Capacity-Tradeoffs in Networks with Mixed-Delay Traffics and Random Arrivals

Michèle Wigger
Telecom Paris

joint work with Jean-Marie Gorce, Homa Nikbakht, Shlomo Shamai, and Vincent Poor

Workshop "Performance Guarantees in Wireless Networks"

9 March, 2023

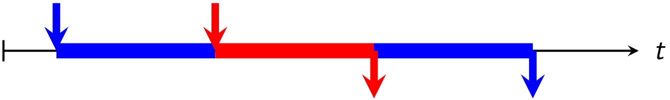
Heterogeneous Traffics in 5G Networks

- **Enhanced Mobile Broadband (eMBB)**:
  streaming, data communication;
  → requires high rates
- **Ultra-Reliable Low-Latency Communication (URLLC)** :
  control applications such as autonomous driving, remote surgery;
  → requires small delays
- **Massive Machine-Type Communications (MTC)**:
  Internet of Things;
  → sporadic and large number of devices

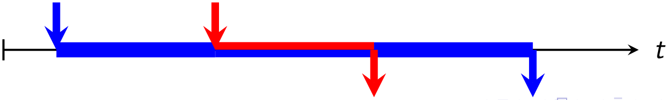
Mixed Delays Networks

Coexistence of eMBB (delay-tolerant) and URLLC (delay-sensitive) on same bandwidth

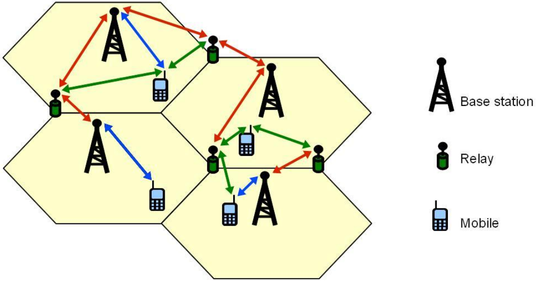
- Key challenge: heterogeneous delays of URRLC and eMBB
- Standard proposition to cope with stringent delay constraint: Smart scheduling of URLLC messages



In this talk: Benefits from joint coding of mixed-delay traffics



High Rates Enabled by Cooperation

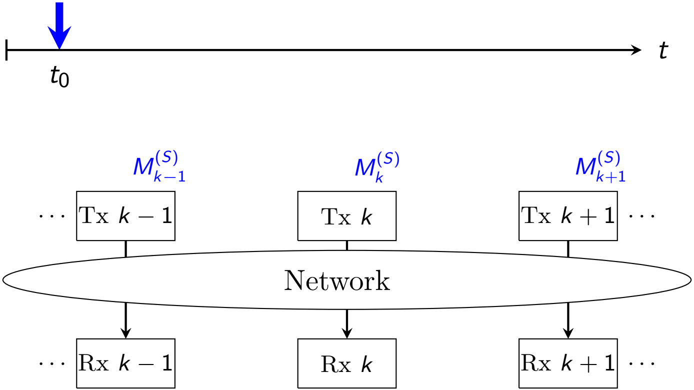


- Cooperation allows for path-diversity and interference mitigation

- Cooperation hops induce additional delays
  $\rightarrow$ Delay-sensitive communications cannot *directly* profit from cooperation

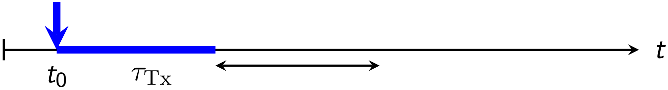
Mixed Delay Traffic in Cooperative Networks

