can tell what $\mu_A/\sigma_A = 0$ might specify in the physical world because A being a non-negative random variable it is impossible to have $\mu_A = 0$ together with $\sigma_A > 0$. Therefore we disagree with [19]: in a Fitts-law plot, whose x axis exhibits an index of difficulty computed from the ratio μ_A/σ_A, there is no rationale for expecting (or even hoping) that the intercept will be close to zero, simply because this intercept is uninterpretable.

Unlike the zero of relative amplitude μ_A/σ_A, the zero of relative spread σ_A/μ_A *does* exist. Just like any coefficient of variation, relative spread σ_A/μ_A (or σ_E/μ_A) zeroes out at the point where the random variability of movement endpoint zeroes out, with $\sigma_A = 0$ while $\mu_A > 0$. The zero of relative spread is simply the theoretical limit where movement amplitude becomes a deterministic quantity [7]. Therefore below we shall use relative spread σ_A/μ_A, rather than relative amplitude μ_A/σ_A, to measure movement accuracy [8]. It is *in*accuracy, not relative accuracy, that σ_A/μ_A measures, but the traditional ratio μ_A/σ_A measures neither.

Thus after our second test a single possibility remains for the measurement of both speed and accuracy, namely μ_T and σ_A/μ_A, respectively. Although we have completed our choice task, let us see how the two surviving candidates stand a third and final test.

2.3 Length/Angle Neutrality

In many aimed-movement tasks A is a length, but in some others it is an angle. We want our speed and accuracy measures to allow performance comparisons between tasks that involve translational and rotational sorts of movement. For example Schmidt et al. [18], who inaugurated the Schmidt paradigm with a stylus-tapping movement, expressed effective width in mm whereas Wright and Meyer [25], who did a replication experiment using a wrist-rotation movement, expressed effective width in angular degrees. Both studies used the Schmidt paradigm, yet their data cannot be plotted together because the accuracy measures (degrees vs. mm) as well as their speed measures (degrees/s vs. cm/s) had different units.

The duration of a movement obviously provides a dimensionless, length/angle-neutral measure of speed — quite unlike average speed μ_A/μ_T or its inverse μ_T/μ_A, which involve an angle or a length. As for accuracy, the dimensionless ratio σ_A/μ_A we have retained has no unit (unlike σ_A, which is either a length or an angle). Thus our third test provides a further argument in favour of the two candidates that already passed the first two tests: mean movement time μ_T for speed and relative spread σ_E/μ_A or σ_A/μ_A for accuracy.

3. A MINIMALIST THEORETICAL FRAMEWORK

In this section we present a small set of pretty obvious axioms regarding the speed and the accuracy variables we have chosen and the trade-off function that relates them. We will see that these axioms suffice to give birth to a parsimonious mathematical model of the speed/accuracy trade-off of aimed movement. The model arises quite straightforwardly from the axioms without the need to theorize about substantive issues such as the information conveyed by the movement (e.g., [5][12][19][9]), or about the cognitive mechanisms of movement programming, execution, and correction (e.g., [3][14]).

Please note that below we will be somewhat disrespectful to an old convention of Fitts-law research: we shall systematically plot movement speed (μ_T) on the horizontal axis, treating it as our x variable, and movement accuracy (σ_A/μ_A) on the vertical axis, treating it as our y variable. When it comes to pointing, we tend to construe accuracy as the independent variable, but this is because we look through the prism of Fitts's highly popular time-minimisation paradigm, in which indeed the accuracy (the i_d) is something experimenters manipulate and the speed (μ_T) some-thing they measure. However, the Fitts paradigm is just one of several possible experimental approaches to our trade-off problem, as will be recalled in Section 4.1, and there is serious reason to assume that ultimately it is the speed of our movements that determines their accuracy rather than the reverse [23].

3.1 Axioms

Let us start with the observation that x and y are both *negative utilities*, meaning quite simply that the shorter the time and the smaller the spread, the better the performance. In the trade-off function we want to model, ideal performance corresponds to the case where the movement would last an average of 0s and would exhibit 0% of relative spread — thus an ideal block of trials would deliver a data point that fell right at the origin of the graph.

The notion of a speed/accuracy trade-off implies a number of prior assumptions [8][16]. In our view no theory of the speed/accuracy trade-off of aimed movement can sensibly avoid any of the following six prior assumptions, or axioms.

3.1.1 An Absolute Minimum of Movement Time

In any particular experimental condition there must be a minimum to the duration T of any individual movement, owing to the limited acceleration and deceleration capabilities of any effector system. On the horizontal axis we have the constraint that

$$x \geq x_0 > 0, \qquad \text{Axiom 1}$$

where x_0 defines the strictly-positive minimum of μ_T, which must be imposed unconditionally on our model. Whenever x_0 can be determined, it will be convenient to express our independent variable as the difference $x-x_0$, rather than x.

Note that the particular value taken by x_0, dependent on an indefinitely large number of parameters (e.g., scale, the musculature involved, the way instructions were formulated and understood by participants, etc.), is uninterpretable per se. The x_0 parameter cannot serve to compare data from different experiments. If, however, the ceteris-paribus condition is satisfied as may be the case within a given experimental design, between-conditions comparisons of x_0 may be useful.

