On Achievability for Downlink Cloud Radio Access Networks with Base Station Cooperation

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***Abstract*—This work investigates the downlink of cloud radio access networks (C-RANs), assuming digital cooperation links among the base stations (BSs). A generalization of the data-sharing scheme is proposed for the case of two BSs and two mobile users. The generalized data-sharing scheme includes a common part and allows full exploitation of correlation among auxiliary codewords. The cooperation links between the BSs are used to exchange and to redirect indices precomputed at the central processor. On the other hand, by simplifying the achievable rate region of the distributed decode–forward (DDF) scheme, it is shown that the DDF scheme for broadcast achieves the capacity region of a downlink N-BS L-user C-RAN with BS cooperation under the memoryless Gaussian model to within a gap of $\frac{L}{2}+\frac{\min\{N, L\log_2 N\}}{2}$ bits per dimension. Numerical evaluations for the memoryless Gaussian model indicate that the generalized data-sharing scheme 1) outperforms the DDF scheme in the low-power regime and when the channel gain matrix is ill-conditioned and 2) benefits more from BS cooperation.**

I. Introduction

Cloud radio access networks (C-RANs) are promising candidates for fifth generation (5G) wireless communication networks. In a C-RAN, the base stations (BSs) are connected to a central processor through digital fronthaul links. Comprehensive surveys on C-RANs can be found in [1], [2]. The two most important coding schemes for downlink C-RANs are

- The *data-sharing* scheme: The central processor splits each message into independent submessages and conveys these independent submessages to one or multiple BSs. The BSs map the received submessages into codewords and transmit these codewords over the interference network. The mobile users decode their intended message parts by treating interference as noise. The data-sharing scheme has been investigated in [3], [4].
- The *compression* scheme: The central processor first precalculates idealized channel inputs and then sends lossy representations of these idealized inputs over the rate-limited fronthaul links to the BSs. The BSs reconstruct the compressed signals and transmit them over the interference network. The compression scheme was investigated by Park, *et al.* [5].

A third scheme, the *reverse compute–forward*, was proposed by Hong and Caire [6], which uses nested lattice codes to perform precalculations in a finite field. The reverse compute–forward scheme can enhance the performance under weak fronthaul links, but it suffers from non-integer penalty in general.

Recently, for the downlink of C-RANs some advanced coding schemes have been developed based on random coding: Liu and Kang [7] generalized the data-sharing scheme to a new scheme, which we will refer to as Liu–Kang scheme. In the Liu–Kang scheme, the central processor maps the message pair (M_1, M_2) into "2-dimensional" Marton codewords: codewords U_1^n, U_2^n for message M_1 and V_1^n, V_2^n for message M_2. The central processor then describes codewords U_1^n, V_1^n to BS 1 and codewords U_2^n, V_2^n to BS 2, where the descriptions are obtained by enumerating all possible pairs of codewords (U_1^n, V_1^n) and (U_2^n, V_2^n). However, the performance analysis in [7] is flawed due to an erroneous application of the mutual covering lemma. This leads to a rate region that is not achievable using the described coding scheme, because of some missing rate constraints.

On the other hand, it was observed in [8] that for the 2-BS 2-user case, distributed decode–forward (DDF) [9] subsumes the compression scheme [5]. The distributed decode–forward scheme precodes every codeword involved in the communication already at the source (the central processor, in our setup). The codewords carry message information in an implicit manner.

In this paper we consider the downlink of C-RANs with BS cooperation and focus on the scenario with two BSs and two mobile users (see Figure 1). The difference from the conventional setup is that now the BSs can also communicate with each other over dedicated digital links. Namely, the BSs act as *conferencing* relays. The main contributions and results of this work are:

1) We modify the Liu–Kang scheme [7] and introduce common codewords to the new *generalized data-sharing (G-DS)* scheme. We use the cooperation links to exchange part of common codewords and to redirect private codewords for asymmetric link or channel conditions.
2) We simplify the achievable rate region of the DDF scheme for downlink C-RANs with BS cooperation. Under the memoryless Gaussian model, we characterize the capacity region of a downlink N-BS L-user C-RAN with BS cooperation to within a gap of $\frac{L}{2}+\frac{\min\{N, L\log_2 N\}}{2}$ bits per dimension.
3) Numerical evaluations for the memoryless Gaussian model show that the G-DS scheme outperforms the DDF scheme in the low-power regime and when the channel gain matrix is ill-conditioned. Furthermore, compared to the DDF scheme, the G-DS scheme benefits more from BS cooperation.

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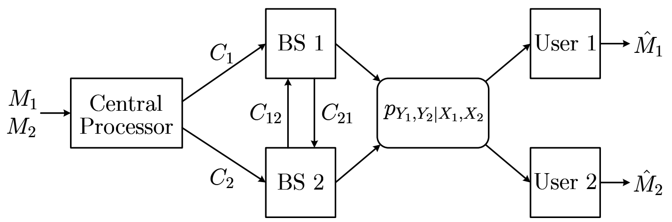


Fig. 1. Downlink C-RAN with BS cooperation: 2 BSs and 2 mobile users.

The rest of the paper is organized as follows. In Section II, we provide the problem statement. Sections III and IV are devoted to the G-DS scheme and the simplification of the DDF scheme, respectively. Finally, in Section V we compare the G-DS scheme and the DDF scheme through evaluation for the memoryless Gaussian model. Due to space limitation, we refer the readers to the full paper [10] for the missing proofs.