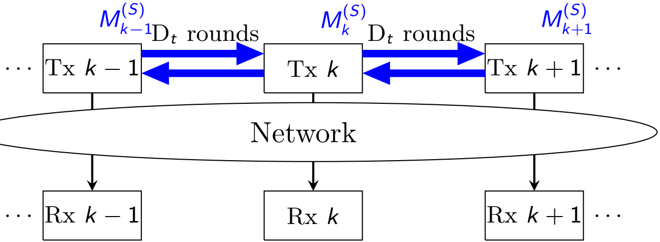
- K transmitter/receiver pairs



Mixed Delay Traffic in Cooperative Networks

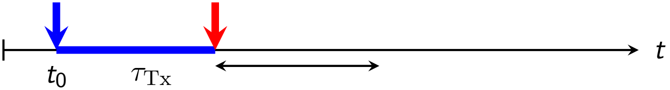
- K transmitter/receiver pairs

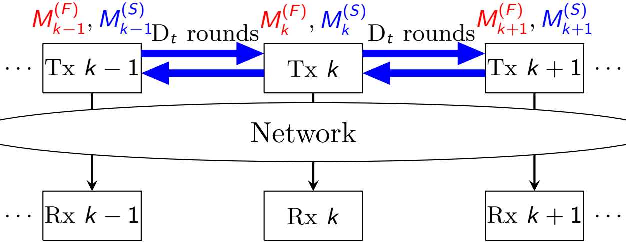




Mixed Delay Traffic in Cooperative Networks

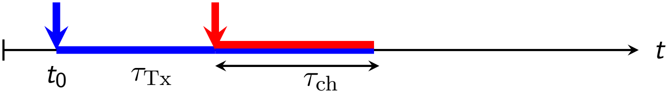
- K transmitter/receiver pairs

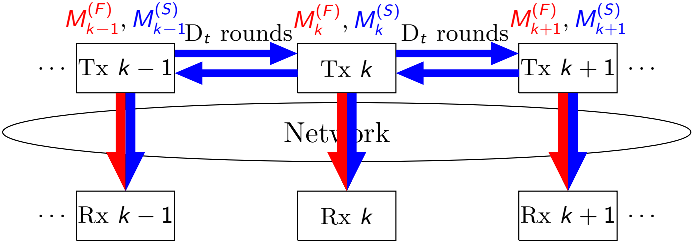




Mixed Delay Traffic in Cooperative Networks

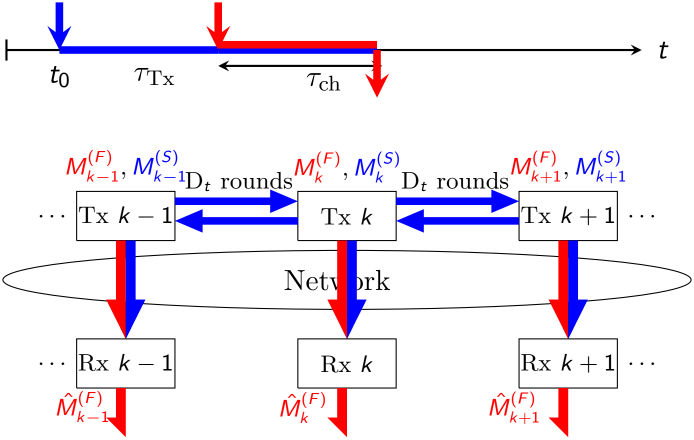
- K transmitter/receiver pairs





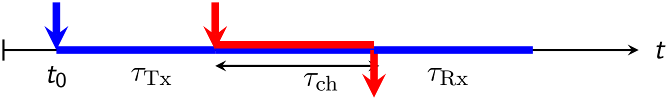
Mixed Delay Traffic in Cooperative Networks

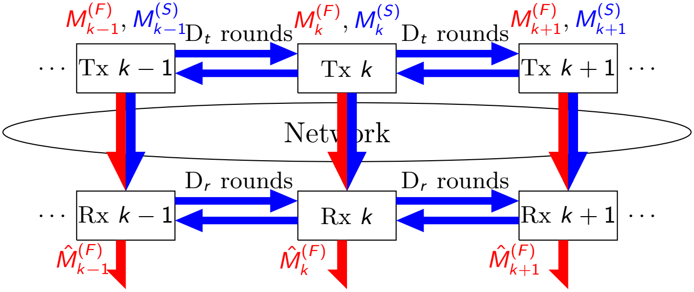
- K transmitter/receiver pairs



Mixed Delay Traffic in Cooperative Networks

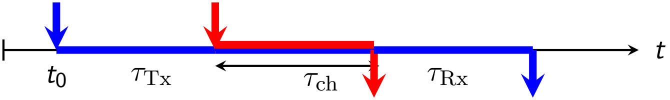
- K transmitter/receiver pairs

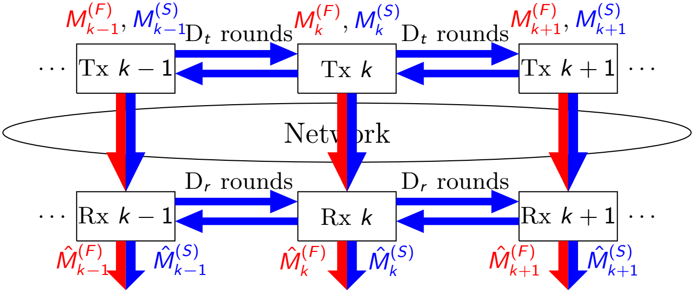




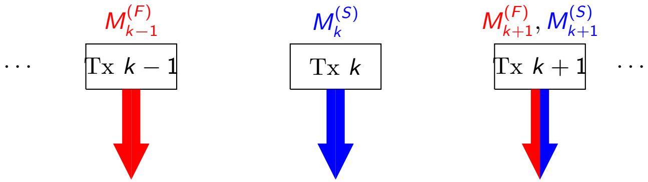
Mixed Delay Traffic in Cooperative Networks

- K transmitter/receiver pairs

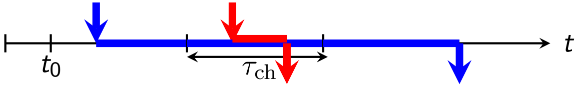




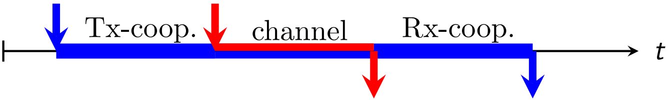
Random Arrivals of Data



- Each Tx might have "slow" or "fast" data to send or both
- Arrivals of new "fast" or "slow" data at random times, not necessarility at beginning of a block ($\rightarrow$ negligible for "slow" messages)
- A typical "fast" transmission time is smaller than the duration of a single block



The "Easiest" Model



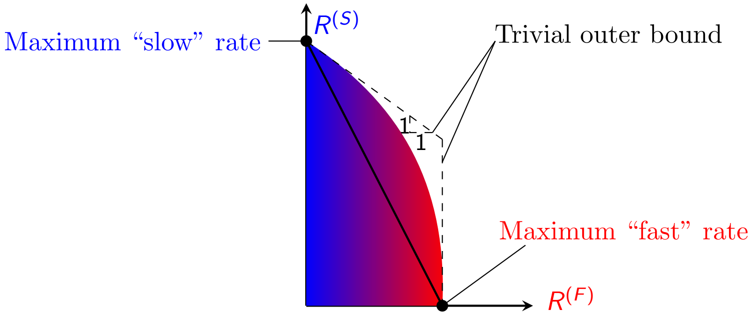
- Each Tx k has "fast" data $M_k^{(F)}$ and "slow" data $M_k^{(S)}$ to send
- Transmission of "fast" data can last an entire block
- Large blocklengths → we are interested in capacity, i.e., we require that probability of error tends to 0 as $n \to \infty$

Questions we wish to address with this model

Best interference mitigation when cooperation only for "slow" data?
Penalty for not being able to cooperate on "fast" data?

Mixed-DelayCapacity Region

- $(R^{(F)}, R^{(S)})$: average achievable "fast" and "slow" rates



Timesharing: large $R^{(F)}$ harms overall performance (sum-rate)
→ Inherent or artefact of time-sharing?

