A Cognitive Network with Clustered Decoding

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***Abstract*—We study the uplink of a linear cellular model featuring short range inter-cell interference. Specifically, we consider a K-transmitter/K-receiver interference network where the signal transmitted by a given transmitter is interfered by the signal sent by the transmitter to its left. We assume that each transmitter has side-information consisting of the messages of the J_ℓ users to its left and the J_r users to its right, and that each receiver can decode its message using the signals received at its own antenna, at the i_ℓ antennas to its left, and at the i_r antennas to its right. For this setting, we characterize the multiplexing gain, i.e., the asymptotic logarithmic growth of the sum-rate capacity at high SNR, and point out interesting duality aspects.**

We also present results on the multiplexing gain of a symmetric version of this network where the signal sent by a given transmitter is interfered by the signals sent by the transmitter to its left and the transmitter to its right.

I. Introduction and Main Result

A. Background

We study a wireless communication scenario where multiple transmitters wish to communicate with multiple receivers, and each message has one intended receiver. The transmitters are assumed to be located on a horizontal line. Opposite each transmitter, on a parallel line, lies the receiver to which the transmitter wishes to send its message. We consider an asymmetric version of Wyner's model [1], where the signal received by a given receiver is a linear combination of the signals sent by its corresponding transmitter and the transmitter to its left, corrupted by Gaussian noise. Moreover, whereas in Wyner's work all receivers are allowed to cooperate, here, we envision a scenario without full cooperation. Hence, our scenario should be modeled by an interference network as in [2], [3], [4], [5].

We further envision that some of the transmitters are located close to each other, and likewise also some of the receivers are located close to each other. To model the vicinity between transmitters we assume *cognition* of messages as in [3], i.e., that each transmitter is cognizant of the messages of nearby located transmitters. To model the vicinity between receivers we allow for *clustered local processing* as in [5], that is, every receiver has access not only to its own receiving antenna but also to nearby located receiving antennas. Clustered local processing is in a way a compromise between the joint (multi-cell) decoding of Wyner in [1] and the single (single-cell) decoding in [2], [3], and it is of practical interest.

We thus study a combination of the models with cognitive transmitters [3] and with clustered local processing [5].

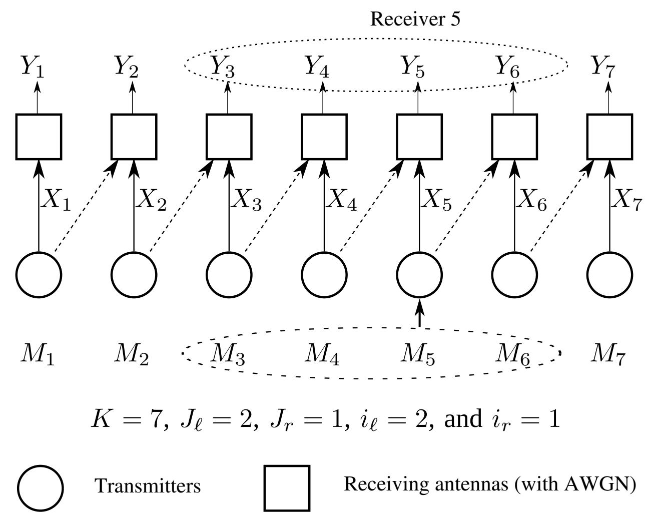

Fig. 1. Problem setting

B. Description of the problem

We consider a situation where K transmitters wish to communicate with K receivers. The transmitters are labeled $\{1, \ldots, K\}$, and similarly, the receivers are labeled $\{1, \ldots, K\}$. Each transmitter and each receiver is equipped with a single antenna. The signal transmitted by a given transmitter is only interfered by its predecessor's (the transmitter to its left) signal; see Figure 1. The time-t symbol $Y_{k,t}$ received by the antenna at Receiver k is given by

$$Y_{k,t} = X_{k,t} + \alpha X_{k-1,t} + N_{k,t}, \qquad 1 \leq k \leq K, \qquad (1)$$

where $X_{k,t}$ denotes the symbol sent by Transmitter k at time t ($X_{0,t} = 0$), α is some non-zero real number, and $\{N_{k,t}\}_{\substack{1 \leq k \leq K \\ 1 \leq t \leq n}}$ are independent and identically distributed (i.i.d.) standard Gaussians. For simplicity, we assume real channel inputs and outputs.