A. Notations

Random variables and their realizations are represented by uppercase letters (e.g., X) and lowercase letters (e.g., x), respectively. We use calligraphic symbols (e.g., $\mathcal{X}$) and the Greek letter Ω to denote sets. The probability distribution of a random variable X is denoted by p_X. Denote by $|\cdot|$ the cardinality of a set and by $\mathbb{1}\{\cdot\}$ the indicator function of an event. We denote $[a] := \{1, 2, \cdots, \lfloor a \rfloor\}$ for all $a \geq 1$, $X^k := (X_1, X_2, \cdots, X_k)$, and $X(\Omega) = (X_i : i \in \Omega)$.

We follow the ϵ–δ notation in [11] and the robust typicality introduced in [12]. Finally, the total correlation among the random variables $X(\Omega)$ is defined as

$$\Gamma(X(\Omega)) := \sum_{i \in \Omega} H(X_i) - H(X(\Omega)). \tag{1}$$

II. Problem Statement

Consider the downlink 2-BS 2-user C-RAN with BS cooperation depicted in Figure 1. The network consists of one central processor, two BSs, and two mobile users. The central processor communicates with the two BSs through individual noiseless bit pipes of finite capacities. Denote by C_k the capacity of the link from the central processor to BS k. In addition, the two BSs can also communicate with each other through individual noiseless bit pipes of finite capacities. Denote by C_{kj} the capacity of the link from BS j to BS k. The network from the BSs to the mobile users is modeled as a discrete memoryless interference channel (DM-IC) $\langle \mathcal{X}_1 \times \mathcal{X}_2, p_{Y_1,Y_2|X_1,X_2}, \mathcal{Y}_1 \times \mathcal{Y}_2 \rangle$ that consists of four finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2$ and a collection of conditional probability mass functions (pmf) $p_{Y_1,Y_2|X_1,X_2}$.

With the help of the two BSs, the central processor wants to communicate two messages M_1 and M_2 to users 1 and 2, respectively. Assume that M_1 and M_2 are independent and uniformly distributed over $[2^{nR_1}]$ and $[2^{nR_2}]$, respectively. In this paper, we restrict attention to information processing on a block-by-block basis. Each block consists of a sequence of n symbols. The entire communication is divided into three successive phases:

1) *central processor to BSs*
The central processor conveys two indices $(W_1, W_2) := f_0(M_1, M_2)$ to BS 1 and BS 2, respectively, where $f_0 : [2^{nR_1}] \times [2^{nR_2}] \to [2^{nC_1}] \times [2^{nC_2}]$ is the encoder of the central processor.
2) *BS to BS conferencing communication*
BS 1 conveys an index $W_{21} := f_1(W_1)$ to BS 2, where $f_1 : [2^{nC_1}] \to [2^{nC_{21}}]$ is the conferencing encoder of BS 1. BS 2 conveys an index $W_{12} := f_2(W_2)$ to BS 1, where $f_2 : [2^{nC_2}] \to [2^{nC_{12}}]$ is the conferencing encoder of BS 2.
3) *BSs to mobile users*
BS 1 transmits a sequence $X_1^n := g_1(W_1, W_{12})$ over the DM-IC, where $g_1 : [2^{nC_1}] \times [2^{nC_{12}}] \to \mathcal{X}_1^n$ is the channel encoder of BS 1. BS 2 transmits a sequence $X_2^n := g_2(W_2, W_{21})$ over the DM-IC, where $g_2 : [2^{nC_2}] \times [2^{nC_{21}}] \to \mathcal{X}_2^n$ is the channel encoder of BS 2.

Upon receiving the sequence $Y_\ell^n \in \mathcal{Y}_\ell^n$, user $\ell \in \{1, 2\}$ finds an estimate $\hat{M}_\ell := d_\ell(Y_\ell^n)$ of message M_ℓ, where $d_\ell : \mathcal{Y}_\ell^n \to [2^{nR_\ell}]$ is the decoder of user ℓ. The collection of the encoders f_0, f_1, f_2, g_1, g_2 and the decoders d_1, d_2 is called a $(2^{nR_1}, 2^{nR_2}, n)$ channel code for the downlink 2-BS 2-user C-RAN model with BS cooperation.

The average error probability is defined as

$$\mathsf{P}_e^{(n)} := \mathbb{P}\left(\bigcup_{\ell=1}^{2} \{\hat{M}_\ell \neq M_\ell\} \right). \tag{2}$$

A rate pair (R_1, R_2) is said to be achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes such that $\lim_{n\to\infty} \mathsf{P}_e^{(n)} = 0$. The capacity region of the downlink C-RAN is the closure of the set of achievable rate pairs.

Finally, we remark that using the discretization procedure [11, Section 3.4.1] and appropriately introducing input costs, our developed results for DM-ICs can be adapted to the Gaussian interference channel with constrained input power. The input–output relation of this channel is

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}, \tag{3}$$

where $X_k \in \mathbb{R}$ is the channel input from BS k, Y_ℓ is the channel output observed at user ℓ, $g_{\ell k} \in \mathbb{R}$ is the channel gain from BS k to user ℓ, and (Z_1, Z_2) are i.i.d. $\mathcal{N}(0, 1)$ and each BS has to satisfy an average power constraint P, i.e., $\frac{1}{n}\sum_{i=1}^{n} x_{ki}^2 \leq P$ for all $k \in \{1, 2\}$.

