An Information-Theoretic Approach to Collaborative Integrated Sensing and Communication for Two-Transmitter Systems

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Abstract

This paper considers information-theoretic models for integrated sensing and communication (ISAC) over multi-access channels (MAC) and device-to-device (D2D) communication. The models are general and include as special cases scenarios with and without perfect or imperfect state-information at the MAC receiver as well as causal state-information at the D2D terminals. For both setups, we propose collaborative sensing ISAC schemes where terminals not only convey data to the other terminals but also state-information that they extract from their previous observations. This state-information can be exploited at the other terminals to improve their sensing performances. Indeed, as we show through examples, our schemes improve over previous non-collaborative schemes in terms of their achievable rate-distortion tradeoffs. For D2D we propose two schemes, one where compression of state information is separated from channel coding and one where it is integrated via a hybrid coding approach.

I. Introduction

Next-generation wireless networks are expected to support several autonomous and intelligent applications that rely heavily on accurate sensing and localization techniques [1]. Important examples are intelligent transport systems, where vehicles continuously sense environmental changes and simultaneously exchange sensing-information and data with already detected vehicles, base stations, or central servers. Such simultaneous sensing and data-communication applications are also the focus of this work. More specifically, we are interested in multi-terminal scenarios where different terminals communicate data with each other and simultaneously exploit the backscattered signals for sensing purposes.

A common but naive approach to address sensing and communication is to separate the two tasks in independent systems and split the available resources such as bandwidth and power between the two systems. In our information-theoretic model, such a system corresponds to resource-sharing (e.g., time-sharing) between communication and sensing. However, the high cost of spectrum and hardware encourages integrating the sensing and communications tasks via a single waveform and a single hardware platform [2], [3]. A large body of works studied integrated sensing and communication (ISAC) scenarios from a communication-theoretic or signal-processing perspective (see, e.g., [4], [5] and references therein), mostly investigating appropriate choices for the employed waveform that in ISAC applications has to serve both the sensing and the communication tasks. Interestingly, different tradeoffs between the communication and sensing performances can be obtained by changing the employed waveform.

The fundamental performance limits of integrated sensing and communication systems were first considered in [6]. Specifically, [6] introduced an information-theoretic model for integrated sensing and communication based on a generalized-feedback model, which captures two underlying assumptions used in radar signal processing. On the one hand, generalized feedback captures the inherently passive nature of the backscattered signal observed at the transmitter (Tx), which cannot be controlled but is determined by its surrounding environment. On the other hand, it models the fact that the backscattered signal depends on the waveform employed by the Tx. It was proposed to use the classical average per-letter block-distortion to measure the Tx's sensing performance on the i.i.d. state-sequence. The authors of [6], see also [7] characterized the exact *capacity-distortion tradeoff* of arbitrary discrete memoryless channels (DMCs) with generalized feedback. This quantity naturally measures the inherent tradeoff between increasing data rate and reducing sensing distortion in such integrated systems. Interestingly, the results show that the optimal tradeoff is achieved by standard random code constructions as used for traditional data communication, where the statistics of the channel inputs (and thus of the codewords) however has to be adapted to meet the desired sensing performance. Notice that this observation is consistent with the signal-processing literature on the search for adequate channel input waveforms which allow to meet the desired sensing performance while still achieving high communication rates. Similar results were also derived for discrete memoryless broadcast channels (DMBCs) [7] where a single transmitter communicates with two receivers. Both the DMC and the DMBC are thus single-Tx networks, and the optimal sensing is a simple per-symbol estimation of the hidden state given the channel inputs and outputs at the sensing terminal. The optimality of such a simple symbol-by-symbol estimator stems from the fact that for a fixed input sequence the generalized feedback channels and the state-sequence both behave in a memoryless manner.

The sensing situation becomes more interesting and challenging when the sensing terminal is not the only terminal feeding inputs to the channel. In this case, the effective disturbance for the sensing is not necessarily memoryless since the inputs from the other terminals also create disturbances and can have memory. In this case, a strategy that first attempts to guess the other Txs' codewords followed by a symbol-wise estimator based on the observations and the guessed codewords can lead to a smaller (and thus better) distortion. This has also been observed in [8], where communication is over a DMC and state estimation is performed at the receiver (Rx) side. In this case, the optimal sensing strategy is first to decode the Tx's codeword and then apply an optimal symbol-by-symbol estimator to this codeword and the observed channel outputs. A similar strategy was applied in the two-transmitter single-Rx multi-access channel (MAC) ISAC scenario of [9] where through the generalized feedback each Tx first decodes part of the data sent by the other Tx and then applies a symbol-by-symbol estimator to the decoded codeword as well as its own channel inputs and outputs. In fact, the ISAC scheme of [9] is based on Willems' scheme for the MAC with generalized feedback, where each Tx encodes its data into two super-positioned codewords, whereof the lower data-layer is decoded by the other Tx. This data is then repeated by both Txs in the next block as part of a third lowest-layer codeword, allowing the two Txs to transmit data cooperatively Somewhat naturally, [9] suggests to use this decoded lower data-layer also for sensing purposes in the sense that each Tx applies the symbol-by-symbol estimator not only to its inputs and outputs but also to this decoded codeword. In this article, which is based on the conference paper [10], we suggest to use this decoded codeword not only to exchange data,but also to exchange sensing information. The concept of exchanging sensing information for ISAC has been studied in the signal processing literature under the paradigm of *collaborative sensing*.

In this sense, we introduce the concept of collaborative sensing for ISAC also to the information-theoretic literature, where we focus on the MAC and the related device-to-device (D2D) communication, i.e., the two-way channel. For the MAC, we naturally extend Willem's coding scheme so as to convey also state-information from one Tx to the other over the communication path that is built over the generalized feedback link. The proposed scheme can be considered as a separate source-channel coding scheme in the sense that each Tx first compresses the obtained outputs and inputs so as to extract state information, and then transmits the compression index using a pure channel code (here Willems' coding scheme) to the other Tx. The proposed scheme obtains a better sensing performance than a previous ISAC scheme [9] without collaborative sensing, and thus a better distortion-capacity tradeoff. For D2D communication, we present a similar collaborative sensing ISAC scheme based on source-channel separation and using Han's two-way channel scheme. Furthermore, we present an improved scheme that is based on joint source-channel coding (JSCC), more specifically on hybrid coding. We show enhanced performances of both simple collaborative sensing schemes. In both the MAC and the D2D scenario, the maximum rates achieved by our proposed scheme for given sensing distortions are strictly concave functions of the distortion pairs, and thus also improve over classical time- or resource-sharing strategies.

Recently, various other information-theoretic works have analyzed the fundamental limits of ISAC systems, such as [11]–[14]. For example, [14] analyzes systems with secrecy constraints, while [11]–[13] study channels that depend on a single fixed parameter and transmitters or sensor nodes wish to estimate this parameter based on backscatter signals. Their model is thus suited for scenarios where the estimation parameters change at a much slower time scale compared to the channel symbol period. Specifically, while in [12] sensing (parameter estimation) is performed at the transmitter, in [11] it is performed at a sensor that is close but not collocated with the transmitter. The study in [13] analyzes the detection-error exponents of open-loop and close-loop coding strategies.

Summary of Contributions and Outline of this Article:

- In Section II we introduce our information-theoretic ISAC MAC model with state-sensing at the Txs. We also show that it is of general nature and in particular can model scenarios with partial or perfect channel state information at the Rx as well as scenarios where the Txs wish to reconstruct functions or distorted versions of the actual state that is governing the channel.
- In Section III we describe our collaborative-sensing ISAC MAC scheme and show at hand of examples that it improves both over simple time-sharing as well as over previous schemes. Notice that our scheme does not employ Wyner-Ziv compression, but the equally strong *implicit binning technique*, as used for example in [15].
- Section IV describes our information-theoretic ISAC D2D model with state-sensing at both terminals. Again, we show that our model is rather general and includes scenarios with strictly-causal perfect or imperfect state-information at the terminals.
- In Section V we propose two collaborative-sensing ISAC D2D schemes. The first is based on a separate source-channel coding approach and the second on an improved JSCC approach using hybrid coding. In both schemes, the transmitted codeword carries not only data but also compression information that the other terminal can exploit for sensing. While the separation-based scheme employs Wyner-Ziv compression to account for the side-information at the other Tx, the JSCC based scheme uses implicity binning as in standard hybrid coding.

Notations: We use calligraphic letters to denote sets, e.g., $\mathcal{X}$. Random variables are denoted by uppercase letters, e.g., X, and their realizations by lowercase letters, e.g., x. For positive integers n, we use $[1:n]$ to denote the set $\{1, \cdots, n\}$, X^n for the tuple of random variables $(X_1, \cdots, X_n)$ and x^n for $(x_1, \ldots, x_n)$. We abbreviate *independent and identically distributed* as *i.i.d.* and *probability mass function* as *pmf*. Logarithms are taken with respect to base 2. We shall use $\mathcal{T}_\epsilon^N(P_{XY})$ to indicate the of strongly jointly-typical sequences $\{(x^n, y^n)\}$ with respect to the distribution P_{XY} as defined in [16]. For an index $k \in \{1, 2\}$, we define $\bar{k} := 3 - k$ and for an event $\mathcal{A}$ we denote its complement by $\bar{\mathcal{A}}$. Moreover, $\mathbb{1}\{\cdot\}$ denotes the indicator function.

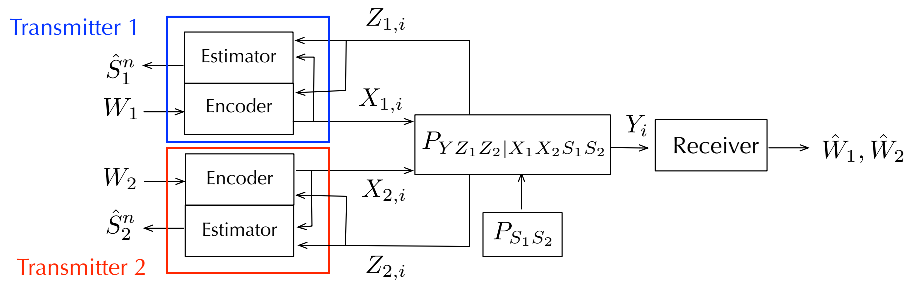

Fig. 1. State-dependent discrete memoryless multiaccess channel with sensing at the transmitters.

II. Two-User MAC with Generalized Feedback: System Model

In this section we consider the two-user multi-access channel (MAC) with generalized feedback, where two Txs wish to convey independent data to a common Rx and through the generalized feedback link they estimate the respective state sequences S_1^n and S_2^n governing the transition law over the MAC and the generalized feedback.

A. System Model

Consider the two-Tx single-Rx MAC scenario in Fig. 1. The model consists of a two-dimensional memoryless state sequence $\{(S_{1,i}, S_{2,i})\}_{i\geq 1}$ whose samples at any given time i are distributed according to a given joint law $P_{S_1S_2}$ over the state alphabets $\mathcal{S}_1 \times \mathcal{S}_2$. Given that at time-$i$ Tx 1 sends input $X_{1,i} = x_1$ and Tx 2 input $X_{2,i} = x_2$ and given state realizations $S_{1,i} = s_1$ and $S_{2,i} = s_2$, the Rx's time-i output Y_i and the Txs' feedback signals $Z_{1,i}$ and $Z_{2,i}$ are distributed according to the time-invariant channel transition law $P_{YZ_1Z_2|S_1S_2X_1X_2}(\cdot,\cdot,\cdot|s_1,s_2,x_1,x_2)$. Input and output alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}, \mathcal{Z}_1, \mathcal{Z}_2, \mathcal{S}_1, \mathcal{S}_2$ are assumed finite.[1] A $(2^{n\mathsf{R}_1}, 2^{n\mathsf{R}_2}, n)$-code consists of

1) two message sets $\mathcal{W}_1 = [1 : 2^{n\mathsf{R}_1}]$ and $\mathcal{W}_2 = [1 : 2^{n\mathsf{R}_2}]$;
2) a sequence of encoding functions $\Omega_{k,i} \colon \mathcal{W}_k \times \mathcal{Z}_k^{i-1} \to \mathcal{X}_k$, for $i = 1, 2, \ldots, n$ and $k = 1, 2$;
3) a decoding function $g \colon \mathcal{Y}^n \to \mathcal{W}_1 \times \mathcal{W}_2$;
4) for each $k = 1, 2$ a state estimator $\phi_k \colon \mathcal{X}_k^n \times \mathcal{Z}_k^n \to \hat{\mathcal{S}}_k^n$, where $\hat{\mathcal{S}}_1$ and $\hat{\mathcal{S}}_2$ are given reconstruction alphabets.

Fix a blocklength n, rates $R_1, R_2 \geq 0$, and a $(2^{nR_1}, 2^{nR_2}, n)$-code $(\{\Omega_{1,i}\}, \{\Omega_{2,i}\}, g, \phi_1, \phi_2)$. Let then the random message W_k be uniformly distributed over the message set $\mathcal{W}_k$, for each

[1] Notice that our results can also be extended to well-behaved continuous channels.