3.1.2 An Absolute Minimum of Relative Spread

Since any effector system at rest suffers some irreducible physiological tremor, and any recording device has a finite resolution [1][22], there necessarily is a strictly positive minimum to the value of spread σ_A and hence of relative spread σ_A/μ_A to be recorded in an experiment. Calling that theoretical minimum y_0, the vertical axis of our function has the constraint that

$$y \geq y_0 > 0. \qquad \text{Axiom 2}$$

Whenever the value of y_0 can be determined, it will be useful to express our dependent variable as the difference $y–y_0$, rather than y. Just like x_0, y_0 is a parameter whose value is of little interest in and of itself, being subject to indefinitely many influences, but it may possibly allow useful comparisons within a controlled experimental design.

3.1.3 A Decreasing Convex Function.

Since the less of one negative utility, the more of the other, the function $y = f(x)$ or $y–y_0 = f(x–x_0)$ must be strictly decreasing and strictly convex, with a vertical asymptote at x_0 (where no more resource is available for the y-minimisation effort), and a horizontal asymptote at y_0 (where no more resource is available for the x-minimisation effort):

$$\left.\begin{array}{l}\text{for } x \to +\infty,\ y \to y_0 > 0\\ \text{for } y \to +\infty,\ x \to x_0 > 0.\end{array}\right\} \qquad \text{Axiom 3}$$

This assumption is consistent with Norman and Bobrow's principle of graceful degradation [15] (p. 44).

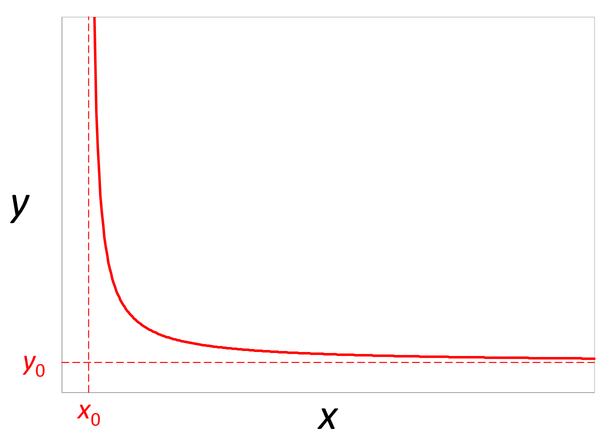

Figure 2. A decreasing and convex trade-off function with asymptotes at x_0 and y_0.

Figure 2 summarises our progress so far.

3.1.4 A Certain Combination of x and y Conserved

The trade-off of speed and accuracy must be supposed to result from the fact that the two concurrent minimisation efforts draw from the same limited resource pool. The content of the hypothetical pool, whose nature is unknown, may be thought to consist of attention or effort. Using the familiar economic analogy, it is assumed that some generic currency is convertible into speed and/or accuracy and that the amount of this currency available to a given individual placed in a given situation is finite [8]. Were 100% of the resource invested in the aimed-movement task, we would have

$$\forall(x, y),\ x \odot y = c, \qquad \text{Axiom 4}$$

where the symbol $\odot$ denotes some as yet unspecified way of combining the two variables, and c denotes some adjustable constant. Notice that since we are combining two negative utilities, c can only work as an estimate of the *scarcity* of the resource — the smaller c, the more resource.

3.1.5 Less-than-Total Investment of the Resource

Although the participants are supposed to invest the totality of their resource to produce their best possible performance in every single experimental condition, human effort is subject to random fluctuations. Only occasionally can participants approach their best possible performance. Of a block of trials (x, y), where $x = \mu_T$ and $y = \sigma_E/\mu_A$, we may say:

$$\left.\begin{array}{l}\text{If } (x, y) \text{ is doable, then,}\\ \forall x' \geq x \text{ and } \forall y' \geq y,\\ (x', y') \text{ is doable.}\end{array}\right\} \qquad \text{Axiom 5}$$

In other words, it is always possible to do worse: All empirical data points must fall above the limiting curve we are looking for — i.e., $y \geq f(x)$ and $x \geq f^{-1}(y)$ — or, equivalently, the curve is necessarily located *below the scatter plot*.

Axiom 5 has one far-reaching implication. Since the empirical function we look for characterizes an upper limit of performance, regression techniques are inadequate to infer the function from empirical scatter plots. A least-squares minimisation procedure delivers an *average* curve summarising all data points, including those obtained in trial blocks with far from complete investment of the resource. But little can be learned from poor performance, and so it is *not* the scatter plot that we want to model, but rather the *South-West quadrant of the convex hull* of the scatter plot — what we call the *convex front of performance* [8].

3.1.6 Resource-Allocation Strategy

Humans can, to an appreciable extent, modulate the proportion in which they allocate their resource to the mutually incompatible speed and accuracy efforts, exhibiting a certain strategic *flexibility*. Little can be learned from the Fitts-law literature about the range of speed/accuracy strategies participants are actually capable of, most studies having used rather narrow ranges of difficulty levels.[1]

At this point we need to introduce a conceptually important distinction between a *curve* in a plane and an *arc* on that curve. A curve corresponds to an infinite function, whereas an arc corresponds to a certain finite interval on a function, specified for example by an x_{min} and an x_{max}. Two extra constraints being required to determine an interval along a given curve, a curvilinear arc conveys more information than a curve.