III. Generalized Data-Sharing Scheme

We first provide a high-level summary of the G-DS scheme. The encoding operation at the central processor is illustrated in Figure 2.

We fix a joint pmf $p_{U_0,V_0,U_1,V_1,U_2,V_2}$ and independently generate the codebooks U_j and V_j from the marginals p_{U_j} and p_{V_j}, respectively, for all $j \in \{0, 1, 2\}$. For $j \in \{0, 1, 2\}$, the codebooks U_j and V_j contain $2^{nR_{\mathsf{u}j}}$ and $2^{nR_{\mathsf{v}j}}$ codewords, respectively. Each message $m_1 \in [2^{nR_1}]$ is associated with a unique bin $\mathcal{B}(m_1)$ of index tuples $(k_0, k_1, k_2) \in [2^{R_{\mathsf{u}0}}] \times [2^{R_{\mathsf{u}1}}] \times [2^{R_{\mathsf{u}2}}]$, which are indices to the codebooks $\mathsf{U}_0, \mathsf{U}_1, \mathsf{U}_2$, respectively. Similarly, each message $m_2 \in [2^{nR_2}]$ is associated with a unique bin $\mathcal{B}(m_2)$ of index tuples $(\ell_0, \ell_1, \ell_2) \in [2^{R_{\mathsf{v}0}}] \times [2^{R_{\mathsf{v}1}}] \times [2^{R_{\mathsf{v}2}}]$, which are indices to the independently generated codebooks $\mathsf{V}_0, \mathsf{V}_1, \mathsf{V}_2$, respectively. Then, given (m_1, m_2), we apply joint typicality encoding to find index tuples $(k_0, k_1, k_2) \in \mathcal{B}(m_1)$ and $(\ell_0, \ell_1, \ell_2) \in \mathcal{B}(m_2)$ such that $(U_0^n(k_0), U_1^n(k_1), U_2^n(k_2), V_0^n(\ell_0), V_1^n(\ell_1), V_2^n(\ell_2))$ are jointly typical.

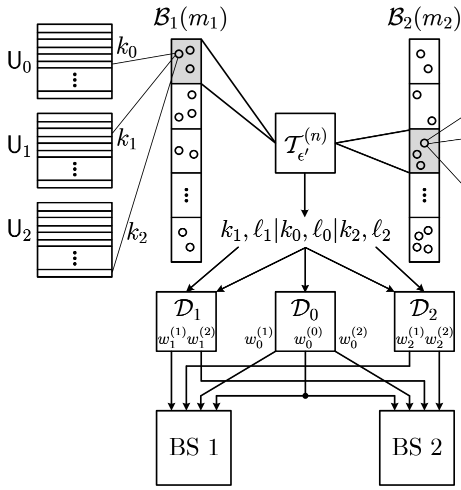


Fig. 2. An illustration of the encoding operation at the central processor in the G-DS scheme.

Remark 1: In addition to including the common auxiliaries U_0 and V_0, as already mentioned in [7], the main difference of our proposed scheme from the Liu–Kang scheme is that we do not enumerate the jointly typical pairs $(U_1^n(k_1), U_2^n(k_2))$ and $(V_1^n(\ell_1), V_2^n(\ell_2))$, which renders the analysis of the success probability of finding jointly typical tuples $(U_1^n(k_1), U_2^n(k_2), V_1^n(\ell_1), V_2^n(\ell_2))$ difficult. $\Diamond$

The next step is to convey $(k_0, \ell_0, k_1, \ell_1)$ to BS 1 and $(k_0, \ell_0, k_2, \ell_2)$ to BS 2. By taking advantage of the following facts, we can reduce the conventional sum rate $R_{\mathsf{u}0} + R_{\mathsf{v}0} + R_{\mathsf{u}j} + R_{\mathsf{v}j}$, $j \in \{1, 2\}$:

1) *Correlated index tuples*
   The index tuple to be sent represents certain jointly typical codewords. As long as U_0, V_0, U_j, V_j are not mutually independent, some members of $[2^{nR_{\mathsf{u}0}}] \times [2^{nR_{\mathsf{v}0}}] \times [2^{nR_{\mathsf{u}j}}] \times [2^{nR_{\mathsf{v}j}}]$ will never be used. Thus, instead of sending $(k_0, \ell_0, k_j, \ell_j)$ separately, we can enumerate all jointly typical codewords and simply convey an enumeration index.
2) *Opportunity of exploiting the cooperation links*
   In the presence of cooperation links, the BSs do not need to learn all the information directly over the link from the central processor, but can learn part of it over the cooperation link.

Finally, user 1 applies joint typicality decoding to recover (k_0, k_1, k_2) and then the message m_1 can be uniquely identified. Similarly, user 2 applies joint typicality decoding to recover (ℓ_0, ℓ_1, ℓ_2) and then the message m_2 can be uniquely identified.

The achieved rate region of the G-DS scheme is presented in the following theorem.