The goal of the communication is that, for each $k \in \{1, \ldots, K\}$, Message M_k is conveyed to Receiver k. The messages $\{M_k\}_{j=1}^{K}$ are assumed to be independent with M_k being uniformly distributed over the set $\mathcal{M}_k \triangleq \{1, \ldots, \lfloor e^{nR_k} \rfloor\}$, where n denotes the block-length of transmission and R_k the rate of transmission of Message M_k.

We assume that each receiver observes the signal received at its own antenna and the signals received at the $i_\ell \geq 0$ antennas to its left and at the $i_r \geq 0$ antennas to its right.

For each $k \in \{1, \ldots, K\}$ we define $\boldsymbol{Y}_k \triangleq (Y_{k,1}, \ldots, Y_{k,n})$. Receiver k, for $k \in \{1, \ldots, K\}$, can guess Message M_k based on the output sequences $\boldsymbol{Y}_{k-i_\ell}, \ldots, \boldsymbol{Y}_{k+i_r}$, where to simplify notation we define $\boldsymbol{Y}_{-i_\ell+1}, \ldots, \boldsymbol{Y}_0$ and $\boldsymbol{Y}_{K+1}, \ldots, \boldsymbol{Y}_{K+i_r}$ to be deterministically 0.

We further assume that, in addition to its own message, each transmitter is also cognizant of the $J_\ell \geq 0$ previous messages and the $J_r \geq 0$ following messages. That means, for each $k \in \{1, \ldots, K\}$, Transmitter k knows messages $M_{k-J_\ell}, \ldots, M_k, \ldots, M_{k+J_r}$, where to simplify notation we define $M_{-J_\ell+1}, \ldots, M_0$ and $M_{K+1}, \ldots, M_{K+J_r}$ to be deterministically zero. Thus, Transmitter k produces its sequence of channel inputs $\boldsymbol{X}_k \triangleq (X_{k,1}, \ldots, X_{k,n})$ as

$$\boldsymbol{X}_k = f_k^{(n)}(M_{k-J_\ell}, \ldots, M_k, \ldots, M_{k+J_r}),$$

for some encoding function

$$f_k^{(n)} \colon \mathcal{M}_{k-J_\ell} \times \cdots \times \mathcal{M}_k \times \cdots \times \mathcal{M}_{k+J_r} \to \mathbb{R}^n. \tag{2}$$

The channel input sequences are subject to symmetric average block-power constraints, i.e., with probability 1 they have to satisfy

$$\frac{1}{n}\|\boldsymbol{X}_k\|^2 \leq P, \quad k \in \{1, \ldots, K\},$$

where $P > 0$ is a constant and $\|\cdot\|$ denotes the Euclidean norm.

We denote by $\mathcal{C}(K, J_\ell, J_r, i_\ell, i_r; P)$ the capacity region of the described network. Thus, $\mathcal{C}(K, J_\ell, J_r, i_\ell, i_r; P)$ denotes the closure of the set of all achievable rate-tuples, where $(R_1, \ldots, R_K)$ is achievable if, as the block-length n tends to infinity, the average probability of error decays to zero. Similarly, we define $\mathcal{C}_\Sigma(K, J_\ell, J_r, i_\ell, i_r; P)$ as the sum-rate capacity, i.e., the supremum of the sum-rate $\sum_{k=1}^K R_k$ over all achievable tuples $(R_1, \ldots, R_K)$. The high-SNR regime of the sum-rate capacity is characterized by the *multiplexing gain*:[1]

$$\mathcal{S}(K, J_\ell, J_r, i_\ell, i_r) \triangleq \varlimsup_{P\to\infty} \frac{\mathcal{C}_\Sigma(K, J_\ell, J_r, i_\ell, i_r; P)}{\frac{1}{2}\log(P)}.$$

Here and throughout $\log(\cdot)$ denotes the natural logarithm.