The trade-off model we are contemplating is an infinite theoretical function extending from $y = +\infty$ at x_0 to $y = y_0$ at $x = +\infty$. Such an infinite function, however, says nothing about the range of strategies actually covered in a given data set. As shown in Figure 3, that range is a finite subset of the function, a curvilinear arc whose localisation on the infinite curve requires two extra parameters: x_{min} (where $y = y_{max}$), and x_{max} (where $y = y_{min}$).

1 One reason why strategic ranges are usually narrow in Fitts-law experimentation is because extreme strategies are difficult to handle within the classic time-minimisation paradigm.

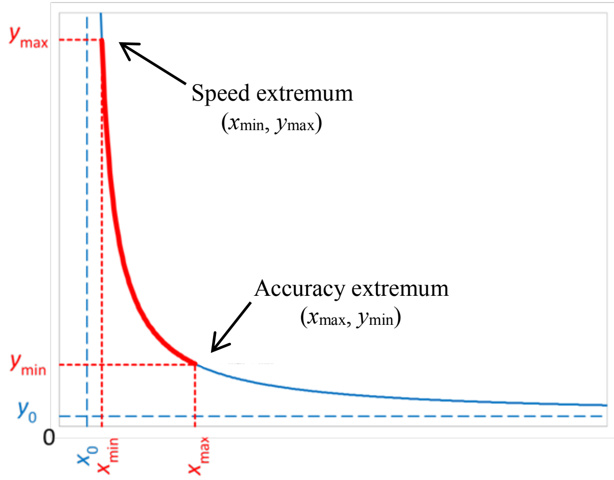

Figure 3. Modelling the theoretical trade-off function with an infinite curve, and the particular subset of the function actually realised in a data set with a finite curvilinear arc.

Thus, facing a set of experimental data, we must have

$$\left.\begin{array}{l} x_{max} > x_{min} \geq x_0 > 0 \\ \\ y_{max} > y_{min} \geq y_0 > 0. \end{array}\right\} \quad \text{Axiom 6}$$

While the function's asymptotes at x_0 and y_0 represent the theoretical minima of Axioms 1-2, the points of coordinates (x_{min}, y_{max}) and (x_{max}, y_{min}) are *empirical* extrema. We will exploit them below to characterize the resource-allocation strategy (Section 4.2.3).

3.2 The Homographic Model

One very simple function that satisfies Axioms 1 through 4 is the so-called homographic function:

$$(y - y_0)\,(x - x_0) = k, \tag{3}$$

where $k > 0$ is an adjustable constant and x_0 and y_0 are the theoretical minima of Axioms 1 and 2.

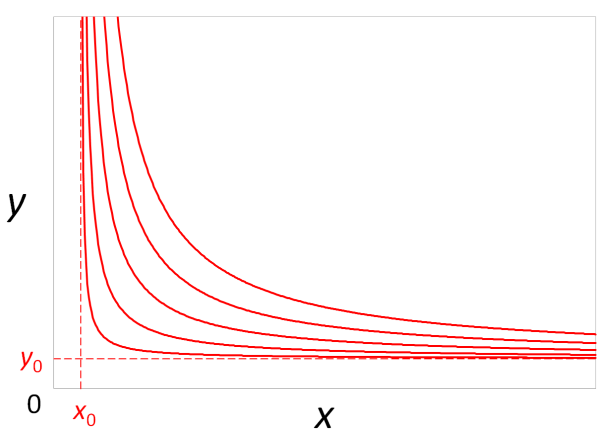

Figure 4. The homographic model with *k* varied from 0.05 (inner curve) to 0.2 (outer curve).

The homographic model (Figure 4) has just the properties we demand. The function is strictly decreasing and strictly convex and it links a vertical asymptote at x_0 to a horizontal asymptote at y_0 (Axioms 1-3). And it conserves the product k, which may serve as a global estimate of the resource (more exactly, of resource scarcity) assumed to be invariant across the variations of the resource allocation (Axiom 4). The model satisfies our axioms while being simplest, in two senses: it involves just one free parameter, and it resorts only to one basic arithmetic operation, the multiplication. The homographic model has just one undesirable property, the curve symmetry with respect to the axes' bisector. Our axioms do not allow us to presuppose in what proportions the speed and the accuracy efforts actually draw on the resource, and so an amendment of the model is in order.

3.3 The Weighted Homographic (WHo) Model

To get rid of the rigid symmetry of the homographic model, we endow it with a free skewness parameter, obtaining what we call the *weighted* homographic (WHo) model (Figure 5):

$$(y - y_0)^{1-\alpha}\,(x - x_0)^{\alpha} = k_\alpha, \tag{4}$$

where the weighting exponent α is an adjustable coefficient ($0 < \alpha < 1$) free to deviate from its neutral value of ½ (symmetry). If α = ½, one is back to Equation 3 whose constant then equals $k_{½}^2$. The role of α is to allow some degree of asymmetry, it being understood that the coefficient should not approach 0 or 1.

