Duality for the Gaussian MAC and BC with Linear-Feedback Codes

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***Abstract*—We establish a duality relationship between the Gaussian multiple-access (MAC) and broadcast (BC) channels with linear-feedback schemes. Our result extends the scalar no-feedback MAC-BC duality by Jindal *et al.* in [1] to the channels with perfect feedback.**

I. Introduction

We focus on the two-user memoryless Gaussian multiple-access (MAC) and broadcast (BC) channels with perfect feedback. In this context, we say that a BC and a MAC are dual if they have the same channel gains, same total power constraint and same noise variance at all receivers.

Our main contribution in this work is to establish a duality relationship for the Gaussian MAC and BC with linear-feedback codes. Duality refers to the fact that the linear-feedback capacity region of the Gaussian BC with power constraint P can be written as the union of the capacity regions of its dual Gaussian MAC, where the union is taken over all individual power constraints whose sum is equal to P. This duality is a linear-feedback extension of the previously no-feedback MAC-BC duality established by Jindal *et al.* in [1]. Using duality, we determine the linear-feedback sum-capacity of the Gaussian BC under equal channel gains.

In the following, a random variable is denoted by an upper-case letter (e.g X, Y, Z) and its realization by a lower-case letter (e.g x, y, z). Similarly, an n-dimensional random column-vector and its realization are denoted by boldface symbols (e.g. $\mathbf{X}$, $\mathbf{x}$). A^{T} denotes the transpose of a matrix A and $\mathrm{tr}(\mathsf{A})$ its trace. I_n denotes the n-by-n identity matrix; $\|\mathbf{x}\|$ denotes the Euclidean norm of the vector $\mathbf{x}$; $\log(\cdot)$ denotes the binary logarithm; $[x]^+$ denotes the maximum between x and 0 and $\mathrm{cl}(\cdot)$ denotes the convex closure operation.

II. Gaussian MAC with Feedback

A. Channel Model

We consider the two-transmitter memoryless Gaussian MAC with feedback depicted in Figure 1. At each time $t \in \mathbb{N}$, if $x_{1,t}$ denotes the symbol sent by Transmitter 1 and $x_{2,t}$ denotes the symbol sent by Transmitter 2, the receiver observes the channel output

$$Y_t = h_1 x_{1,t} + h_2 x_{2,t} + Z_t, \tag{1}$$

where h_1 and h_2 are constant channel coefficients and $\{Z_t\}$ is a sequence of i.i.d. zero-mean unit-variance Gaussian random variables describing the noise.

The goal of communication is that Transmitters 1 and 2 convey independent messages M_1 and M_2 to a common receiver. The messages M_1 and M_2 are independent of the noise sequence $\{Z_t\}$ and uniformly distributed over the sets $\mathcal{M}_1 \triangleq \{1, \dots, \lfloor 2^{nR_1} \rfloor\}$ and $\mathcal{M}_2 \triangleq \{1, \dots, \lfloor 2^{nR_2} \rfloor\}$, where R_1 and R_2 are called the rates of transmission and n the blocklength. The two transmitters observe perfect feedback from the channel outputs. Thus, Transmitter i's, $i \in \{1, 2\}$, channel input at time t, $X_{i,t}$, can depend on the prior output symbols Y^{t-1} and its message M_i:

$$X_{i,t} = f_{i,t}^{(n)}(M_i, Y^{t-1}), \quad t \in \{1, \dots, n\}, \tag{2}$$

for some encoding functions of the form:

$$f_{i,t}^{(n)} : \mathcal{M}_i \times \mathbb{R}^{t-1} \to \mathbb{R}. \tag{3}$$

The channel input sequences $\{X_{1,t}\}_{t=1}^n$ and $\{X_{2,t}\}_{t=1}^n$ have to satisfy an *expected average total block-power constraint* P:

$$\frac{1}{n} \sum_{t=1}^{n} \left(\mathbf{E}[X_{1,t}^2] + \mathbf{E}[X_{2,t}^2] \right) \leq P, \tag{4}$$

where the expectation is over the messages and the realizations of the channel.

The receiver decodes the messages (M_1, M_2) by means of a decoding function $\Phi^{(n)}$ of the form

$$\Phi^{(n)} \colon \mathbb{R}^n \to \mathcal{M}_1 \times \mathcal{M}_2. \tag{5}$$

This means, based on the output sequence Y^n, the receiver produces its guess $(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) = \Phi^{(n)}(Y^n)$.

