# SPECTRAL INTERPOLATION CODER FOR IMPULSE NOISE CANCELLATION OVER A BINARY SYMETRIC CHANNEL

*Abraham Gabay*, Pierre Duhamel and Olivier Rioul*
Depts. Traitement du Signal et des Images & COMELEC,
Ecole Nationale Supérieure des Télécommunications,
46 rue Barrault, 75634 Paris Cedex 13, FRANCE
Tel: +33 1 45817502; fax: +33 1 45887935
e-mail: gabay@tsi.enst.fr; duhamel@tsi.enst.fr; rioul@com.enst.fr

**ABSTRACT**

In this paper, a spectral interpolation coder (SIC) and decoder are investigated for simultaneous source coding and impulse noise cancellation. For simplicity of the analysis, we restrict ourselves to the framework of scalar quantization of a memoryless gaussian source to be transmitted over binary symmetric channel (BSC).

Our approach is to make a carefully designed interpolation of the data in the spectral domain prior to quantization and transmission. The SIC decoder then exploits the properties of SIC codes in order to analyse, detect and correct (or reduce) erroneous data.

A nice feature of this procedure is that the decoder deals simultaneously with the quantization noise and impulse channel noise; therefore it is able to reduce distortion introduced not only by the transmission channels errors but also by the quantizer.

A comparaison study is also investigated in this paper: Simulations show that we obtain a 3dB improvement in SNR over the classical TSC scheme for a global rate of 8.2 transmitted bits per source sample and small BSC crossover probability.

## 1 Introduction

It is known that the performance of a scalar quantizer can be degraded if used over a noisy channel. While quantization produces errors of small amplitude in the reconstructed data, channel errors have the effect of producing impulse noise of larger amplitude.

A classical approach to solving this problem is to use *channel coding*: one inserts redundancy in the output of the source encoder to make it easier for the receiver to detect and/or correct the erroneously received data. Thus protecting against errors results in an increase in bandwith.

This classical approach does not take full advantage of the redundancy introduced by the channel coder: if no error occured in the channel this redundancy is wasted whereas it could have been useful to reduce the quantization noise by increasing the quantizer's precision.

*This work has been supported by the french space agency (CNES) and Alcatel Space Industries Toulouse.

In this paper, we follow the approach suggested by Sayood and Borkenhagen [1] which combines both channel and source statistics in the design of the source coder. Our approach inserts a carefully designed redundancy prior to quantization and exploits the knowledge of the channel characteristics and quantization to restore the received data.

The underlying theory in our approach is related to frame expansions [2] and to spectral analysis [3] in the presence of background noise. The SIC design can also be rephrased in terms of Bose-Chaudhuri-Hocquenghem (BCH) codes in the field of real numbers [4]. Compared to a classical tandem source and channel coding (TSC) scheme, in which a binary BCH coder would take place *after* quantization, our approach makes use of BCH coding *prior to* quantization, thus allowing *joint* source and channel decoding.

## 2 Transmission Scheme

The proposed transmission scheme is depicted in fig. 1. Each source word $\underline{i}$ ($k$ gaussian samples) is first encoded using a SIC coder that produces a codeword $\underline{c}$ on $n$ samples, which is then quantized on $b$ bits per sample using a Lloyd-Max quantizer. Natural index assignement is

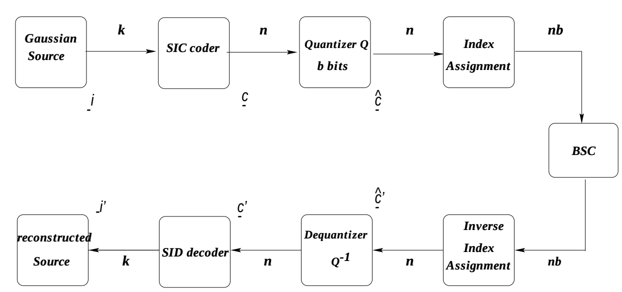


Figure 1: Principle of the SIC transmission scheme

then applied to $\hat{\underline{c}} = Q(\underline{c})$ and the resulting $n \times b$ bits are transmitted over the BSC defined by its crossover (bit error) probability $\epsilon$.