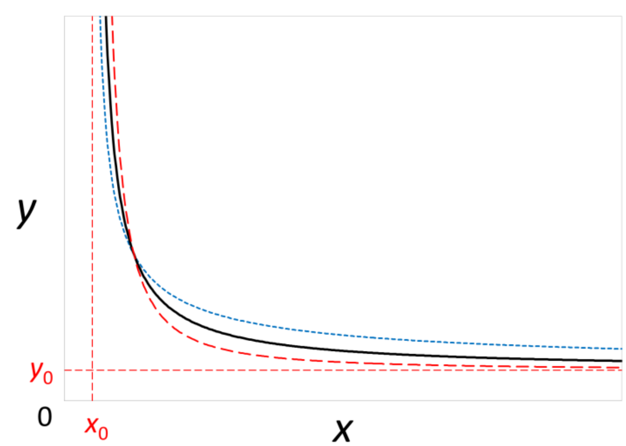

Figure 5. The WHo model with *k* = 0.15 and α = .4 (dotted line), .5 (continuous line), and .6 (dashed line).

4. DATA FROM THE THREE PARADIGMS

In this section we consider a selection of speed/accuracy trade-offs of the literature and replot all of them with μ_T (s) shown on the x axis and σ_A/μ_A (or σ_A/d, if μ_A is unknown) shown on the y axis. We assume a negligible difference between σ_A/d and σ_A/μ_A on the ground that pointing experiment produce generally little or no aiming bias.

We will consider the famous tapping data of Fitts [5], the two relevant data sets of Schmidt et al. [18], and the more recent data reported by Guiard et al. [8]. These data sets are commonly supposed to be governed by different laws because they emanate from three different paradigms.

4.1 Three Possible Experimental Paradigms

The speed/accuracy trade-off of aimed movement has been investigated using three experimental paradigms — three different ways to experimentally handle the same crucial statistics μ_T, μ_A, and σ_A (see Figure 1). Note that in all three paradigms a certain target level of amplitude is prescribed to the participants, who have to produce samples of movement such that $\mu_A \approx d$ or $\mu_E \approx 0$ (i.e., they must aim at target centres).

In the *time-minimisation* paradigm of Fitts [5], the participants are to minimise the duration of their movements at various pre-specified levels of endpoint spread. The Fitts paradigm thus treats mean movement time as the dependent measure and endpoint spread as a constraint. Target width is manipulated with the hope that σ_A will remain about proportional to it so that the frequency of target misses will remain approximately fixed — which, however, involves some wishful thinking, as has been known since the nineteen-fifties [2].

The *spread-minimisation* paradigm of Schmidt [18] goes the other way round, asking the participants to minimise their endpoint spread at various pre-specified levels of movement time. The Schmidt paradigm thus treats the spread as the dependent measure and movement time as a constraint, experimenters hoping that μ_T will approximately equal the recommended value of t across all the range of nominal target times — wishful thinking again, as an inspection of the published data clearly reveals.[2]

The third possibility is the *dual-minimisation* paradigm recently explored by Guiard et al. [8], who asked their participants to minimise both μ_T and σ_A with various degrees of imbalance. The participants were encouraged by verbal instructions to 'push' their data points in various down-left directions corresponding to a number of different speed/accuracy compromises. Such instructions amount to asking them to produce data points located as close as possible to various regions of the limiting curve which constitutes their trade-off function. In this paradigm any pretence to have direct experimental control over either the speed or the accuracy of participants' movements is renounced. Notice that here neither the x nor the y can be considered an 'independent' variable: involved in the trade-off are two participant-dependent random variables.

4.2 Data from the Dual-Minimisation Paradigm

For convenience we start with the data of Guiard et al. [8], which will allow us to illustrate in finer detail the new methodology we have developed in compliance with Axioms 1-6.

The authors asked their 16 participants to perform a fixed-amplitude movement (d = 15cm), trying to minimise μ_T and σ_A concurrently, though in variable proportions. Five sets of instructions served to encourage the participants to cover their full spectrum of resource-allocation strategy. The instructions ranged from a recommendation to perform at maximum speed to a recommendation to perform with maximal accuracy (zero pixel error).

Of course we fitted the WHo model separately for each participant, but for brevity here we will only consider pooled data. The shortest movement time value of the fastest participant will serve here as an estimate of the theoretical minimum of μ_T — i.e., we will set x_0 = 0.092s. The movement being recorded on a digitising tablet with higher resolution than the screen, we will compute the theoretical minimum of spread as the uncertainty entailed by screen discretisation— i.e., we will set y_0 = 0.0018.

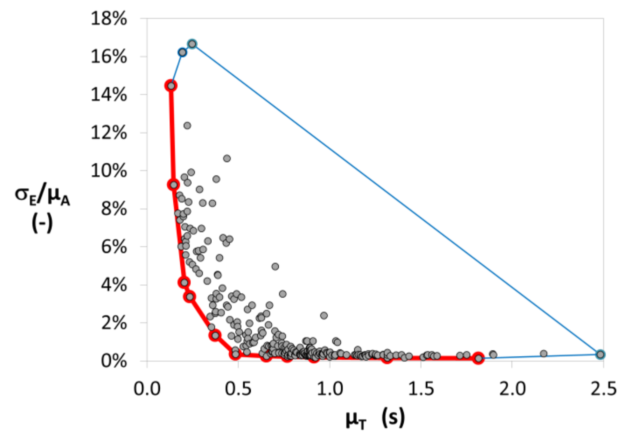

Figure 6. Extracting the convex front of performance from pooled data. The data points of the convex hull are circled. The subset of them that form the critical South-West quadrant are connected with thicker line segments.