$k = 1, 2$, and the generate the inputs according to the encoding function $X_{k,i} = \Omega_{k,i}(W_k, Z_k^{i-1})$, for $i = 1, \ldots, n$. The Txs' state estimates are obtained as $\hat{S}_k^n := (\hat{S}_{k,1}, \cdots, \hat{S}_{k,n}) = \phi_k(X_k^n, Z_k^n)$ and the Rx's guess of the messages as $(\hat{W}_1, \hat{W}_2) = g(Y^n)$. We shall measure the quality of the state estimates $\hat{S}_k^n$ by bounded per-symbol distortion functions $d_k \colon \mathcal{S}_k \times \hat{\mathcal{S}}_k \mapsto [0, \infty)$, and consider *expected average block distortions*

$$\Delta_k^{(n)} := \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[d_k(S_{k,i}, \hat{S}_{k,i})], \quad k = 1, 2. \tag{1}$$

The probability of decoding error is defined as:

$$P_e^{(n)} := \Pr\Big(\hat{W}_1 \neq W_1 \quad \text{or} \quad \hat{W}_2 \neq W_2\Big). \tag{2}$$

Definition 1. *A rate-distortion tuple* $(\mathsf{R}_1, \mathsf{R}_2, \mathsf{D}_1, \mathsf{D}_2)$ *is achievable if there exists a sequence (in* n*) of* $(2^{n\mathsf{R}_1}, 2^{n\mathsf{R}_2}, n)$ *codes that simultaneously satisfy*

$$\lim_{n\to\infty} P_e^{(n)} = 0 \tag{3a}$$

$$\overline{\lim_{n\to\infty}} \, \Delta_k^{(n)} \leq \mathsf{D}_k, \quad \text{for } k = 1, 2. \tag{3b}$$

Definition 2. *The capacity-distortion region $\mathcal{CD}$ is the closure of the set of all achievable tuples* $(\mathsf{R}_1, \mathsf{R}_2, \mathsf{D}_1, \mathsf{D}_2)$.

Remark 1 (On the States)**.** *Notice that the general law $P_{S_1S_2}$ governing the states S_1^n and S_2^n allows to model various types of situations including scenarios where the state sequences are highly correlated (even identical) or scenarios where the state-sequences are independent.*

Our model also includes a scenario where the channel is governed by an internal i.i.d. state sequence S^n of pmf P_S and the states S_1^n, S_2^n are related to S^n over an independent memoryless channel $P_{S_1S_2|S}$. For example, the states S_1^n and S_2^n can be imperfect or noisy versions of the actual state sequence S^n. To see that this scenario can be included in our model, notice that since no terminal observes S^n nor attempts to reconstruct S^n, both the distortions and the error *probabilities only depend on the conditional law*

$$P_{YZ_1Z_2|X_1X_2S_1S_2}(y,z_1,z_2|x_1,x_2,s_1,s_2) = \sum_s P_{YZ_1Z_2|X_1X_2S}(y,z_1,z_2|x_1,x_2,s)\frac{P_S(s)P_{S_1S_2|S}(s_1,s_2|s)}{P_{S_1S_2}(s_1,s_2)}, \quad (4)$$

where $P_{S_1S_2}(s_1,s_2) = \sum_s P_S(s)P_{S_1S_2|S}(s_1,s_2|s)$ *denotes the joint pmf of the two states. Computing the channel law in* (4) *and plugging it into our results in the next section, thus immediately also provides results for the described setup where the actual state is* S^n *and the states* S_1^n *and* S_2^n *are noisy versions thereof.*

Remark 2 (State-Information)**.** *Our model also includes scenarios with perfect or imperfect state-information at the Rx. In fact, considering our model with an output* $Y = (T, Y')$ *where* Y' *denotes the actual MAC output and* T *the Rx's imperfect channel state-information about the states* S_1^n *and* S_2^n *. Notice that in our model, the Rx observes the state-information* T^n *only in a causal manner. Causality is however irrelevant here since the Rx only has to decode the messages at the end of the entire transmission. Therefore, plugging the choice* $Y = (T, Y')$ *into our results for* T *the Rx state-information and* Y' *the actual MAC output, our results in the following section directly lead to results for this related setup with Rx state-information.*

Remark 3 (The Relay-Channel)**.** *The MAC with generalized-feedback model includes the relay-channel as a special case. It suffices to restrict* $R_2 = 0$*, in which case Tx 2 degenerates to a relay terminal. The results we elaborate in the following section does immediately apply also to the relay channel.*

III. A Collaborative ISAC Scheme for the MAC

Before describing our collaborative ISAC scheme for the MAC, we review literature on the MAC and in particular Willem's scheme for the MAC with generalized feedback, which acts as a building block for our scheme.

While the capacity region of the MAC without feedback was determined in [17], [18], single-letter expressions for the capacity region are only known in special cases such as the two-user

Gaussian MAC with perfect feedback [19] or a class of semi-deterministic MACs [20] with one-sided perfect feedback. In [21], Kramer derived a multi-letter characterization of the capacity region of a general MAC with perfect feedback. For most channels it seems however challenging to evaluate this multi-letter characterization even numerically. In contrast, various inner and outer bounds on the capacity region of the MAC with generalized or perfect feedback are known. Outer bounds are typically based on the dependence balance bound idea by Hekstra and Willems [22], see also [23]. Various inner bounds were proposed based on schemes that each Tx decodes part of the data sent by the other Tx, which allows the two Txs to cooperatively resend these data parts in the next block using a more efficient coding scheme, see [22], [24]–[27]. The one most relevant to our work is Willems's inner bound [25], which we explain in more detail in the following subsection.

A. Willems' Coding Scheme with Generalized Feedback and the ISAC extension

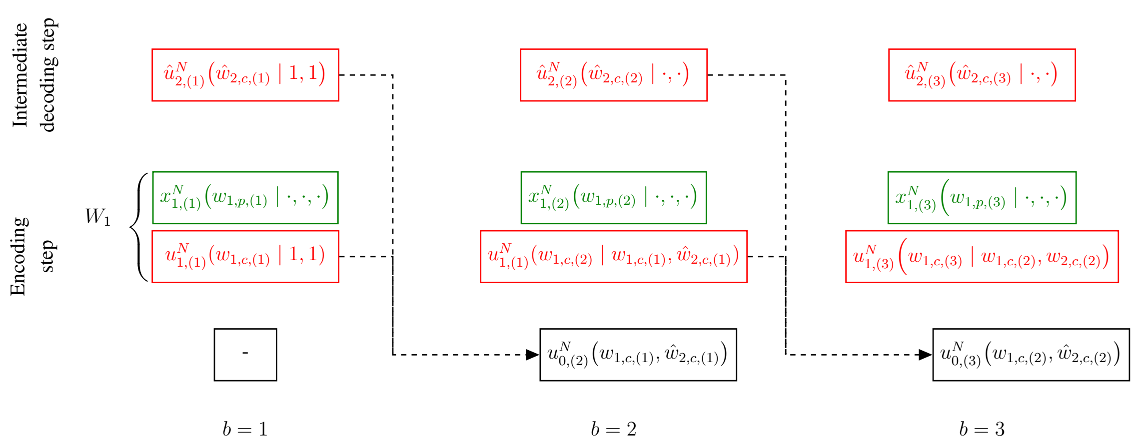

Fig. 2. Operations at Tx 1 in Willems' scheme during the first three blocks. After each block b Tx 1 decodes message $W_{2,c,(b)}$ based on its generalized feedback output $Z^N_{1,(b)}$. The decoded message is then retransmitted in block $b+1$ jointly with $W_{1,c,(b)}$.

Willems' scheme splits the blocklength n into $B+1$ blocks of length $N = n/(B+1)$ each. Accordingly, throughout, we let $X^N_{1,(b)}, X^N_{2,(b)}, S^N_{1,(b)}, S^N_{2,(b)}, Z^N_{1,(b)}, Z^N_{2,(b)}, Y^N_{(b)}$ denote the block-b inputs, states and outputs, e.g., $S^N_{1,(b)} := (S_{1(b-1)N+1}, \ldots, S_{1,bN})$. We also represent the two messages W_1 and W_2 in a one-to-one way as the $2B$-length tuples

$$W_k = (W_{k,c,(1)}, \ldots, W_{k,c,(B)}, W_{k,p,(1)}, \ldots, W_{k,p,(B)}), \qquad k \in \{1,2\}, \tag{5}$$

where all pairs $(W_{k,c,(b)}, W_{k,p,(b)})$ are independent and uniformly distributed over $\left[2^{N\bar{R}_{k,c}}\right] \times \left[2^{N\bar{R}_{k,p}}\right]$ for $\bar{R}_{k,c} \triangleq \frac{B+1}{B} R_{k,c}$ and $\bar{R}_{k,p} \triangleq \frac{B+1}{B} R_{k,p}$ and $R_{k,c} + R_{k,p} = R_k$.

An independent superposition code is constructed for each block b (see also Figure 2):

- A lowest-level code $\mathcal{C}_{0,(b)}$ consisting of $2^{N\bar{R}_{1,c}} \cdot 2^{N\bar{R}_{2,c}}$ codewords $u_{0,(b)}(w_{1,c}, w_{2,c})$ is constructed by drawing all entries i.i.d. according to a auxiliary pmf P_{U_0}.
- At the lowest level of encoding, we apply superposition coding to combine two codebooks $\{u^N_{k,(b)}(w'_{k,c} \mid w_{1,c}, w_{2,c})\}$ onto each codeword $u^N_{0,(b)}(w_{1,c}, w_{2,c})$, for $k \in \{1, 2\}$ and $w'_{k,c} \in [2^{NR_{k,c}}]$, by drawing the i-th entry of each codeword according to $P_{U_k|U_0}(\cdot \mid u_0)$ where u_0 denotes the i-th entry of $u^N_0(w_{1,c}, w_{2,c})$.
- For each second-layer codeword $u^N_{k,(b)}(w'_{k,c}|w_{1,c}, w_{2,c})$, we apply superposition coding by drawing the i-th entry of a codebook $x^N_{k,(b)}(w'_{k,p}|w'_{k,c}, w_{1,c}, w_{2,c})$ according to $P_{X_k|U_0U_k}(\cdot \mid u_0, u_k)$, where $k \in 1, 2$ and $w'_{k,p} \in [2^{NR_{k,p}}]$ and u_k represents the i-th entry of $u^N_{k,(b)}(w'_{k,c} \mid w_{1,c}, w_{2,c})$.

As depicted in Figure 2, in Willems' scheme, Tx 1 sends the following block-b channel inputs

$$x^N_{1,(b)} = x^N_{1,(b)}\left(W_{1,p,(b)} \middle| W_{1,c,(b)}, W_{1,c,(b-1)}, \hat{W}_{2,c,(b-1)}\right), \qquad b \in \{1, \ldots, B+1\}, \tag{6}$$

where $\hat{W}_{2,c,(b-1)}$ denotes the message part that Tx 1 decodes after reception of the block-$(b-1)$ generalized feedback signal $Z^N_{1,(b-1)}$, e.g., through a joint typicality decoding rule. Also, we set throughout $W_{k,c,(0)} = \hat{W}_{k,c,(0)} = W_{k,p,(B+1)} = 1$, for $k \in \{1, 2\}$.

Decoding at the Rx is performed backwards, starting with the last block $B+1$ based on which the Rx decodes the pair of common messages $(W_{1,c,(B)}, W_{2,c,(B)})$ using for example a joint-typicality decoder. It then uses knowledge of these common messages and the outputs in block B to decode the block-B private messages $(W_{1,p,(B)}, W_{2,p,(B)})$ and the block $(B-1)$ common messages $(W_{1,c,(B-1)}, W_{2,c,(B-1)})$, etc. The backward decoding procedure is also depicted in Figure 3.

As Willems showed, his scheme can achieve the following rate-region.

Theorem 1 (Willems' Achievable Region [25])**.** *Any nonnegative rate-pair* (R_1, R_2) *is achievable*

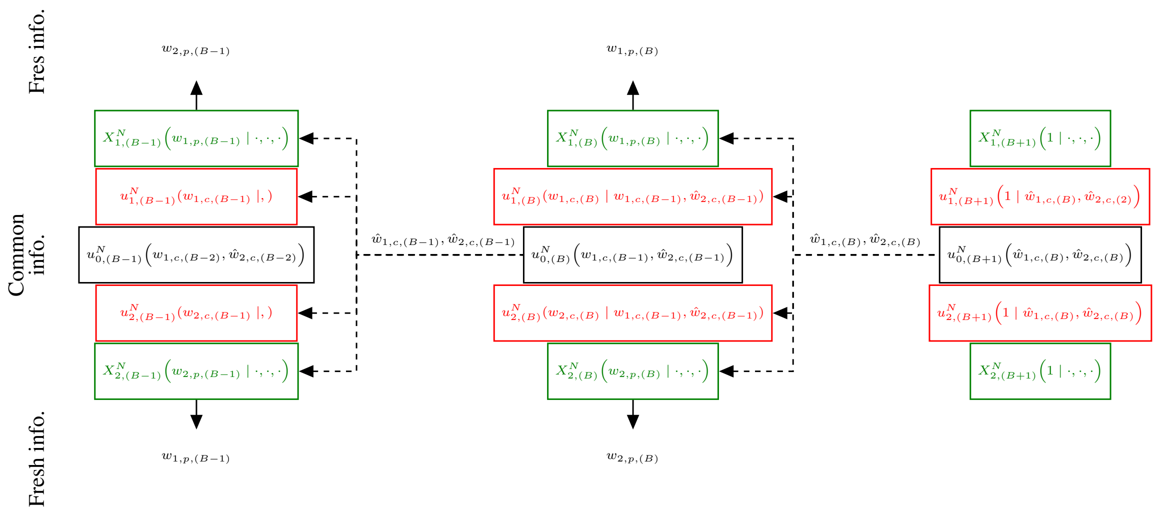

Fig. 3. Backward decoding procedure at the Rx in Willems' scheme. The pair of common messages $(W_{1,c,(b-1)}, W_{2,c,(b-1)})$ and private messages $(W_{1,p,(b)}, W_{2,p,(b)})$ are jointly decoded based on the block-b outputs $Y_{(b)}^N$ and using the previously decoded $(\hat{W}_{1,c,(b)}, \hat{W}_{2,c,(b)})$.