C. Main result

Definition 1. *Define the integer γ as*

$$\gamma \triangleq \left\lceil \frac{K - J_\ell - i_\ell - 1}{J_\ell + J_r + i_\ell + i_r + 2} \right\rceil. \tag{3}$$

Theorem 1. *The multiplexing gain of the described network is given by*

$$\mathcal{S}(K, J_\ell, J_r, i_\ell, i_r) = K - \gamma. \tag{4}$$

Specializing Theorem 1 to the case where $i_\ell = i_r = J_r = 0$, so that in particular, each receiver has access only to its own receiving antenna, recovers the result in [3].

Remark 1. *Notice that in Expression* (4)*, J_ℓ and i_ℓ (resp. J_r and i_r) play the same role. This shows an equivalence between cognition of messages at the transmitters and clustered local decoding at the receivers.*

As a corollary to Theorem 1 we can derive the *asymptotic multiplexing gain per-user*:

$$\mathcal{S}_\infty(J_\ell, J_r, i_\ell, i_r) \triangleq \varlimsup_{K\to\infty} \frac{\mathcal{S}(K, J_\ell, J_r, i_\ell, i_r)}{K}.$$

Corollary 1. *The asymptotic multiplexing gain per-user of the described network is given by*

$$\mathcal{S}_\infty(J_\ell, J_r, i_\ell, i_r) = \frac{J_\ell + J_r + i_\ell + i_r + 1}{J_\ell + J_r + i_\ell + i_r + 2}. \tag{5}$$

Specializing Corollary 1 to the case where $J_\ell = J_r = 0$, so each transmitter knows only its own message, recovers the result in [5].

Remark 2. *The asymptotic multiplexing gain per-user in* (5) *depends on the parameters J_ℓ, J_r, i_ℓ, and i_r only through their sum. Thus, in the considered setup the asymptotic multiplexing gain per-user only depends on the total amount of side-information at the transmitters and receivers and not on how the side-information is distributed. In particular, cognition of messages at the transmitters and clustered local decoding at the receivers are equally valuable, and—despite the asymmetry of the interference network—also left and right side-information are equally valuable.*

D. The symmetric interference network

In this subsection, we present results for a different network with symmetric interference. For proofs, see [6].

We consider a similar communication scenario as presented in Section I-B but where the channel law (1) is replaced by

$$Y_{k,t} = \alpha X_{k-1,t} + X_{k,t} + \alpha X_{k+1,t} + N_{k,t}, \qquad 1 \leq k \leq K.$$

(Here, $X_{0,t} = X_{K+1,t} = 0$, for all $t \in \{1, \ldots, n\}$.)

For all integers $p \geq 1$, we denote by $H_p(\alpha)$ the $p \times p$ matrix with value 1 on the diagonal, value α above and below the diagonal, and value 0 elsewhere.

Theorem 2. *If the parameters i_ℓ, i_r, J_ℓ, J_r are such that $i_\ell + J_\ell = i_r + J_r$ and $\det\left(H_{i_\ell+J_\ell+1}(\alpha)\right) \neq 0$, then the asymptotic multiplexing gain per-user of the symmetric interference network is given by*

$$\mathcal{S}_\infty^{\text{sym}}(J_\ell, J_r, i_\ell, i_r) = \frac{i_\ell + J_\ell + 1}{i_\ell + J_\ell + 2}.$$

Remark 3.
1) *Like the original asymmetric network, the symmetric interference network exhibits an equivalence between cognition of messages at the transmitters and clustered local decoding at the receivers.*
2) *The hypothesis $\det\left(H_{i_\ell+J_\ell+1}(\alpha)\right) \neq 0$ in Theorem 2 reflects the fact that, unlike in the original asymmetric network, the asymptotic multiplexing gain per-user for the symmetric setup depends on the specific value of $\alpha \neq 0$. This issue is further addressed in [6].*
3) *In the symmetric interference network double the amount of side-information is required to achieve the same multiplexing gain as in the original network.*

[1] The multiplexing gain is also referred to as the "high-SNR slope", "pre-log", or "degrees of freedom"

II. Proof

Before proving Theorem 1, we provide some intuition about our upper and lower bounds. The idea of the lower bound is to silence γ transmitters and thereby split the network into non-interfering subnets which can be treated separately. In each subnet, some of the transmitters use simple single-user encoding schemes and some of the transmitters use dirty-paper coding (DPC) [7] to mitigate the interference at the corresponding receiver. Accordingly, some of the receivers apply successive interference cancellation and some of the receivers apply dirty-paper decoding.