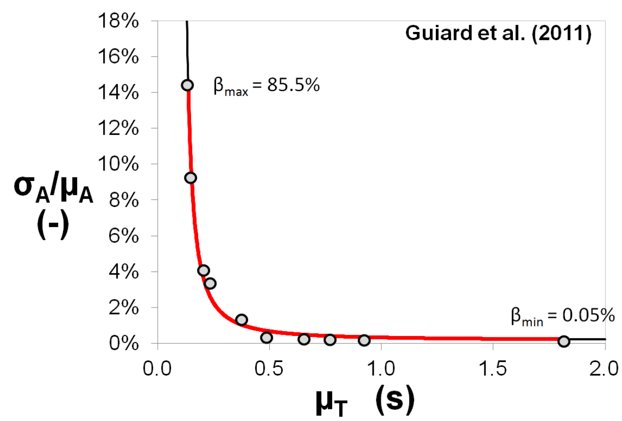

Figure 7. The WHo model fitted to the Guiard et al. data [8], with the CFP arc marked with a thicker red line.—

4.2.1 *Convex Front of Performance (CFP)*

Figure 6 plots 253 blocks of trials from all 16 participants (15-20 movements per block). As expected, the mass of data points tend to cluster against the South-West quadrant of the convex hull of the scatter plot, the CFP being the subset of the convex hull that belong to this critical quadrant (11 points here).

4.2.2 *Fitting the WHo model to the CFP*

One manual fit of the WHo model to the CFP of Figure 6 is shown in Figure 7, where α = .60 and k = 0.070. Note that usual regression procedures are of no help in the present case and we will report no r^2 scores.[3] Figure 7 should suffice to show that the model accommodates the data fairly well.

[2] Guiard, Y. & Rioul, O. (in preparation). The Schmidt law, a false alarm.

[3] Equation 4 can be linearised in log-log coordinates and so linear regression is doable, but in this context the linear r^2 has no sense mathematically. On the other hand, standard non-linear regression is

4.2.3 *Resource, Strategic Style, and Flexibility*

The parameter k is an estimate of the overall level of performance of Guiard et al.'s participants (see Equation 4). But Figure 7 shows not just a curve, it shows a finite curvilinear arc whose location and extent reflect the various strategies that were actually explored by the participants. Relying on Lagrangian minimisation, we developed a simple index to capture this aspect of the data. Letting $\lambda = -dy/dx$, then $\beta = \lambda/(1+\lambda)$ is a quantity that varies from 1 (or 100%) at x_0 to 0 (or 0%) at $x = +\infty$. Given that the CFP of Figure 7 extends from β = 85.5% at x_{min} to β = 0.05% at x_{max}, the *strategic style* for that data set can be characterized as an average of these two values, $\mu_\beta = ½ (\beta_{min} + \beta_{max})$ = 42.8%. But we may also compute a no less useful index of *strategic flexibility* as the difference between these two extrema, $\Delta_\beta = \beta_{max} - \beta_{min}$ = 85.5%. While the index of strategic style μ_β is an indication of the *location* of the arc on the trade-off curve, the index of strategic flexibility Δ_β is an indication of the *extent* of the arc.

4.3 Data from the Fitts Paradigm

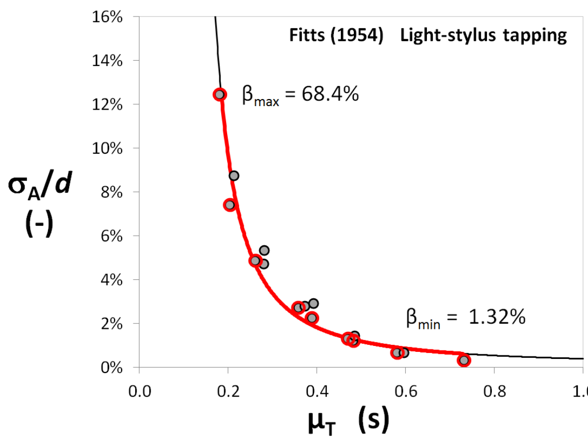

Figure 8. The WHo model fitted to the Fitts data [5]. The data points of the CFP are circled.

The numerical data reported by Fitts [5] consist of 16 tabulated averages corresponding to four d levels times four w levels. All participants being collapsed, obviously we could not use the method of CFP extraction described in Section 4.2.1, in which we considered all the individual trial blocks of the Guiard et al. experiment. We estimated the CFP for the Fitts data by determining which of the 16 data points belong to the convex hull of the graph: nine data points happen to satisfy this criterion. As visible in Figure 8, we obtained an excellent manual fit of the WHo model to Fitts's CFP, assuming x_0 = 0.092s and y_0 = 0.001 and setting α = .62 and k = 0.103.

Notice that the β index reaching less extreme values in the Fitts data (68.4 – 1.32 = 67.1%) than in the Guiard et al. data (85.5 – 0.05 = 85.4%), meaning less strategic flexibility. This is not surprising as Fitts's experiment included neither a max-speed nor a max-accuracy condition. As for the average strategic style of Fitts' participants, they favoured accuracy more than did the participants of Guiard et al. (μ_β = 34.9% to be compared with 42.8%). Again this outcome is easy to understand as Fitts explicitly emphasized accuracy in his task instructions whereas Guiard et al. used instructions aimed at covering the whole spectrum of speed/accuracy strategies.

