# Equivalence of Transmitter and Receiver Cooperation in Interference Networks

Michèle Wigger

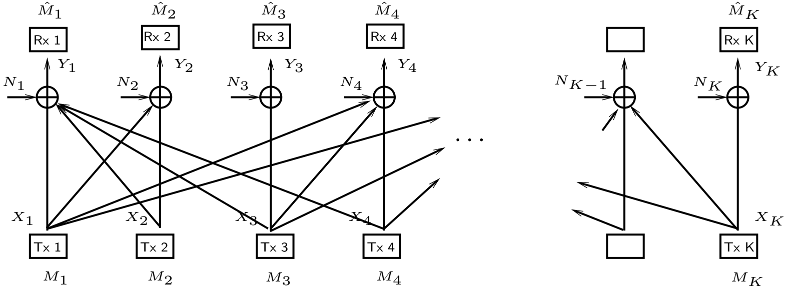
joint work with A. Lapidoth, N. Levy, and S. Shamai (Shitz)

wigger@ece.ucsd.edu

Qualcomm, 13 November 2009

# Interference Networks

- $K$ Transmitter/Receiver pairs
- Communications interfere with each other
- Constant (*not time-varying*) Gaussian linear channel
- Single-antenna transmitters/receivers



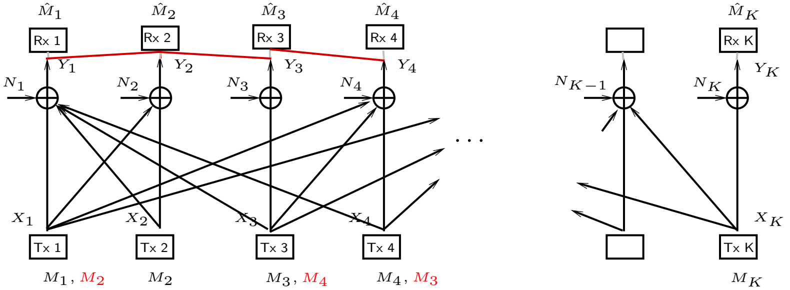
# Two Types of Cooperation in Interference Networks

Receivers cooperation:

- Clustered decoding, i.e., access to other antennas (e.g., over backhaul)

Transmitters cooperation:

- Message cognition (e.g., through bluetooth)



# Prelog $S$

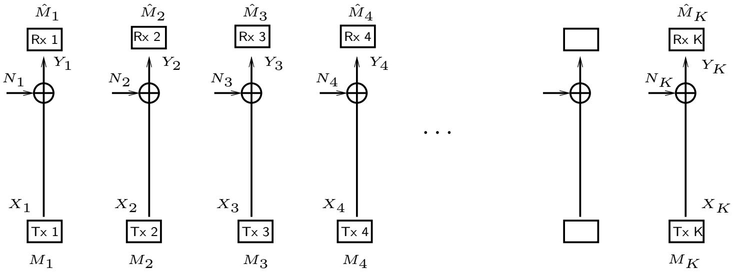
- Also called *degrees of freedom* or *multiplexing gain*

$$C_\Sigma \approx S \cdot C_{\mathsf{AWGN}}, \qquad \mathsf{SNR} \gg 1.$$

$C_\Sigma$: Sum-rate capacity of network; $C_{\mathsf{AWGN}}$: AWGN-channel capacity

- Intuition: Number of interference-free channels in system
- $1 \leq S \leq K$

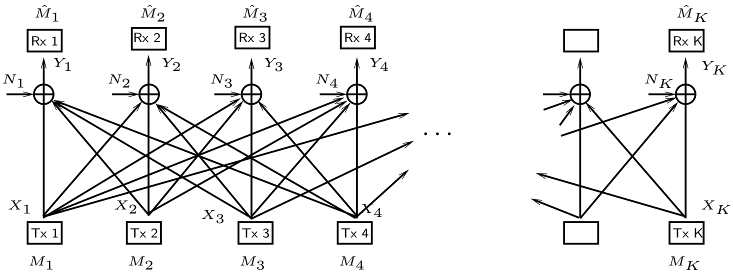
# Interference-Free Network - A Well Known Prelog




Without transmitters cooperation/receivers cooperation:

- Prelog is $K$

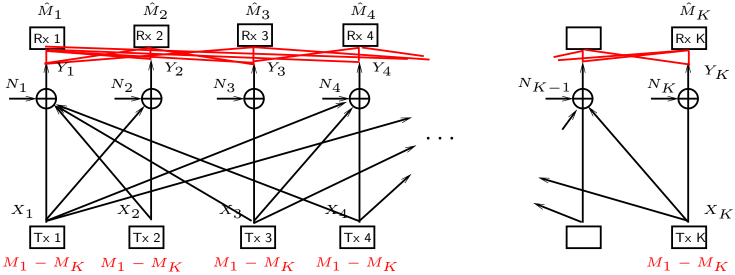
# Example II: Full-Interference - A "Hot" Prelog




Without transmitters cooperation/receivers cooperation:

- For some channels, prelog is 1
- For some channels, prelog K/2 achievable with interference alignment

# Example III: Full Coop. - Another Well Known Prelog

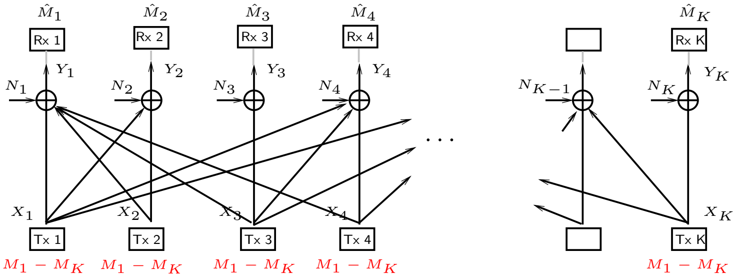



All transmitters know all messages & all receivers observe all antennas
$\rightarrow$ corresponds to MIMO channel

- Prelog = rank(channel matrix)
- For non-degenerate channels, prelog $= K$

Cooperation at transmitters & receivers can greatly increase prelog!