For our upper bound we extend Sato's Multi-Access Channel (MAC) bound [8] to more general interference networks with cognitive transmitters and multi-antenna receivers having side-information (see also [3], [4]). More specifically, in our upper bound, this extended MAC bound is preceded by the following two steps. We first partition the K receivers into groups A and B, and within each group we allow the receivers to cooperate. Then, we let a genie reveal specific linear combinations of the noise sequences to the receivers in Group A. These linear combinations are such that whenever the receivers in Group A have decoded their intended messages correctly, jointly they can reconstruct the outputs observed at the receivers in Group B (Remark 6 ahead). Obviously, these two enhancements can only increase the capacity region, and thus the multiplexing gain. For the resulting interference network (with cognitive transmitters and multi-antenna receivers having side-information) we apply our extended MAC bound. That means, we show that the capacity region of the resulting interference network is included in the capacity region of a MAC (with cognitive transmitters) where the receiver is formed by the union of Group A receivers (thus having multiple antennas and side-information) and is required to decode all messages $M_1, \ldots, M_K$, i.e., also the messages intended to the receivers in Group B. We conclude our upper bound by proving that the multiplexing gain of this cognitive multi-antenna MAC is upper bounded by $K - \gamma$.

A. Lower bound

We derive a lower bound by giving an appropriate coding scheme based on silencing certain transmitters, on Costa's dirty-paper coding, and on successive interference cancellation. We silence γ transmitters and let the remaining $(K-\gamma)$ transmitters send their messages at rates $\frac{1}{2}\log(1+P)$ or $\frac{1}{2}\log(1+\alpha^2 P)$. Such a scheme achieves the claimed multiplexing gain.

Before explaining which transmitters are silenced, we define

$$\beta \triangleq J_\ell + J_r + i_\ell + i_r + 2, \tag{6}$$

and $q \triangleq (K - \beta \lfloor K/\beta \rfloor)$. For each $p \in \{1, \ldots, \lfloor K/\beta \rfloor\}$, we silence Transmitter $p\beta$, and if $q > (i_\ell + J_\ell + 1)$ also Transmitter K.

This splits the network into $\lceil K/\beta \rceil$ non-interfering subnets (sub-networks). The first $\lfloor K/\beta \rfloor$ subnets all have the same topology. They consist of $(J_\ell + J_r + i_\ell + i_r + 1)$ active transmitting antennas and $(J_\ell + J_r + i_\ell + i_r + 2)$ receiving antennas. We refer to these subnets as *generic* subnets. If K is not a multiple of β, there is an additional last subnet with

$$\begin{cases} q \text{ active transmitting antennas,} & \text{if } q \leq (J_\ell + i_\ell + 1), \\ (q-1) \text{ active transmitting antennas,} & \text{if } q > (J_\ell + i_\ell + 1), \end{cases}$$

and with q receiving antennas. We refer to such a subnet as a *reduced* subnet.

As we shall see, in our scheme each transmitter ignores the part of its side-information pertaining to the messages transmitted in other subnets. Likewise, each receiver ignores the outputs of antennas outside its own subnets. Therefore, we can describe our scheme for each subnet separately.

We first describe our scheme for a generic subnet. For simplicity, we assume that the parameters $K, J_\ell, J_r, i_\ell, i_r$ are such that the first subnet is generic and describe the scheme for the first subnet. Moreover, we assume $i_r > 0$. When $i_r = 0$ a similar scheme can be applied, see [6].

In the first subnet, we wish to transmit Messages $M_1, \ldots, M_{J_\ell + J_r + i_\ell + i_r + 1}$. Define the sets

$$\begin{aligned} \mathcal{G}_1 &= \{1, \ldots, i_\ell + 1\}, \\ \mathcal{G}_2 &= \{i_\ell + 2, \ldots, i_\ell + J_\ell + 1\}, \\ \mathcal{G}_3 &= \{i_\ell + J_\ell + 2, \ldots, i_\ell + J_\ell + J_r + 1\}, \\ \mathcal{G}_4 &= \{i_\ell + J_\ell + J_r + 2, \ldots, i_\ell + J_\ell + J_r + i_r + 1\}. \end{aligned}$$

Messages $1, \ldots, (i_\ell + 1)$ are transmitted as follows.