Mixed-Delay High-SNR Capacity for Hexagonal Model

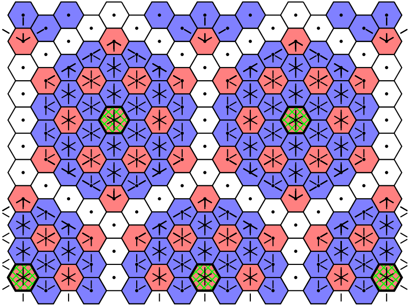
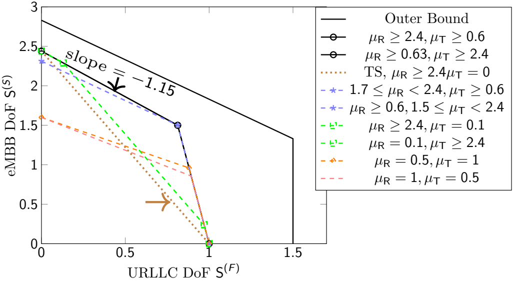


Figure: Hexagonal network.



(Plots assume three antennas at each terminal.)

Mixed-Delay DoF Region for Sectorized Hex. Model

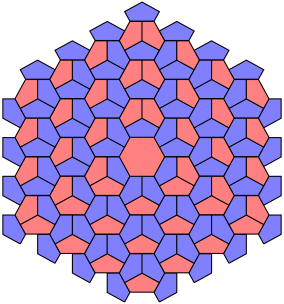
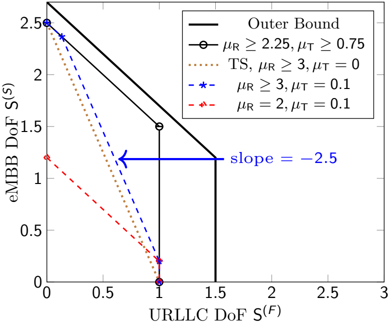


Figure: Sectorized Hexagonal Network.



Mixed-Delay Capacity of the Wyner Symmetric Model

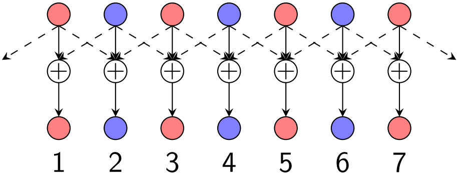
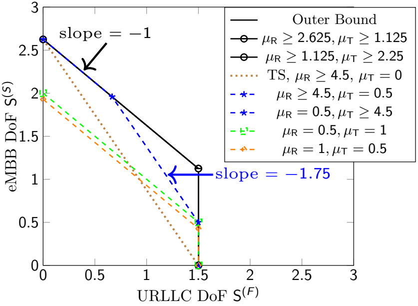
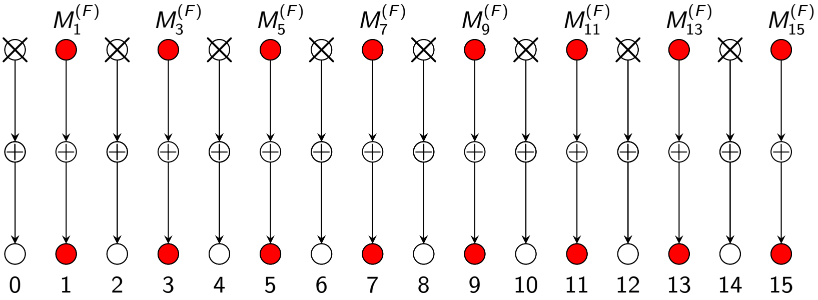


Figure: Symmetric Wyner Network.



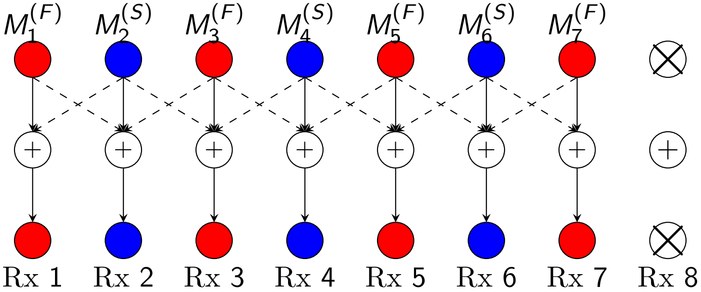
The Coding Scheme for the Wyner-Network

- K transmitter/receiver pairs
- Deactivate every other transmitter
- $\mathsf{S}^{(F)} = \frac{1}{2}$



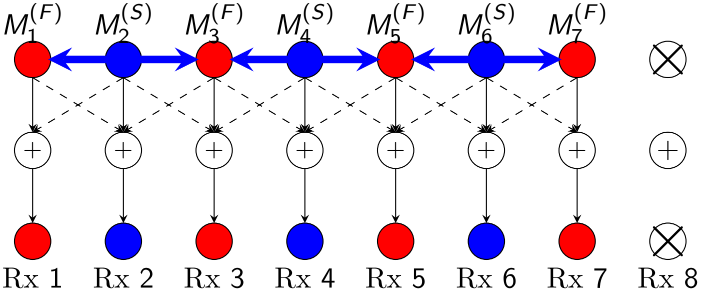
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $D_t = 1$, $D_r = 5$.



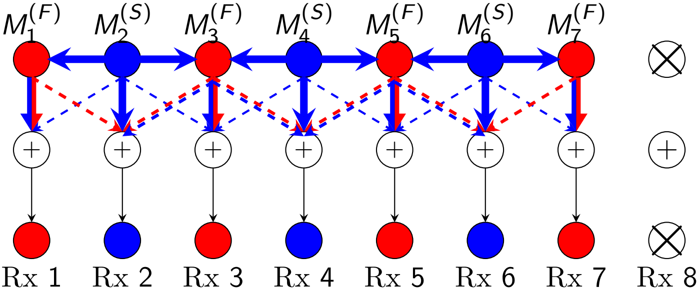
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $\mathrm{D}_t = 1$, $\mathrm{D}_r = 5$.