Theorem 1: A rate pair (R_1, R_2) is achievable for the downlink 2-BS 2-user C-RAN with BS cooperation if there exist some rates $R_{\mathsf{u}j}, R_{\mathsf{v}j} \geq 0$, $j \in \{0, 1, 2\}$, some joint pmf $p_{U_0, V_0, U_1, V_1, U_2, V_2}$, and some functions $x_k(u_0, v_0, u_k, v_k)$, $k \in \{1, 2\}$, such that for all $\Omega_{\mathsf{u}}, \Omega_{\mathsf{v}} \subseteq \{0, 1, 2\}$ satisfying $|\Omega_{\mathsf{u}}| + |\Omega_{\mathsf{v}}| \geq 2$,

$$\mathbb{1}\{|\Omega_{\mathsf{u}}| = 3\} R_1 + \mathbb{1}\{|\Omega_{\mathsf{v}}| = 3\} R_2 \tag{4}$$

$$< \sum_{i \in \Omega_{\mathsf{u}}} R_{\mathsf{u}i} + \sum_{j \in \Omega_{\mathsf{v}}} R_{\mathsf{v}j} - \Gamma(U(\Omega_{\mathsf{u}}), V(\Omega_{\mathsf{v}})); \tag{5}$$

for all non-empty $\Omega_{\mathsf{u}}, \Omega_{\mathsf{v}} \subseteq \{0, 1, 2\}$,

$$\sum_{i \in \Omega_{\mathsf{u}}} R_{\mathsf{u}i} < I(U(\Omega_{\mathsf{u}}); U(\Omega_{\mathsf{u}}^c), Y_1) + \Gamma(U(\Omega_{\mathsf{u}})), \tag{6}$$

$$\sum_{j \in \Omega_{\mathsf{v}}} R_{\mathsf{v}j} < I(V(\Omega_{\mathsf{v}}); V(\Omega_{\mathsf{v}}^c), Y_2) + \Gamma(V(\Omega_{\mathsf{v}})); \tag{7}$$

and

$$R_{\mathsf{u}0} + R_{\mathsf{v}0} + R_{\mathsf{u}1} + R_{\mathsf{v}1} < C_1 + C_{12} + \Gamma(U_0, V_0, U_1, V_1), \tag{8}$$

$$R_{\mathsf{u}0} + R_{\mathsf{v}0} + R_{\mathsf{u}2} + R_{\mathsf{v}2} < C_2 + C_{21} + \Gamma(U_0, V_0, U_2, V_2), \tag{9}$$

$$\sum_{i=0}^{2} R_{\mathsf{u}i} + \sum_{j=0}^{2} R_{\mathsf{v}j} < C_1 + C_2 + \Gamma(U_0, V_0, U_1, V_1) + \Gamma(U_0, V_0, U_2, V_2) - \Gamma(U_0, V_0). \tag{10}$$

Unfortunately, the rate region in Theorem 1 is hard to evaluate. Besides, we find it insightful to learn the effects of different code components. Thus, now we present three corollaries to Theorem 1 where we restrict the correlation structure:

1) Corollary 1: $U_j = V_j = \emptyset$ and $R_{\mathsf{u}j} = R_{\mathsf{v}j} = 0$, $j \in \{1, 2\}$,
2) Corollary 2: $p_{U_0, V_0, U_1, V_1, U_2, V_2} = \prod_{j=1}^{2} p_{U_j} p_{V_j}$,
3) Corollary 3: $U_0 = V_0 = \emptyset$ and $R_{\mathsf{u}0} = R_{\mathsf{v}0} = 0$.

In all the corollaries, the auxiliaries $(R_{\mathsf{u}j}, R_{\mathsf{v}j} : j \in \{0, 1, 2\})$ are eliminated through the Fourier–Motzkin elimination.

Corollary 1 (Scheme I): A rate pair (R_1, R_2) is achievable for the downlink 2-BS 2-user C-RAN with BS cooperation if

$$R_1 < I(U_0; Y_1), \tag{11}$$

$$R_2 < I(V_0; Y_2), \tag{12}$$

$$R_1 + R_2 < I(U_0; Y_1) + I(V_0; Y_2) - I(U_0; V_0), \tag{13}$$

$$R_1 + R_2 < \min\{C_1 + C_{12}, C_2 + C_{21}, C_1 + C_2\}, \tag{14}$$

for some joint pmf p_{U_0, V_0} and some functions $x_k(u_0, v_0)$, $k \in \{1, 2\}$.

Corollary 2 (Scheme II): Let $D_1 := C_1 + C_{12}$ and $D_2 := C_2 + C_{21}$. A rate pair (R_1, R_2) is achievable for the downlink 2-BS 2-user C-RAN with BS cooperation if

$$R_1 < D_1 + I(U_2; Y_1 | U_0, U_1), \tag{15}$$

$$R_1 < D_2 + I(U_1; Y_1 | U_0, U_2), \tag{16}$$

$$R_1 < I(U_0, U_1, U_2; Y_1), \tag{17}$$

$$R_2 < D_1 + I(V_2; Y_2 | V_0, V_1), \tag{18}$$

$$R_2 < D_2 + I(V_1; Y_2 | V_0, V_2), \tag{19}$$

$$R_2 < I(V_0, V_1, V_2; Y_2), \tag{20}$$

$$R_1 + R_2 < C_1 + C_2, \tag{21}$$

$$R_1 + R_2 < D_1 + I(U_2; Y_1 | U_0, U_1) + I(V_2; Y_2 | V_0, V_1), \tag{22}$$

$$R_1 + R_2 < D_2 + I(U_1; Y_1 | U_0, U_2) + I(V_1; Y_2 | V_0, V_2), \tag{23}$$

$$R_1 + 2R_2 < D_1 + D_2 + I(V_1, V_2; Y_2 | V_0), \tag{24}$$

$$2R_1 + R_2 < D_1 + D_2 + I(U_1, U_2; Y_1 | U_0), \tag{25}$$

$$2R_1 + 2R_2 < D_1 + D_2 + I(U_1, U_2; Y_1 | U_0) + I(V_1, V_2; Y_2 | V_0), \tag{26}$$

for some joint pmf $\prod_{j=0}^{2} p_{U_j} p_{V_j}$ and some functions $x_k(u_0, v_0, u_k, v_k)$, $k \in \{1, 2\}$.

