Coordination in State-Dependent Distributed Networks: The Two-Agent Case

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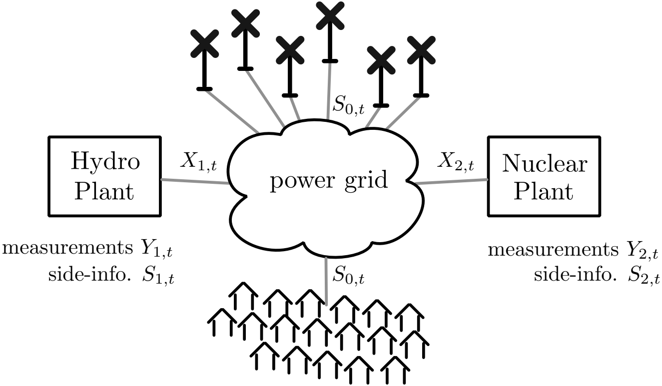
Motivating Example



- Wish to coordinate inputs and state:

$$\frac{1}{T}\sum_{t=1}^{T} P_{S_{0,t}X_{1,t},X_{2,t}} \to \bar{Q} \quad \text{as } T \to \infty$$

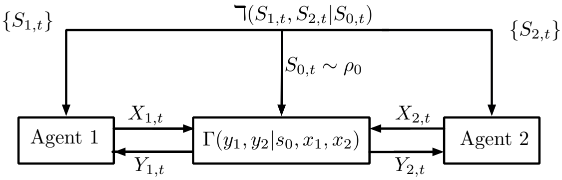
Game-Theoretic Motivation: Infinitively Repeated Games

- Agents' actions $X_{1,t}$ and $X_{2,t}$
- Payoff-functions $\omega_1(s_{0,t}, x_{1,t}, x_{2,t})$ and $\omega_2(s_{0,t}, x_{1,t}, x_{2,t})$
- Average expected payoff:

$$\overline{\omega}_k \triangleq \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[\omega_k(S_{0,t}, X_{1,t}, X_{2,t})\right]$$

$$= \sum_{(s_0,x_1,x_2)} \omega_k(s_0, x_1, x_2) \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} P_{S_{0,t}X_{1,t}X_{2,t}}(s_0, x_1, x_2).$$

Setup and Implementable Distributions



- Causal or non-causal SI:

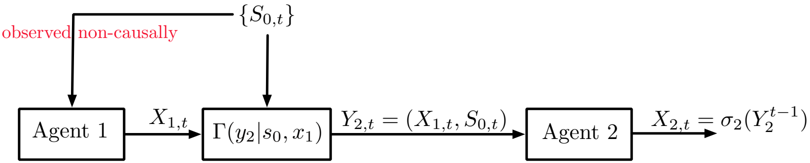
$$X_{k,t} = \sigma^{(\mathrm{c})}_{k,t}(S^t_k, Y^{t-1}_k) \qquad \text{or} \qquad X_{k,t} = \sigma^{(\mathrm{nc})}_{k,t}(S^T_k, Y^{t-1}_k)$$

Implementable distributions $\overline{Q}$

$\forall \epsilon > 0$ there exist T and encodings, s.t.

$$\left| \frac{1}{T} \sum_{t=1}^{T} P_{S_{0,t}X_{1,t}X_{2,t}}(s_0, x_1, x_2) - \overline{Q}(s_0, x_1, x_2) \right| \leq \epsilon.$$