over the MAC with generalized feedback if it satisfies the following inequalities

$$R_k \leq I(X_k; Y \mid X_{\bar{k}} U_k U_0) + I(U_k; Z_{\bar{k}} \mid X_{\bar{k}} U_0), \qquad k \in \{1, 2\}, \tag{7}$$

$$R_1 + R_2 \leq I(X_1 X_2; Y), \tag{8}$$

$$R_1 + R_2 \leq I(X_1 X_2; Y \mid U_0 U_1 U_2) + I(U_1; Z_2 \mid X_2 U_0) + I(U_2; Z_1 \mid X_1 U_0), \tag{9}$$

for some choice of pmfs $P_{U_0}, P_{U_1|U_0}, P_{U_2|U_0}, P_{X_1|U_0U_1}, P_{X_2|U_0U_2}$*, and where above mutual informations are calculated according to the pmf* $P_{U_0}P_{U_1|U_0}P_{U_2|U_0}P_{X_1|U_0U_1}P_{X_2|U_0U_2}P_{S_1S_2}$ $P_{YZ_1Z_2|S_1S_2X_1X_2}$*. One hereby can restrict to auxiliary variables over alphabets of sizes* $|\mathcal{U}_k| \leq (|\mathcal{X}_k| + 1)|\mathcal{U}_0|$*, for* $k = 1, 2$*, and* $|\mathcal{U}_0| \leq |\mathcal{X}_1||\mathcal{X}_2| + 1$*.*

Kobayashi et al. [9] extended Willems' scheme to a ISAC scenario by adding a state estimator at the two Txs. Specifically, for any block b each Tx k applies the symbol-per-symbol estimation

$$\hat{s}_{k,(b)}^N = \tilde{\phi}_k^{*\otimes N}\left(x_{k,(b)}^N, z_{k,(b)}^N, u_{\bar{k},(b)}^N\left(W_{\bar{k},c,(b)} \;\middle|\; W_{k,c,(b-1)}, \hat{W}_{\bar{k},c,(b-1)}\right)\right), \qquad b \in \{1, \ldots, B\}, \tag{10}$$

where $\tilde{\phi}_k^*$ denotes the optimal estimator of S_k based on the tuple $(X_k, Z_k, U_{\bar{k}})$:

$$\tilde{\phi}_k^*(x_k, z_k, u_{\bar{k}}) := \arg\min_{s'_k \in \hat{\mathcal{S}}_k} \sum_{s_k \in \mathcal{S}_k} P_{S_k|X_k Z_k U_{\bar{k}}}(s_k|x_k, z_k, u_{\bar{k}})\, d_k(s_k, s'_k). \tag{11}$$

Thus, any of the two Txs bases its state-estimation not only on its inputs and outputs of a given block but also on the codeword that it decoded from the other Tx.

For the last block $B+1$, Tx k can produce any trivial estimate, e.g., $\hat{s}^N_{k,(B+1)}$ because its influence on the average distortion vanishes as the number of blocks grows, $B \to \infty$.

Combining the described state-estimation with Willems' scheme, the following rate-distortion region can be shown to be achievable.

Theorem 2. *[Kobayashi et al.'s ISAC region [9]] A rate-distortion tuple (R_1, R_2, D_1, D_2) is achievable if it satisfies (7)–(9) and*

$$\mathrm{E}\Big[d_k\left(S_k, \tilde{\phi}_k^*\left(X_k, Z_k, U_{\bar{k}}\right)\right)\Big] \leq D_k, \quad k = 1, 2, \tag{12}$$

for some choice of pmfs $P_{U_0}, P_{U_1|U_0}, P_{U_2|U_0}, P_{X_1|U_1U_0}, P_{X_2|U_2U_0}$.

B. Our Proposed Collaborative ISAC Scheme

We present our collaborative ISAC scheme. It extends the scheme in [9] in that the second-layer codeword of Willems' code construction is not only used to transmit data but also compression information useful for state sensing. Each Tx generates compression information, which is primarily intended to be used by the other Tx to improve its sensing performance. In our scheme, the Rx however also decodes this information and uses it to improve its decoding performance.

1) Code construction: Choose pmfs $P_{U_0}, P_{U_1|U_0} P_{U_2|U_0}, P_{X_1|U_1U_0}, P_{X_2|U_2U_0}$, and define the pmf

$$P_{U_0U_1U_2X_1X_2S_1S_2YZ_1Z_2V_1V_2} = P_{U_0}P_{U_1|U_0}P_{U_2|U_0}P_{X_1|U_1U_0}P_{X_2|U_2U_0}P_{S_1S_2}P_{YZ_1Z_2|X_1X_2S_1S_2} P_{V_1|X_1Z_1U_2U_0}P_{V_2|X_2Z_2U_1U_0}. \tag{13}$$

Employ Willems' three-level superposition code construction for the given choice of pmfs, except that each second-layer codeword is indexed by a pair of indices. We thus denote the second-layer codewords by $u^N_{k,(b)}(w'_{1,c}, j_1 \mid w_{1,c}, w_{2,c})$ and $u^N_{2,(b)}(w'_{2,c}, j_2 \mid w_{1,c}, w_{2,c})$

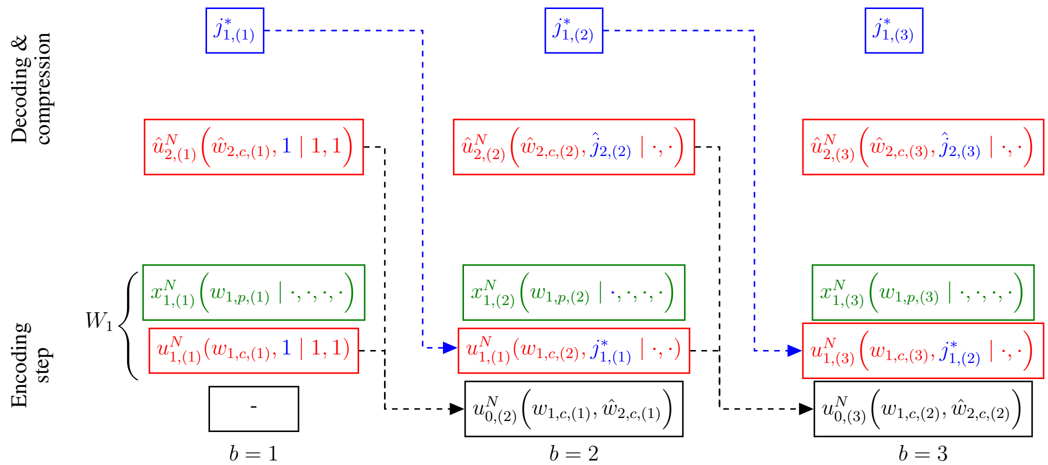

Fig. 4. Our proposed scheme at Tx 1 during the firts three blocks

and accordingly the corresponding third-layer codewords by $x^N_{1,(b)}(w'_{1,p}|w'_{1,c}, j_1, w_{1,c}, w_{2,c})$ and $x^N_{1,(b)}(w'_{2,p}|w'_{2,c}, j_2, w_{1,c}, w_{2,c})$, where the indices j_1 and j_2 take value in the sets $[2^{nR'_1}]$ and $[2^{nR'_2}]$ for some positive auxiliary rates $R_{1,v}$ and $R_{2,v}$.

We further construct a compression codebook for each block and each of the two Txs, For each $b \in \{1, \dots, B\}$ and each sixtuple $(w_{1,c}, w_{2,c}, w'_{1,c}, j_1 w'_{2,c}, j_2) \in [2^{NR_{1,c}}] \times [2^{NR_{2,c}}] \times [2^{NR_{1,c}}] \times [2^{NR_{1,v}}] \times [2^{NR_{2,c}}] \times [2^{NR_{2,v}}]$ we generate a sequence $v^N_{1,(b)}(j'_1 \mid w'_{1,c}, j_1, w'_{2,c}, j_2, w_{1,c}, w_{2,c})$ for each $j'_1 \in [2^{NR_{1,v}}]$ and a sequence $v^N_{2,(b)}(j'_2 \mid w'_{1,c}, j_1, w'_{2,c}, j_2, w_{1,c}, w_{2,c})$ for each $j'_2 \in [2^{NR_{2,v}}]$. The sequences $v^N_{1,(b)}(j'_1 \mid w'_{1,c}, j_1, w'_{2,c}, j_2, w_{1,c}, w_{2,c})$ and $v^N_{2,(b)}(j'_2 \mid w'_{1,c}, j_1, w'_{2,c}, j_2, w_{1,c}, w_{2,c})$ are obtained by drawing their i-th entries according to $P_{V_1|U_0U_1U_2}(\cdot \mid u_0, u_1, u_2)$ and $P_{V_2|U_0U_1U_2}(\cdot \mid u_0, u_1, u_2)$, respectively, for u_0, u_1, u_2 denoting the i-th entries of the sequences $u^N_{0,(b)}(w_{1,c}, w_{2,c})$, $u^N_{1,(b)}(w'_{1,c}, j_1 \mid w_{1,c}, w_{2,c})$, and $u^N_{2,(b)}(w'_{2,c}, j_2 \mid w_{1,c}, w_{2,c})$.

2) Operations at the Txs: In each block b, Tx k sends the block-b sequence

$$X^N_{k,(b)} = x^N_{k,(b)}\left(W_{k,p,(b)} \,\middle|\, W_{k,c,(b)}, J^*_{k,(b-1)}, W_{k,c,(b-1)}, \hat{W}^{(k)}_{\bar{k},c,(b-1)}\right), \tag{14}$$

where Tx k generates the indices $J^*_{k,(b-1)}$ and $\hat{W}_{\bar{k},c,(b-1)}$ during a joint decoding and compression step at the end of block $b-1$ as follows. (For convenience we again set $W_{k,p,(B+1)} = W_{k,c,(0)} = \hat{W}^k_{\bar{k},c,(0)} = J^{\bar{k}}_{k,(B+1)} = 1$.)

After receiving the generalized feedback signal $Z^N_{k,(b-1)}$, Tx k looks for a triple of indices j^*_k,

$$\Big(u_{0,(b-1)}^N\Big(W_{1,c,(b-2)},\hat{W}_{2,c,(b-2)}^{(1)}\Big),\; u_{1,(b-1)}^N\Big(W_{1,c,(b-1)},J_{1,(b-2)}^*\;\Big|\;W_{1,c,(b-2)},\hat{W}_{2,c,(b-2)}^{(1)}\Big)$$
$$u_{2,(b-1)}^N\Big(\hat{w}_2,\hat{j}_2\;\Big|\;W_{1,c,(b-2)},\hat{W}_{2,c,(b-2)}^{(1)}\Big),$$
$$x_{1,(b-1)}^N\Big(W_{1,p,(b-1)}\;\Big|\;W_{1,c,(b-1)},J_{1,(b-2)}^*,W_{1,c,(b-2)},\hat{W}_{2,c,(b-2)}^{(1)}\Big),$$
$$v_{1,(b-1)}^N\Big(j_1^*\;\Big|\;J_{1,(b-2)}^*,W_{1,c,(b-1)},\hat{w}_2,\hat{j}_2,W_{1,c,(b-2)},\hat{W}_{2,c,(b-2)}^{(1)}\Big),Z_{1,(b-1)}^N\Big)\in\mathcal{T}_\epsilon^N(P_{U_0U_1U_2X_1V_1Z_1}) \tag{15}$$

$$\Big(u_{0,(b-2)}^N\Big(W_{1,c,(b-3)},\hat{W}_{2,c,(b-3)}^{(1)}\Big),\; u_{1,(b-2)}^N\Big(W_{1,c,(b-2)},J_{1,(b-2)}^*\;\Big|\;W_{1,c,(b-3)},\hat{W}_{2,c,(b-3)}^{(1)}\Big),$$
$$u_{2,(b-2)}^N\Big(\hat{W}_{2,c,(b-2)}^{(1)},\hat{J}_{2,(b-3)}^{(1)}\;\Big|\;W_{1,c,(b-3)},\hat{W}_{2,c,(b-3)}^{(1)}\Big),$$
$$x_{1,(b-2)}^N\Big(W_{1,p,(b-2)}\;\Big|\;W_{1,c,(b-2)},J_{1,(b-3)}^*,W_{1,c,(b-3)},\hat{W}_{2,c,(b-3)}^{(1)}\Big),$$
$$v_{2,(b-2)}^N\Big(\hat{j}_2\;\Big|\;W_{1,c,(b-2)},J_{1,(b-3)}^*,\hat{W}_{2,c,(b-2)}^{(1)},\hat{J}_{2,(b-3)}^{(1)},W_{1,c,(b-3)},\hat{W}_{2,c,(b-3)}^{(1)}\Big),$$
$$Z_{1,(b-2)}^N\Big)\in\mathcal{T}_\epsilon^N(P_{U_0U_1U_2X_1V_2Z_1}). \tag{16}$$

$\hat{w}_{\bar{k}}$, and $\hat{j}_{\bar{k}}$ satisfying the joint typicality check (15),

and if $b>2$ also the typicality check (16), which are displayed on top of the page. It randomly picks one of these triples and sets

$$J_{1,(b-1)}^*=j_1^*,\qquad \hat{W}_{2,(b-1)}^{(1)}=\hat{w}_2,\qquad \hat{J}_{2,(b-2)}^{(1)}=\hat{j}_2. \tag{17}$$

Tx k also produces the block-b state estimate

$$\hat{s}_{k,(b)}^n=\phi_k^{*\otimes N}\Big(x_{k,(b)}^N\left(W_{k,p,(b)}|W_{k,c,(b)},J_{k,(b-1)},W_{k,c,(b-1)},\hat{W}_{\bar{k},c,(b-1)}^{(k)}\right),$$
$$z_{k,(b)}^N,u_{\bar{k},(b)}^N\left(W_{k,c,(b)}\mid J_{k,(b-1)},W_{k,c,(b-1)},\hat{W}_{\bar{k},c,(b-1)}^{(k)}\right),$$
$$v_{\bar{k},(b)}^N\left(J_{k,(b)}\mid W_{k,c,(b)},J_{k,(b-1)},W_{k,c,(b-1)},\hat{W}_{\bar{k},c,(b-1)}^{(k)}\right)\Big) \tag{18}$$

where

$$\phi_k^*(x_k,z_k,u_{\bar{k}},v_{\bar{k}}):=\arg\min_{s_k'\in\hat{\mathcal{S}}_k}\sum_{s_k\in\mathcal{S}_k}P_{S_k|X_kZ_kU_{\bar{k}}V_{\bar{k}}}(s_k|x_k,z_k,u_{\bar{k}},v_{\bar{k}})\;d_k(s_k,s_k'). \tag{19}$$

Without loss in performance as $B\to\infty$, the estimate in the last block $B+1$ can again be set to a dummy sequence.