4.4 Data from the Schmidt Paradigm

In their notorious stylus-pointing experiment on fast discrete movements that gave birth to the Schmidt law, Schmidt et al. [18] used nominal movement times in the 140-200ms range. The data are shown in Figure 9. With only four data points in the CFP to constrain the fit of the WHo model, obviously our finding of α = .45 and k = 0.090 is tentative.

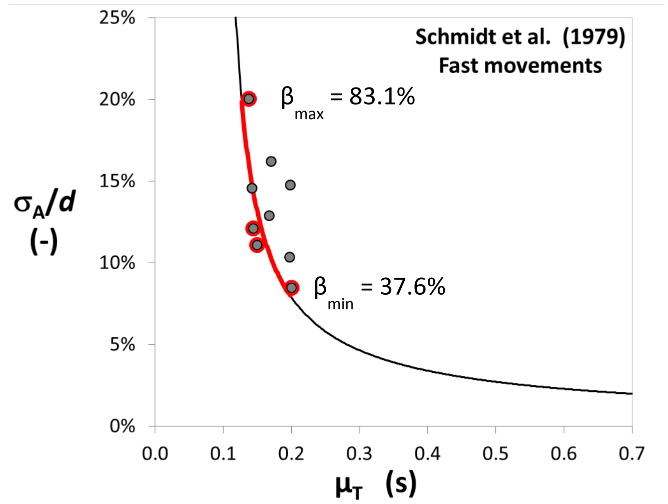

Figure 9. The WHo model fitted to the fast-movement data of Schmidt et al. [18]

The authors also did an experiment, less successful in their view, with nominal times in the 200-500ms range. Figure 10 shows a manual fit of the WHo model for these slower movements, in which we obtained α = .43 and k = 0.073.

Comparing the curvilinear arcs of Figure 9 and 10 one can see a more speedy strategic style in the fast-movement (μ_β = 60.4%) than slower-movement experiment (μ_β = 45.5%), in keeping with the authors' intention. However, the most pronounced difference was in the flexibility of the speed/accuracy strategy, with Δ_β = 45.6% for fast movements and 84.0% for slower movements. In fact, rather surprisingly, it is in the experiment on "slower" movements that the shortest values of μ_T were recorded. These differences make sense if it is realized that apparently the performance benefited from more resource in the slower-movement experiment (k = 0.073, to be compared with k = 0.090).

inapplicable in practice due to the model's vertical asymptote: in the region of low x values, where the slope is very steep, a minute horizontal error may entail a huge residual in the vertical dimension — the only dimension taken into account in ordinary least squares methods — yielding in some cases a strongly negative r^2. We have no space here to present and justify our current solution, which combines non-linear regression with a total least squares minimisation technique (Rioul, Guiard, & Gori, in preparation).

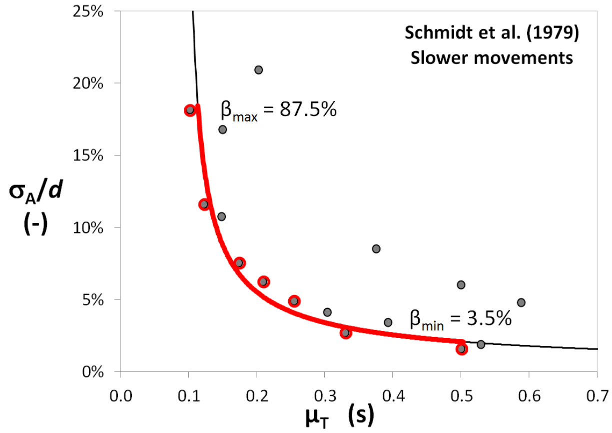

Figure 10. The WHo model fitted to the slower-movement data of Schmidt et al. [18].

Figure 11 offers a superposed view of the fours CFPs, as the WHo model idealises them. Dropping the data points to avoid cluttering, we just retain the arcs. The merit of such arcs is that they not only outline a certain function, they also specify a certain range along the function, thus answering both the question of the resource and the question of the range of resource allocation.

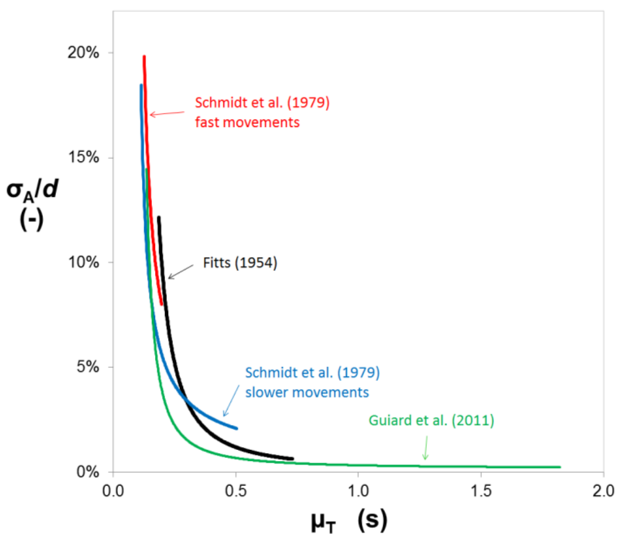

Figure 11. Plotting the four curvilinear CFP arcs together.