- For each $k \in \mathcal{G}_1$, Transmitter k ignores the interference and encodes its message M_k as for a Gaussian single-user channel using a Gaussian codebook of power P.
- Receiver 1 decodes Message M_1 based on the interference-free outputs $\boldsymbol{Y}_1$.
  If $i_\ell > 0$, Receiver 2 first decodes Message M_1 also based on $\boldsymbol{Y}_1$. Then, it reconstructs $\boldsymbol{X}_1$ (which is a function of M_1 only), and subtracts α times its reconstruction from the output sequence $\boldsymbol{Y}_2$. It finally decodes Message M_2 based on this difference. We refer to such a procedure as *successive interference cancellation*.
  More generally, for each $k \in \mathcal{G}_1$, Receiver k uses successive interference cancellation to first decode Message M_1, followed by Message M_2, etc. up to Message M_k.
- Notice that, for each $k \in \mathcal{G}_1$, if the previous messages $M_1, \ldots, M_{k-1}$ were decoded correctly, then Message M_k can be decoded based on the interference-free signal $\boldsymbol{X}_k + \boldsymbol{N}_k$, where $\boldsymbol{N}_k \triangleq (N_{k,1}, \ldots, N_{k,n})$. Thus, in the proposed scheme, Messages $M_1, \ldots, M_{i_1+1}$ can be communicated with arbitrary small average probability of error at rates $R_1 = \ldots = R_{i_\ell+1} = \frac{1}{2}\log(1+P)$.

If $J_\ell \geq 1$, Messages $(i_\ell+2), \ldots, (i_\ell + J_\ell + 1)$ are transmitted as follows.

- For each $k \in \mathcal{G}_2$, Transmitter k can use its side-information to compute the interference term $\alpha \boldsymbol{X}_{k-1}$. Indeed, in our scheme the input sequence $\boldsymbol{X}_{k-1}$ depends only on messages $M_{i_\ell+1}, \ldots, M_{k-1}$, and these messages are known also to Transmitter k, because $(k-(i_\ell+1)) \leq J_\ell$, for all $k \in \mathcal{G}_2$.

- For each $k \in \mathcal{G}_2$, Transmitter k uses a dirty-paper code of power P and rate $R_k = \frac{1}{2}\log(1+P)$ to transmit its message M_k and mitigate the interference $\alpha \boldsymbol{X}_{k-1}$ experienced at the antenna of Receiver k.
- For each $k \in \mathcal{G}_2$, Receiver k decodes Message M_k based on the output sequence $\boldsymbol{Y}_k$ using dirty-paper decoding.

If $J_r \geq 1$, Messages $(i_\ell + J_\ell + 2), \ldots, (i_\ell + J_\ell + J_r + 1)$ are transmitted as follows.

- For each $k \in \mathcal{G}_3$, Transmitter k can use its side-information to compute the signal sent by its right neighbor $\boldsymbol{X}_{k+1}$. Indeed, in our scheme the transmitted sequence $\boldsymbol{X}_{k+1}$ depends only on messages $M_{k+1}, \ldots, M_{i_\ell+J_\ell+J_r+2}$, and where these messages are known to Transmitter k, because $((i_\ell + J_\ell + J_r + 2) - k) \leq J_r$, for all $k \in \mathcal{G}_3$.
- For each $k \in \mathcal{G}_3$, Transmitter k uses a dirty-paper code of power $\alpha^2 P$ and rate $R_k = \frac{1}{2}\log(1+\alpha^2 P)$ to encode its message M_k and mitigate the "interference" $\boldsymbol{X}_{k+1}$ experienced at the antenna of Receiver $(k+1)$. Denoting the resulting dirty-paper sequence by $\tilde{\boldsymbol{X}}_k$, Transmitter k sends the scaled sequence $\boldsymbol{X}_k = \frac{1}{\alpha}\tilde{\boldsymbol{X}}_k$ over the channel. Since the sequence $\tilde{\boldsymbol{X}}_k$ is average block-power constrained to $(\alpha^2 P)$, the transmitted sequence $\boldsymbol{X}_k$ is average block-power constrained to P.
- Recall that by our assumption $i_\ell \geq 1$, for each $k \in \mathcal{G}_3$, Receiver k has access to the antenna of Receiver $(k+1)$. Therefore, Receiver k can use dirty-paper decoding to decode Message M_k based on the output sequence $\boldsymbol{Y}_{k+1} = \boldsymbol{X}_{k+1} + \alpha \boldsymbol{X}_k + \boldsymbol{N}_{k+1}$, which in our scheme is given by $\boldsymbol{X}_{k+1} + \tilde{\boldsymbol{X}}_k + \boldsymbol{N}_{k+1}$.