When applied to the memoryless Gaussian model (3), Corollary 2 with $C_{12} = C_{21} = 0$ recovers the rate region of the scheme of Zakhour and Gesbert [3, Proposition 1].

Corollary 3 (Scheme III): A rate pair (R_1, R_2) is achievable for the downlink 2-BS 2-user C-RAN with BS cooperation if

$$R_1 < C_1 + C_{12} + I(U_2; U_1, Y_1) - I(U_2; U_1, V_1), \quad (27)$$

$$R_1 < C_2 + C_{21} + I(U_1; U_2, Y_1) - I(U_1; U_2, V_2), \quad (28)$$

$$R_1 < I(U_1, U_2; Y_1) + \min\left\{ \begin{array}{l} 0, \\ I(V_1; V_2, Y_2) - I(V_1; U_1, U_2), \\ I(V_2; V_1, Y_2) - I(V_2; U_1, U_2) \end{array} \right\}, \quad (29)$$

$$R_2 < C_1 + C_{12} + I(V_2; V_1, Y_2) - I(V_2; U_1, V_1), \quad (30)$$

$$R_2 < C_2 + C_{21} + I(V_1; V_2, Y_2) - I(V_1; U_2, V_2), \quad (31)$$

$$R_2 < I(V_1, V_2; Y_2) + \min\left\{ \begin{array}{l} 0, \\ I(U_2; U_1, Y_1) - I(U_2; V_1, V_2), \\ I(U_1; U_2, Y_1) - I(U_1; V_1, V_2) \end{array} \right\}, \quad (32)$$

$$R_1 + R_2 < I(U_1, U_2; Y_1) + I(V_1, V_2; Y_2) - I(U_1, U_2; V_1, V_2), \quad (33)$$

$$R_1 + R_2 < C_1 + C_2 - I(U_1, V_1; U_2, V_2), \quad (34)$$

and (A) and (B) (on the bottom of this page) hold for some joint pmf p_{U_1,V_1,U_2,V_2} and some functions $x_k(u_k, v_k)$, $k \in \{1, 2\}$ such that

$$I(U_1; V_1) < I(U_1; U_2, Y_1) + I(V_1; V_2, Y_2), \quad (35)$$

$$I(U_2; V_2) < I(U_2; U_1, Y_1) + I(V_2; V_1, Y_2), \quad (36)$$

$$I(U_1; V_2) < I(U_1; U_2, Y_1) + I(V_2; V_1, Y_2), \quad (37)$$

$$I(U_2; V_1) < I(U_2; U_1, Y_1) + I(V_1; V_2, Y_2). \quad (38)$$

A. Examples

Now let us consider two special cases with simpler topologies.

Example 1 (1 BS and 2 users): The downlink 1-BS 2-user C-RAN can be considered as a special case of the downlink 2-BS 2-user C-RAN with $C_2 = C_{12} = C_{21} = 0$ and $p_{Y_1,Y_2|X_1,X_2} = p_{Y_1,Y_2|X_1}$. We fix a joint pmf $p_{U,V}$ and substitute $(U_1, V_1) = (U, V)$, $U_j = V_j = \emptyset$, and $R_{\mathsf{u}j} = R_{\mathsf{v}j} = 0$, $j \in \{0, 2\}$, in Theorem 1. Then, after removing $R_{\mathsf{u}1}$ and $R_{\mathsf{v}1}$ by the Fourier–Motzkin elimination, we have the following corollary.

Corollary 4: A rate pair (R_1, R_2) is achievable for the downlink 1-BS 2-user C-RAN if there exist some pmf $p_{U,V}$ and some function $x_1(u, v)$ such that

$$R_1 < I(U; Y_1), \quad (39)$$

$$R_2 < I(V; Y_2), \quad (40)$$

$$R_1 + R_2 < I(U; Y_1) + I(V; Y_2) - I(U; V), \quad (41)$$

$$R_1 + R_2 < C_1. \quad (42)$$

Thus, the achieved rate region is essentially Marton's inner bound [13] with the additional constraint (42) due to the fact that the digital link is of finite capacity. $\Diamond$

Example 2 (2 BSs and 1 user): The downlink 2-BS 1-user C-RAN is a class of *diamond networks* [14], [15], which can be considered as a special case of the downlink 2-BS 2-user C-RAN by setting $R_2 = 0$. We fix a joint pmf p_{U,X_1,X_2} and substitute $(U_0, U_1, U_2) = (U, X_1, X_2)$, $V_j = \emptyset$, and $R_{\mathsf{v}j} = 0$, $j \in \{0, 1, 2\}$, in Theorem 1. Then, after removing $R_{\mathsf{u}0}$, $R_{\mathsf{u}1}$, and $R_{\mathsf{u}2}$ by the Fourier–Motzkin elimination, we have the following corollary.