The average probability of error is defined as

$$P_{e,MAC}^{(n)} \triangleq \Pr\left[(\hat{M}_1, \hat{M}_2) \neq (M_1, M_2) \right]. \tag{6}$$

A $(\lfloor 2^{nR_1} \rfloor, \lfloor 2^{nR_2} \rfloor, n)$ *MAC feedback-code of blocklength n and total power P* is a triple $\left(\{f_{1,t}^{(n)}\}_{t=1}^n, \{f_{2,t}^{(n)}\}_{t=1}^n, \Phi^{(n)} \right)$ where $\{f_{1,t}^{(n)}\}_{t=1}^n$ and $\{f_{2,t}^{(n)}\}_{t=1}^n$ are of the form (3) and satisfy (4) and $\Phi^{(n)}$ is as in (5).

We say that a rate-pair (R_1, R_2) is achievable over the Gaussian MAC with feedback and total power constraint P, if there exists a sequence of $(\lfloor 2^{nR_1} \rfloor, \lfloor 2^{nR_2} \rfloor, n)$ MAC feedback-codes such that the average probability of a decoding error $P_{e,MAC}^{(n)}$ tends to zero as the blocklength n tends to infinity. The closure of the union of all achievable regions is called *the capacity region*. We denote it by $\mathscr{C}_M^{fb}(h_1, h_2, P)$.

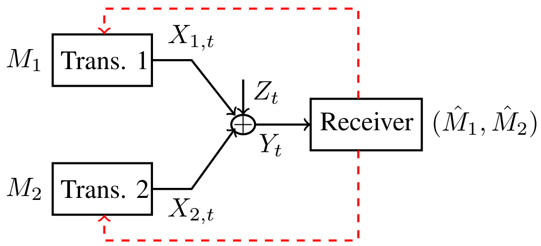

Fig. 1. Two-user Gaussian MAC with feedback.

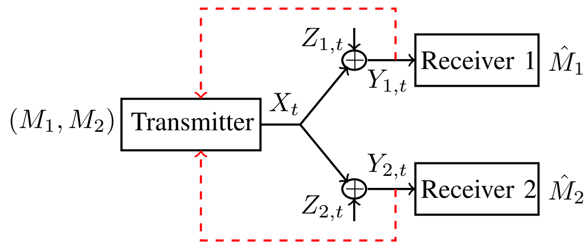

Fig. 2. Two-user Gaussian BC with feedback.

The supremum of the sum $R_1 + R_2$, where (R_1, R_2) are in $\mathscr{C}_M^{fb}(h_1, h_2, P)$ is called the sum-capacity and is denoted $C_{\Sigma,M}^{fb}(h_1, h_2, P)$.

Remark 1. *Assuming that the noise variance in* (1) *is equal to one entails no loss of generality because otherwise the receiver can simply scale its outputs appropriately, which does not change the set of achievable rates.*

B. Capacity Region

Given channel coefficients h_1 and h_2 and non-negative numbers $P_1, P_2 \geq 0$, let $\mathscr{C}_M^{fb,ind}(h_1, h_2, P_1, P_2)$ denote Ozarow's capacity region of the Gaussian MAC with feedback under individual expected power constraints P_1 and P_2 at the two transmitters [2] and let $C_{\Sigma,M}^{fb,ind}(h_1, h_2, P_1, P_2)$ denote its corresponding sum-capacity.

Proposition 1. *The capacity region of the Gaussian MAC under a total power constraint P is*

$$\mathscr{C}_M^{fb}(h_1, h_2, P) = \bigcup_{\substack{P_1 + P_2 = P \\ P_1, P_2 \geq 0}} \mathscr{C}_M^{fb,ind}(h_1, h_2, P_1, P_2). \tag{7}$$

The sum-capacity is

$$C_{\Sigma,M}^{fb}(h_1, h_2, P) = \max_{\substack{P_1 + P_2 = P \\ P_1, P_2 \geq 0}} C_{\Sigma,M}^{fb,ind}(h_1, h_2, P_1, P_2). \tag{8}$$