3) Decoding at the Rx: Decoding at the Rx is similar to Willems' scheme and uses backward decoding. The difference is that the Rx in block b not only decodes the message tuple $(W_{1,p,(b)}, W_{2,p,(b)}, W_{1,c,(b-1)}, W_{2,c,(b-1)})$ but also the compression indices $J^*_{1,(b-1)}$ and $J^*_{2,(b-2)}$. Specifically, in a generic block $b \in \{2, \ldots, B\}$, the Rx looks for a unique sixtuple $(w_{1,p}, w_{2,p}, w_{1,c}, w_{2,c}, j_1, j_2) \in [2^{NR_{1,p}}] \times [2^{NR_{2,p}}] \times [2^{NR_{1,c}}] \times [2^{NR_{2,c}}] \times [2^{NR_{1,v}}] \times [2^{NR_{2,v}}]$ satisfying

$$\Big(u^N_{0,b}(w_{1,c}, w_{2,c}),\ u^N_{1,(b)}\Big(\hat{W}_{1,c,(b)}, j_1 \,\Big|\, w_{1,c}, w_{2,c}\Big),\ u^N_{2,(b)}\Big(\hat{W}_{2,c,(b)}, j_2 \,\Big|\, w_{1,c}, w_{2,c}\Big),$$
$$x^N_{1,(b)}\Big(w_{1,p} \,\Big|\, \hat{W}_{1,c,(b)}, j_1, w_{1,c}, w_{2,c}\Big),\ x^N_{2,(b)}\Big(w_{2,p} \,\Big|\, \hat{W}_{2,c,(b)}, j_2, w_{1,c}, w_{2,c}\Big),$$
$$v^N_{1,(b)}\left(\hat{J}_{1,(b)} \,\Big|\, \hat{W}_{1,c,(b)}, j_1, \hat{W}_{2,c,(b)}, j_2, w_{1,c}, w_{2,c}\right),$$
$$v^N_{2,(b)}\left(\hat{J}_{2,(b)} \,\Big|\, \hat{W}_{1,c,(b)}, j_1, \hat{W}_{2,c,(b)}, j_2, w_{1,c}, w_{2,c}\right),\ Y^N_{(b)}\Big) \in \mathcal{T}^N_{2\epsilon}(P_{U_0U_1U_2X_1X_2Y}) \tag{20}$$

If such a unique sixtuple exists, it sets $\hat{W}_{1,c,(b-1)} = w_{1,c}$, $\hat{W}_{1,p,(b)} = w_{1,p}$, $\hat{W}_{2,c,(b-1)} = w_{2,c}$, $\hat{W}_{2,p,(b)} = w_{2,p}$, $\hat{J}_{1,(b-1)} = j_1$, and $\hat{J}_{2,(b-1)} = j_2$. Otherwise it declares an error.

The Rx finally declares the messages $\hat{W}_1$ and $\hat{W}_2$ that correspond to the produced guesses $\{(\hat{W}_{k,p,(b)}, \hat{W}_{k,c,(b)})\}$.

In [28] we show that as $N \to \infty$ and $B \to \infty$, the described scheme achieves vanishing probabilities of error, the compressions are successful with probability 1, and the asymptotic expected distortions are bounded by D_1 and D_2 whenever B is sufficiently large and

$$R_{k,v} > I(V_k; X_k Z_k \mid \underline{U}) \tag{21a}$$

$$R_{\bar{k},v} + R_{k,c} < I(U_k V_{\bar{k}}; X_{\bar{k}} Z_{\bar{k}} \mid U_0 U_{\bar{k}}) \tag{21b}$$

$$R_{1,v} + R_{2,v} + R_{k,c} < I(U_k V_{\bar{k}}; X_{\bar{k}} Z_{\bar{k}} \mid U_0 U_{\bar{k}}) + I(V_k; X_{\bar{k}} Z_{\bar{k}} \mid \underline{U}) \tag{21c}$$

$$R_{k,p} < I(X_k; Y V_1 V_2 \mid \underline{U} X_{\bar{k}}) \tag{21d}$$

$$R_{k,v} + R_{k,p} < I(X_k; Y \mid U_0 X_{\bar{k}}) + I(V_2; X_1 X_2 Y V_1 \mid \underline{U}) + I(V_1; X_1 X_2 Y \mid \underline{U}) \tag{21e}$$

$$R_{k,v} + R_{k,p} + R_{\bar{k},p} < I(X_1 X_2; Y \mid U_0 U_{\bar{k}}) + I(V_2; X_1 X_2 Y V_1 \mid \underline{U}) + I(V_1; X_1 X_2 Y \mid \underline{U}) \tag{21f}$$

$$R_{1,p} + R_{2,p} < I(X_1 X_2; Y V_1 V_2 \mid \underline{U}) \tag{21g}$$

$$R_{1,v} + R_{1,p} + R_{2,v} + R_{2,p} < I(X_1X_2; Y \mid U_0) + I(V_1; X_1X_2Y \mid \underline{U}) + I(V_2; X_1X_2YV_1 \mid \underline{U}) \tag{21h}$$

$$R_{1,v} + R_1 + R_{2,v} + R_2 < I(X_1X_2; Y) + I(V_1; X_1X_2Y \mid \underline{U}) + I(V_2; X_1X_2YV_1 \mid \underline{U}), \tag{21i}$$

where $\underline{U} \triangleq (U_0, U_1, U_2)$ and

$$\mathrm{E}[d_k(S_k, \phi_k^*(X_k, Z_k, U_{\bar{k}}, V_{\bar{k}})] \leq D_k, \quad k = 1, 2, \tag{21j}$$

for ϕ_k^* defined in (19).

Using the Fourier-Motzkin Elimination (FME) algorithm it can be shown, see [28], that such a choice of rates is possible under the rate-constraints (22).

Theorem 3. *The capacity-distortion region $\mathcal{CD}$ includes any rate-distortion tuple (R_1, R_2, D_1, D_2) that for some choice of pmfs $P_{U_0}, P_{U_1|U_0}, P_{U_2|U_0}, P_{X_1|U_0U_1}, P_{X_2|U_0U_2}, P_{V_1|U_0U_2X_1Z_1}, P_{V_2|U_0U_1X_2Z_2}$ and pmf $P_{U_0U_1U_2X_1X_2S_1S_2YZ_1Z_2V_1V_2}$ as defined in (13), satisfies Inequalities (22) on top of the next page (where $\underline{U} := (U_0, U_1, U_2)$) as well as the distortion constraints (21j). It suffices to consider auxiliary random variables with alphabets of sizes $|\mathcal{U}_0| \leq |\mathcal{X}_1||\mathcal{X}_2| + 9$ and for $k = 1, 2$: $|\mathcal{U}_k| \leq (|\mathcal{X}_k| + 9)|\mathcal{U}_0|$ and $|\mathcal{V}_k| \leq (|\mathcal{X}_k||\mathcal{Z}_k||\mathcal{U}_{\bar{k}}||\mathcal{U}_0| + 9)$.*

Notice that Theorem 3 recovers the previous achievable region in Theorem 2 through the choice $V_1 = V_2 =$constants, which removes the collaborative sensing between the two Txs.

Remark 4 (Wyner-Ziv Coding)**.** *In our scheme, no binning as in Wyner-Ziv coding is used for the compression of the V_1- and V_2-codewords. Instead, decoder side-information is taken into account through the additional typicality check* (16) *and by including the V_1- and V_2-codewords in the typicality check* (20)*. These strategies are known as implicit binning and allow multiple decoders to exploit different levels of side-information, see [15].*

C. Examples

The following two examples show the improvement of Theorem 3 over Theorem 2. In the first example, one of the transmitting nodes directly receives state information through its feedback, and allows to easily illustrate the concept of collaborative sensing. The second example presents

$$
\begin{aligned}
R_k \leq\ & I(U_k;X_{\bar{k}}Z_{\bar{k}} \mid U_0U_{\bar{k}}) + I(V_k;X_{\bar{k}}Z_{\bar{k}} \mid \underline{U}) - I(V_k;X_kZ_k \mid \underline{U}) + \min\{\\
& I(X_k;Y \mid U_0X_{\bar{k}}) + I(V_k;X_1X_2Y \mid \underline{U}) + I(V_{\bar{k}};X_1X_2YV_k \mid \underline{U})\\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -I(V_k;X_kZ_k \mid \underline{U}),\\
& I(X_1X_2;Y \mid U_0U_k) + I(V_k;X_1X_2Y \mid \underline{U}) + I(V_{\bar{k}};X_1X_2YV_k \mid \underline{U})\\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -I(V_{\bar{k}};X_{\bar{k}}Z_{\bar{k}} \mid \underline{U}),\\
& I(X_1X_2;Y \mid U_0) + I(V_k;X_1X_2Y \mid \underline{U}) + I(V_{\bar{k}};X_1X_2YV_k \mid \underline{U})\\
& \quad -I(V_k;X_kZ_k \mid \underline{U}) - I(V_{\bar{k}};X_{\bar{k}}Z_{\bar{k}} \mid \underline{U}),\ I(X_k;YV_1V_2 \mid \underline{U}X_{\bar{k}})\},\ k=1,2,
\end{aligned} \tag{22a}
$$

$$
\begin{aligned}
R_1+R_2 \leq\ & I(U_2;X_1Z_1 \mid U_0U_1) + I(V_2;X_1Z_1 \mid \underline{U}) - I(V_2;X_2Z_2 \mid \underline{U})\\
& +I(U_1;X_2Z_2 \mid U_0U_2) + I(V_1;X_2Z_2 \mid \underline{U}) - I(V_1;X_1Z_1 \mid \underline{U}) + \min\{\\
& I(X_1X_2;Y \mid U_0U_2) + I(V_1;X_1X_2Y \mid \underline{U}) + I(V_2;X_1X_2YV_1 \mid \underline{U}) - I(V_1;X_1Z_1 \mid \underline{U}),\\
& I(X_1X_2;Y \mid U_0U_1) + I(V_1;X_1X_2Y \mid \underline{U}) + I(V_2;X_1X_2YV_1 \mid \underline{U}) - I(V_2;X_2Z_2 \mid \underline{U}),\\
& I(X_1X_2;Y \mid U_0) + I(V_1;X_1X_2Y \mid \underline{U}) + I(V_2;X_1X_2YV_1 \mid \underline{U})\\
& \qquad\qquad\qquad\qquad\qquad\qquad -I(V_1;X_1Z_1 \mid \underline{U}) - I(V_2;X_2Z_2 \mid \underline{U}),\\
& I(X_1X_2;YV_1V_2 \mid \underline{U})\}
\end{aligned} \tag{22b}
$$

$$
\begin{aligned}
R_1+R_2 \leq\ & I(X_1X_2;Y) + I(V_1;X_1X_2Y \mid \underline{U}) - I(V_1;X_1Z_1 \mid \underline{U})\\
& \qquad\qquad\qquad +I(V_2;X_1X_2YV_1 \mid \underline{U}) - I(V_2;X_2Z_2 \mid \underline{U})
\end{aligned} \tag{22c}
$$

and for $k=1,2$

$$
I(U_k;X_{\bar{k}}Z_{\bar{k}} \mid U_0U_{\bar{k}}) + I(V_k;X_{\bar{k}}Z_{\bar{k}} \mid \underline{U}) \geq I(V_k;X_kZ_k \mid \underline{U}), \tag{22d}
$$

$$
\begin{aligned}
I(X_1X_2;Y \mid U_0) + I(V_1;X_1X_2Y \mid \underline{U}) + I(V_2;X_1X_2YV_1 \mid \underline{U}) \geq\ & I(V_1;X_1Z_1 \mid \underline{U})\\
& +I(V_2;X_2Z_2 \mid \underline{U})
\end{aligned} \tag{22e}
$$

$$
I(X_k;Y \mid U_0X_{\bar{k}}) + I(V_1;X_1X_2Y \mid \underline{U}) + I(V_2;X_1X_2YV_1 \mid \underline{U}) \geq I(V_k;X_kZ_k \mid \underline{U}). \tag{22f}
$$

---

a more realistic model, and provides a more practical implementation of our collaborative sensing scheme.