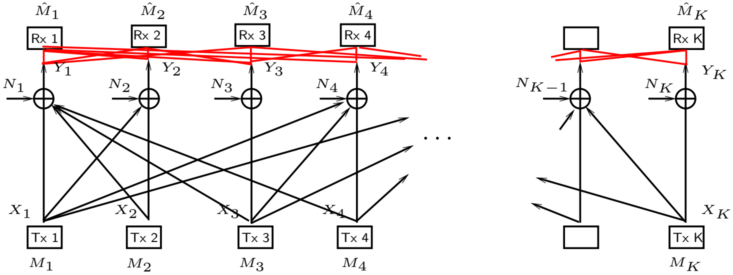
# Example III: Full Coop. - Another Well Known Prelog




All transmitters know all messages & receivers observe only own antennas
$\rightarrow$ corresponds to Broadcast channel

- Prelog $=$ rank(channel matrix)
- For non-degenerate channels, prelog $= K$

# Example III: Full Coop. - Another Well Known Prelog




Transmitters know only own message & all receivers observe all antennas
$\rightarrow$ corresponds to MAC channel

- Prelog $=$ rank(channel matrix)
- For non-degenerate channels, prelog $= K$

Full transmitters cooperation equivalent to full receivers cooperation!

# Questions on Prelog

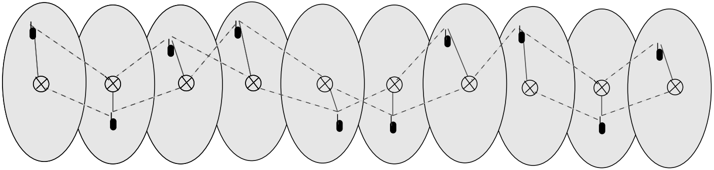
- What are gains from given transmitters cooperation/receivers cooperation?
- What is better: transmitters cooperation or receivers cooperation?
- How much transmitters cooperation/receivers cooperation is needed to achieve 75% of ideal (interference-free) performance?

# In this talk

- General prelog-problem seems difficult!
- $\rightarrow$ Restrict attention to specific networks...

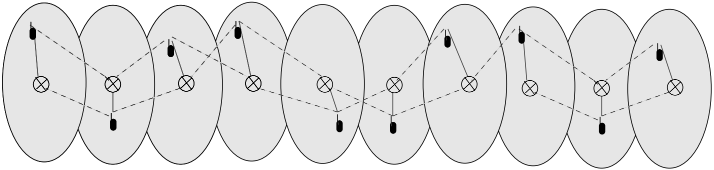
# Two Network Models Studied in this Talk

- Based on Wyner's one-dim. linear cellular model '94 (uplink)



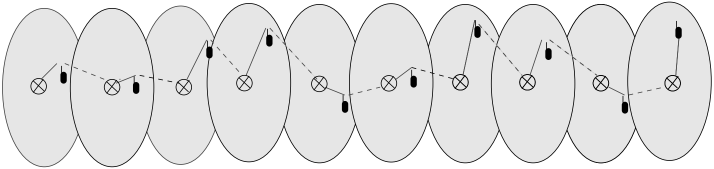
- Linear array of cells (see also Hanly/Whitening'93)
- Interference only from mobiles in adjacent cells
  (Assumption: interference from other mobiles below noise-level)
- Only interested in Prelog $\Rightarrow$ Suffices to consider one mobile per cell

# Two Network Models Studied in this Talk



- Wyner's regular model: symmetric interference
- Soft-handoff model: asymmetric interference

# Two Network Models Studied in this Talk

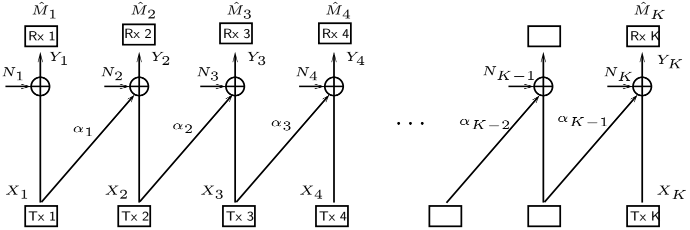


- Wyner's regular model: symmetric interference
- Soft-handoff model: asymmetric interference

# Asymmetric Model

(Soft-Handoff Model)

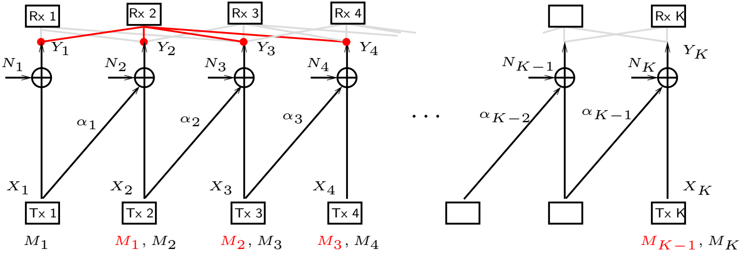
# Asymmetric Channel Model




- Interference only from left: $Y_{k,t} = \alpha_{k-1} X_{k-1,t} + X_{k,t} + N_{k,t}$
- $\alpha_1, \ldots, \alpha_{K-1} \neq 0$ $\quad \{N_{k,t}\}_{\substack{1 \le k \le K \\ 1 \le t \le n}}$ IID $\sim \mathcal{N}\big(0, \sigma^2\big)$
- Equal power constraints: $(\frac{1}{n} \sum_{t=1}^{n} x_{k,t}^2) \le P$
- Goal: Receiver $k$ learns Message $M_k$ where $M_k \sim U\{1, \ldots, \lfloor e^{nR_k} \rfloor\}$