C. Linear-Feedback Schemes for the MAC

We focus on the class of coding schemes where the feedback is used in a linear way. We say that a blocklength-n scheme is a *linear-feedback scheme*, if for each $i \in \{1, 2\}$, the vector $\mathbf{X}_i \triangleq (X_{i,1}, \ldots, X_{i,n})^\mathsf{T}$, of Transmitter i's channel inputs can be written as:

$$\mathbf{X}_i = \mathbf{U}_i + \mathsf{C}_i \mathbf{Y}, \tag{9}$$

where $\mathbf{Y} \triangleq (Y_1, \ldots, Y_n)^\mathsf{T}$ is the channel output vector, C_1 and C_2 are n-by-n strictly lower-triangular matrices and $\mathbf{U}_i \triangleq (U_{i,1}, \ldots, U_{i,n})^\mathsf{T}$ is an n-dimensional information-carrying vector of the form:

$$\mathbf{U}_i = \varphi_i(M_i), \tag{10}$$

where $\varphi_i : \mathcal{M}_i \to \mathbb{R}^n, \quad i \in \{1, 2\}$, are arbitrary functions. There is no constraint on the decoding function Φ.
The strict-lower-triangularity of the matrices C_1 and C_2 ensures that the feedback is used in a strictly causal way.

The set of all rate-pairs that are achievable using a linear-feedback scheme is called the linear-feedback capacity region. Since the capacity-achieving scheme by Ozarow [2] is a linear-feedback scheme, the feedback-capacity and the linear-feedback capacity are the same.

III. Gaussian BC with Feedback

A. Channel Model

We consider the two-receiver memoryless Gaussian BC with feedback depicted in Figure 2. At each time $t \in \mathbb{N}$, if x_t denotes the channel input symbol sent by the transmitter, Receiver $i \in \{1, 2\}$ observes the channel output

$$Y_{i,t} = h_i x_t + Z_{i,t}, \tag{11}$$

where h_1 and h_2 are constant channel coefficients and $\{Z_{1,t}\}_{t=1}^n$ and $\{Z_{2,t}\}_{t=1}^n$ model the additive noise at Receivers 1 and 2. The noise sequence $\{Z_{1,t}\}_{t=1}^n$ and $\{Z_{2,t}\}_{t=1}^n$ are independent sequences of i.i.d. centered Gaussian random variables of unit-variance[1].

The transmitter wishes to convey a message M_1 to Receiver 1 and an independent message M_2 to Receiver 2. The messages are independent of the noise sequences $\{Z_{1,t}\}_{t=1}^n$ and $\{Z_{2,t}\}_{t=1}^n$ and uniformly distributed over the sets $\mathcal{M}_1 \triangleq \{1, \ldots, \lfloor 2^{nR_1} \rfloor\}$ and $\mathcal{M}_2 \triangleq \{1, \ldots, \lfloor 2^{nR_2} \rfloor\}$, where R_1 and R_2 are called the rates of transmission and n the blocklength.

The transmitter observes causal, noise-free output feedback from both receivers. Thus, the time-t channel input X_t can depend on all previous channel outputs Y_1^{t-1} and Y_2^{t-1} and the messages M_1 and M_2:

$$X_t = g_t^{(n)}(M_1, M_2, Y_1^{t-1}, Y_2^{t-1}), \quad t \in \{1, \ldots, n\}, \tag{12}$$

for some encoding function of the form:

$$g_t^{(n)} : \mathcal{M}_1 \times \mathcal{M}_2 \times \mathbb{R}^{t-1} \times \mathbb{R}^{t-1} \to \mathbb{R}. \tag{13}$$

We impose an *expected average block-power constraint*

$$\frac{1}{n} \sum_{t=1}^{n} \mathbf{E}[X_t^2] \leq P, \tag{14}$$

where the expectation is over the messages and the realizations of the channel.

Each Receiver i decodes its corresponding message M_i by means of a decoding function $\phi_i^{(n)}$ of the form

$$\phi_i^{(n)} : \mathbb{R}^n \to \mathcal{M}_i, \quad i \in \{1, 2\}. \tag{15}$$

That means, based on the output sequence Y_i^n, Receiver i produces the guess $\hat{M}_i^{(n)} = \phi_i^{(n)}(Y_i^n)$.

[1] As for the MAC, there is no loss of generality in assuming that $Z_{1,t}$ and $Z_{2,t}$ have variance one.