Example 1. *Consider a MAC with binary input, output, and state alphabets* $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y} = \mathcal{S}_2 = \{0,1\}$. *State* $S_2 \sim Ber(p_s)$, *while* $S_1 = 0$ *is a constant. The channel input-output relation is described by*

$$
Y = S_2X_2, \qquad (Z_1,Z_2) = (S_2,X_1). \tag{23}
$$

For this channel, the tuple $(\mathsf{R}_1,\mathsf{R}_2,\mathsf{D}_1,\mathsf{D}_2) = (0,0,0,0)$ *lies in the achievable region of Theorem 3 through the choice* $V_1 = Z_1 = S_2$ *and* $(\hat{S}_2 = V_1, \hat{S}_1 = 0)$. *Distortion* $\mathsf{D}_2 = 0$ *is* *however not achievable in Theorem 2 because* S_2 *is independent of* $(U_1, U_2, U_0, X_1, X_2)$ *and thus of* (X_2, U_1, Z_2)*, and the optimal estimator is the trivial estimator* $\hat{S}_2 = \psi_2^*(X_2, Z_2, U_1) = \mathbb{1}\{p_s > 1/2)$ *which achieves distortion* $\mathsf{D}_2 = \min\{1 - p_s, p_s\}$.

Example 2. *Consider binary noise, states and channel inputs* $B_0, B_k, S_k, X_k \in \{0, 1\}$*. The noise to the receiver* B_0 *is Bernoulli-*t_0*, and* B_k*, the noise on the feedback to Tx* k*, is Bernoulli-*t_k*. All noises are independent and also independent of the states* S_1, S_2*, which are i.i.d. Bernoulli-*p_s*. We can then des are described the channel as*

$$Y' = S_1X_1 + S_2X_2 + B_0, \qquad Y = (Y', S_1, S_2), \tag{24}$$

$$Z_1 = S_1X_1 + S_2X_2 + B_1, \qquad Z_2 = S_1X_1 + S_2X_2 + B_2. \tag{25}$$

where the summation operators '$+$*' denote real additions. In this example the Rx thus has perfect channel state-information, see also Remark 2. Hamming distance is considered as a distortion measure:* $d(s, \hat{s}) = s \oplus \hat{s}$ *where the operator '*$\oplus$*' is a binary operation representing module-2 addition.*

We compare Theorems 2 and 3 on the following choices of random variables. Let

$$X_k = \underbrace{U_0 \oplus \Sigma}_{\triangleq U_1} \oplus \theta_k, \quad \text{for } k \in \{1, 2\} \tag{26}$$

where $U_0, \Sigma_1\Sigma_2, \theta_1, \theta_2$ *are all independent Bernoulli random variables of parameters* p, q_1, q_2, r_1, r_2*. For the evaluation of Theorem 3 we further choose the compression random variables*

$$V_k = \begin{cases} \mathbb{1}\{Z_k = 1\} + 2 \cdot \mathbb{1}\{Z_k = 2\} & \text{if } E_k = 0 \\ \text{``?''} & \text{if } E_k = 1 \end{cases} \qquad \forall k = \{1, 2\} \tag{27}$$

for a binary E_k *independent of* $(S_1, S_2, B_0, B_1, B_2, U_0, U_1, U_2, \Sigma_1, \Sigma_2, \theta_1, \theta_2)$*. For this choice, Tx* k *conveys information about* Z_k *to Tx* $\bar{k}$*, which helps this latter to better estimate its state* $S_{\bar{k}}$*. For instance, when* $E_1 = 0$*, Tx-2 receives another noisy observation of the output which helps* *it to better estimate its state, because*

$$Y = \begin{cases} 0 & \text{if } Z_2 \in \{0,1\}, V_1 = 0 \\ 1 & \text{if } V_1 = 1 \\ 2 & \text{if } Z_2 \in \{2,3\}, V_1 = 0 \end{cases} . \tag{28}$$

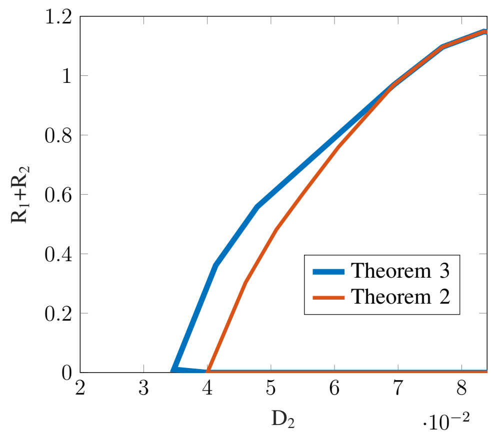

Fig. 5. Sum-rate distortion tradeoff achieved by Theorems 2 and 3 in Example 2 for given channel parameters $p_s = 0.9$, $t_0 = 0.3$, $t_1 = 0.1$ and $t_2 = 0.1$.

For channel parameters $p_s = 0.9$, $t_0 = 0.3$, $t_1 = 0.1$ and $t_2 = 0.1$ and above choices of random variables, Figure 5 shows the maximum sum-rate $R_1 + R_2$ in function of distortion D_2 achieved by Theorems 3 and 2, where recall that for the region in Theorem 2 we set $V_1 = V_2 = 0$. Notice that both curves are strictly concave and thus improve over classic time- and resource sharing strategies. The minimum distortions achieved by Theorems 3 and 2 are $D_{2,\min} = 0.035$ and $D_{2,\min} = 0.04$.

IV. Device-to-Device Communication (The Two-Way Channel)

In this section, we consider the ISAC two-way channel, where two terminals exchange data over a common channel and based on their inputs and outputs also wish to estimate the state-sequences that govern the two-way channel.

A. System Model

Consider the two-terminal two-way communication scenario in Fig. 6. The model consists of a two-dimensional memoryless state sequence $\{(S_{1,i}, S_{2,i})\}_{i\geq 1}$ whose samples at any given time i are distributed according to a given joint law $P_{S_1 S_2}$ over the state alphabets $\mathcal{S}_1 \times \mathcal{S}_2$. Given that at time-$i$ Tx 1 sends input $X_{1,i} = x_1$ and Tx 2 input $X_{2,i} = x_2$ and given state realizations $S_{1,i} = s_1$ and $S_{2,i} = s_2$, the Txs' time-i feedback signals $Z_{1,i}$ and $Z_{2,i}$ are distributed according to the stationary channel transition law $P_{Z_1 Z_2|S_1 S_2 X_1 X_2}(\cdot, \cdot|s_1, s_2, x_1, x_2)$. Input and output alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}, \mathcal{Z}_1, \mathcal{Z}_2, \mathcal{S}_1, \mathcal{S}_2$ are assumed finite.[2]

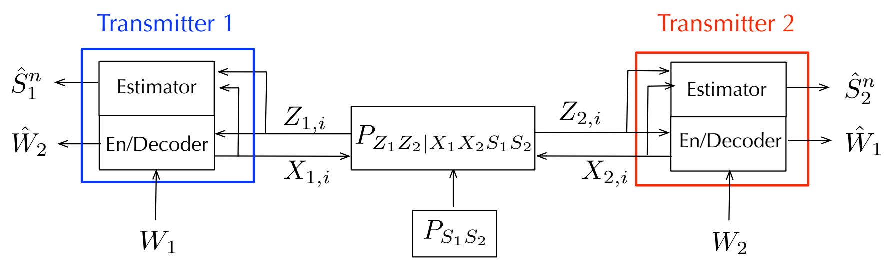

Fig. 6. State-dependent discrete memoryless two-way channel with sensing at the terminals.

A $(2^{n\mathsf{R}_1}, 2^{n\mathsf{R}_2}, n)$ code consists of

1) two message sets $\mathcal{W}_1 = [1 : 2^{n\mathsf{R}_1}]$ and $\mathcal{W}_2 = [1 : 2^{n\mathsf{R}_2}]$;
2) sequences of encoding functions $\Omega_{k,i} \colon \mathcal{W}_k \times \mathcal{Z}_k^{i-1} \to \mathcal{X}_k$, for $i = 1, 2, \ldots, n$ and $k = 1, 2$;
3) decoding functions $g_k \colon \mathcal{Z}^n \to \mathcal{W}_k$, for $k = 1, 2$;
4) state estimators $\phi_k \colon \mathcal{X}_k^n \times \mathcal{Z}_k^n \to \hat{\mathcal{S}}_k^n$, for $k = 1, 2$, where $\hat{\mathcal{S}}_1$ and $\hat{\mathcal{S}}_2$ are given reconstruction alphabets.

Fix a blocklength n, rates $R_1, R_2 \geq 0$, and a $(2^{nR_1}, 2^{nR_2}, n)$-code $(\{\Omega_{1,i}\}, \{\Omega_{2,i}\}, g_1, g_2, \phi_1, \phi_2)$. Let then the random message W_k be uniformly distributed over the message set $\mathcal{W}_k$, for each $k = 1, 2$, and generate the inputs according to the encoding function $X_{k,i} = \Omega_{k,i}(W_k, Z_k^{i-1})$, for $i = 1, \ldots, n$. Tx $k \in \{1, 2\}$ obtains its state estimate as $\hat{S}_k^n := (\hat{S}_{k,1}, \cdots, \hat{S}_{k,n}) = \phi_k(X_k^n, Z_k^n)$ and its message guess as $\hat{W}_{3-k} = g_k(Z_k^n, W_k)$

[2] The results can be extended to well-behaved continuous channels.

We shall measure the quality of the state estimates $\hat{S}_k^n$ by bounded per-symbol distortion functions $d_k\colon \mathcal{S}_k \times \hat{\mathcal{S}}_k \mapsto [0,\infty)$, and consider *expected average block distortions*

$$\Delta_k^{(n)} := \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}\left[d_k\left(S_{k,i}, \hat{S}_{k,i}\right)\right], \quad k=1,2. \tag{29}$$

The probability of decoding error is defined as:

$$P_e^{(n)} := \Pr\Big(\hat{W}_1 \neq W_1 \quad \text{or} \quad \hat{W}_2 \neq W_2\Big). \tag{30}$$

Definition 3. *A rate-distortion tuple* $(\mathsf{R}_1, \mathsf{R}_2, \mathsf{D}_1, \mathsf{D}_2)$ *is achievable if there exists a sequence (in* n*) of* $(2^{n\mathsf{R}_1}, 2^{n\mathsf{R}_2}, n)$ *codes that simultaneously satisfy*

$$\lim_{n\to\infty} P_e^{(n)} = 0 \tag{31a}$$

$$\overline{\lim_{n\to\infty}}\, \Delta_k^{(n)} \leq \mathsf{D}_k, \quad \text{for } k=1,2. \tag{31b}$$

Definition 4. *The capacity-distortion region* $\mathcal{CD}$ *is the closure of the set of all achievable tuples* $(\mathsf{R}_1, \mathsf{R}_2, \mathsf{D}_1, \mathsf{D}_2)$.

Remark 5 (State-Information at the Terminals)**.** *Considering a two-way channel where*

$$Z_k = (S_{\bar{k}}, Z'_k), \qquad k \in \{1,2\}, \tag{32}$$

for some output Z'_k*. This models a situation where each terminal obtains strictly causal state-information about the other terminal's state. Inner bounds for this setup with strictly causal state-information can immediately be obtained from our results presented in the next section by plugging in the choice in* (32)*. The same remark applies also to imperfect strictly-causal state-information in which case the output should be modelled as*

$$Z_k = (T_k, Z'_k), \qquad k \in \{1,2\}, \tag{33}$$

where Z'_k *again models the actual channel output and* T_k *models the strictly causal imperfect state-information at Terminal* k*. Alternatively,* T_k *could even be related to the desired channel state* S_k *and not only to the other terminal's state* $S_{\bar{k}}$*. Plugging the choice* (33) *into our results for* *an appropriate choice of T_k leads to results for this related setup with imperfect or generalized state-information at the terminals.*

In contrast, our model does not include causal or non-causal state-information. These are interesting extensions of our work, but left for future research. They would certainly require new tools such as dirty-paper coding [29].

V. A Collaborative ISAC Scheme for Device-to-Device Communication

We first review Han's scheme for pure data communication over the two-way channel and then include the collaborative sensing idea in Han's scheme. Finally we integrate collaborative sensing and communication through joint source-channel coding (JSCC).

A. Han's Two-Way Coding Scheme

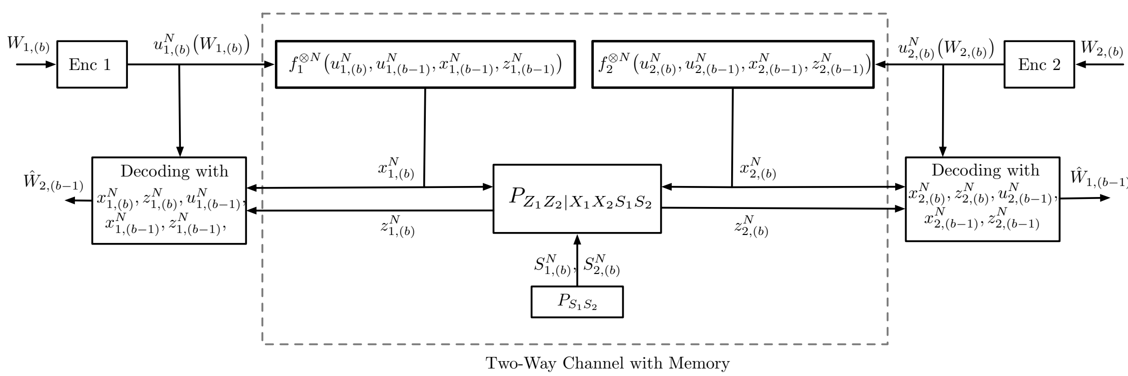

Fig. 7. Han's coding scheme in a given block b. Encoders transform the discrete-memoryless two-way channel into a channel with memory so as to be able to correlate the inputs of the two terminals. Encoding is then performed through the independent codewords $u_{1,(b)}^N$ and $u_{2,(b)}^N$. Decoding of block-$(b-1)$ messages is performed based on the inputs/outputs in the two consecutive blocks $b-1$ and b.