Remembering that Fitts-law students have believed since 1979 [18] that the data from the Schmidt paradigm are of a special nature that motivates the conjuring up of another law, our result is good news: from the moment the data are processed in a unified fashion the quantitative patterns are strikingly similar. It is not just that the shapes of the curves are all describable in the minimalist terms of the WHo model. Consider the *spielraum* of x and y in Figure 11: we might summarise things by saying that our speed/accuracy problem is pretty much a matter of 20% of relative inaccuracy that trade with 2 seconds of time. While the four (infinite) curves have similar shapes and fall roughly in the same region of the space of relevance, there is quite some variability in the location and extent of the (finite) curvilinear arcs, and we have seen that this variability sensibly reflects the different palettes of speed/accuracy instructions used in the different experiments.

5. CONCLUSION

5.1 Implications for Basic Research

The above data provide reassuring evidence in favour of the view that human aimed movements are indeed governed by a single speed/accuracy trade-off, and that that trade-off is what it is regardless of the experimental technique with which it is demonstrated. The high quality of the fits is not surprisingly, bearing in mind that the WHo model arises from six obvious axioms. Theorising out of carefully-chosen sets of truisms is an old recipe of good science and it is certainly the least risky of all — truisms, as far as we can see, are true propositions.

If the present work involves some theorizing, its potential contribution has little to do with the substantial theory of the subject. Here our concerns are essentially methodological and empirical, our primary focus being the particular shape of the trade-off under study. The WHo model says nothing whatsoever about the real-time cognitive, physiological, or physical mechanisms that might possibly explain the speed/accuracy trade-off of aimed movement. For example, this work does not address the question of whether qualitatively different phases, one ballistic and the other monitored under the control of vision, take place in the course of an aimed movement viewed as a continuous kinematic event [4][14][17]. We believe that every step forward in the quest for an accurate and robust mathematical description of the empirical regularity psychologists have been concerned with since Woodworth (1899) [24] may be valuable, if only because an improved description of the observables should facilitate the work of substantive theorists.

5.2. Implications for HCI

Suppose that a team of HCI researchers, practitioners of the Fitts law who care about methodology and scrupulously follow the recommendations of the ISO standard [10], design and carry out an experiment to compare a promising novel pointing technique with some traditional baseline. Also suppose that the research takes place in an industrial context where safety is critical. Alas, they find no throughput difference (in our language no difference in the amount of resource users have at their disposal, that is, similar values of k). According to the ISO standard, the researchers' intuitions were false and they unluckily wasted their time. But there is reason to be sceptical.

The odds of two qualitatively different techniques yielding non-different throughputs in a pointing experiment are what they are, but there is no question that the odds of the two techniques yielding not just similar throughputs but also the same range of speed/accuracy strategies, given standardised task instructions, are much lower. Suppose the innovation induces in its users more careful strategies (as revealed by a systematic rightward and downward shift of the β index), in comparison with performance with the baseline technique. If so (recalling that in our hypothetical scenario safety is a critical concern) the novel technique must certainly be judged preferable but the ISO standard, which claims that only the throughput matters, is an invitation to miss that important conclusion. Improved experimental procedures and finer analytic tools should allow HCI researchers to save experimentation time and eventually take better informed decisions.

6. REFERENCES

[1] Chapuis, O., & Dragicevic, P. (2011). Effects of motor scale, visual scale, and quantization on small target acquisition difficulty. *ACM Transactions on Computer-Human Interaction* (*TOCHI*), *18*(3), 13.

[2] Crossman, E. R. F. W. (1957). The speed and accuracy of hand movements. In: Crossman, E. R. F. W., Seymour, W. D. (Eds.), *The Nature and Acquisition of Industrial Skill*. Unpublished report to the MRC and DSIR Joint Committee on Individual Efficiency in Industry.

[3] Crossman, E. R. F. W., & Goodeve, P. J. (1963/1983). Feedback control of hand-movement and Fitts' law. *Quarterly Journal of Experimental Psychology, 35*(2), 251-278.

[4] Elliott, D., Helsen, W. F., & Chua, R. (2001). A century later: Woodworth's (1899) two-component model of goal-directed aiming. *Psychological bulletin*,*127*(3), 342.

[5] Fitts, P. M. (1954). The information capacity of the human motor system in controlling the amplitude of movement. *Journal of Experimental Psychology*, 47, 381-391. Reprinted in *J. Exp. Psychol*.: *Gen*., 1992, 121, 262-269.

[6] Guiard, Y. (2009). The problem of consistency in the design of Fitts' law experiments: Consider either target distance and width or movement form and scale. In *Proceedings of CHI 2009* (pp. 1809-1818). ACM.

[7] Guiard, Y., Olafsdottir, H. B. (2011). On the measurement of movement difficulty in the standard approach to Fitts' law. *PLoS one*, *6*(10): e24389.

[8] Guiard, Y., Olafsdottir, H. B., Perrault, S.T. (2011). Fitts' law as an explicit time/error trade-off. In *Proceedings of CHI 2011* (pp. 1609-1628). ACM.