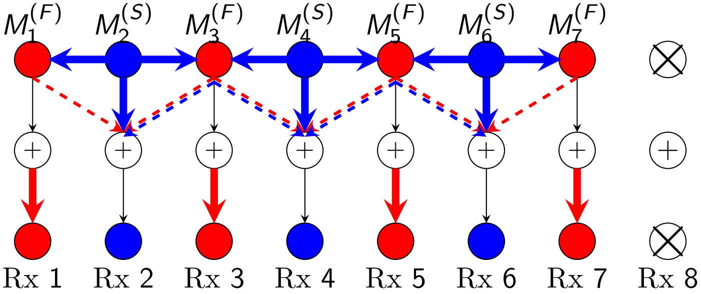
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $D_t = 1$, $D_r = 5$.



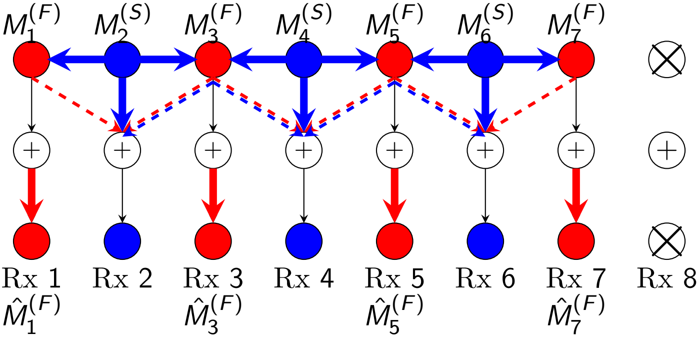
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $\mathrm{D}_t = 1$, $\mathrm{D}_r = 5$.



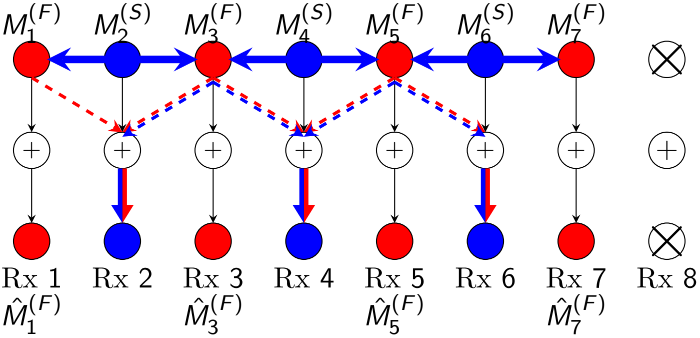
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $\mathrm{D}_t = 1$, $\mathrm{D}_r = 5$.



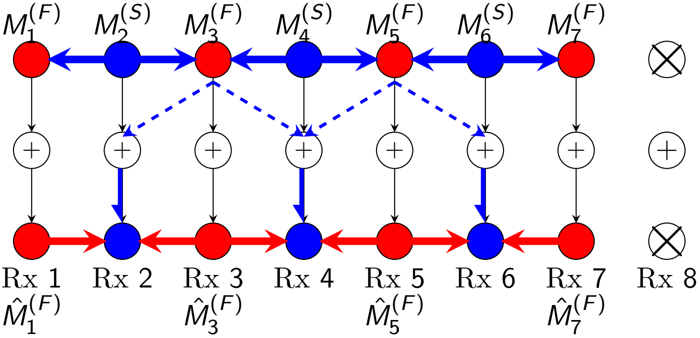
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $D_t = 1$, $D_r = 5$.



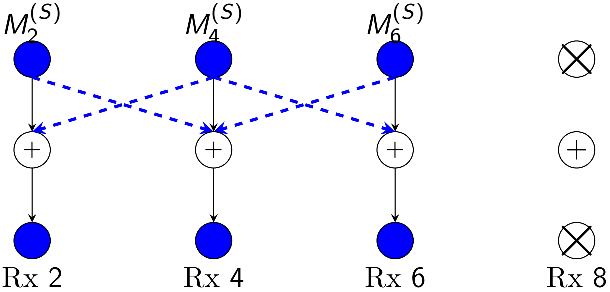
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $\mathrm{D}_t = 1$, $\mathrm{D}_r = 5$.



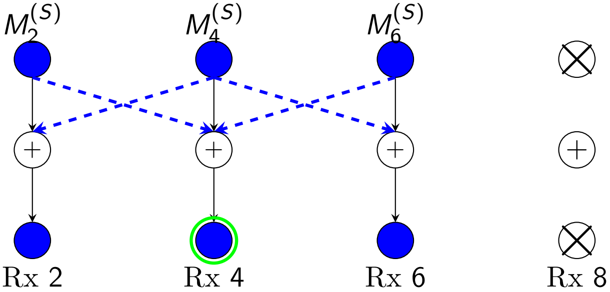
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $\mathrm{D}_t = 1$, $\mathrm{D}_r = 5$.



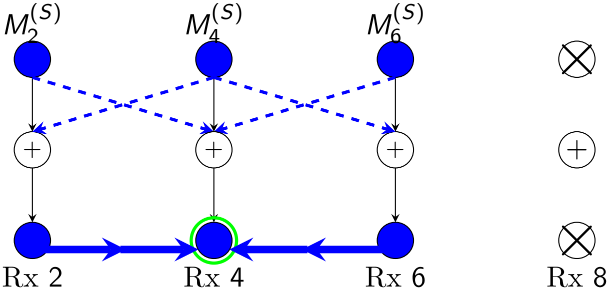
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $\mathrm{D}_t = 1$, $\mathrm{D}_r = 5$.



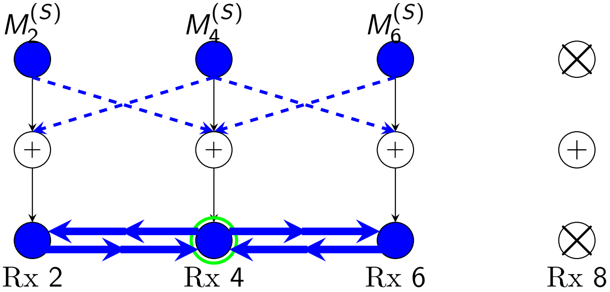
The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $\mathrm{D}_t = 1$, $\mathrm{D}_r = 5$.