The capacity region of the two-way channel, and thus the optimal coding scheme is still open for general channels. Various inner and outer bounds on the capacity region have been proposed. For example, Schalkwijk proposed an interesting inner bound for a binary multiplier channel based on a method that iteratively describes message points, an approach that is reminiscent of single-user feedback schemes. Han [30] and Kramer [31] proposed schemes that correlate the inputs of the two terminals in a block-fashion. While for Han's coding scheme the correlation ensures a stationary distribution of the inputs and outputs across the blocks and thus still allows for single-letter rate-expressions, Kramer has to resort to multi-letter rate-expressions based on directed mutual informations. An interesting outer bound on the capacity region was proposed by Hekstra and Willems [22] again based on the dependence-balance idea, similar to the MAC with feedback.

The ISAC scheme we present in this manuscript is based on Han's coding scheme, which is depicted in Figure 7 and described in the following. For convenience of notation, define

$$P_{Z_1Z_2|X_1X_2}(z_1,z_2|x_1,x_2) = \sum_{s_1\in\mathcal{S}_1,s_2\in\mathcal{S}_2} P_{S_1S_2}(s_1,s_2)P_{Z_1Z_2|X_1X_2S_1S_2}(z_1,z_2|x_1,x_2,s_1,s_2). \tag{34}$$

Han's scheme splits the blocklength n into $B+1$ blocks of length $N = n/(B+1)$ each. Accordingly, throughout, we let $X_{1,(b)}^N, X_{2,(b)}^N, S_{1,(b)}^N, S_{2,(b)}^N, Z_{1,(b)}^N, Z_{2,(b)}^N$ denote the block-b inputs, states and outputs, e.g., $S_{1,(b)}^N := (S_{1(b-1)N+1},\dots,S_{1,bN})$. We also represent the two messages W_1 and W_2 in a one-to-one way as the B-length tuples

$$W_k = (W_{k,(1)},\dots,W_{k,(B)}), \qquad k\in\{1,2\}, \tag{35}$$

where each $W_{k,(b)}$ is independent and uniformly distributed over $\left[2^{N\bar{R}_k}\right]$ for $\bar{R}_k \triangleq \frac{B+1}{B}R_k$.

Construct an independent code $\mathcal{C}_{k,(b)} = \left\{u_{k,(b)}^N(1),\dots,u_{k,(b)}^N\left(2^{n\bar{R}_k}\right)\right\}$ for each of the two terminals by picking entries i.i.d. according to some pmf P_{U_k}. As shown in Figure 7, Terminal k encodes Message $W_{k,(b)}$ by means of the codeword $u_{k,(b)}^N(W_{k,(b)})$ and sends the sequence

$$X_{k,(b)}^N = f_k^{\otimes N}\left(u_{k,(b)}^N(W_{k,(b)}),\; u_{k,(b-1)}^N(W_{k,(b-1)}),\; x_{k,(b-1)}^N,\; z_{k,(b-1)}^N\right) \tag{36}$$

over the channel during block b. Notice that by applying the function f_k to the block-b codeword symbols as well as to the symbols of the block-$(b-1)$ codeword $u_{k,(b-1)}^N(W_{k,(b-1)})$ and the block-$(b-1)$ channel inputs and outputs $x_{k,(b-1)}^N$ and $z_{k,(b-1)}^N$, the terminals introduce memory to the channel. An interesting point of view is to consider the transition of the codewords $u_{1,(b)}^N$ and $u_{2,(b)}^N$ to the channel outputs $z_{1,(b)}^N$ and $z_{2,(b)}^N$ as a virtual two-way channel with block-memory over which one can code and decode. Naturally, decoding of each message part $W_{k,(b)}$ is not based only on the signals in block (b) because other blocks depend on this message as well. In Han's scheme, decoding is over two consecutive blocks. Specifically, Terminal k decodes the block-b message $W_{\bar{k},(b)}$ using a joint-typicality decoder based on the block-b inputs, outputs, and own transmitted codewords $x^N_{k,(b)}, z^N_{k,(b)}$ and $u^N_{k,(b)}$, as well as on the block-$(b+1)$ inputs and outputs $x^N_{k,(b+1)}$ and $z^N_{k,(b+1)}$.

Notice that without any special care, the rate-region that is achievable with above scheme has to be described with a multi-letter expression because the joint pmf of the tuple $x^N_{1,(b+1)}, z^N_{1,(b+1)}, u^N_{1,(b)}, x^N_{1,(b)}, z^N_{1,(b)}$ that Terminal 1 uses to decode codeword $u^N_{2,(b)}(W_{k,(b)})$ varies with the block b. However, if one chooses a joint pmf $P_{U_1U_2X_1X_2Z_1Z_2}$ satisfying the stationarity condition

$$\begin{aligned} &P_{U_1U_2X_1X_2Z_1Z_2}(u_1,u_2,x_1,x_2,z_1,z_2) \\ &\quad= \sum_{\tilde{u}_1,\tilde{u}_2,\tilde{x}_1,\tilde{x}_2,\tilde{z}_1,\tilde{z}_2} P_{Z_1Z_2|X_1X_2}(z_1,z_2|x_1,x_2)\mathbb{1}\{x_1=f_1(u_1,\tilde{u}_1,\tilde{x}_1,\tilde{z}_1)\} \\ &\qquad \mathbb{1}\{x_2=f_2(u_2,\tilde{u}_2,\tilde{x}_2,\tilde{z}_2)\}\cdot P_{U_1}(u_1)P_{U_2}(u_2)P_{U_1U_2X_1X_2Z_1Z_2}(\tilde{u}_1,\tilde{u}_2,\tilde{x}_1,\tilde{x}_2,\tilde{z}_1,\tilde{z}_2), \end{aligned} \tag{37}$$

where P_{U_1} and P_{U_2} are the marginals of $P_{U_1U_2X_1X_2Z_1Z_2}$, then the pmf of the tuple of sequences $x^N_{1,(b+1)}, x^N{}_{2,(b+1)}, z^N_{1,(b+1)}, z^N_{2,(b+1)}, u^N_{1,(b)}, u^N_{2,(b)}, x^N_{1,(b)}, x^N_{2,(b)}, z^N_{1,(b)}, z^N_{2,(b)}$ is independent of the block index b. This allows to characterize the rate region achieved by the described coding scheme using a single-letter expression. All rate-pairs (R_1,R_2) are achievable that satisfy

$$R_1 \leq I(U_1;X_2,Z_2,\tilde{U}_2,\tilde{X}_2,\tilde{Z}_2) \tag{38a}$$

$$R_2 \leq I(U_2;X_1,Z_1,\tilde{U}_1,\tilde{X}_1,\tilde{Z}_1), \tag{38b}$$

where $(U_1,U_2,X_1,X_2,Z_1,Z_2,\tilde{U}_1,\tilde{U}_2,\tilde{X}_1,\tilde{X}_2,\tilde{Z}_1,\tilde{Z}_2)$ are distributed according to the pmf

$$\begin{aligned} &P_{U_1U_2X_1X_2Z_1Z_2\tilde{U}_1\tilde{U}_2\tilde{X}_1\tilde{X}_2\tilde{Z}_1\tilde{Z}_2}(u_1,u_2,x_1,x_2,z_1,z_2,\tilde{u}_1,\tilde{u}_2,\tilde{x}_1,\tilde{x}_2,\tilde{z}_1,\tilde{z}_2) \\ &\quad= P_{Z_1Z_2|X_1X_2}(z_1,z_2|x_1,x_2)\mathbb{1}\{x_1=f_1(u_1,\tilde{u}_1,\tilde{x}_1,\tilde{z}_1)\}\mathbb{1}\{x_2=f_2(u_2,\tilde{u}_2,\tilde{x}_2,\tilde{z}_2)\} \\ &\qquad \cdot P_{U_1}(u_1)P_{U_2}(u_2)P_{U_1U_2X_1X_2Z_1Z_2}(\tilde{u}_1,\tilde{u}_2,\tilde{x}_1,\tilde{x}_2,\tilde{z}_1,\tilde{z}_2). \end{aligned} \tag{39}$$

This recovers Han's theorem:

Theorem 4 (Han's Achievable Region for Two-Way Channels [30])**.** *Any nonnegative rate-pair* (R_1,R_2) *is achievable over the two-way channel if it satisfies Inequalities* (38) *for some choice of pmf* $P_{U_1U_2X_1X_2Z_1Z_2}$ *and functions* f_1 *and* f_2 *satisfying the stationarity condition* (37).

For certain cases the above theorem can be simplified, and for certain channels the simplified region even coincides with capacity. The simplification is obtained by choosing the two functions f_1 and f_2 to simply produce the codewords $u^N_{1,(b-1)}$ and $u^N_{2,(b-1)}$ from the previous block[3] and ignore the other arguments. In this case, the set of rates that can be achieved coincides with the following inner bound that was first proposed by Shannon [32].

Theorem 5 (Shannon's Inner Bound, [32])**.** *A pair of nonnegative pairs* (R_1, R_2) *is achievable if it satisfies*

$$R_1 \leq I(X_1; Z_2|X_2) \tag{40a}$$

$$R_2 \leq I(X_2; Z_1|X_1), \tag{40b}$$

for some input pmfs P_{X_1} *and* P_{X_2} *and where* $(X_1, X_2, Z_1, Z_2) \sim P_{X_1} P_{X_2} P_{Z_1 Z_2|X_1 X_2}$.

B. Collaborative Sensing and Communication based on Han's Two-Way Coding Scheme

We extend Han's coding scheme to include also collaborative sensing, that means each terminal compresses its block-b inputs and outputs so as to capture information about the other terminal's state and sends this state-information in the next-following block. In this first collaborative sensing and communication scheme that we present here, the sensing (compression) does not affect the communication (except possibly for the choice of the pmf $P_{U_1 U_2 X_1 X_2 Z_1 Z_2}$). In fact, we again use Han's encodings and decodings as described in the previous subsection, except that the block-b codeword not only encodes message $W_{k,(b)}$ but also a compression index $J^*_{k,(b-1)}$ that carries information about the block-$(b-1)$ state $S_{\bar{k},(b-1)}$. This compression index is then decoded at Terminal $\bar{k}$ after block $(b+1)$ simultaneously with message $W_{k,(b)}$. See Figure 8.

The analysis of the communication-part of our ISAC scheme is similar as in Han's scheme. Since the compression indices take parts of the place reserved for ordinary messages in Han's scheme, their rates $R_{\text{WZ},1}$ and $R_{\text{WZ},2}$ have to be subtracted from Han's communication rates.

[3]The delay of a block introduced in this scheme is not crucial, it simply comes from the fact that Han's scheme decodes the block-$(b-1)$ codewords based on the block-b outputs. In this special case without adaptation, Han's scheme could be simplified by transmitting and decoding the codewords $u^N_{1,(b-1)}$ and $u^N_{2,(b-1)}$ directly in block $b-1$ without further delay.

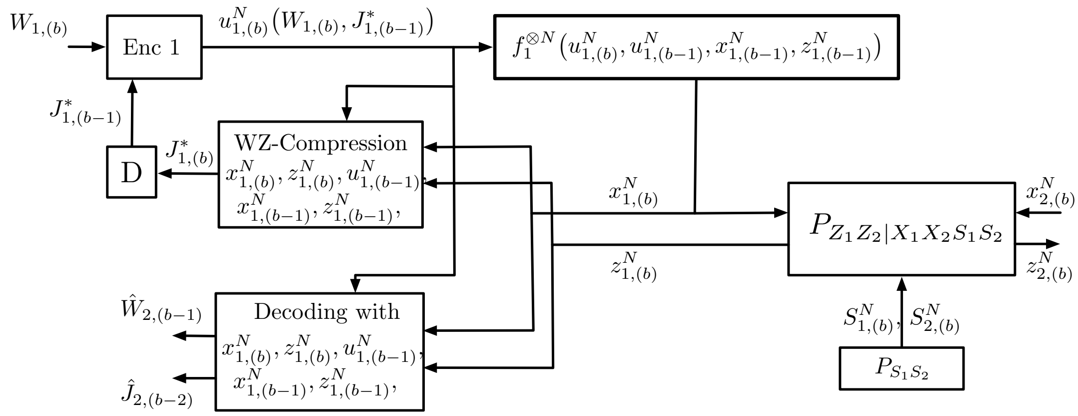

Fig. 8. A first collaborative-sensing version of Han's coding scheme. The figure illustrates the encoding and decoding operations in a given block b at Terminal 1; Terminal 2 behaves analogously. To facilitate sensing at Terminal 2, Terminal 1 compresses its block-b channel inputs and outputs, together with its inputs, outputs, and codeword from the previous block $(b-1)$ (which are all resent in block b) using Wyner-Ziv compression [33] to account for the side-information at Terminal 2.