Messages $(i_\ell + J_\ell + J_r + 2), \ldots, (i_\ell + J_\ell + J_r + i_r + 1)$ are transmitted as follows.

- For each $k \in \mathcal{G}_4$, Transmitter k encodes its message as for an interference-free Gaussian single-user channel using Gaussian codebooks of power P.
- For each $k \in \mathcal{G}_4$, Receiver k applies successive interference cancellation, but starting with the last output sequence $\boldsymbol{Y}_{i_\ell+J_\ell+J_r+i_r+2}$ and the last Message $M_{i_\ell+J_\ell+J_r+i_r+1}$. More precisely, for each $k \in \mathcal{G}_4$, Receiver k repeatedly applies the successive interference cancellation procedure and decodes messages $M_{i_\ell+J_\ell+J_r+i_r+1}$, $M_{i_\ell+J_\ell+J_r+i_r}$, etc., up to M_k.
- For each $k \in \mathcal{G}_4$, if the previous messages $M_{i_\ell+J_\ell+J_r+i_r+1}, \ldots, M_{k+1}$ were decoded correctly, Message M_k can be decoded based on the interference-free, but attenuated, signal $\alpha \boldsymbol{X}_k + \boldsymbol{N}_{k+1}$. Therefore, Message M_k can be transmitted with arbitrary small average probability of error at a rate $\frac{1}{2}\log(1+\alpha^2 P)$.

This coding scheme achieves a multiplexing gain $(J + i_\ell + i_r + 1)$ over a generic subnet. A similar scheme over a reduced subnet achieves multiplexing gain q, when $q \leq (J_\ell + i_\ell + 1)$, and multiplexing gain $(q-1)$, when $q > (J_\ell + i_\ell + 1)$.Thus, over the entire network our scheme achieves a multiplexing gain of $K - \gamma$, which concludes the lower bound.

B. Upper Bound

If $\gamma = 0$, then the upper bound follows from the Max-Entropy Theorem [9]. Thus, in the following we assume that $K, J_\ell, J_r, i_\ell, i_r$ are such that $\gamma \geq 1$.

We first briefly sketch the derivation of our upper bound, followed by a proof in Subsections II-B1—II-B4. To prove our desired upper bound we introduce a *Cognitive MAC Network*, whose capacity region $\mathcal{C}_{\text{MAC}}(K, J_\ell, J_r, i_\ell, i_r; P)$ satisfies

$$\mathcal{C}(K, J_\ell, J_r, i_\ell, i_r; P) \subseteq \mathcal{C}_{\text{MAC}}(K, J_\ell, J_r, i_\ell, i_r; P), \quad (7)$$

and whose multiplexing gain $\mathcal{S}_{\text{MAC}}(K, J_\ell, J_r, i_\ell, i_r)$ satisfies

$$\mathcal{S}_{\text{MAC}}(K, J_\ell, J_r, i_\ell, i_r) \leq K - \gamma. \quad (8)$$

We describe the Cognitive MAC Network in Subsections II-B1–II-B3. Specifically, we first enhance our original network to a *Rx-Cooperative Network*, then we enhance this latter network to a *Genie-Aided Network*, and finally we modify the latter to the Cognitive MAC Network.

1) Rx-Cooperative Network: The Rx-Cooperative Network is defined as the original network described in Section I with the following enhancement. We partition the set of receivers into Group A and Group B, as described shortly, and let all receivers within a group cooperate. Group A is defined as the set of all Receivers k, for which k lies in $\mathcal{A} \triangleq \bigcup_{m=0}^{\gamma-1} \mathcal{A}_m$, where

$$\mathcal{A}_m \triangleq \begin{cases} \{(\gamma-1)\beta + i_\ell + 2, \ldots, K\} & m = \gamma - 1 \\ \{m\beta + i_\ell + 2, \ldots, (m+1)\beta - i_r\} & m < \gamma - 1, \end{cases}$$

and where recall that β is defined as $(J_\ell + J_r + i_\ell + i_r + 2)$. Group B consists of all other receivers, i.e., it includes each Receiver k, for which $k \in \mathcal{B} \triangleq (\{1, \ldots, K\} \setminus \mathcal{A})$.