The Coding Scheme for the Wyner-Network

- Coordinated multi-point reception: $D_t = 1$, $D_r = 5$.



The Coding Arrangement for Hexagonal Networks

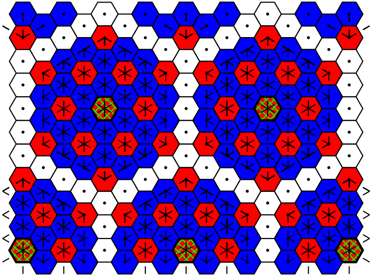


Figure: Hexagonal network.

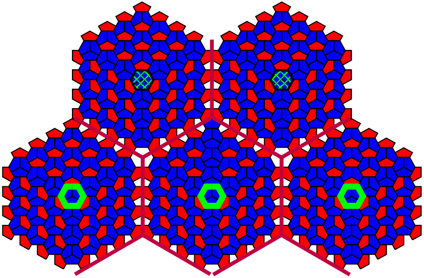
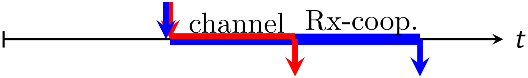
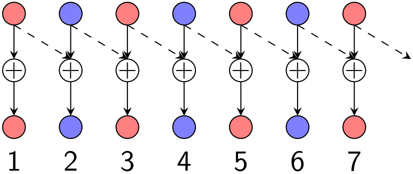
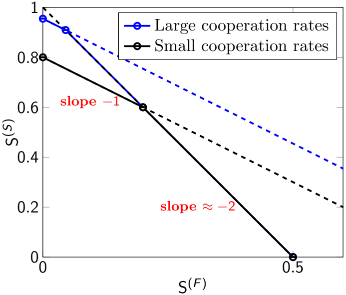


Figure: Sectorized hexagonal model.

Receiver Cooperation Only

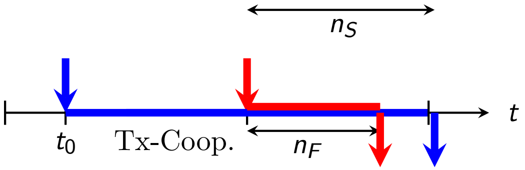






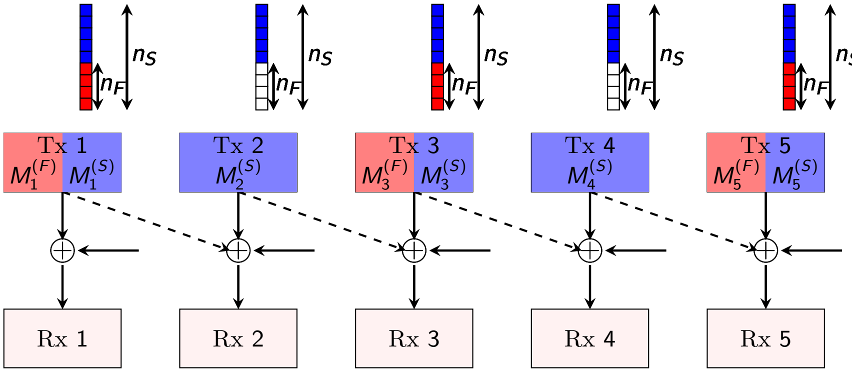
- Small $S^{(F)}$: sum-rate not decreased by insisting on fast decoding
- Large $S^{(F)}$: 1 "fast" bit costs 2 "slow" bits

A More Complicated Model: Finite Blocklengths



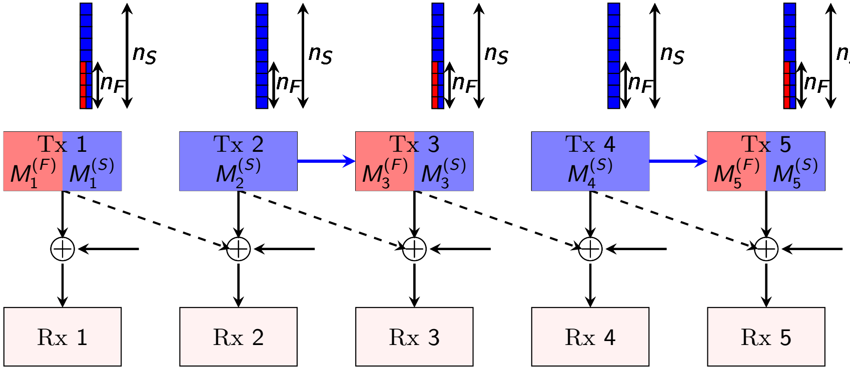
- "Fast" data has to be decoded after n_F channel uses
- "Slow" data can be decoded after $n_S > n_F$ channel uses
- All transmissions start at time $t = 1$
- "Slow" data can also be shared with neighbouring Txs
- We fix rates $R^{(F)}$ and $R^{(S)}$ and power P
- Performance is measured by the error probabilities ϵ_F and ϵ_S.

Scheduling: URLLC and eMBB Assignment



- Odd Txs send "fast" data over the first n_F channel uses and "slow" data over the remaining $n_S - n_F$ channel uses.
- To send "fast" data interference-free, even Txs send "slow" data only over the last $n_S - n_F$ channel uses.