We thus have the following constraints for reliable communication and reliable decoding of the compression indices:

$$R_1 + R_{\text{WZ},1} \leq I(U_1; X_2, Z_2, \tilde{U}_2, \tilde{X}_2, \tilde{Z}_2) \tag{41a}$$

$$R_2 + R_{\text{WZ},2} \leq I(U_2; X_1, Z_1, \tilde{U}_1, \tilde{X}_1, \tilde{Z}_1). \tag{41b}$$

It remains to explain the compression and state estimation in more details. In our scheme, the index $J^*_{k,(b-1)}$ is obtained by means of a Wyner-Ziv compression [33] that lossily compresses the tuple $(x^N_{k,(b-1)}, z^N_{k,(b-1)}, u^N_{k,(b-2)}, x^N_{k,(b-2)}, z^N_{k,(b-2)})$ for a decoder that has side-information $(x^N_{\bar{k},(b-1)}, z^N_{\bar{k},(b-1)}, u^N_{\bar{k},(b-2)}, x^N_{\bar{k},(b-2)}, z^N_{\bar{k},(b-2)})$. In order for the decoder to be able to correctly reconstruct the compression codeword, the Wyner-Ziv codes need to be of rates at least [33]

$$R_{\text{WZ},k} > I(V_k; X_k, Z_k, \tilde{U}_k, \tilde{X}_k, \tilde{Z}_k | X_{\bar{k}}, Z_{\bar{k}}, \tilde{U}_{\bar{k}}, \tilde{X}_{\bar{k}}, \tilde{Z}_{\bar{k}}), \qquad k \in \{1, 2\}, \tag{42}$$

where the tuple $(U_1, U_2, X_1, X_2, Z_1, Z_2, \tilde{U}_1, \tilde{U}_2, \tilde{X}_1, \tilde{X}_2, \tilde{Z}_1, \tilde{Z}_2)$ refers to the auxiliary random variables chosen by Han's scheme of joint pmf as in (39) and V_1 and V_2 can be any random variables satisfying the Markov chains:

$$V_k \to (X_k, Z_k, \tilde{U}_k, \tilde{X}_k, \tilde{Z}_k) \to (X_{\bar{k}}, Z_{\bar{k}}, \tilde{U}_{\bar{k}}, \tilde{X}_{\bar{k}}, \tilde{Z}_{\bar{k}}, S_k, S_{\bar{k}}). \tag{43}$$

In Wyner-Ziv coding, the encoder produces a codeword that is then reconstructed also at the receiver. We shall denote these codewords by $v^N_{k,(b-1)}(J^*_{k,(b-1)}, \ell_{k,(b-1)})$, for $k \in \{1,2\}$, where $\ell_{k,(b-1)}$ denotes a binning-index that does not have to be conveyed to the Terminal $\bar{k}$ because this latter can recover it from its side-information. Thus, after block $(b+1)$ and after decoding index $J^*_{k,(b-1)}$, with high probability Terminal $\bar{k}$ can reconstruct the codeword $v^N_{k,(b-1)}(J^*_{k,(b-1)}, \ell_{k,(b-1)})$ chosen at Terminal k.

Terminal k can wait arbitrarily long to produce an estimate of the state-sequence S^N_k. We propose that it waits after the block-$(b+1)$ decoding to reconstruct the block-b state $S^N_{k,(b)}$ by applying an optimal symbol-by-symbol estimator to the related sequences of inputs, outputs, and channel codewords of blocks $b-1$ and b, as well as on the compression codeword $v^N_{\bar{k},(b)}$:

$$\hat{S}^N_{k,(b)} = \tilde{\phi}^{*\otimes N}_{2,k}\left(v^N_{\bar{k},(b)}, x^N_{k,(b)}, z^N_{k,(b)}, \hat{u}_{\bar{k},(b)}, u^N_{k,(b-1)}, x^N_{k,(b-1)}, z^N_{k,(b-1)}, \hat{u}_{\bar{k},(b-1)}\right), \tag{44}$$

where

$$\tilde{\phi}^*_{2,k}(v_{\bar{k}}, x_k, z_k, u_{\bar{k}}, \tilde{u}_k, \tilde{x}_k, \tilde{z}_k, \tilde{u}_{\bar{k}}) := \arg\min_{s'_k \in \hat{\mathcal{S}}_k} \sum_{s_k \in \mathcal{S}_k} P_{S_k|X_k Z_k U_{\bar{k}}}(s_k|x_k, z_k, u_{\bar{k}})\, d_k(s_k, s'_k). \tag{45}$$

By (41) and (42) and standard typicality arguments, one obtains the following theorem. (The theorem is a special case of the next-following theorem, for which we provide a detailed analysis in the extended version [28].)

Theorem 6 (Inner Bound via Separate Source-Channel Coding)**.** *Any nonnegative rate-distortion quadruple* (R_1, R_2, D_1, D_2) *is achievable if it satisfies the following two rate-constraints*

$$R_1 \leq I(U_1; X_2, Z_2, \tilde{U}_2, \tilde{X}_2, \tilde{Z}_2) - I(V_1; X_1, Z_1, \tilde{U}_1, \tilde{X}_1, \tilde{Z}_1 | X_2, Z_2, \tilde{U}_2, \tilde{X}_2, \tilde{Z}_2) \tag{46a}$$

$$R_2 \leq I(U_2; X_1, Z_1, \tilde{U}_1, \tilde{X}_1, \tilde{Z}_1) - I(V_2; X_2, Z_2, \tilde{U}_2, \tilde{X}_2, \tilde{Z}_2 | X_1, Z_1, \tilde{U}_1, \tilde{X}_1, \tilde{Z}_1), \tag{46b}$$

and the two distortion constraints

$$\mathsf{E}\Big[d_1\big(S_1,\ \tilde{\phi}^*_{2,1}(V_2, X_1, Z_1, U_2, \tilde{U}_1, \tilde{X}_1, \tilde{Z}_1, \tilde{U}_2)\big)\Big] \leq D_1 \tag{46c}$$

$$\mathsf{E}\Big[d_2\big(S_2,\ \tilde{\phi}^*_{2,2}(V_1, X_2, Z_2, U_1, \tilde{U}_2, \tilde{X}_2, \tilde{Z}_2, \tilde{U}_2)\big)\Big] \leq D_2 \tag{46d}$$

for some choice of pmf $P_{U_1U_2X_1X_2Z_1Z_2}$ *and functions* f_1 *and* f_2 *satisfying the stationarity condition* (37) *and* V_1, V_2 *satisfying the Markov chains* (43)*.*

Similarly to Shannon's inner bound, we can obtain the following corollary by setting $X_k = \tilde{U}_k$.

Corollary 1 (Inner Bound via Non-Adaptive Coding)**.** *Any nonnegative rate-distortion quadruple* (R_1, R_2, D_1, D_2) *is achievable if it satisfies the following two rate-constraints*

$$R_1 \leq I(X_1; X_2, Z_2) - I(V_1; X_1, Z_1 | X_2, Z_2) \tag{47a}$$

$$R_2 \leq I(X_2; X_1, Z_1) - I(V_2; X_2, Z_2 | X_1, Z_1), \tag{47b}$$

and the two distortion constraints

$$\mathsf{E}\Big[d_1\big(S_1,\ \tilde{\phi}^*_{2,1}(V_2, X_1, X_2, Z_1)\big)\Big] \leq D_1 \tag{47c}$$

$$\mathsf{E}\Big[d_2\big(S_2,\ \tilde{\phi}^*_{2,2}(V_1, X_1, X_2, Z_2)\big)\Big] \leq D_2 \tag{47d}$$

for some choice of pmfs P_{X_1}*,* P_{X_2}*,* $P_{V_1|X_1,Z_1}$*, and* $P_{V_2|X_2,Z_2}$*.*

As the following example shows, above corollary achieves the fundamental rate-distortion tradeoff for some channels.

Example 3. *Consider the following state-dependent two-way channel*

$$Z_1 = X_1 \oplus X_2 \oplus S_2 \qquad \text{and} \qquad Z_2 = X_1 \oplus X_2 \oplus S_1, \tag{48a}$$

where inputs, outputs, and states are binary and S_1 *and* S_2 *are independent Bernoulli-*p_1 *and* p_2 *random variables, for* $p_1, p_2 \in [0, 1/2]$*. Notice that Terminal 1's outputs depend on the state desired at Terminal 2 and Terminal 2's outputs on the state desired at Terminal 1, which calls for collaborative sensing.*

Whenever $D_{\bar{k}} < p_{\bar{k}}$*, we chooose* $V_k = Z_k \oplus X_k \oplus B_k = X_{\bar{k}} \oplus S_{\bar{k}} \oplus B_k$ *where* B_k *is an independent Bernoulli-*D_k *random variable. If* $D_k \geq p_k$*, choose* V_k *a constant. Inputs* X_1 *and* X_2 *are chosen independent Bernoulli-*$1/2$*, i.e., capacity-achieving on channels with Bernoulli-noises. When* $D_{\bar{k}} < p_{\bar{k}}$*, the optimal symbo-by-symbol state-estimator is*

$$\tilde{\phi}^*_{2,\bar{k}}(v_k, x_1, x_2, z_{\bar{k}}) = v_k \oplus x_{\bar{k}} \tag{49}$$

and otherwise it is the constant estimator $\tilde{\phi}^*_{2,\bar{k}}(v_k, x_1, x_2, z_{\bar{k}}) = 0$.

For the described choice of random variables, Corollary 1 evaluates to the set of rate-distortion tuples (R_1, R_2, D_1, D_2) *satisfying*

$$R_k \leq 1 - H_b(p_k) - \max\{0,\ H_b(p_{\bar{k}}) - H_b(D_{\bar{k}})\}, \qquad k \in \{1, 2\}, \tag{50}$$

and achieves the fundamental rate-distortion region as we show through a converse in Appendix A. The region in (50) *is concave (because the rate-distortion function* $\max\{0,\ H_b(p_{\bar{k}}) - H_b(D_{\bar{k}})\}$ *is convex), and thus improves over classic time- and resource-sharing schemes. It also improves over a similar ISAC scheme without collaborative sensing where the compression codewords* V_1 *and* V_2 *are set to constants. In this latter case, only rate-distortion tuples are possible that satisfy* $D_k \geq p_k$*, for* $k \in \{1, 2\}$.

Remark 6. *For certain channels and state-distributions Theorem 6 can be improved with the idea of coded time-sharing. The same applies for Theorem 7 in the next-following section.*

C. Collaborative Sensing and JSCC Scheme

In this scheme, we fully integrate the compression into the communication scheme, in a similar way that hybrid coding [34] uses a single codeword for compression and channel coding in source-channel coding applications.

Encoding and decoding in block b of the new scheme are depicted in Figure 9. The main difference compared to the scheme in the previous subsection is that here the block-b codeword $u_{1,(b)}^N$ is *correlated* with the inputs and outputs in the previous block $(b-1)$.[4] This correlation introduces additional dependence between blocks, which was previously missing because of the independence of the compression codewords and the codewords used for channel coding in the next block. To still obtain a stationary distribution on the codewords and channel inputs/outputs, which then allows for a single-letter characterization of the performance of the scheme, one has

[4] In the previous scheme, the compression codeword $v_{1,(b)}^N$ was correlated with the block-$(b-1)$ signals but not the channel coding codeword $u_{1,(b)}^N$. Now the codeword $u_{1,(b)}^N$ acts both as a compression codeword and as a channel coding codeword.

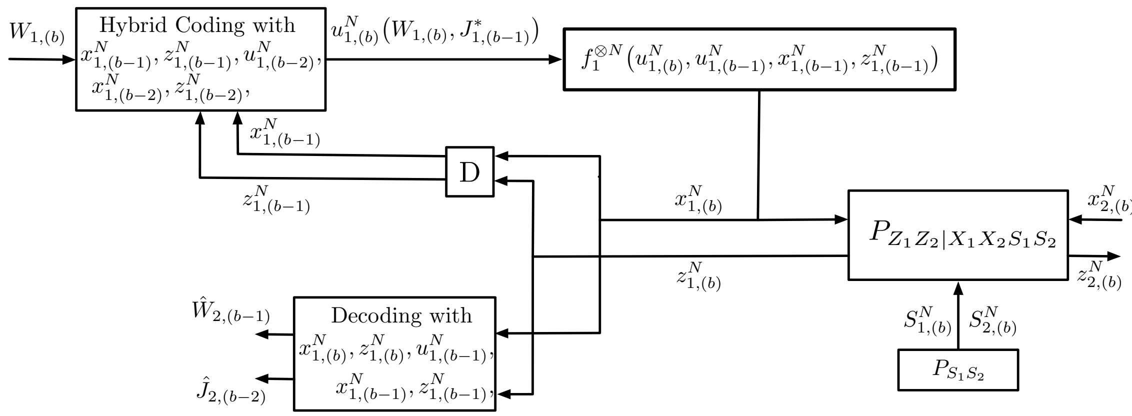

Fig. 9. A ISAC scheme integrating collaborative sensing for D2D into Han's two-way coding scheme by means of hybrid coding. A single codeword is used both for compression and for channel coding.

to choose a joint pmf $P_{U_1'U_2'Z_1Z_2X_1X_2U_1U_2}$, conditional pmfs $P_{U_1'|X_1Z_1\tilde{U}_1\tilde{X}_1\tilde{Z}_1}$ and $P_{U_2'|X_2Z_2\tilde{U}_2\tilde{X}_2\tilde{Z}_2}$ as well as functions f_1 and f_2 on appropriate domains satisfying the new stationarity condition

$$\begin{aligned}
&P_{U_1'U_2'Z_1Z_2X_1X_2U_1U_2}(u_1', u_2', z_1, z_2, x_1, x_2) \\
&\quad = \sum_{\tilde{u}_1,\tilde{u}_2,\tilde{x}_1,\tilde{x}_2,\tilde{z}_1,\tilde{z}_2} P_{U_1'|X_1Z_1\tilde{U}_1\tilde{X}_1\tilde{Z}_1}(u_1'|u_1, x_1, z_1, \tilde{u}_1, \tilde{x}_1, \tilde{z}_1) P_{U_2'|X_2Z_2\tilde{U}_2\tilde{X}_2\tilde{Z}_2}(u_2'|x_2, z_2, u_2, \tilde{u}_2, \tilde{x}_2, \tilde{z}_2) \\
&\qquad \cdot P_{Z_1Z_2|X_1X_2}(z_1, z_2|x_1, x_2) \mathbb{1}\{x_1 = f_1(u_1, \tilde{u}_1, \tilde{x}_1, \tilde{z}_1)\} \mathbb{1}\{x_2 = f_2(u_2, \tilde{u}_2, \tilde{x}_2, \tilde{z}_2)\} \\
&\qquad \cdot P_{U_1'U_2'Z_1Z_2X_1X_2U_1U_2}(u_1, u_2, \tilde{z}_1, \tilde{z}_2, \tilde{x}_1, \tilde{x}_2, \tilde{u}_1, \tilde{u}_2), \qquad (51)
\end{aligned}$$

In the following, all mentioned conditional and marginal pmfs are with respect to the joint pmf $P_{U_1'U_2'Z_1Z_2X_1X_2U_1U_2\tilde{U}_1\tilde{U}_2\tilde{W}_1\tilde{W}_2\tilde{V}_1\tilde{V}_2}$ indicated by the summand in (51).