# Transmitters Cooperation / Receivers Cooperation

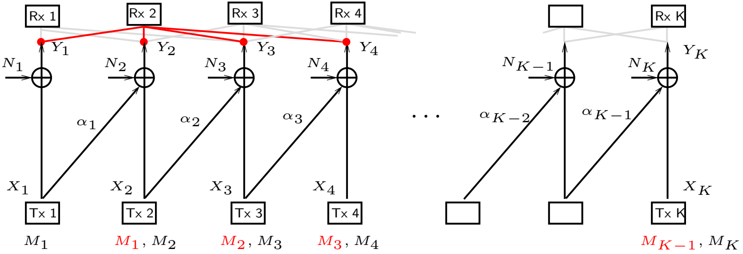
Example: $t_\ell = 1, t_r = 0, r_\ell = 1, r_r = 2$




- Each transmitter knows $t_\ell$ Messages to left and $t_r$ to right
- Each receiver has access to $r_\ell$ antennas to left and $r_r$ to right

# Transmitters Cooperation / Receivers Cooperation

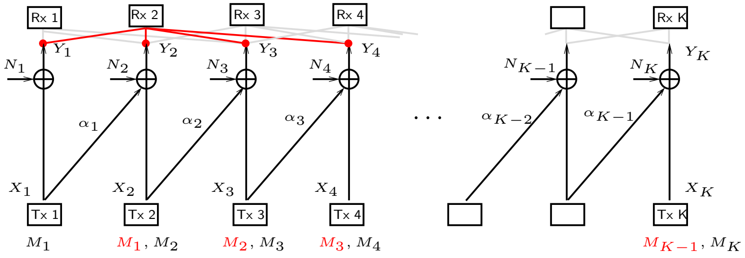
Example: $t_\ell = 1, t_r = 0, r_\ell = 1, r_r = 2$




- Transmitters know other messages, not necessarily other signals!
- Receivers observe other antennas, cannot necessarily decode other messages!

# Transmitters Cooperation / Receivers Cooperation

Example: $t_\ell = 1, t_r = 0, r_\ell = 1, r_r = 2$




## Side-Information:

Bits (Messages) at transmitters / real symbols (output signals) at receivers!

# Relevant Parameters

- $K$: # transmitter/receiver pairs
- $t_\ell$: # *messages* to the left known to each transmitter
- $t_r$: # *messages* to the right known to each transmitter
- $r_\ell$: # *antennas* to the left observed by each receiver
- $r_r$: # *antennas* to the right observed by each receiver

# Prelog / Prelog Per-User

- Sum-rate capacity $C_\Sigma(K, t_\ell, t_r, r_\ell, r_r; P)$

- Prelog $S(K, t_\ell, t_r, r_\ell, r_r)$:

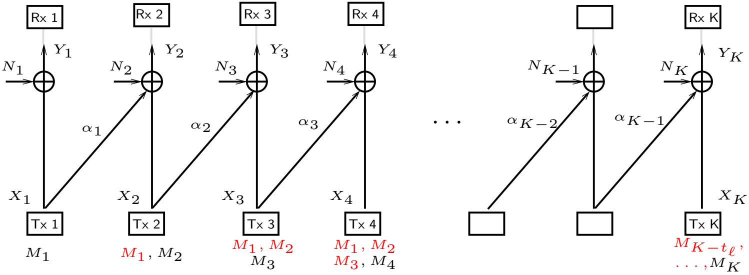
$$C_\Sigma \approx S \cdot \frac{1}{2} \log\left(1 + \frac{P}{N}\right), \qquad \frac{P}{N} \gg 1$$

- Asymptotic prelog per-user $S_\infty(t_\ell, t_r, r_\ell, r_r)$:

$$C_\Sigma \approx S_\infty \cdot K \cdot \frac{1}{2} \log\left(1 + \frac{P}{N}\right), \qquad \frac{P}{N} \gg 1,\ K \gg 1$$

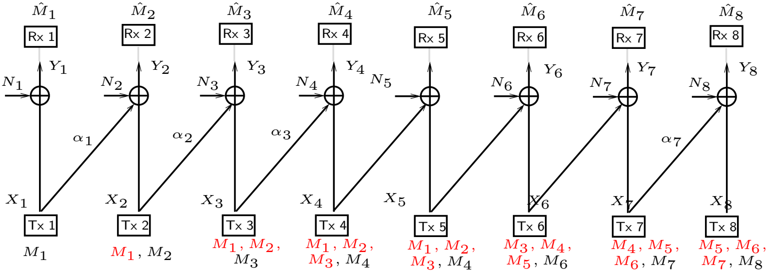
# Only Transmitter Cooperation/No Receivers Cooperation

- Each receiver has access only to its own antenna
- Each transmitter knows Messages of $t_\ell$ transmitters to its left




Lapidoth/Shamai/Wigger'07: $$S(K, t_\ell, 0, 0, 0) = K - \left\lfloor \frac{K}{t_\ell + 2} \right\rfloor$$

# Example $t_\ell = 3$: How to achieve $S = K - \left\lfloor \frac{K}{t_\ell+2} \right\rfloor$




- Silence every $t_\ell + 2$nd transmitter $\rightarrow$ splits network into subnets
- Each active transmitter achieves Prelog $1 \Rightarrow S = K - \left\lfloor \frac{K}{t_\ell+2} \right\rfloor$

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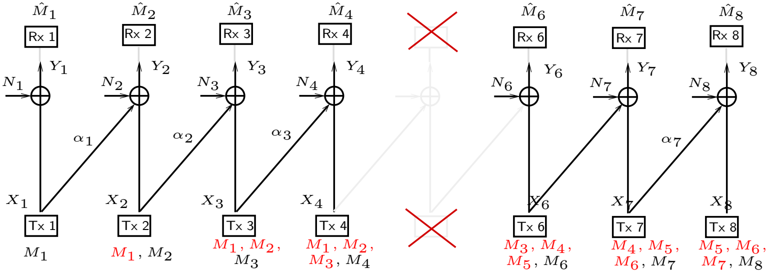