Corollary 5: Any rate R_1 is achievable for the downlink 2-BS 1-user C-RAN with BS cooperation if there exists some pmf p_{U,X_1,X_2} such that

$$R_1 < \min\left\{ \begin{array}{l} C_1 + C_2 - I(X_1; X_2|U), \\ C_1 + C_{12} + I(X_2; Y_1|U, X_1), \\ C_2 + C_{21} + I(X_1; Y_1|U, X_2), \\ I(X_1, X_2; Y_1), \\ \frac{1}{2}[C_1 + C_2 + C_{12} + C_{21} \\ \qquad + I(X_1, X_2; Y_1|U) - I(X_1; X_2|U)] \end{array} \right\}. \quad (43)$$

Remark 2: Considering diamond networks with an orthogonal broadcast channel, the proposed G-DS scheme recovers the achievability results in [14, Theorem 2] and [15, Theorem 1]. Furthermore, the G-DS scheme recovers the achievability result in [16, Theorem 2] for the scenario with relay cooperation. $\Diamond$

IV. Simplification of Distributed Decode–Forward

The DDF scheme for broadcast [9], which is developed for general memoryless broadcast relay networks, in particular applies to downlink C-RAN with arbitrary N BSs and L users.§ The following theorem states its performance in this setup. For convenience, we denote $\tilde{X} = (W_1, \cdots, W_N)$, $\check{X}_j = (X_j, (W_{kj} : k \neq j))$, $j \in [N]$, and $\check{Y}_k = (W_k, (W_{kj} : j \neq k))$, $k \in [N]$.

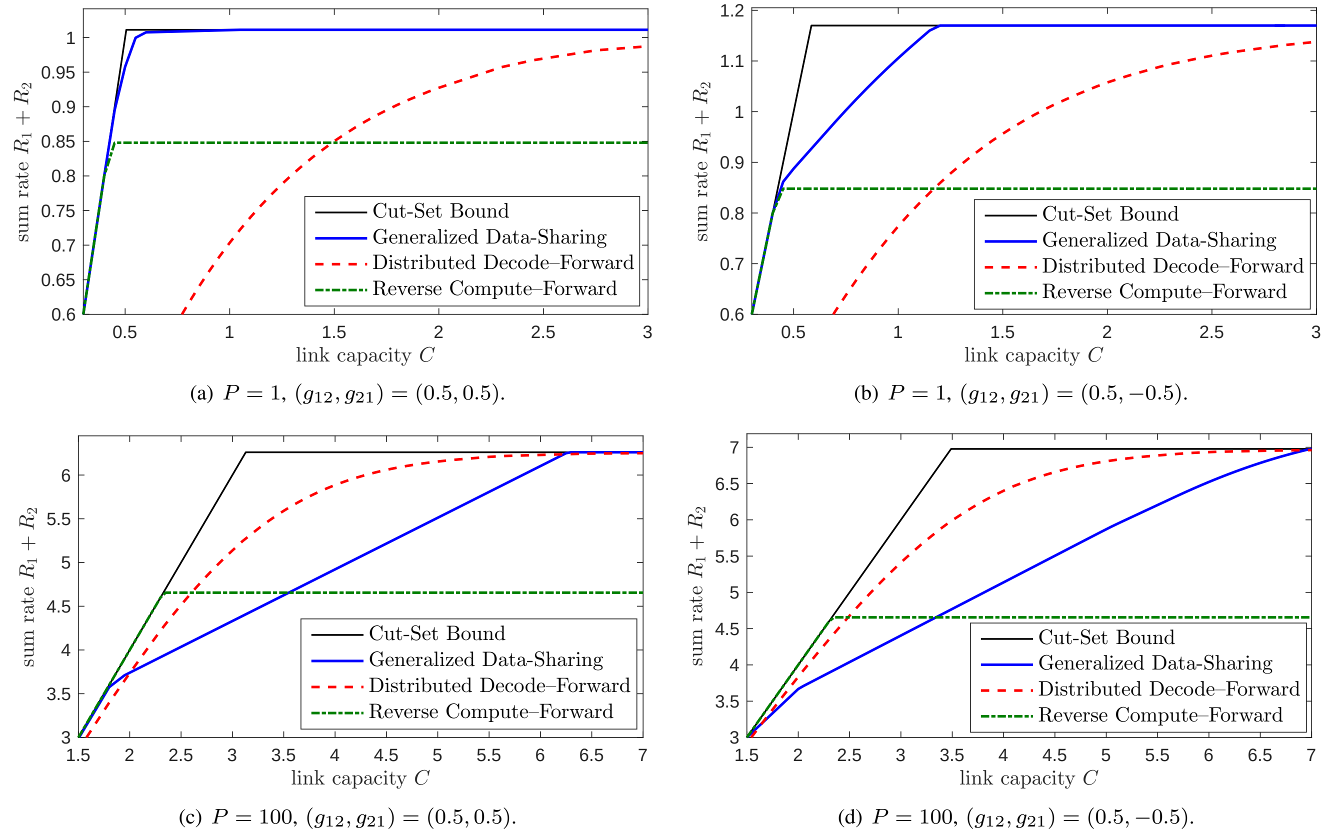
Theorem 2 (Lim, Kim, Kim [9]): A rate tuple $(R_1, \cdots, R_L)$ is achievable for the downlink N-BS L-user C-RAN with BS cooperation if

$$\begin{aligned}\sum_{\ell \in \mathcal{D}} R_\ell < {} & I(\tilde{X}, \check{X}(\mathcal{S}); \tilde{U}(\mathcal{S}^c), U(\mathcal{D}) | \check{X}(\mathcal{S}^c)) \\ & - \sum_{k \in \mathcal{S}^c} \Big[I(\tilde{U}_k; \tilde{U}(\mathcal{S}_k^c), \tilde{X}, \check{X}^N | \check{X}_k, \check{Y}_k) + I(\check{X}_k; \check{X}(\mathcal{S}_k^c)) \Big] \\ & - \sum_{\ell \in \mathcal{D}} I(U_\ell; U(\mathcal{D}_\ell), \tilde{U}(\mathcal{S}^c), \tilde{X}, \check{X}^N | Y_\ell), \end{aligned} \quad (44)$$