## 3 Spectral Interpolation Encoder

### 3.1 Description

We describe SIC codes in the framework of BCH codes [5], whose definition and properties can be investigated using the Discrete Fourier Transform (DFT). The BCH coder diagram is shown in fig. 2. A block $\underline{I}$ of $k$ spectral components is computed from the original data block $\underline{i} = (i_0, i_1, \ldots, i_{k-1})$ by applying a length-$k$ DFT. This block is then padded with $n-k$ consecutives zeros in such a way that Hermitian symmetry is preserved (see fig. 2), and a length-$n$ inverse DFT is applied, resulting in a real encoded signal $\underline{c} = (c_0, c_1, \ldots, c_{n-1})$. This codeword is normalized so that energy is preserved: $||\underline{c}||^2 = ||\underline{i}||^2$. Compared to a classical systematic BCH coding scheme as described by Blahut [4], this encoding procedure has the nice feature that it roughly preserves the amplitude range of data samples.

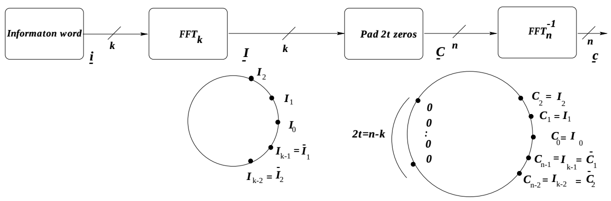


Figure 2: SIC coding from the information word to the expanded SIC encoded signal

### 3.2 Relation to frame expansion methods

The SIC encoding procedure is a special case of the expansion-quantization-reconstruction scenario depicted in [2].

In our case the encoding equation can be written as

$$\underbrace{\underline{c}}_{n\times 1} = \underbrace{\underline{\underline{W_n}}^{-1}}_{n\times n} \underbrace{\underline{\underline{\mathcal{P}}}}_{n\times k} \underbrace{\underline{\underline{W_k}}}_{k\times k} \times \underbrace{\underline{i}}_{k\times 1}$$

where $\underline{\underline{W_l}}$ is the length-$l$ DFT matrix, $\underline{\underline{\mathcal{P}}}$ is the $n-k$ zero-padding matrix.

The codeword components $c_j$, $j = 0, 1, \ldots, n-1$ are then quantized as in [2]. The problem is how to use the redundancy present in the quantized codeword to correct errors introduced by the channel. This is solved in the next section using the language of BCH codes.

## 4 Spectral Interpolation Decoder

### 4.1 Description

The decoding problem can be rephrased as follows. Given a noisy codeword $\underline{\hat{c'}}$ input to the decoder, estimate the transmitted codeword $\underline{c}'$ in order to finally reconstruct the initial source word $\underline{i}'$.

From fig. 1 and the assumption that the quantizer is scalar and the BSC channel is memoryless, we find that each codeword sample is affected (independently of the others) according to

$$\hat{c}'_j = c_j + n_j + e_j \qquad j = 0, 1, \ldots, n-1$$

where $n_j$ is the quantization noise (assumed white) and $e_j$ is the impulse "error" due to the BSC channel. The impulse error probability that $e_j \neq 0$ is $p = 1 - (1-\epsilon)^b$ where $b$ is the number of quantized bits per sample and $\epsilon$ is the BSC bit error probability. Typically when $e_j \neq 0$, $e_j$ takes larger values than $n_j$.

We use the redundancy introduced by SIC coding to simultaneously localize and correct impulse noise samples *and* reduce quantization noise. The SIC decoding algorithm is based upon the fact that $n-k$ consecutives DFT components of the codeword $\underline{c}$ vanish. After quantization and transmission, the corresponding components of $\underline{\hat{c'}}$ will no longer take zero values even when no channel errors have been introduced. These components are first computed in the spectral domain by the SIC decoder. They constitute the so-called *syndrome* [4] and is used as a "signature" of the impulse noise to be removed in the presence of the background noise.

The decoding algorithm is then in three steps: (1) evaluate the number of impulses considered as "errors," (2) find the error locations and (3) find and correct the error values to recover $\underline{c}'$. Blahut [4] describes several efficient algorithms for doing this, but unfortunately these are very sensitive to quantization noise. Therefore, we have followed the approach taken in [6], which is a modified version of the classical Peterson-Gorenstein-Zierler algorithm adapted to the real number case:

(1) The number of "error impulses" is first determined as the rank of a suitable "syndrome matrix," taking the statistical contribution of the quantization noise into account.

(2) Then, we solve a Yule-Walker system to compute the error-locator polynomial, whose roots give the location of the impulses.

(3) Finally, from the estimated locations we solve an overdetermined Van der Monde system in the least squares sense to estimate the impulse amplitudes.