First related model and result: Gossner et al. 2006



- $\{X_{1,t}\}$ communicates $\{S_{0,t}\}$ to Agent 2

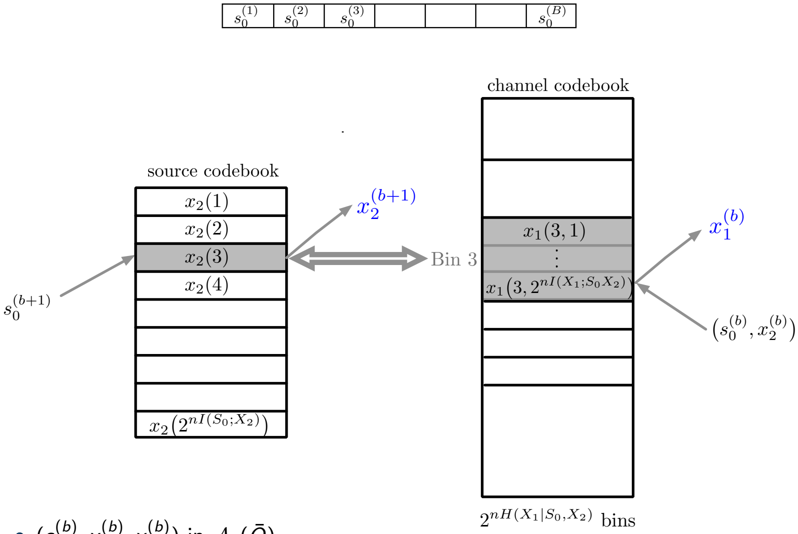
Theorem

$\overline{Q}(s_0, x_1, x_2)$ *implementable iff* $\quad I(S_0; X_2) \leq H(X_1|S_0, X_2)$

[1] O. Gossner, P. Hernandez, and A. Neyman, "Optimal use of communi-cation resources," Econometrica 2006.

[2] P. Cuff and L. Zhao, "Coordination using implicit communication," in Proc. of ITW 2011.

Gossner-Hernandez-Neyman coding scheme



- $(s_0^{(b)}, x_1^{(b)}, x_2^{(b)})$ in $\mathcal{A}_\epsilon(\bar{Q})$

More previous works

- Solution when Agent 1 has S_0^T, Agent 2 observes $Y_{2,t} = (\tilde{Y}_{2,t}, S_{0,t})$
- Achievability when Agent 1 has S_0^T, Agent 2 observes general $Y_{2,t}$
- Empirical coordination: source coding equivalent

[3] B. Larrousse, S. Lasaulce, and M. Bloch, "Coordination in distributed networks via coded actions with application to power control," *submitted to IEEE Trans. on Inform. Theory*, 2014.

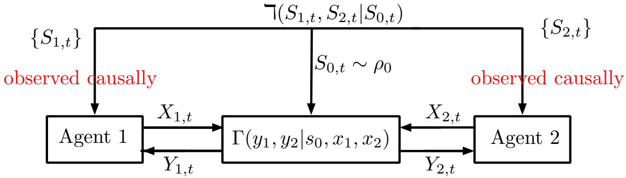
[4] M. Le Treust, "Empirical coordination for joint source-channel coding," *submitted to the IEEE Trans. on Inform. Theory*, 2014.

[5] P. H. Cuff, H. H. Permuter, T. Cover, "Coordination capacity," *IEEE Trans. on Inform. Theory*, 2010.

[6] M. Le Treust, ISIT 2015.

New Results

New Results under Causal SI at both Agents: $X_{k,t} = \sigma^{(c)}_{k,t}(S^t_k, Y^{t-1}_k)$

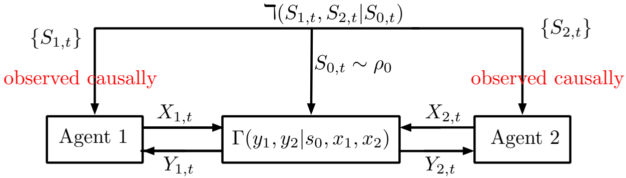


Theorem

$\overline{Q}$ *implementable iff it factorizes as*

$$\overline{Q}(s_0, x_1, x_2) = \sum_{u, s_1, s_2} \left[\rho_0(s_0) \daleth(s_1, s_2|s_0) P_U(u) \prod_{k=1}^{2} P_{X_k|US_k}(x_k|u, s_k) \right]$$

New Results under Causal SI at both Agents: $X_{k,t} = \sigma^{(c)}_{k,t}(S_k^t, Y_k^{t-1})$



Theorem

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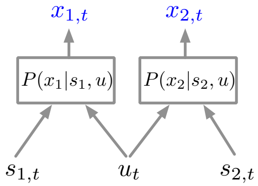
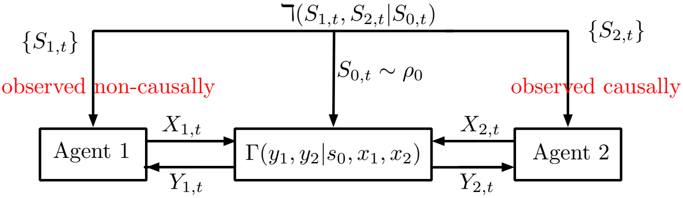
- No coding/communication required:



New Results for Mixed Causal/Non-Causal SI



Theorem

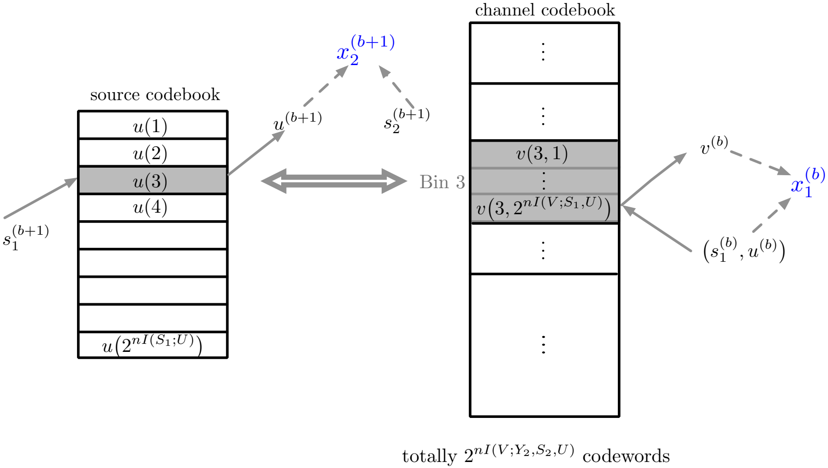
$\overline{Q}(s_0, x_1, x_2)$ *implementable iff it is marginal of some*

$$Q(s_0, s_1, s_2, u, v, x_1, x_2, y_1, y_2)$$
$$= \rho_0(s_0)\daleth(s_1, s_2|s_0)P_{UVX_1|S_1}(u, v, x_1|s_1)P_{X_2|US_2}(x_2|u, s_2)\Gamma(y_1, y_2|s_0, x_1, x_2)$$

satisfying

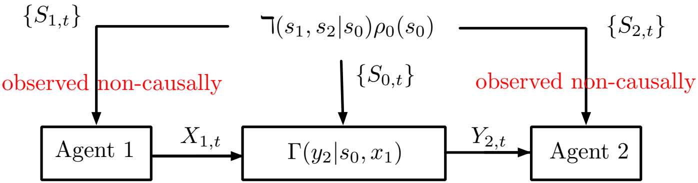
$$I(S_1; U) \leq I(V; Y_2, S_2|U) - I(V; S_1|U)$$

Coding Scheme for Mixed SI



- Successful if $I(S_1;U) \leq I(V;Y_2,S_2,U) - I(V;S_1,U)$

Non-Causal State-Info at both Agents: Point-to-Point Channel



Theorem

$\overline{Q}(s_0, x_1, x_2)$ *implementable iff it is marginal of some*

$$
\begin{aligned}
&Q(s_0, s_1, s_2, u, v, x_1, x_2, y_2)\\
&= \rho_0(s_0)\daleth(s_1, s_2|s_0)P_{UVX_1|S_1}(u, v, x_1|s_1)P_{X_2|US_2}(x_2|u, s_2)\Gamma(y_2|s_0, x_1)
\end{aligned}
$$

satisfying

$$I(S_1; U|S_2) \leq I(V; Y_2, S_2|U) - I(V; S_1|U)$$

- Wyner-Ziv codeword $u^{(b)}$

Converse for Non-Causal SI and Point-to-Point Channel

- Let

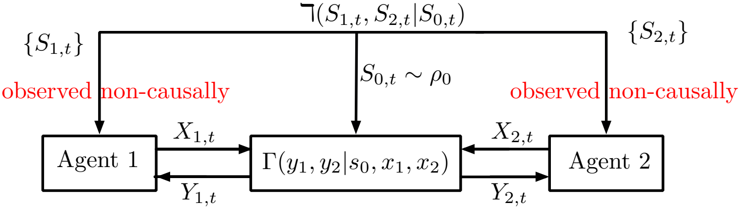
$$U_t \triangleq (Y_2^{t-1}, S_2^{t-1}, S_{2,t+1}^T); \qquad V_t \triangleq S_{1,t+1}^T; \qquad U \triangleq (U_Z, Z)$$