§The problem statement in Section II has to be expanded to general number of BSs and users and to allow symbol-wise operations.

$$R_1 + R_2 < C_1 + C_{12} - I(U_1, V_1; U_2, V_2) + \min\left\{ \begin{array}{l} I(U_2; U_1, Y_1) + I(V_2; V_1, Y_2) - I(U_2; V_2), \\ 2I(U_2; U_1, Y_1) + I(V_1, V_2; Y_2) - I(U_2; V_1) - I(U_2; V_2) + I(V_1; V_2), \\ I(U_1, U_2; Y_1) + 2I(V_2; V_1, Y_2) - I(U_1; V_2) - I(U_2; V_2) + I(U_1; U_2) \end{array} \right\}, \quad (A)$$

$$R_1 + R_2 < C_2 + C_{21} - I(U_1, V_1; U_2, V_2) + \min\left\{ \begin{array}{l} I(U_1; U_2, Y_1) + I(V_1; V_2, Y_2) - I(U_1; V_1), \\ 2I(U_1; U_2, Y_1) + I(V_1, V_2; Y_2) - I(U_1; V_1) - I(U_1; V_2) + I(V_1; V_2), \\ I(U_1, U_2; Y_1) + 2I(V_1; V_2, Y_2) - I(U_1; V_1) - I(U_2; V_1) + I(U_1; U_2) \end{array} \right\}, \quad (B)$$



(a) $P = 1$, $(g_{12}, g_{21}) = (0.5, 0.5)$.

(b) $P = 1$, $(g_{12}, g_{21}) = (0.5, -0.5)$.

(c) $P = 100$, $(g_{12}, g_{21}) = (0.5, 0.5)$.

(d) $P = 100$, $(g_{12}, g_{21}) = (0.5, -0.5)$.

Fig. 3. Achieved sum-rates of the G-DS scheme, the DDF scheme for broadcast, and the reverse compute–forward scheme with power control under the symmetric memoryless Gaussian model. Here $T = 0$ and $g_{11} = g_{22} = 1$.

for all $\mathcal{S} \subseteq [N]$, $\mathcal{D} \subseteq [L]$ for some pmf $p_{\tilde{U}^N, U^L, \tilde{X}, \check{X}^N}$, where $\mathcal{S}_k^c = \mathcal{S}^c \cap [k-1]$ and $\mathcal{D}_\ell = \mathcal{D} \cap [\ell - 1]$.

The following proposition shows that it is without loss in optimality to restrict to auxiliaries and input distributions that divide the network into three separate phases: central processor to BSs, BS conferencing, and BSs to mobile users.

Proposition 1: A rate tuple $(R_1, \cdots, R_L)$ lies in the achieved rate region (44) of the DDF scheme for the downlink N-BS L-user C-RAN with BS cooperation if and only if there exists some joint pmf p_{X^N, U^L} such that

$$\sum_{\ell \in \mathcal{D}} R_\ell < \sum_{k \in \mathcal{S}^c} C_k + \sum_{j \in \mathcal{S}} \sum_{k \in \mathcal{S}^c} C_{kj} + \sum_{\ell \in \mathcal{D}} I(U_\ell; Y_\ell) - \Gamma(X(\mathcal{S}^c), U(\mathcal{D})), \tag{45}$$

for all $\mathcal{S} \subseteq [N]$ and $\mathcal{D} \subseteq [L]$ such that $|\mathcal{D}| \geq 1$.

Since downlink C-RAN is a special instance of memoryless broadcast relay networks, the DDF scheme achieves any point in the capacity region of a N-BS L-user C-RAN to within a gap of $\frac{1+N+L}{2}$ bits per dimension under the memoryless Gaussian model [9, Corollary 8]. The following theorem tightens this gap for downlink C-RANs.

Theorem 3: Consider the downlink of any N-BS L-user C-RAN with BS cooperation. Under the memoryless Gaussian model, the DDF scheme for broadcast achieves within $\frac{L}{2} + \frac{\min\{N, L \log_2 N\}}{2}$ bits per dimension from the capacity region.

V. Comparison and Numerical Evaluation

We first present a concrete example showing that the G-DS scheme is optimal but the DDF scheme for broadcast can be strictly suboptimal. Then, we provide numerical results for the memoryless Gaussian model.

A. Example: One BS and One User

Consider the special case with only one BS and one mobile user. Our model reduces to this scenario when the DM-IC is of the form $p_{Y_1, Y_2|X_1, X_2} = p_{Y_1, Y_2|X_1}$ and when $C_2 = R_2 = 0$. Decode-and-forward [17] is optimal in this special case and rate R_1 is achievable whenever

$$R_1 < \min\left\{C_1, \max_{p_{X_1}} I(X_1; Y_1)\right\}. \tag{46}$$

Furthermore, compress-and-forward [17] is optimal since the first hop is noiseless. This performance is also recovered by the G-DS scheme; see Corollary 4 specialized to $R_2 = 0$ and the choice of auxiliaries $V = \emptyset$ and $X_1 = U$.