[9] Hoffmann, E. R. (2013). Which version/variation of Fitts' law? A critique of information-theory models. *Journal of Motor Behavior*, *45*(3), 205-215.

[10] ISO (2002). Reference Number: ISO 9241-9:2000(E). Ergonomic requirements for office work with visual display terminals (VDTs)—Part 9—Requirements for non-keyboard input devices (ISO 9241-9) (Vol. February 15, 2002): International Organisation for Standardisation.

[11] McMahon, T. A., & Bonner, T. (1983). *On Size and Life*. New York: Scientific American Library.

[12] MacKenzie, I. S. (1989). A note on the information-theoretic basis for Fitts' law. *Journal of Motor Behavior*, *21*(3), 323-330.

[13] Meehl, P. E. (1997). The problem is epistemology, not statistics: Replace significance tests by confidence intervals and quantify accuracy of risky numerical predictions. In L.L. Harlow, S.A. Mulaik, & J. H. Steiger (Eds.), *What if there were no significance tests?* (pp. 393-425). Psychology Press.

[14] Meyer, D. E., Abrams, R. A., Kornblum, S., Wright, C. E., & Smith, J. E. K. (1988). Optimality in human motor performance: Ideal control of rapid aimed movements. *Psychological Review*, *95*, 340-370.

[15] Norman, D. A., & Bobrow, D. G. (1975). On data-limited and resource-limited processes. *Cognitive psychology, 7*(1), 44-64.

[16] Norman, D. A., & Bobrow, D. G. (1976). On the analysis of performance operating characteristics. *Psychological Review*, *83*, 508-510.

[17] Rioul, O., & Guiard, Y. (2012). Power vs. logarithmic model of Fitts' law: A mathematical analysis. *Mathematics and social sciences*, *199*, 85-96.

[18] Schmidt, R. A., Zelaznik, H. N., Hawkins, B., Frank, J. S., & Quinn, J. T. (1979). Motor-output variability: A theory for the accuracy of rapid motor acts. *Psychological Review*, *86*, 415-451.

[19] Soukoreff, R. W., & MacKenzie, I. S. (2004). Towards a standard for pointing device evaluation: Perspectives on 27 years of Fitts' law research in HCI. *International Journal of Human-Computer Studies*, *61*, 751-789.

[20] Viviani, P., & Terzuolo, C. (1982). Trajectory determines movement dynamics. *Neuroscience*, *7*(2), 431-437.

[21] Warren Jr, W. H., & Shaw, R. E., Eds. (2013). *Persistence and change: Proceedings of the First International Conference on Event Perception*. Psychology Press.

[22] Welford, A. T., Norris, A. H., & Shock, N. W. (1969). Speed and accuracy of movement and their changes with age. *Acta psychological*, *30*, 3-15.

[23] Wickelgren, W. A. (1977). Speed-accuracy tradeoff and information processing dynamics. *Acta psychologica*, *41*(1), 67-85.

[24] Woodworth, R. S. (1899). Accuracy of voluntary movement. *Psychological Review: Monograph Supplements*, *3*(3), i.

[25] Wright, C. E., & Meyer, D. E. (1983). Conditions for a linear speed-accuracy trade-off in aimed movements. *Quarterly Journal of Experimental Psychology*, *35*(2), 279-296.

[26] Zhai, S., Kong, J., & Ren, X. (2004). Speed-accuracy tradeoff in Fitts' law tasks — on the equivalency of actual and nominal pointing precision. *International Journal of Human-Computer Studies*, *61*, 823–856.

APPENDIX 1: MATHEMATICAL NOTATION AND TERMINOLOGY

Symbol	*Variable name*	*Physical dimension*	*Practical unit*
Task Parameters under Experimenter Control			
d	target distance	[L] or [-]	cm or degree
w	target width or tolerance	[L] or [-]	cm or degree
t	nominal movement duration	[T]	s
d / w	relative target distance	[-]	-
w / d	relative target tolerance	[-]	%
$v = d / t$	nominal average speed	$[LT^{-1}]$ or $[T^{-1}]$	cm/s or deg./s
i_d	index of task difficulty	[-]	bit
Movement Measures Subject to Random Variability			
A	movement amplitude	[L] or [-]	cm or degree
T	movement duration	[T]	s
$E = A - d$	endpoint error	[L] or [-]	cm or degree
$V = A / T$	average movement speed	$[LT^{-1}]$ or $[T^{-1}]$	cm/s or deg./s
I_{de}	index of effective difficulty	[-]	bit
Movement Statistics for Blocks of Repeated Movements or Higher Aggregates			
μ_A	mean amplitude	[L] or [-]	cm or degree
μ_T	mean movement duration	[T]	s
$\mu_V = \mu_a / \mu_T$	mean average speed	$[LT^{-1}]$ or $[T^{-1}]$	cm/s or deg./s
$\mu_E = \mu_A - d$	constant error or aiming bias	[L] or [-]	cm or degree
$\sigma_A = \sigma_E$	variable error or endpoint spread	[L] or [-]	cm or degree
$\sigma_A / \mu_A = \sigma_E / \mu_A$	relative endpoint spread	[-]	%
$\mu_A / \sigma_A = \mu_A / \sigma_E$	mean relative amplitude	[-]	-