- $$\begin{aligned}\frac{1}{T}\sum_{t=1}^{T} I(S_{1,t}; Y_2^{t-1}|S_{1,t+1}^T, S_2^T) &= \frac{1}{T}\sum_{t=1}^{T} I(S_{1,t}; Y_2^{t-1}, S_{1,t+1}^T S_2^{t-1}, S_{2,t+1}^T|S_{2,t}) \\ &= I(S_{1,Z}; U_Z, V_Z|S_{2,Z}, Z) \\ &= I(S_1; U, V|S_2),\end{aligned}$$

- $$\begin{aligned}\frac{1}{T}\sum_{t=1}^{T} I(S_{1,t+1}^T; Y_{2,t}|Y_2^{t-1}, S_2^T) &= I(V_Z; Y_{2,Z}|U_Z, S_{2,Z}, Z) \\ &= I(V; Y_2|U),\end{aligned}$$

- Csiszar telescoping identity

General Two-Way Channel with Non-Causal SI at both Agents: Open!

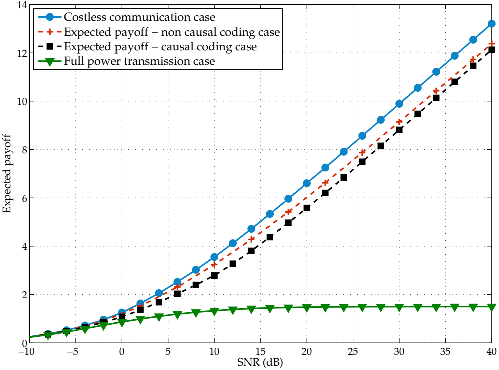


- Preceeding result remains valid if $S_2 = f(S_1)$
  $\rightarrow$ all relevant comm. from Agent 1 to 2

- New scheme required with symmetric operations and two-way source and channel codes

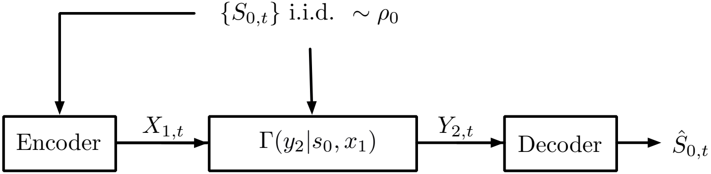
Application: Power Control for the Two-User Fading IC

- State $\{S_{0,t}\}$: channel gains $G_{11}, G_{12}, G_{21}, G_{22} \in \{0.1, 2\}$ w.p. 0.5/0.5 or 0.9/0.1
- Actions $X_{1,t}$ and $X_{2,t}$: transmit powers $\{0, \mathrm{SNR}\}$
- Global payoff function $\frac{1}{2}\log(1 + \mathrm{SINR}_1) + \frac{1}{2}\log(1 + \mathrm{SINR}_1)$
- $\Gamma = \mathrm{BSC}(0.05)$



State-Communication with Strictly-Causal Decoding

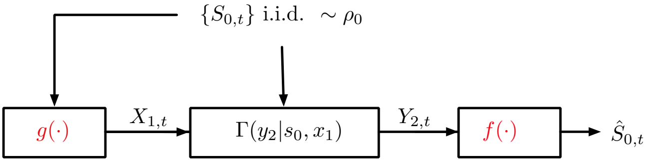
State-Communication: Problem Setup and Connection to Coordination



Distortion D achievable if $\overline{\lim}_{T\to\infty} \left(\frac{1}{T} \sum_{t=1}^{T} \mathsf{E}\left[d(S_{0,t}, \hat{S}_{0,t}) \right] \right) \leq D$

- Achievable distortions D obtained by linear projection of coordination set $\bar{Q}$