Thus, the average probability of error is

$$P_{e,BC}^{(n)} \triangleq \Pr\big[(\hat{M}_1 \neq M_1) \text{ or } (\hat{M}_2 \neq M_2)\big]. \tag{16}$$

A $(\lfloor 2^{nR_1} \rfloor, \lfloor 2^{nR_2} \rfloor, n)$ *BC feedback-code of blocklength n and power P* is composed of a sequence of encoding functions $\{g_t^{(n)}\}_{t=1}^n$ as in (13) and satisfying (14) and of two decoding functions $\phi_1^{(n)}$ and $\phi_2^{(n)}$ as in (15).

We say that a rate-pair (R_1, R_2) is achievable over the Gaussian BC with feedback and power constraint P, if there exists a sequence of $(\lfloor 2^{nR_1} \rfloor, \lfloor 2^{nR_2} \rfloor, n)$ BC feedback-codes such that the average probability of error $P_{e,BC}^{(n)}$ tends to zero as the blocklength tends to infinity. The closure of the union of all achievable regions is called *the capacity region*. We denote it by $\mathscr{C}_B^{fb}(h_1, h_2, P)$. The supremum of the sum $R_1 + R_2$, where (R_1, R_2) are in $\mathscr{C}_B^{fb}(h_1, h_2, P)$ is called the sum-capacity and is denoted $C_{\Sigma,B}^{fb}(h_1, h_2, P)$.

The capacity region $\mathscr{C}_B^{fb}(h_1, h_2, P)$ and the sum-capacity $C_{\Sigma,B}^{fb}(h_1, h_2, P)$ are unknown. Achievable regions that can improve over the no-feedback capacity region have been proposed in Ozarow&Leung [3], Elia [4], Wu *et al.* [5], Ardestanizadeh *et al.* [6] and Gastpar *et al.* [7].

B. Linear-Feedback Schemes for the BC

We will consider the class of coding schemes where the feedback is only used linearly. More precisely, we say that a blocklength-n BC feedback scheme is a linear-feedback scheme, if the channel input vector $\mathbf{X} \triangleq \big(X_1, \ldots, X_n\big)^\intercal$ can be written as:

$$\mathbf{X} = \mathbf{U} + \mathsf{B}_1 \mathbf{Z}_1 + \mathsf{B}_2 \mathbf{Z}_2, \tag{17}$$

where $\mathbf{Z}_i \triangleq \big(Z_{i,1}, \ldots, Z_{i,n}\big)^\intercal$ represents the noise vector at Receiver i, $i \in \{1,2\}$, $\mathbf{U} \triangleq \big(U_1, \ldots, U_n\big)^\intercal$ is an n-dimensional information-carrying vector that results from an arbitrary mapping

$$\xi : \mathcal{M}_1 \times \mathcal{M}_2 \to \mathbb{R}^n \tag{18}$$
$$(M_1, M_2) \mapsto \mathbf{U}, \tag{19}$$

and B_1 and B_2 are strictly lower-triangular matrices. Notice that instead of using the channel output vectors $\mathbf{Y}_1$ and $\mathbf{Y}_2$ as feedback symbols, the transmitter uses the noise vectors $\mathbf{Z}_1$ and $\mathbf{Z}_2$ which can be learnt from the noiseless feedback. The strict lower-triangularity of the matrices B_1 and B_2 ensures the causality of the feedback. The decoding operations ϕ_1 and ϕ_2 are arbitrary.

The set of rate-pairs that are achievable using a sequence of BC linear-feedback codes is called the *linear-feedback capacity region*. We denote it by $\mathscr{C}_B^{lin}(h_1, h_2, P)$. The supremum of the sum-rates that are achievable using a linear-feedback scheme is called the linear-feedback sum-capacity. We denote it by $C_{\Sigma,B}^{lin}(h_1, h_2, P)$.

IV. Main Results

In this section, we state our main results on MAC-BC duality with linear-feedback schemes. We need the following definitions to establish Proposition 2 and 3.