Remark 4. *The set $\mathcal{A}_{\gamma-1}$ has at least $(J_\ell + 1)$ and at most $(\beta + J_\ell)$ elements.*

Remark 5. *The union of receivers in Group A observes all output sequences, except for output sequences $\{\boldsymbol{Y}_{1+m\beta}\}_{m=0}^{\gamma-1}$.*

2) Genie-Aided Network: The Genie-Aided Network is defined as the Rx-Cooperative Network with the following enhancement. It is assumed that a genie reveals to the receivers in Group A the sequences $\boldsymbol{V}_0, \ldots, \boldsymbol{V}_{\gamma-1}$, where

$$\boldsymbol{V}_0 \triangleq \boldsymbol{N}_1 + \sum_{\nu=1}^{J_\ell+i_\ell+1} \left(-\frac{1}{\alpha}\right)^{\nu} \boldsymbol{N}_{1+\nu},$$

and, for $m \in \{1, \ldots, \gamma-1\}$:

$$\boldsymbol{V}_m \triangleq \boldsymbol{N}_{1+m\beta} + \sum_{\nu=1}^{J_\ell+i_\ell+1} \left(-\frac{1}{\alpha}\right)^{\nu} \boldsymbol{N}_{1+m\beta+\nu} + \sum_{\nu=1}^{J_r+i_r} (-\alpha)^{\nu} \boldsymbol{N}_{1+m\beta-\nu}.$$

Remark 6. *The genie information $\boldsymbol{V}_0, \ldots, \boldsymbol{V}_{\gamma-1}$ is such that, for given encoding functions $f_1^{(n)}, \ldots, f_K^{(n)}$ as in* (2)*, (i.e., for encoding functions exploiting the cognition at the transmitters) the output sequences observed at the receivers in Group B can* *be reconstructed from the messages $\{M_k\}_{k\in\mathcal{A}}$ intended to the receivers in Group A, the output sequences observed at the receivers in Group A, and the genie information.*

Proof of Remark 6: By Remark 5, it suffices to show that the output sequences $\{\boldsymbol{Y}_{1+m\beta}\}_{m=0}^{\gamma-1}$ can be perfectly reconstructed from the messages $\{M_k\}_{k\in\mathcal{A}}$, all other channel output sequences, and the genie information.

We first notice that by the definition of the set $\mathcal{A}$ and by Remark 4, the set $\{M_k\}_{k\in\mathcal{A}}$ includes messages $\{M_{i_\ell+2+\nu+m\beta}\}_{\substack{0\leq\nu\leq J_\ell+J_r\\ 0\leq m\leq\gamma-1}}$, where out of range indices should be ignored. Thus, based on the messages $\{M_k\}_{k\in\mathcal{A}}$ the input sequences $\{\boldsymbol{X}_{J_\ell+i_\ell+2+m\beta}\}_{m=0}^{\gamma-1}$, can be reconstructed as:

$$\begin{aligned}&\boldsymbol{X}_{i_\ell+J_\ell+2+m\beta}\\&\quad= f^{(n)}_{i_\ell+J_\ell+2+m\beta}(M_{i_\ell+2+m\beta},\ldots,M_{i_\ell+J_\ell+J_r+2+m\beta}).\end{aligned}$$