At each step of the algorithm, we are able to detect probable malfunction of the decoder. If malfunction is detected, and if we insist on correcting errors there will be a significant increase in distortion due to the decoder because additionnal errors will be introduced. Therefore, in this case, the algorithm stops and the noisy input data $\underline{\hat{c'}}$ is directly output as $\underline{c'}$.

As a final step, the corrected word is "projected back to the code": the $n-k$ spectral components are removed in order to recover the source word $\underline{i}'$ by inverting the encoding process of section 3. Notice that when no "impulse" is detected at the reception, this last step will always reduce the quantization distortion. This, as we have already noticed, cannot occur in a classical "tandem" scheme.

### 4.2 Relation to Spectral Analysis

Our SIC decoding algorithm is closely related with the multiple frequency estimation problem in mixed-spectrum time series [3], with the difference that time becomes frequency and frequency becomes time.

In this spectral analysis context, the syndrome is seen as a sum of complex sinusoids in additive noise, where each sinusoid correspond to one impulse error introduced by the channel. It satisfies a $\nu$th order autoregressive (AR) equation, where $\nu$ is the number of sinusoids (errors). Our error localization routine can be rephrased as a Prony algorithm for estimating the sinusoids' frequencies [3].

A noticable difference with what usually happens in spectral analysis is that we are given only $n-k$ observations of the noisy data (the syndrome components), which limits the performance of our location estimator.

## 5 Product SIC codes on the reals

The concept of real SIC product codes [7] is a simple and relatively efficient method to construct powerful codes capable of solving the decoder malfunction by iterating the decoding algortihm.

Given a code $\mathcal{C}=(n,k)$, the product code is obtained by:

- placing $(k \times k)$ information samples in a matrix,
- coding the $k$ rows by the code $\mathcal{C}$,
- coding the $n$ columns using the code $\mathcal{C}$.

The resulting product codeword is a $n \times n$ matrix. On receiving the matrix, the first decoder performs the decoding of the columns (and rows)of the matrix, estimates and correct the errors when no malfunction is detected, and gives as output to the next decoder the resulting decoded matrix [6].

In fact, a very simple loop procedure is used to achieve decoding: A first pass is made on the lines of the matrix, a second pass is then performed on the columns. Next iteration the same procedure is repeated. Even when only a few impulses are suitably corrected in the beginning of the algorithm, such correction greatly reduces the task of the following step, which is performed in the other direction in the matrix.

## 6 Simulation results

We compare our method (fig. 1) to the classical TSC scheme depicted in fig. 3.

This tandem scheme consists of quantizing the source on $b$ bits, index assignment and binary BCH coding with the BCH code of parameters $(N=n\times b, K=k\times b)$. The binary coded flow is then transmitted over the BSC.

Both transmission schemes use the same global rate $nb/k = N/K \times b$ transmitted bits per source sample. The comparison is also made with the same quantizer, index assignment, and sample delay $k$.

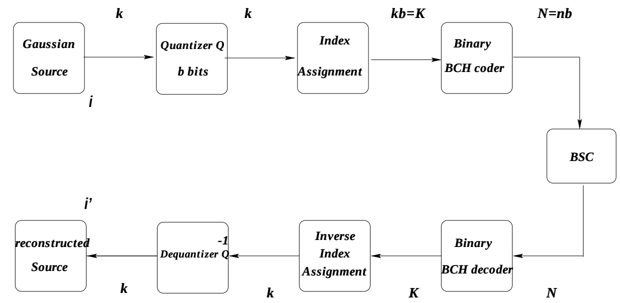


Figure 3: Principle of the TSC scheme

Figure 4 gives the end-to-end SNR in the reconstructed source $\_i'$ relative to the initial source $\_i$, as a function of the BSC bit error probability $\epsilon$ for both SIC and TSC schemes. The numerical values were $b=5$ bits/sample for the scalar Lloyd-Max quantizer, delay $k=31$ samples, $n=51$, and $(N=255, K=155, t=13)$ binary BCH code, giving a global rate of 8.2 bits/sample. Also shown in fig. 4 are the SIC coding