State-Communication: State of the Art and Contributions

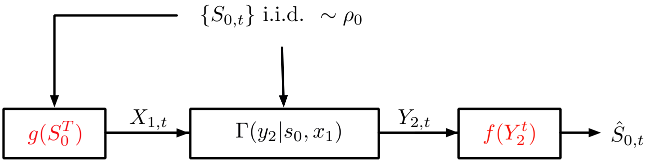


		encoding $g(\cdot)$		
		non-causal	causal	strictly causal
dec. $f(\cdot)$	non-causal	$g(S_0^T)$ & $f(Y_2^T)$ XXX	$g(S_0^t)$ & $f(Y_2^T)$	$g(S_0^{t-1})$ & $f(Y_2^T)$
	causal	$g(S_0^T)$ & $f(Y_2^t)$	$g(S_0^t)$ & $f(Y_2^t)$	$g(S_0^{t-1})$ & $f(Y_2^t)$
	strictly causal	$g(S_0^T)$ & $f(Y_2^{t-1})$	$g(S_0^t)$ & $f(Y_2^{t-1})$	$g(S_0^{t-1})$ & $f(Y_2^{t-1})$

[7] C. Choudhuri, Y.-H. Kim, and U. Mitra, "Causal state communication," *IEEE Trans. on Inform. Theory*, 2013.

[8] A. Sutivong, M. Chiang, T. Cover, and Y. H. Kim, "Channel capacity and state estimation for state-dependent Gaussian channels," *IEEE Trans. on Inform. Theory*, 2005.

State-Communication: Non-Causal Encoding / Causal Decoding



Theorem

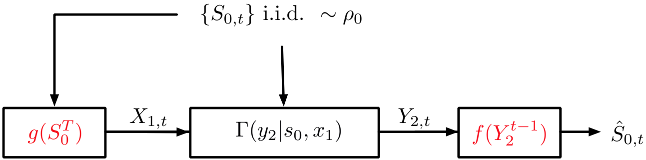
D achievable iff,

$$\mathsf{E}[S_0, \phi(U, Y_2)] \leq D$$

for some $P_{UVX_1|S_0}(u, v, x_1|s_0)$ and $\phi(\cdot, \cdot)$:

$$\underbrace{I(U; S_0)}_{\text{src coding with causal SI}} \leq \underbrace{I(V; Y_2|U) - I(V; S_0|U)}_{\text{Gel'fand}-\text{Pinsker}}$$

State-Communication: Non-Causal Encoding / Causal Decoding



Theorem

D achievable iff,

$$\mathsf{E}\left[S_0, \hat{S}_0\right] \le D$$

for some $P_{U\hat{S}_0X_1|S_0}(u, \hat{s}_0, x_1|s_0)$:

$$\underbrace{I(S_0; \hat{S}_0)}_{\text{standard src coding}} \quad \le \underbrace{I(V; Y_2|X_2) - I(V; S_0|X_2)}_{\text{Gel'fand}-\text{Pinsker}}$$

Impacts of Causality Constraints at Encoder and Decoder

	non-causal	causal	strictly causal
encoding	GP coding		
decoding	WZ coding	coding for causal SI	standard source coding

[9] Y. Steinberg, "Coding and Common Reconstruction," *IEEE Trans. on Inform. Theory*, 2009

[10] R. Timo and B. N. Vellambi, "Two lossy source coding problems with causal side-information," in *Proc. of ISIT 2009*

[11] A. Maor and N. Merhav, "On successive refinement with causal side information at the decoders," *IEEE Trans on Inform. Theory*, 2008.

[12] T. Weissman and A. El Gamal, "Source coding with limited-look ahead side information at the decoder," *IEEE Trans on Inform. Theory*, 2006.

Impacts of Causality Constraints at Encoder and Decoder

	non-causal	causal	strictly causal
encoding	GP coding	Shannon strategies	standard coding
decoding	WZ coding		

[9] Y. Steinberg, "Coding and Common Reconstruction," *IEEE Trans. on Inform. Theory*, 2009

[10] R. Timo and B. N. Vellambi, "Two lossy source coding problems with causal side-information," in *Proc. of ISIT 2009*

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Summary/Outlook

- Coordination of agents over state-dependent networks
- Set of implementable distributions when agents have causal SI
  $\rightarrow$ extends to arbitrary number of agents
- Set of implementable distributions for mixed causal/non-causal SI
- Set of implementable distributions for all non-causal SI with one-way channel
- State-communication with causal and strictly-causal decoding