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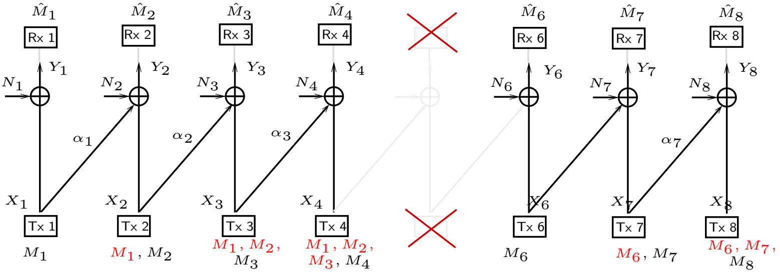
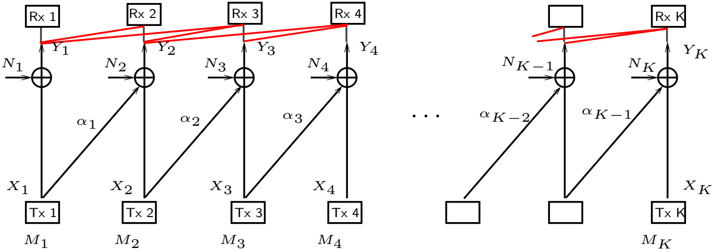



- Each transmitter ignores messages outside subnet
- Each transmitter can compute signal sent to its left
- Use dirty-paper coding to cancel interference from left

→ Each transmitter can communicate as over an interference-free channel!

# No Transmitter Cooperation/Only Receivers Cooperation

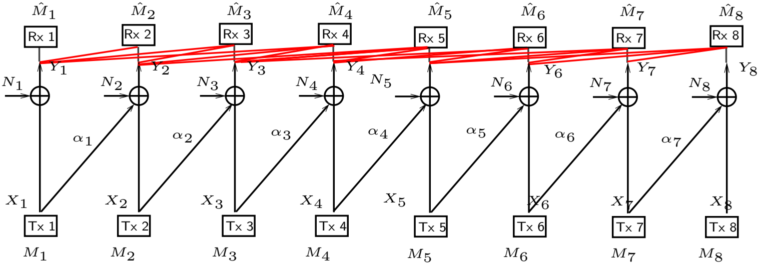
- Each receiver has also access to $r_\ell$ antennas to its left
- Each transmitter knows only its own message




Levy/Shamai'08: $$S(K,0,0,r_\ell,0) = K - \left\lfloor \frac{K}{r_\ell + 2} \right\rfloor$$

Access to left antennas and cognition of left messages are equivalent!

# Example $r_\ell = 3$: How to achieve $S = K - \left\lfloor \frac{K}{r_\ell+2} \right\rfloor$

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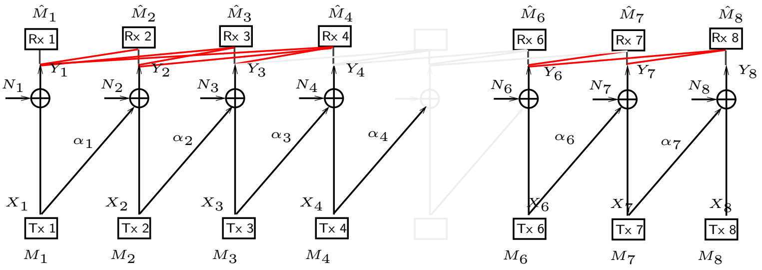
# Example $r_\ell = 3$: How to achieve $S = K - \left\lfloor \frac{K}{r_\ell+2} \right\rfloor$




- Each active transmitter: point-to-point encoding $X_k^n(M_k)$
- Each receiver observes all antennas in its subnet to its left
- Decoding: successive interference cancellation starting from left

→ Each message decoded while seeing an interference-free channel!

# Prelog for General $t_\ell, t_r, r_\ell, r_r$

### Prelog for all $\alpha_1, \ldots, \alpha_{K-1} \neq 0$

$$S(K, t_\ell, t_r, r_\ell, r_r) = K - \left\lceil \frac{K - t_\ell - r_\ell - 1}{t_\ell + t_r + r_\ell + r_r + 2} \right\rceil$$

- Depends only on total left $(t_\ell + r_\ell)$ and total right $(t_r + r_r)$ side-info.

### Prelog Per-User for all $\alpha_1, \ldots, \alpha_{K-1} \neq 0$

$$S_\infty(t_\ell, t_r, r_\ell, r_r) = \frac{t_\ell + t_r + r_\ell + r_r + 1}{t_\ell + t_r + r_\ell + r_r + 2}$$

- Depends only on total side-information $(t_\ell + t_r + r_\ell + r_r)$

# Answers to our Questions for Asymmetric Model

- Gains depend only on total left and total right side-info.!
  Large networks: Gains depend only on total side-info.!
- Transmitters side-info. $\equiv$ receivers side-info.
- Large networks: left side-info. $\equiv$ right side-info.
- $(t_\ell + t_r + r_\ell + r_r) = 2$ required for 75% of interference-free prelog

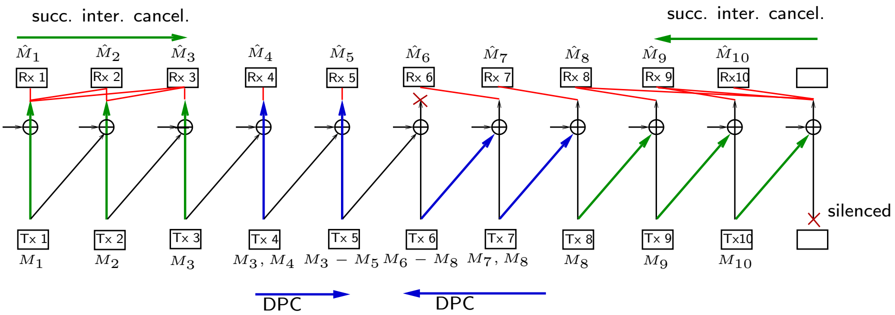
# Achievability

- Silence every $(t_\ell + t_r + r_\ell + r_r + 2)$-th transmitter
  $\Rightarrow$ splits network into unconnected subnets

- In each subnet (except maybe smaller last one)
  - $(t_\ell + t_r + r_\ell + r_r + 1)$ active transmitters, each sends at prelog 1!