Definition 1. *Given $\eta \in \mathbb{N}$ and strictly lower-triangular η-by-η matrices D_1 and D_2, let Q_1 and Q_2 be the positive square roots of the positive definite η-by-η matrices*

$$\mathsf{M}_1 \triangleq (\mathsf{I}_n + h_1 \mathsf{D}_1)^\intercal (\mathsf{I}_n + h_1 \mathsf{D}_1) + h_1^2 \mathsf{D}_2^\intercal \mathsf{D}_2, \tag{20a}$$
$$\mathsf{M}_2 \triangleq h_2^2 \mathsf{D}_1^\intercal \mathsf{D}_1 + (\mathsf{I}_n + h_2 \mathsf{D}_2)^\intercal (\mathsf{I}_n + h_2 \mathsf{D}_2). \tag{20b}$$

Let $\mathcal{R}_M(\eta, \mathsf{D}_1, \mathsf{D}_2, h_1, h_2, P)$ denote the capacity region as determined by Cheng and Verdu [8] of the MIMO MAC (without feedback):

$$\mathbf{Y}^M \triangleq h_1 \mathsf{Q}_1^{-1} \mathbf{V}_1 + h_2 \mathsf{Q}_2^{-1} \mathbf{V}_2 + \mathbf{Z}, \tag{21}$$

where $\mathbf{Z}$ is a centered Gaussian vector of covariance matrix I_η and where $\mathbf{V}_1$ and $\mathbf{V}_2$ are the input vectors that have to satisfy the total power constraint

$$\mathbf{E}\left[\|\mathbf{V}_1\|^2 + \|\mathbf{V}_2\|^2\right] \leq [\eta P - \text{tr}(\mathsf{D}_1 \mathsf{D}_1^\intercal) - \text{tr}(\mathsf{D}_2 \mathsf{D}_2^\intercal)]^+. \tag{22}$$

Definition 2. *Given $\eta \in \mathbb{N}$ and strictly lower-triangular η-by-η matrices B_1 and B_2 are, let S_1 and S_2 be the positive square roots of the positive definite η-by-η matrices*

$$\mathsf{A}_1 \triangleq (\mathsf{I} + h_1 \mathsf{B}_1)(\mathsf{I} + h_1 \mathsf{B}_1)^\intercal + h_1^2 \mathsf{B}_2 \mathsf{B}_2^\intercal, \tag{23a}$$
$$\mathsf{A}_2 \triangleq h_2^2 \mathsf{B}_1 \mathsf{B}_1^\intercal + (\mathsf{I} + h_2 \mathsf{B}_2)(\mathsf{I} + h_2 \mathsf{B}_2)^\intercal. \tag{23b}$$

Let $\mathcal{R}_B(n, \mathsf{B}_1, \mathsf{B}_2, h_1, h_2, P)$ denote the private-message capacity region (dirty-paper region) as described in [9], of the MIMO BC (without feedback)

$$\mathbf{Y}_i^B \triangleq h_i \mathsf{S}_i^{-1} \mathbf{U} + \mathbf{Z}_i^B, \quad \text{for } i \in \{1,2\}, \tag{24}$$

where $\mathbf{Z}_1^B$ and $\mathbf{Z}_2^B$ are independent centered Gaussian vectors, each of covariance matrix I_η and where $\mathbf{U}$ is the input vector that has to satisfy the power constraint:

$$\mathbf{E}\left[\|\mathbf{U}\|^2\right] \leq [\eta P - \text{tr}(\mathsf{B}_1 \mathsf{B}_1^\intercal) - \text{tr}(\mathsf{B}_2 \mathsf{B}_2^\intercal)]^+. \tag{25}$$

Proposition 2. *The set of all rate-pairs that are achievable with a linear-feedback scheme for the MAC with noise variance one, channel coefficients h_1, h_2 and total power constraint P can be written as:*

$$\mathscr{C}_M^{fb}(h_1, h_2, P) = \text{cl}\left(\bigcup_{(\eta, \mathsf{D}_1, \mathsf{D}_2)} \frac{1}{\eta} \mathcal{R}_M(\eta, \mathsf{D}_1, \mathsf{D}_2, h_1, h_2, P)\right), \tag{26}$$

where the union is over all positive integers η and strictly lower-triangular η-by-η dimensional matrices D_1 and D_2.

Proposition 3. *The set of all rate-pairs that are achievable with a linear-feedback scheme for the BC with noise variance one, channel coefficients h_1, h_2 and power constraint P can be written as:*

$$\mathscr{C}_B^{lin}(h_1, h_2, P) = \text{cl}\left(\bigcup_{(\eta, \mathsf{B}_1, \mathsf{B}_2)} \frac{1}{\eta} \mathcal{R}_B(\eta, \mathsf{B}_1, \mathsf{B}_2, h_1, h_2, P)\right), \tag{27}$$

where the union is over all positive integers η and strictly lower-triangular η-by-η dimensional matrices B_1 and B_2.