Joint Coding Scheme: Encoding at the Txs in $\mathcal{K}\backslash\mathcal{K}_{\mathrm{U}}$

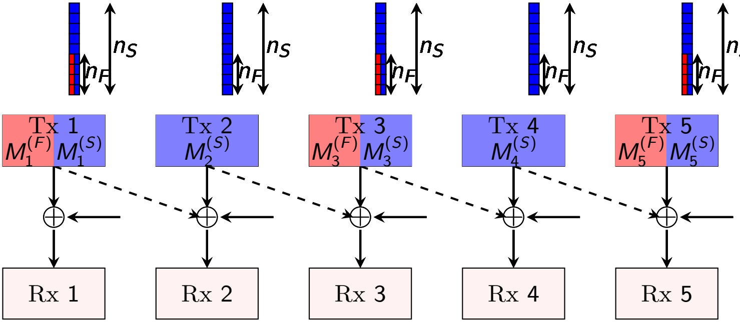


- Txs 2, 4: Power allocation: $\beta_S \in [0, 1]$

$$||\boldsymbol{X}_k^{(S,1)}||^2 = n_F \beta_S \mathsf{P}, \quad ||\boldsymbol{X}_k^{(S,2)}||^2 = (n_S - n_F)(1 - \beta_S)\mathsf{P}$$

- Txs 2 and 4 also describe their input signal $\boldsymbol{X}_k^{(S,1)}$ to their right neigbours

Joint Coding Scheme: Encoding at the Txs in $\mathcal{K}_{\text{U}}$



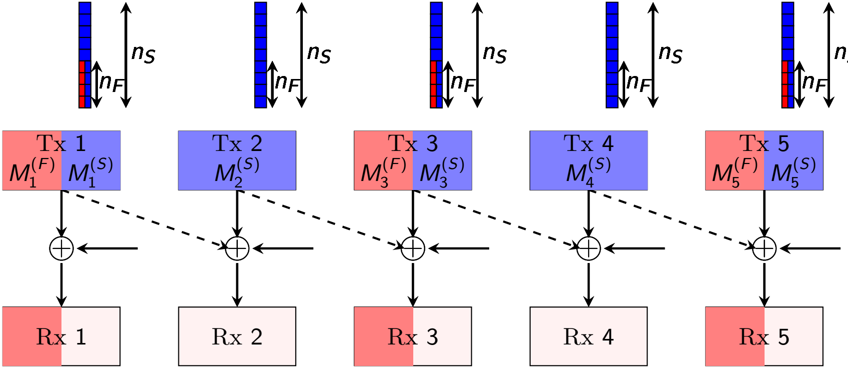
- Power allocation: $\beta_F,\ \beta_{S,1},\ \beta_{S,2} \in [0,1]$ such that $\beta_F + \beta_{S,1} + \beta_{S,2} = 1$

$$||\boldsymbol{X}_k^{(S,1)}||^2 = n_F \beta_{S,1} \mathsf{P}, \quad \boldsymbol{X}_k^{(F)}, \quad ||\boldsymbol{X}_k^{(S,2)}||^2 = (n_S - n_F)\beta_{S,2}\mathsf{P}$$

- Dirty paper coding to encode "fast" data:
$\boldsymbol{X}_k^{(F)} := \boldsymbol{V}_k - \alpha_{k,1}\boldsymbol{X}_k^{(S,1)} - \alpha_{k,2}\boldsymbol{X}_{k-1}^{(S,1)}$

Decoding "Fast" messages

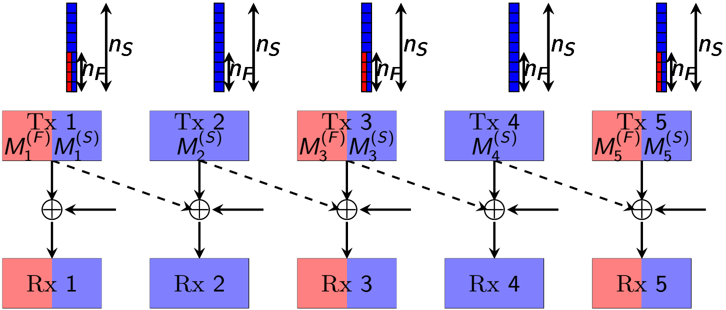
- Decoding "fast" messages $M_k^{(F)}$ with $k \in \{1, 3, 5\}$



- After n_F channel uses: $\boldsymbol{Y}_{k,1} = h_{k,k}(\boldsymbol{X}_k^{(F)} + \boldsymbol{X}_k^{(S,1)}) + h_{k-1,k}\boldsymbol{X}_{k-1}^{(S,1)} + \boldsymbol{Z}_{k,1}$
- Rx k estimates $M_k^{(F)}$ as an index m for which $\boldsymbol{v}_k(m, i)$ maximizes

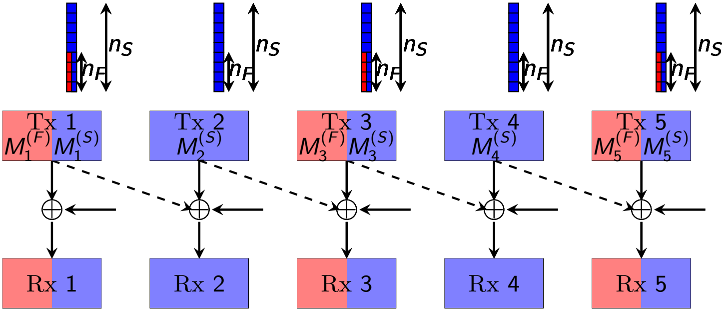
$$i(\boldsymbol{v}_k; \boldsymbol{y}_{k,1}) := \ln \frac{f(\boldsymbol{y}_{k,1}|\boldsymbol{v}_k)}{f(\boldsymbol{y}_{k,1})}, \quad \text{among all codewords } \boldsymbol{v}_k = \boldsymbol{v}_k(m', j).$$

- Decoding "Slow" data $M_k^{(S)}$



- The first n_F channel uses:
  $\boldsymbol{Y}_{k,1} = h_{k,k}(\boldsymbol{X}_k^{(F)} + \boldsymbol{X}_k^{(S,1)}) + h_{k-1,k}\boldsymbol{X}_{k-1}^{(S,1)} + \boldsymbol{Z}_{k,1}$
- The last $n_S - n_F$ channel uses: $\boldsymbol{Y}_{k,2} = h_{k,k}\boldsymbol{X}_k^{(S,2)} + h_{k-1,k}\boldsymbol{X}_{k-1}^{(S,2)} + \boldsymbol{Z}_{k,2}$,