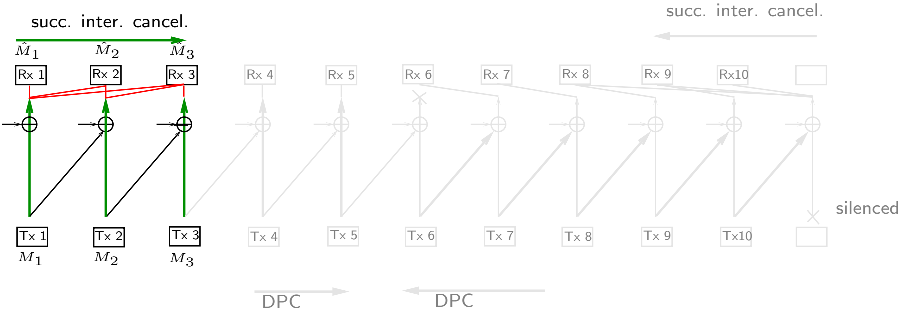
    $\rightarrow$ Prelog $K - \left\lfloor \frac{K}{t_\ell+t_r+r_\ell+r_r+2} \right\rfloor$ achievable

# Scheme in each Subnet, Ex.: $t_\ell = 2, t_r = 2, r_\ell = 2, r_r = 3$




- Send $(t_\ell + t_r + r_\ell + r_r + 1)$ messages at prelog 1!
- Intuition: each message “uses” one side-info.
- Depending on “used” side-info. different scheme is applied

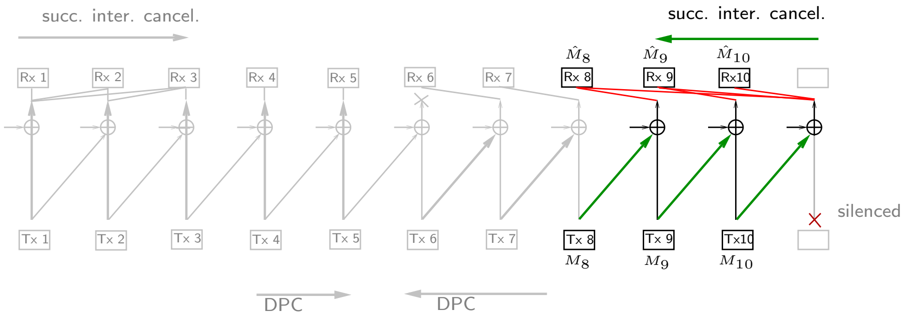
# Scheme in each Subnet, Ex.: $t_\ell = 2, t_r = 2, r_\ell = 2, r_r = 3$




First group with $(r_\ell + 1)$ messages

- Each receiver observes all antennas to its left
- Point-to-point encoding $X_k^n(M_k)$
- Decoding: successive interference cancellation starting from first antenna
  - Each message decoded based on interference-free link → prelog 1

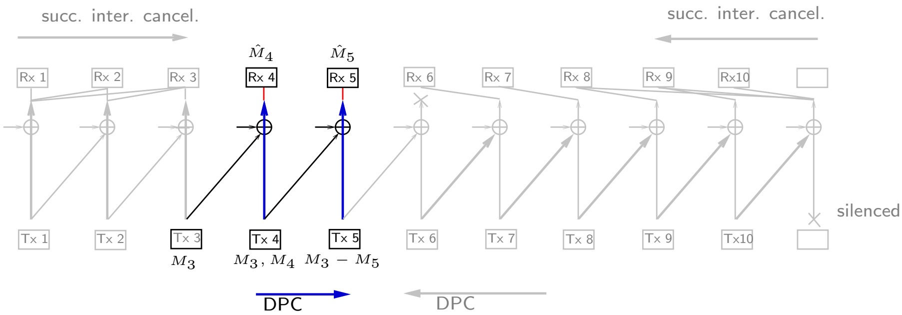
# Scheme in each Subnet, Ex.: $t_\ell = 2, t_r = 2, r_\ell = 2, r_r = 3$




Last group with $r_r \geq 1$ messages

- Each receiver observes all antennas to its right
- Each message decoded based on antenna to right (assumption $r_r \geq 1$)
- Decoding: succ. inter. cancel. starting from last antenna

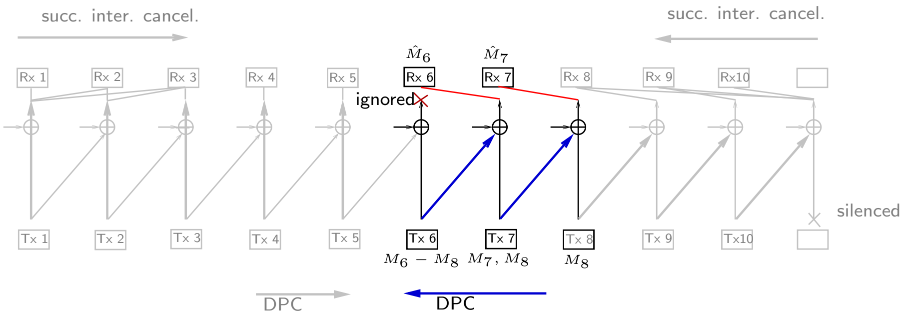
# Scheme in each Subnet, Ex.: $t_\ell = 2, t_r = 2, r_\ell = 2, r_r = 3$




Second group with $t_\ell$ messages

- Each transmitter knows signal sent by left-neighbor
- Dirty-paper coding canceling interference from left $\rightarrow$ prelog 1

# Scheme in each Subnet, Ex.: $t_\ell = 2, t_r = 2, r_\ell = 2, r_r = 3$




Third group with $t_r$ messages

- Each transmitter knows signal sent by right-neighbor
- Dirty-paper coding canceling "interference" from right $\rightarrow$ prelog 1
- Each message decoded based on antenna to right (assumption $r_r \geq 1$)