Recall that a Gaussian MAC and BC are said to be dual if they have the same channel gains, same noise variance at all receivers and same total power constraint. We use Proposition 2 and 3 to prove that:

Theorem 1. *The linear-feedback capacity regions of dual Gaussian MAC and BC under the same total power constraint P are equal:*

$$\mathscr{C}_M^{fb}(h_1, h_2, P) = \mathscr{C}_B^{lin}(h_1, h_2, P). \tag{28}$$

Theorem 1 implies the following corollary on sum-capacities:

Corollary 1.

$$C_{\Sigma,M}^{fb}(h_1, h_2, P) = C_{\Sigma,B}^{lin}(h_1, h_2, P). \tag{29}$$

We use Corollary 1 and the definition of $C_{\Sigma,M}^{fb}(h_1, h_2, P)$, to derive the linear-feedback sum-capacity of the Gaussian BC under equal channel gains:

Theorem 2. *Under equal channel gains $h_1 = h_2 = h$, the linear-feedback sum-capacity of the Gaussian BC with power constraint P and noise variance one is:*

$$C_{\Sigma,B}^{lin}(h, h, P) = \frac{1}{2}\log\left(1 + h^2 P(1+\rho^\star)\right), \tag{30}$$

where $\rho^\star$ is the unique solution in $[0, 1]$ to

$$1 + h^2 P(1+\rho) = \left(1 + h^2 \frac{P}{2}(1-\rho^2)\right)^2. \tag{31}$$

V. Proofs

A. Proof of Theorem 1

We have shown that given a positive integer η and two η-by-η strictly lower-triangular matrices B_1 and B_2, choosing two η-by-η strictly lower-triangular matrices D_1 and D_2 as[2]

$$\mathsf{D}_i = \bar{\mathsf{B}}_i, \quad i \in \{1, 2\}, \tag{32}$$

the η-by-η MIMO MAC with capacity region $\mathcal{R}_M(\eta, \mathsf{D}_1, \mathsf{D}_2, h_1, h_2, P)$ is the dual channel of the η-by-η MIMO BC with capacity region $\mathcal{R}_B(\eta, \mathsf{B}_1, \mathsf{B}_2, h_1, h_2, P)$ and vice-versa. The choice in (32) implies that the total power constraint is the same for both channels. Notice that instead of considering the MIMO MAC in (21), we consider a reversed version of this channel obtained by inverting the order of the antennas at each end which does not change the set of achievable rates. The details of this step are omitted due to lack of space.

Hence, MIMO MAC-BC duality in [10] gives

$$\mathcal{R}_M(\eta, \mathsf{D}_1, \mathsf{D}_2, h_1, h_2, P) = \mathcal{R}_B(\eta, \mathsf{B}_1, \mathsf{B}_2, h_1, h_2, P). \tag{33}$$

Now, taking the union over all positive integers η and strictly lower-triangular η-by-η dimensional matrices B_1 and B_2 and since the mapping $\mathsf{A} \mapsto \bar{\mathsf{A}}$ is one-to-one over the set of strictly lower-triangular matrices, using Proposition 2 and 3 we establish the result.

[2] $\bar{\mathsf{B}}_i$ is the *reversed image* of B_i which is obtained by reversing all rows and columns of B_i^T.

B. Proof of Theorem 2

We consider the dual Gaussian MAC and BC under equal channel gains i.e. $h_1 = h_2 = h$. Using Corollary 1 and (8), we have

$$C_{\Sigma,B}^{lin}(h, h, P) = \max_{\substack{P_1+P_2=P \\ P_1,P_2 \geq 0}} C_{\Sigma,M}^{fb,ind}(h, h, P_1, P_2). \tag{34}$$

The maximization problem in (34) corresponds to finding the optimal power allocation among users in terms of sum-rate. We have shown that the optimal strategy is to use Ozarow's coding scheme [2] with $P_1 = P_2 = \frac{P}{2}$ which establishes the proof. The details are omitted due to lack of space.

VI. Conclusion

In this paper, we investigated a duality relationship between Gaussian MAC and BC with linear-feeback schemes. We dealt exclusively with the scalar Gaussian MAC and BC. In a related journal paper, we show that our results can be extended to MIMO channels.

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