- Decoding “Slow” data $M_k^{(S)}$



- Rx k estimates $M_k^{(S)}$ as an index m for which its codewords maximize

$$i_2(\mathbf{x}_k^{(S,1)}, \mathbf{x}_k^{(S,2)}; \mathbf{y}_{k,1}, \mathbf{y}_{k,2}) := \ln \frac{f(\mathbf{y}_{k,1}|\mathbf{x}_k^{(S,1)}) f(\mathbf{y}_{k,2}|\mathbf{x}_k^{(S,2)})}{f(\mathbf{y}_{k,1}) f(\mathbf{y}_{k,2})}$$

Lemma

Consider the vector $\boldsymbol{Y} = a_1\boldsymbol{X}_1 + a_2\boldsymbol{X}_2 + a_3\boldsymbol{X}_3 + \boldsymbol{Z}$ *where* $||\boldsymbol{X}_i||^2 = nP_i$ *for* $i \in \{1,2,3\}$, $\boldsymbol{Z} \sim \mathcal{N}(0, \sigma^2_{z^*} I_n)$, *and* $a_i s$ *with* $i \in \{1,2,3\}$ *are constants. Let* $f_{\boldsymbol{Y}}(\boldsymbol{Y})$ *be the pdf of* $\boldsymbol{Y}$ *and*

$$\begin{aligned} \tilde{Q}_{\boldsymbol{Y}}(\boldsymbol{y}) &\sim \mathcal{N}(\boldsymbol{y}; 0, \sigma^2_{z^*} I_n), \\ Q_{\boldsymbol{Y}}(\boldsymbol{y}) &\sim \mathcal{N}(\boldsymbol{y}; 0, (a_1^2 P_1 + a_2^2 P_2 + a_3 P_3 + \sigma^2_{z^*}) I_n). \end{aligned}$$

One can prove that

$$\begin{aligned} \frac{f_{\boldsymbol{Y}}(\boldsymbol{Y})}{\tilde{Q}_{\boldsymbol{Y}}(\boldsymbol{y})} &\geq 2^{\frac{3(n-2)}{2}} (a_1 a_2 a_3)^{(n-2)} e^{-\frac{n}{2}\left(\frac{a_1^2 P_1}{\sigma^2_{z_1}} + \frac{a_2^2 P_2}{\sigma^2_{z_2}} + \frac{a_3^2 P_3}{\sigma^2_{z_3}}\right)}, \\ \frac{f_{\boldsymbol{Y}}(\boldsymbol{Y})}{Q_{\boldsymbol{Y}}(\boldsymbol{y})} &\leq e^{\kappa} e^{\frac{e^{c_\Gamma} a_2^2 P_2}{\sqrt{2\pi a_1^2 P_1}}}, \end{aligned}$$

where $\kappa := \left(\ln(\frac{1}{2}) + c_\Gamma + \ln(\sqrt{\frac{\pi}{8}}) - 2\ln(a_3)\right)$ *with* $c_\Gamma \leq 2$, *and* $\sigma^2_{z_1} + \sigma^2_{z_2} + \sigma^2_{z_3} = \sigma^2_{z^*}$.

Simulation Results

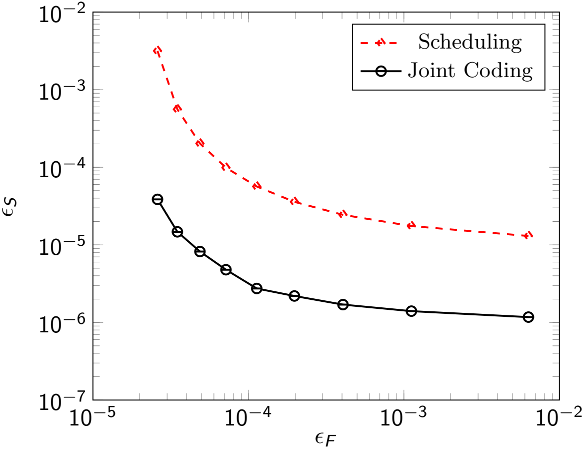
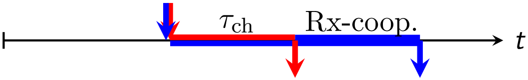


Figure: ϵ_S vs ϵ_F for $\mathsf{P} = 10$, $n_S = 100$ and n_F from 90 to 10 with steps of 10.

The values of the parameters β_S, β_F, $\beta_{S,1}$, $\beta_{S,2}$, $\alpha_{k,1}$ and $\alpha_{k,2}$ are optimized to minimize ϵ_S for a given ϵ_F.

Random Arrival Models

- Transmitters can be inactive (no data arrived)
- Active transmitters can have “slow” or “fast” data to transmit
- Back to large blocklengths and capacity
- “Fast” transmissions can last an entire block but cannot profit from cooperation
- Cooperation only at Rx-side (for decoding of “slow” data)

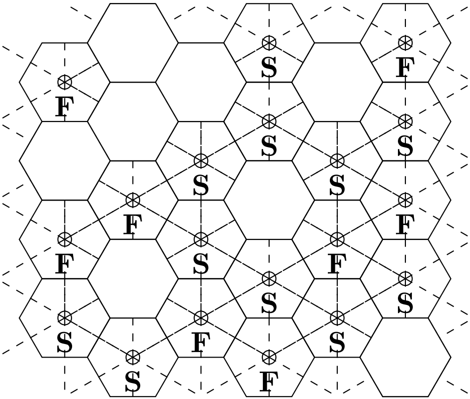


Model 1: Active Txs Have "Slow" *or* "Fast" data

For $\rho \in [0, 1]$ and $\rho_f \in [0, 1]$:

- Tx k is active with probability ρ.
- Active Txs send with prob. ρ_f "fast" data at rate $R^{(F)}$; otherwise it sends "slow" data at user-dependent rate $R_k^{(S)}$.
- Interested in average expected "slow" rate $\bar{R}^{(S)}$.