The DDF scheme for broadcast achieves all rates R_1 that satisfy:

$$R_1 < I(U_1; Y_1), \tag{47}$$

$$R_1 < C_1 + I(U_1; Y_1) - I(U_1; X_1) \tag{48}$$

$$= C_1 - I(U_1; X_1 | Y_1), \tag{49}$$

for some pmf p_{U_1, X_1} s.t. $U_1 \multimap X_1 \multimap Y_1$ form a Markov chain. If the second hop is deterministic, i.e., Y_1 is a deterministic function of X_1, then the DDF scheme with $U_1 = Y_1$ achieves the capacity. However, it can be shown that for general noisy channels $p_{Y_1|X_1}$, $I(U_1; X_1|Y_1) = 0$ only if $I(U_1; Y_1) = 0$ as well. The details can be found in [10]. Thus, the DDF scheme for broadcast is not capacity achieving for this simple topology.

B. Numerical Evaluation for the Memoryless Gaussian Model

Under the memoryless Gaussian model, we compare the G-DS scheme (time sharing among the G-DS schemes I, II, and III) with the DDF scheme for broadcast and the reverse compute–forward scheme with power allocation. We restrict attention to the symmetric case, i.e., $C_1 = C_2 = C$, $C_{12} = C_{21} = T$, $g_{11} = g_{22} = 1$, and $|g_{12}| = |g_{21}|$. The achievable sum rate $R_1 + R_2$ can be upper bounded using the cut-set bound as

$$R_1 + R_2 < \min\{2C, R^\star_{\mathsf{sum}}\}, \tag{50}$$

where $R^\star_{\mathsf{sum}}$ denotes the optimal sum rate assuming $C = \infty$, which can be computed by evaluating the corresponding Gaussian MIMO broadcast channel. We will use the cut-set bound (50) as a reference for comparison. Our choice of auxiliary random variables for the various schemes can be found in the full paper [10]. Except for the G-DS scheme II, all other schemes are evaluated based on dirty paper coding.

In Figure 3, we fix $g_{12} = 0.5$ and consider $(P, g_{21}) \in \{1, 100\} \times \{0.5, -0.5\}$. From the evaluation results, we make the following observations and remarks for the considered setup:

- The G-DS scheme achieves the optimal sum rate when the link capacity C is relatively small or relatively large. The range of optimality depends on the power and the channel conditions. In general, in the low-power regime and/or when the channel gain matrix is ill-conditioned, the G-DS scheme is more advantageous than the other two schemes.
- The DDF scheme achieves a better performance in the high-power regime. As P increases, the DDF scheme outperforms the other two schemes in the middle range of link capacity.
- The reverse compute–forward performs well when the link capacity C is relatively small, especially when P is large. However, it suffers from non-integer penalty and thus its achieved sum rate cannot reach $R^\star_{\mathsf{sum}}$ even if the link capacity C is large.

Finally, we consider BS cooperation[‡], i.e., the case where $T > 0$. Figure 4 plots the achieved sum rates for the case of $(P, g_{12}, g_{21}) = (100, 0.5, -0.5)$. It turns out that for the symmetric case, only the G-DS scheme can benefit from the cooperation links. In particular, as the link capacity T increases to two, the G-DS scheme already outperforms the DDF scheme for all values of C. Recall that the G-DS scheme I achieves the sum rate $\min\{C+T, 2C, R^\star_{\mathsf{sum}}\}$. Since the cut-set bound is $\min\{2C, R^\star_{\mathsf{sum}}\}$, we see that increasing T is beneficial when $R_1 + R_2 < C + T$ is the dominating constraint. By contrast, for the symmetric case the DDF scheme cannot benefit from the cooperation links because the dominating rate constraints do not involve C_{12} and C_{21}:

$$R_1 + R_2 < I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2), \tag{51}$$

$$\begin{aligned} R_1 + R_2 < C_1 + C_2 &- I(U_1; X_0, X_1, X_2|Y_1) \\ &- I(U_2; U_1, X_0, X_1, X_2|Y_2) - I(X_1; X_2|X_0). \end{aligned} \tag{52}$$

[‡]We note that the reverse compute–forward has not been extended to the scenario with BS cooperation. We only include it here as a reference.

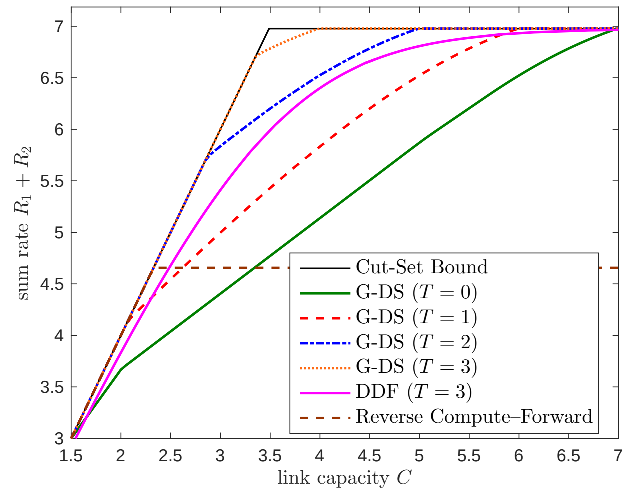


Fig. 4. Achieved sum-rates under the symmetric memoryless Gaussian model. Here $P = 100$ and $\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ -0.5 & 1 \end{bmatrix}$.

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