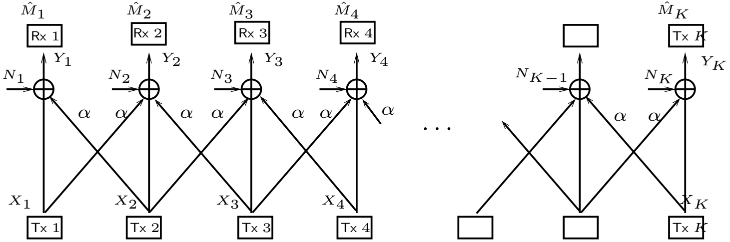
# Converse: Generalized Sato MAC-Bound

- Merge subset of receivers into *Big Receiver*
- Reveal genie-information (linear combination of the noises) to *Big Receiver*
- Show that *Big Receiver* can decode messages at least as good as original receivers
- Find an upper bound on prelog to *Big Receiver*

# Symmetric Model

(Regular Wyner Model)

# Symmetric Channel Model

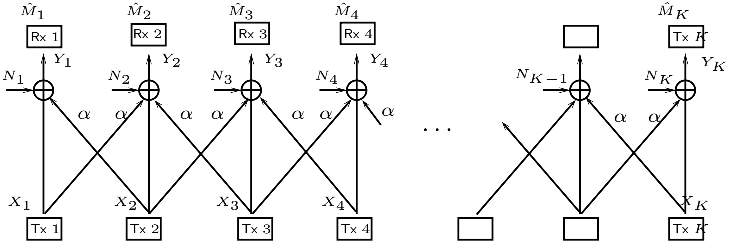



$$Y_{k,t} = \alpha X_{k-1,t} + X_{k,t} + \alpha X_{k+1,t} + N_{k,t}, \qquad t \in \{1,\ldots,n\},\ k \in \{1,\ldots,K\}$$

- $\alpha \neq 0$ (same $\alpha$ for all interferences)

- $K \times K$-dimensional matrix $\mathsf{H}_K \triangleq \begin{pmatrix} 1 & \alpha & 0 & 0 & \ldots & 0 \\ \alpha & 1 & \alpha & 0 & \ldots & 0 \\ 0 & \alpha & 1 & \alpha & \ldots & 0 \\ & & \ldots & \ldots & & \\ 0 & \ldots & 0 & \alpha & 1 & \alpha \\ 0 & \ldots & 0 & 0 & \alpha & 1 \end{pmatrix}$

# Symmetric Channel Model




- Message cognition at txs: $t_\ell$ messages to left and $t_r$ messages to right
- Clustered decoding at rxs: $r_\ell$ antennas to left and $r_r$ antennas to right

# Results for $t_\ell + r_\ell = t_r + r_r$

### Prelog and Prelog Per-User for most $\alpha \neq 0$:

If $\alpha$ such that $\det(\mathsf{H}_{t_\ell+r_\ell+1}) \neq 0$ and $\det(\mathsf{H}_{t_\ell+r_\ell}) \neq 0$, then

$$\begin{aligned} S(K, t_\ell, t_r, r_\ell, r_r) &= K - \left\lfloor \frac{K}{t_\ell + r_\ell + 2} \right\rfloor \\ S_\infty(t_\ell, t_r, r_\ell, r_r) &= \frac{t_\ell + r_\ell + 1}{t_\ell + r_\ell + 2} \end{aligned}$$

- Depend only on total left side-info. $(t_\ell + r_\ell)$

# Answers for Symmetric Model when $t_\ell + r_\ell = t_r + r_r$

## Answers to Initial Questions

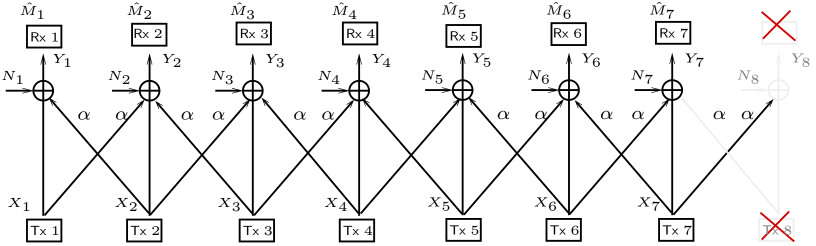
- Gains only depend on total side-info.!
- Transmitters side-info. $\equiv$ receivers side-info.
- $(t_\ell + t_r + r_\ell + r_r) = 4$ required for 75% high-SNR ideal performance

  $\longrightarrow$ Need double the side-information as in asymmetric case!

# Coding Scheme for $t_\ell + r_\ell = t_r + r_r$

- Silence every $(t_\ell + r_\ell + 2)$-th tx $\rightarrow$ Splits network in subnets
- In each subnet, achieve prelog $t_\ell + r_\ell + 1$

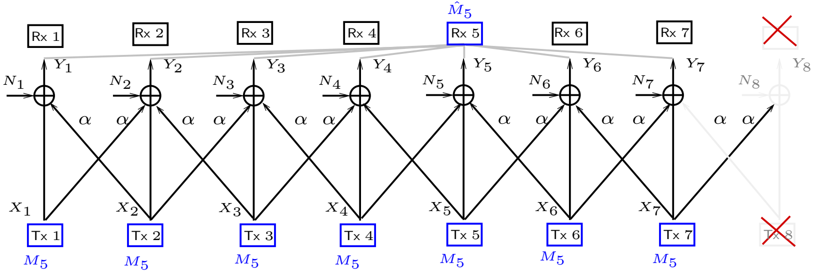
Example: $t_\ell + r_\ell = t_r + r_r = 6$