We next explain the code construction, encodings and decodings.

For each $k \in \{1, 2\}$, for each block $b \in \{1, \ldots, B+1\}$, and each message $m_k \in [2^{N\bar{R}_k}]$, choose a subcodebook $\{u^N_{k,(b)}(m_k, j) \colon j \in [2^{NR_k'}]\}$ by picking all entries i.i.d. $P_{U_k'}$. Terminal k then picks the codeword $u^N_{k,(b)}(W_{k,(b)}, j)$ so that the following joint-typicality check is satisfied for some fixed $\epsilon > 0$:

$$\left(u^N_{k,(b)}(W_{k,(b)}, j), x^N_{k,(b-1)},\, z^N_{k,(b-1)},\, u^N_{k,(b-2)},\, x^N_{k,(b-2)},\, z^N_{k,(b-2)}\right) \in \mathcal{T}^N_\epsilon\left(P_{U_k'X_kZ_k\tilde{U}_k\tilde{X}_k\tilde{Z}_k}\right), \qquad (52)$$

and sets $J^*_{k,(b-1)} = j$. By standard arguments, such an index j exists with probability tending to

1 as $N \to \infty$ if

$$R'_k \geq I(U'_k; X_k, Z_k, \tilde{U}_k, \tilde{X}_k, \tilde{Z}_k), \quad k \in \{1, 2\}. \tag{53}$$

Terminal k then sends the block-b input sequence

$$X^N_{k,(b)} = f_k^{\otimes N} \left(u^N_{k,(b)}\big(W_{k,(b)} J^*_{k,(b-1)}\big), u^N_{k,(b-1)}, x^n_{k,(b-1)}, z^N_{k,(b-1)} \right). \tag{54}$$

Decoding is again performed using a joint-typicality decoder. At the end of block b, Terminal k looks for indices $\hat{w}_{\bar{k}}$ and $\hat{j}_{\bar{k}}$ satisfying the two typicality checks

$$\left(u^N_{\bar{k},(b-1)}(\hat{w}_{\bar{k}}, \hat{j}_{\bar{k}}), x^N_{k,(b)},\, z^N_{k,(b)},\, u^N_{k,(b-1)}\, x^N_{k,(b-1)},\, z^N_{k,(b-1)}\right) \in \mathcal{T}^N_\epsilon \left(P_{\tilde{U}_{\bar{k}} X_k Z_k \tilde{U}_k \tilde{X}_k \tilde{Z}_k} \right) \tag{55}$$

and

$$\left(u^N_{\bar{k},(b-1)}(\hat{w}_{\bar{k}}, \hat{j}_{\bar{k}}), x^N_{k,(b-2)},\, z^N_{k,(b-2)},\, u^n_{k,(b-3)},\, x^N_{k,(b-3)},\, z^N_{k,(b-3)}\right) \in \mathcal{T}^N_\epsilon \left(P_{U_{\bar{k}} X_k Z_k \tilde{U}_k \tilde{X}_k \tilde{Z}_k} \right). \tag{56}$$

If a unique pair of such element exists, set $\hat{W}_{\bar{k},(b-1)} = w_{\bar{k}}$ and $\hat{u}^N_{\bar{k},(b-1)} \triangleq u^N_{\bar{k},(b-1)}(\hat{w}_{\bar{k}}, \hat{j}_{\bar{k}})$. Decoding is successful with probability tending to 0 as $N \to \infty$ if

$$\bar{R}_{\bar{k}} + R'_{\bar{k}} \leq I(\tilde{U}_{\bar{k}}; X_k, Z_k, \tilde{U}_k, \tilde{X}_k, \tilde{Z}_k) + I(U_{\bar{k}}; X_k, Z_k, \tilde{U}_k, \tilde{X}_k, \tilde{Z}_k), \quad k \in \{1, 2\}. \tag{57}$$

State-estimation is similar to (44), but where Terminal k replaces the compression codeword $v^N_{k,(b)}$ by the joint source-channel codeword $u^N_{k,(b+1)}$ and similarly to hybrid coding also uses the inputs/outputs corresponding to the block where the codeword $u^N_{1,(b+1)}$ is sent, i.e., inputs and outputs in block $b+1$. Thus, Terminal k computes its estimate of the block-b state as:

$$\hat{s}^N_{k,(b)} = \phi^{*\otimes N}_{2,k} \big(\hat{u}^N_{\bar{k},(b+1)}, x^N_{k,(b+1)}, z^N_{k,(b+1)}, \hat{u}^N_{\bar{k},(b)}, x^N_{k,(b)}, z^N_{k,(b)}, u^N_{k,(b-1)}, x^N_{k,(b-1)}, z^N_{k,(b-1)}, \hat{u}^N_{\bar{k},(b-1)}\big), \tag{58}$$

where

$$\phi^*_{2,k}(u'_{\bar{k}}, x'_k, z'_k, u_{\bar{k}}, x_k, z_k, \tilde{u}_k, \tilde{x}_k, \tilde{z}_k, \tilde{u}_{\bar{k}}) := \text{arg} \min_{s'_k \in \hat{\mathcal{S}}_k} \sum_{s_k \in \mathcal{S}_k} P_{S_k|X_k Z_k U_{\bar{k}}}(s_k | x_k, z_k, u_{\bar{k}})\; d_k(s_k, s'_k). \tag{59}$$

By standard arguments and because of the stationarity condition in (51) the probability of violating the distortion constraints tends to 0 as $N \to \infty$ if

$$\mathsf{E}\Big[d_k\big(S_k,\ \phi^*_{2,k}(U'_{\bar{k}}, X'_k, Z'_k, U_{\bar{k}}, X_k, Z_k, \tilde{U}_k, \tilde{X}_k, \tilde{Z}_k, \tilde{U}_{\bar{k}})\big)\Big] \leq D_k, \qquad k \in \{1,2\}, \tag{60a}$$

where $X'_1 = f_1(U'_1, U_1, X_1, Z_1)$ and $X'_2 = f_2(U'_2, U_2, X_2, Z_2)$ and the outputs Z'_1 and Z'_2 are obtained from X'_1 and X'_2 via the channel transition law $P_{Z_1Z_2|X_1X_2}$.

From above considerations and by eliminating the dummy rates R'_1 and R'_2, we obtain the following theorem.

Theorem 7 (Inner Bound via Joint Source-Channel Coding)**.** *Any nonnegative rate-distortion quadruple (R_1, R_2, D_1, D_2) is achievable if it satisfies the following two rate-constraints*

$$R_k \leq I(\tilde{U}_k; X_{\bar{k}}, Z_{\bar{k}}, \tilde{U}_{\bar{k}}, \tilde{X}_{\bar{k}}, \tilde{Z}_{\bar{k}}) - I(U_k; X_k, Z_k, \tilde{U}_k, \tilde{X}_k, \tilde{Z}_k | X_{\bar{k}}, Z_k, \tilde{U}_{\bar{k}}, \tilde{X}_{\bar{k}}, \tilde{Z}_{\bar{k}}), \quad k \in \{1,2\} \tag{61}$$

and the two distortion constraints in (60) *for some choice of pmf $P_{U'_1U'_2Z_1Z_2X_1X_2U_1U_2}$ and functions f_1 and f_2 satisfying the stationarity condition* (37).

Remark 7. *We notice that the described compression technique does not use binning as in Wyner-Ziv coding [33]. Instead, decoder side-information is taken into account via the joint typicality check in* (56)*.*

Remark 8. *For the choice $U'_k = (U''_k, V_k)$ with $U''_k \sim P_{U_k}$ independent of all other random variables and V_1 and V_2 satisfying the Markov chains in* (43)*, the inner bound in Theorem 7 achieved by our joint source-channel coding scheme specializes to the inner bound Theorem 6 achieved by separate source-channel coding. For above choice of auxiliary random variables, the reconstruction functions g_1 and g_2 can restrict their first arguments only to the V_1- and V_2-components without loss in performance.*

VI. Summary and Outlook

We considered integrated sensing and communication (ISAC) over multi-access channels (MAC) and device-to-device (D2D) communication, where different terminals help each other to improve sensing. We reviewed related communication schemes and proposed adaptations that fully integrate the collaborative sensing into information-theoretic data communication schemes. For D2D communication, we also proposed a joint source-channel coding (JSCC) scheme to integrate compression and coding into a single codeword as in hybrid coding. Various interesting future research directions arise. As already mentioned, the JSCC scheme proposed for ISAC D2D communication could be integrated into our ISAC MAC scheme. Another interesting research direction for the MAC scheme is to include state-estimation at the Rx. In this respect, it would be interesting to include an additional superposition compression layer to generate compression information that is only decoded by the Rx but not the other Tx. For D2D communication an interesting extension would be to consider specific channel models and to replace Han's result by two-way communication schemes that are tailored to these specific channels.

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APPENDIX A
CONVERSE TO EXAMPLE 3

By the independence of the messages and Fano's Inequality, we obtain for some function ϵ_n that vanishes as $n \to \infty$,

$$
\begin{aligned}
R_k &\leq \frac{1}{n} I(W_k; Y_{\bar{k}}^n | W_{\bar{k}}) + \epsilon_n && (62)\\
&= \frac{1}{n} I(W_k S_{\bar{k}}^n; Y_{\bar{k}}^n | W_{\bar{k}}) - \frac{1}{n} I(S_{\bar{k}}^n; Y_{\bar{k}}^n | W_k W_{\bar{k}}) + \epsilon_n && (63)\\
&= \frac{1}{n} \left[\sum_{i=1}^{n} I(Y_{\bar{k},i}; W_k S_{\bar{k}}^n | Y_{\bar{k}}^{i-1} W_{\bar{k}}) - I(S_{\bar{k},i}; Y_{\bar{k}}^n | W_k W_{\bar{k}} S_{\bar{k}}^{i-1}) \right] + \epsilon_n && (64)\\
&\stackrel{(a)}{\leq} \frac{1}{n} \left[\sum_{i=1}^{n} I(Y_{\bar{k},i}; X_{k,i} W_k S_{\bar{k}}^n | X_{\bar{k},i} Y_{\bar{k}}^{i-1} W_{\bar{k}}) - I(S_{\bar{k},i}; Y_{\bar{k}}^n W_k W_{\bar{k}} S_{\bar{k}}^{i-1}) \right] + \epsilon_n && (65)\\
&\stackrel{(b)}{\leq} \frac{1}{n} \left[\sum_{i=1}^{n} I(Y_{\bar{k},i}; X_{k,i}) - I(S_{\bar{k},i}; \hat{S}_{\bar{k},i}) \right] + \epsilon_n && (66)\\
&\stackrel{(c)}{\leq} nC_k - R_{\bar{k}}(D_{\bar{k}}) + \epsilon_n, && (67)
\end{aligned}
$$

where C_k denotes the capacity of the point-to-point channel from X_k to $X_k + S_k$ and $R_{\bar{k}}(\cdot)$ denotes the rate-distortion function of source $S_{\bar{k}}$. In our example, $C_{\bar{k}} = 1 - H_{\rm b}(p_k)$ and

$R_{\bar{k}}(D_{\bar{k}}) = [H_{\mathrm{b}}(p_k) - H_{\mathrm{b}}(D_k)]^+$. Justification for above inequalities are as follows: (a) holds because conditioning cannot increase entropy, because $X_{\bar{k},i}$ is a function of $W_{\bar{k}}$ and $Y_{\bar{k}}^{i-1}$, and by the i.i.d.ness of the source sequence $S_{\bar{k}}^n$; (b) holds because of the Markov chain $Y_{\bar{k},i} \to (X_{k,i}, X_{\bar{k},i}) \to (W_k, W_{\bar{k}}, S_{\bar{k}}^n, Y_{\bar{k}}^{i-1})$ and because $\hat{S}_{\bar{k},i}$ is a function of $Y_{\bar{k}}^n$ and again because conditioning cannot increase entropy; (c) holds by the definition of the rate-distortion function $R_{\bar{k}}(\cdot)$ and because $R_{\bar{k}}(\cdot)$ is convex and monotonic.

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