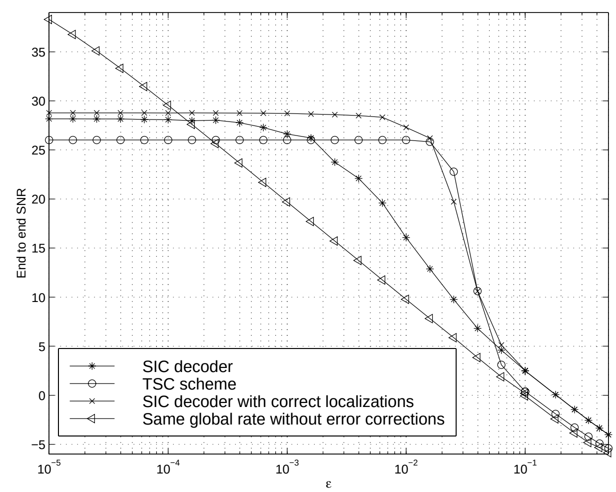


Figure 4: End-to-end SNR comparison of SIC coding and Binary BCH (TSC) coding schemes.

scheme with side information about the correct localization of impulses, and a simple Lloyd-Max quantization scheme on 8 bits/sample without channel coding.

We observe that, for noisy channels ($\epsilon > 10^{-4}$), the SIC scheme (as well as the TSC scheme) always outperform the case of simple quantization without channel coding, for the same global rate, as was to be expected. Moreover, the SIC scheme is more robust to channel noise than the TSC scheme for a large range of $\epsilon$ values, especially when the correct localization of errors are known at the decoder.

For example, we obtain a 3dB improvement in SNR over the classical TSC scheme for very small BSC crossover probability.

Further improvement of the SIC decoding algorithm is achieved, as expected, using the product SIC code ($n = 19, k = 15$) and the same quantizer $b = 5$. As shown in fig. 5, this method allows to recover almost all the error locations for a crossover probability around $10^{-2}$

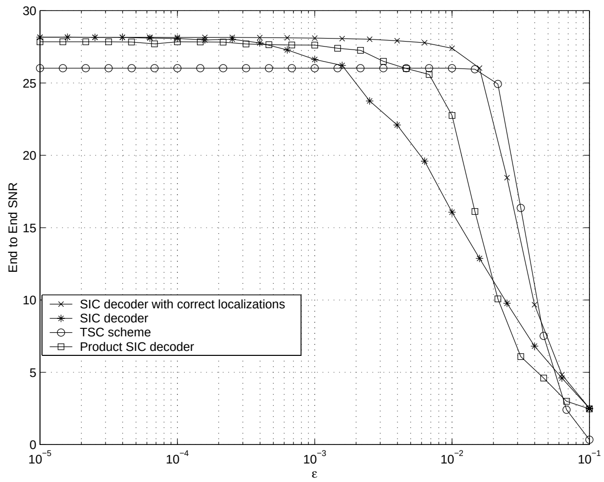


Figure 5: End-to-end SNR comparison of SIC coding and product SIC coding schemes.

## 7 Conclusion

The presented method of SIC coding prior to quantization and the decoding algorithm is able to reduce distortion introduced not only by the transmission channels errors but also by the quantizer. The decoding algorithm is therefore suitable to increase the end-to-end SNR since it deals simultaneously with the quantization noise and impulse channel noise.

Compared to classical TSC scheme, our SIC coding approach is more robust to channel noise for a large range of crossover propability, thus allowing efficient *joint* source and channel decoding.

## References


[1] K. Sayood J.C. Borkehagen, "Use of Residual Redundancy in the Design of Joint Source/Channel Coders," *IEEE Trans. on Comm.*, Vol. 39, No 6, pp. 838–846, June 1991.

[2] V. K. Goyal, M. Vetterli and N. T. Thao, "Quantized Overcomplete Expansions in $R^N$: Analysis, Synthesis, and Algorithms," *IEEE Trans. Inf. Theory*, Vol. 44, No 1, pp. 16–31, January 1998.

[3] S-M. Kay, and A.K Shaw, " Frequency estimation by principal component AR spectral estimation method without eigendecomposition," *IEEE Trans. Acoust., Speech, Signal Process.*, Vol. 36, No 1, pp. 95–101, 1988.

[4] R.E. Blahut, *Algebric methods for signal processing and communications coding*, Signal Processing and Digital Filtering, C.S. Burrus ed., Springer Verlag: New York, 1992.

[5] J. K. Wolf, "Redundancy, the discrete Fourier transform, and impulse noise cancellation," *IEEE Trans. on Comm.*, Vol. COM-31, No. 3, pp. 458-461, March 1983.

[6] O. Rioul, "A spectral algorithm for removing salt and pepper from images," *Proc. 1996 IEEE Digital Signal Processing Workshop*, Loen, Norway, September 1-4 1996.

[7] F. J. Macwilliams and N.J.A Sloane, *The theory of error correcting codes*, North-Holland publishing compagny,1978 p. 567–580.