# Coding Scheme for $t_\ell + r_\ell = t_r + r_r$

- In each subnet: if $t_\ell + t_r = r_\ell + r_r$ → use a MIMO scheme

Example: $t_\ell = 2, t_r = 4, r_\ell = 4, r_r = 2$

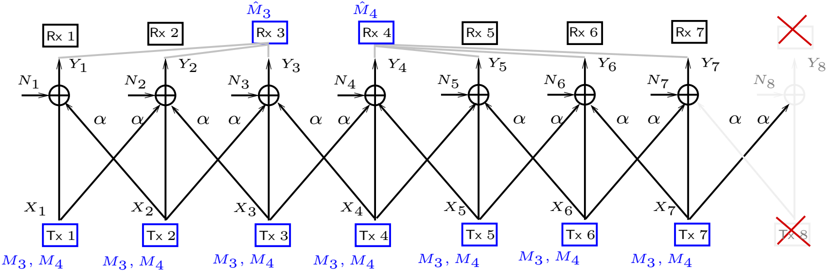



Algoet-Cioffi:
Optimal MIMO-scheme achieves prelog $(t_\ell + r_\ell + 1)$ if $\det(\mathsf{H}_{t_\ell + r_\ell + 1}) \neq 0$

# Coding Scheme for $t_\ell + r_\ell = t_r + r_r$

- In each subnet: if $t_\ell + t_r > r_\ell + r_r$ → use a Broadcast scheme

Example: $t_\ell = 4, t_r = 3, r_\ell = 2, r_r = 3$

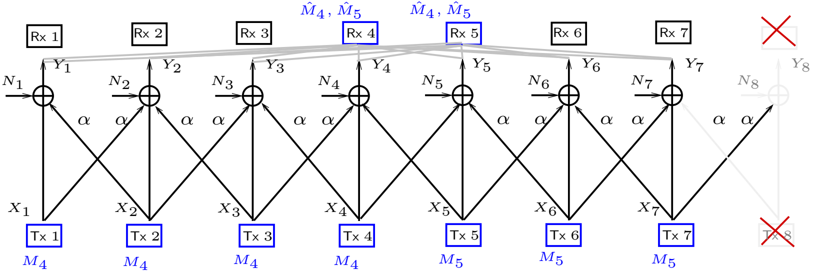



Caire-Shamai:
Optimal BC-scheme achieves prelog $(t_\ell + r_\ell + 1)$ if $\det(\mathsf{H}_{t_\ell + r_\ell + 1}) \neq 0$

# Coding Scheme for $t_\ell + r_\ell = t_r + r_r$

- In each subnet: if $t_\ell + t_r < t_\ell + t_r$ $\rightarrow$ use a MAC scheme

Example: $t_\ell = 2, t_r = 3, r_\ell = 4, r_r = 3$




Optimal MAC-scheme achieves prelog $(t_\ell + r_\ell + 1)$ if $\det(\mathsf{H}_{t_\ell + r_\ell + 1}) \neq 0$

# Cooperation only at the Receivers $t_\ell = t_r = 0$

**Prelog Per-User**

If $\alpha$ such that $\det(\mathsf{H}_{r_\ell+1}) = 0$ and $\det(\mathsf{H}_{r_\ell+r_r+1}) \neq 0$, then

$$S_\infty(0,0,r_\ell,r_r) = \frac{r_\ell + r_r + 1}{r_\ell + r_r + 3}$$

Recall: If $r_\ell = r_r$ and $\alpha$ such that $\det(\mathsf{H}_{r_\ell+1}) \neq 0$, then

$$S_\infty(0,0,r_\ell,r_r) = \frac{r_\ell + 1}{r_\ell + 2}$$

**For symmetric model**

Prelog per-user (and prelog) depends on specific value $\alpha$!

# Summary

Asymmetric model:

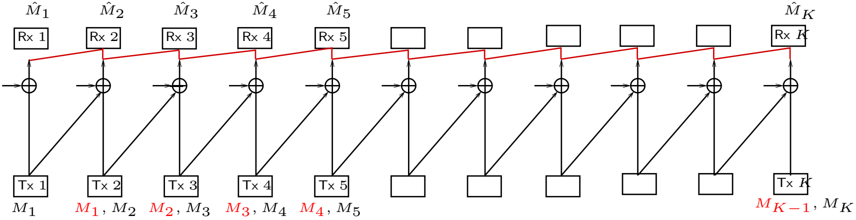
- Determined prelog and prelog per-user
- Message cognition $\equiv$ clustered decoding
- Large networks: left side-info. $\equiv$ right side-info.

Symmetric model:

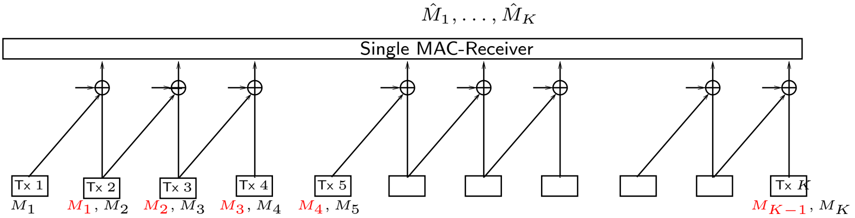
- Determined prelog and prelog per-user for most $\alpha$ when $t_\ell + r_\ell = t_r + r_r$
- Message cognition $\equiv$ clustered decoding
- Prelog varies with $\alpha \neq 0$!

Symmetric network needs double side-info. for same prelog as asym. network

# Converse: Generalized Sato MAC-Bound

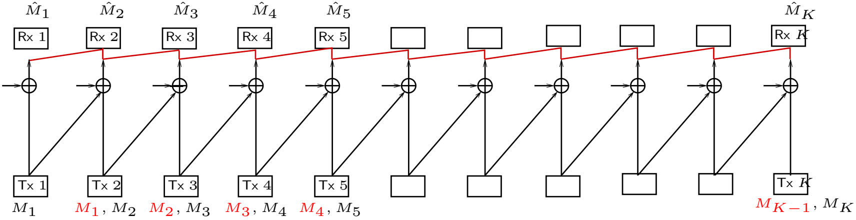



$\leq$ (in prelog)

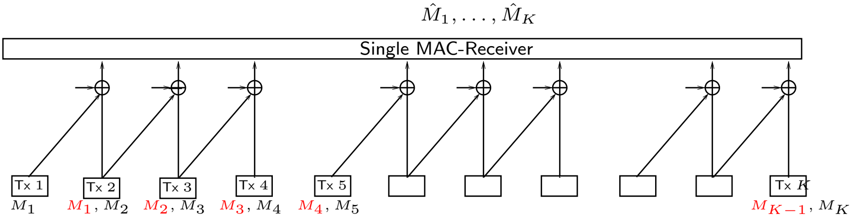



- MAC-Receiver does not observe every $(J_\ell + J_r + i_\ell + i_r + 2)$-th antenna!