Using these reconstructed input sequences, the output sequences observed at the receivers in Group A, and the genie-information $\{\boldsymbol{V}_m\}_{m=0}^{\gamma-1}$ it is then possible to reconstruct the channel outputs $\{\boldsymbol{Y}_{1+m\beta}\}_{m=0}^{\gamma-1}$ as follows:

$$\begin{aligned}\boldsymbol{Y}_1 = &-\sum_{\nu=1}^{J_\ell+i_\ell+1}\left(-\frac{1}{\alpha}\right)^{\nu}\boldsymbol{Y}_{1+\nu}\\&+\left(-\frac{1}{\alpha}\right)^{J_\ell+i_\ell+1}\boldsymbol{X}_{J_\ell+i_\ell+2}+\boldsymbol{V}_0\end{aligned}$$

and, for $m\in\{1,\ldots,\gamma-1\}$,

$$\begin{aligned}&\boldsymbol{Y}_{1+m\beta}\\&=-\sum_{\nu=1}^{J_\ell+i_\ell+1}\left(-\frac{1}{\alpha}\right)^{\nu}\boldsymbol{Y}_{1+m\beta+\nu}-\sum_{\nu=1}^{i_r+J_r}(-\alpha)^{\nu}\boldsymbol{Y}_{1+m\beta-\nu}\\&\quad+\left(-\frac{1}{\alpha}\right)^{J_\ell+i_\ell+1}\boldsymbol{X}_{J_\ell+i_\ell+2+m\beta}\\&\quad-(-\alpha)^{J_r+i_r+1}\boldsymbol{X}_{J_\ell+i_\ell+2+(m-1)\beta}+\boldsymbol{V}_m.\end{aligned}$$

■

3) Cognitive MAC Network: The Cognitive MAC Network is obtained from the Genie-Aided Network by eliminating the receivers in Group B and by requiring that the receivers in Group A decode *all* the messages $M_1,\ldots,M_K$. Since the receivers in Group A can cooperate and the transmitters are unchanged compared to the original network, the Cognitive MAC Network is indeed a MAC with cognitive transmitters. In the following, we shall refer to the union of Group A receivers as *the Group A receiver*.

4) Analysis: As previously outlined, the desired upper bound on $\mathcal{S}(K,J_\ell,J_r,i_\ell,i_r)$ follows by Inclusion (7) and Upper Bound (8). We first sketch the proof of (7), followed by the proof of (8) in Lemma 2 ahead. For details, see [6].

Towards proving (7), we notice that cooperation and genie-information at the receivers can only increase the capacity region. Therefore, the capacity regions of the Rx-Cooperative Network, $\mathcal{C}_{\text{Coop}}(K,J_\ell,J_r,i_\ell,i_r;P)$, and of the Genie-Aided Network, $\mathcal{C}_{\text{Genie}}(K,J_\ell,J_r,i_\ell,i_r;P)$, trivially satisfy

$$\begin{aligned}\mathcal{C}(K,J_\ell,J_r,i_\ell,i_r;P)&\subseteq\mathcal{C}_{\text{Coop}}(K,J_\ell,J_r,i_\ell,i_r;P)\\&\subseteq\mathcal{C}_{\text{Genie}}(K,J_\ell,J_r,i_\ell,i_r;P).\end{aligned}$$

Combined with the following Lemma 1, this establishes (7).

Lemma 1. *The capacity region of the Genie-Aided Network is included in the capacity region of the Cognitive MAC Network:*

$$\mathcal{C}_{\text{Genie}}(K,J_\ell,J_r,i_\ell,i_r;P)\subseteq\mathcal{C}_{\text{MAC}}(K,J_\ell,J_r,i_\ell,i_r;P).$$

Proof: Follows by proving that every coding scheme for the Genie-Aided Network can be modified to a coding scheme for the Cognitive MAC Network such that whenever the original scheme is successful (i.e, all messages are decoded correctly), then so is the modified scheme.

The idea of the modified scheme is to use the same encodings as in the original scheme and to let the Group A receiver apply the following three decoding steps: 1.) it first decodes Messages $\{M_k\}_{k\in\mathcal{A}}$ as in the original scheme; 2.) it then attempts to reconstruct the channel output sequences observed by the Group B receivers in the Genie-Aided Network (see Remark 6); and 3.) finally, based on these reconstructions it decodes the remaining messages in the same way as the Group B receivers in the original scheme. ■

Lemma 2. *The multiplexing gain of the Cognitive MAC Network satisfies*

$$\mathcal{S}_{\text{MAC}}(K,J_\ell,J_r,i_\ell,i_r)\leq K-\gamma.$$

Proof: Based on showing that the multiplexing gain is not increased by the genie-information, and thus it is upper bounded by the number of antennas observed at the Group A receiver. ■

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