# Converse: Generalized Sato MAC-Bound

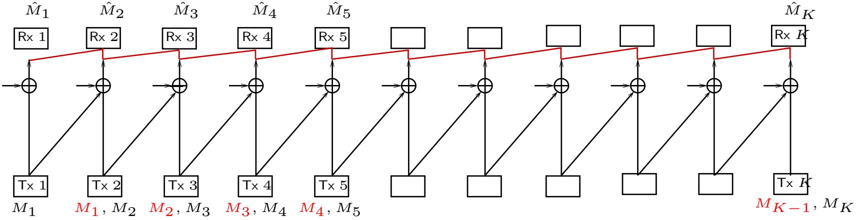



$\leq$ (in prelog)

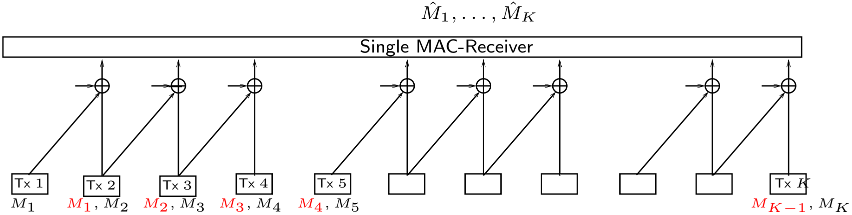



- $S(K, J_\ell, J_r, i_\ell, i_r) \leq \#$ antennas at MAC-Receiver

# Converse: Generalized Sato MAC-Bound

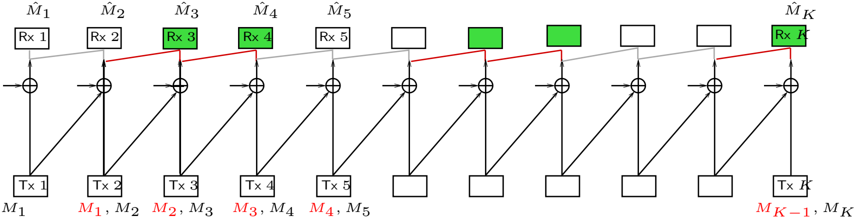



$\leq$ (in prelog)

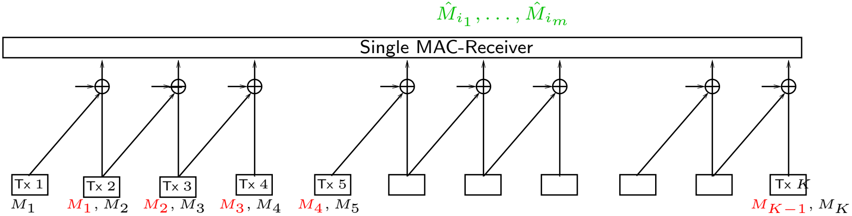



- Proof idea: MAC-Rx can decode at least as well as original receivers

# Converse: Generalized Sato MAC-Bound

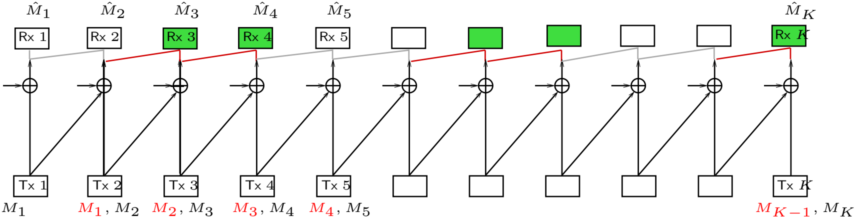



$\leq$ (in prelog)

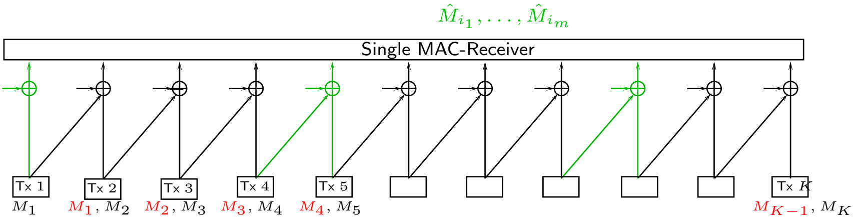



- MAC-Receiver decodes in 3 steps

# Converse: Generalized Sato MAC-Bound

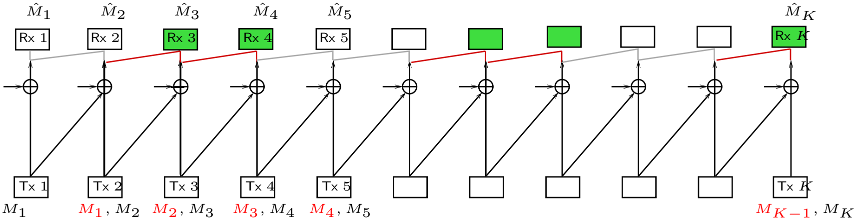



$\leq$ (in prelog)

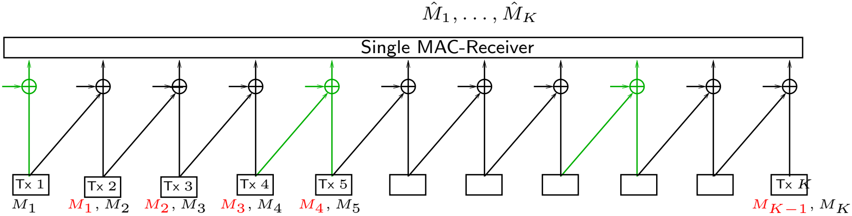



- MAC-Receiver decodes in 3 steps

# Converse: Generalized Sato MAC-Bound




$\leq$ (in prelog)




- MAC-Receiver decodes in 3 steps