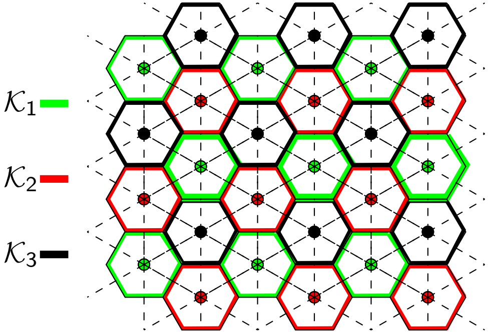
Random Arrivals in the Hexagonal Model



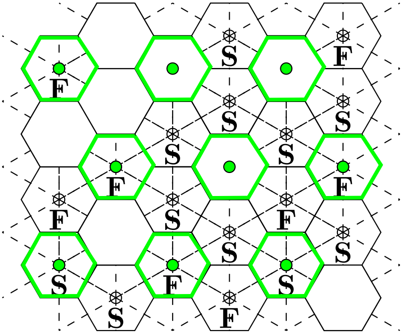
Random arrivals can change network structure

Coding Scheme for Hexagonal Model

- Partition $\mathcal{K} = \{1, \ldots, K\}$ into 3 subsets $\mathcal{K}_1, \mathcal{K}_2$ and $\mathcal{K}_3$
- Divide the total transmission time into 3 equally-sized phases.

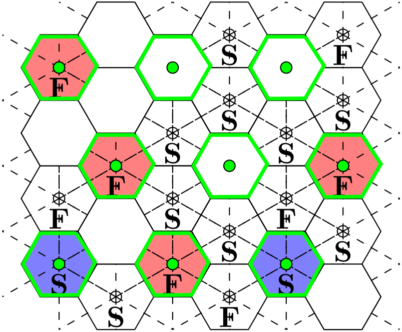


Coding Scheme in the First Phase



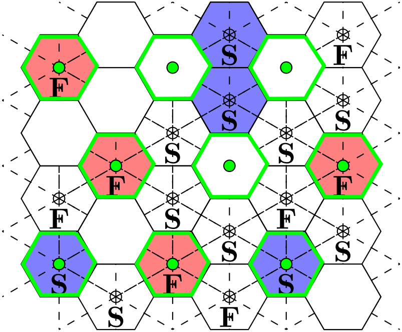
Coding Scheme in the First Phase

- Schedule all users in $\mathcal{K}_1$



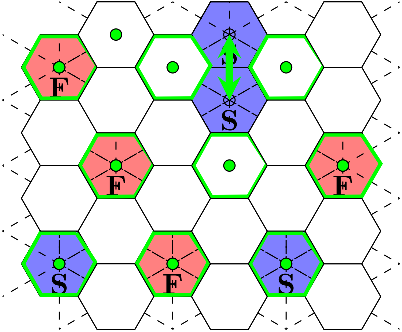
Coding Scheme in the First Phase

- Schedule all users in $\mathcal{K}_1$
- Schedule all "slow" users not interfering "fast" transmissions



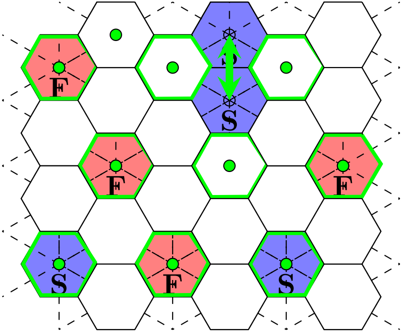
Coding Scheme in the First Phase

- Schedule all users in $\mathcal{K}_1$
- Schedule all "slow" users not interfering "fast" transmissions
- Jointly decode "slow" messages → DoF 1.



Coding Scheme in the First Phase

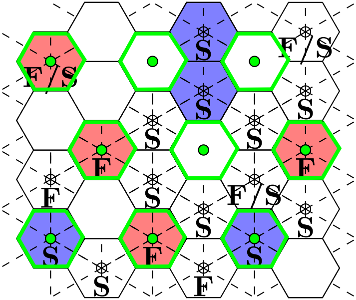
- Schedule all users in $\mathcal{K}_1$
- Schedule all "slow" users not interfering "fast" transmissions
- Jointly decode "slow"messages $\rightarrow$ DoF 1.



Penalty of transmitting "fast"messages on sum DoF increases with ρ

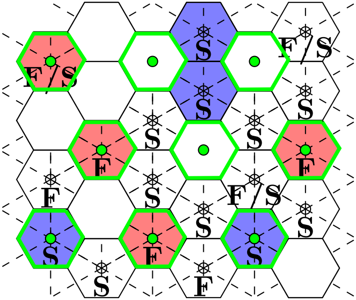
Variations on the Coding Scheme

- If all active Txs had "slow" messages to send → can add additional "slow" Txs
-

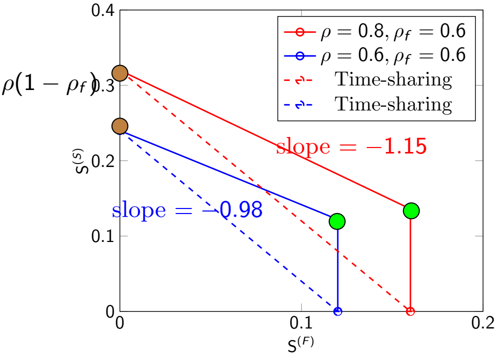


Variations on the Coding Scheme

-
- If Txs and Rxs could cooperate on "slow" data → no need to cancel Txs around "fast" Txs



Statistical DoF Regions: Hexagonal Network



- Max. Sum-DoF at $S^{(F)} = 0$ or $S^{(F)}_{\max} = \frac{\rho\rho_f}{3}$
- $\rho \ll 1$: Silence few "slow" Txs

The following pairs $(S^{(F)}, S^{(S)})$ are achievable

$$S^{(F)} \leq \frac{\rho\rho_f}{3}, \quad \underbrace{2\rho(1-\rho_f)(3-3\rho\rho_f+\rho^2\rho_f^2)}_{\approx 6\rho(1-\rho_f) \text{ if } \rho_f \ll 1} S^{(F)} + S^{(S)} \leq \rho(1-\rho_f).$$

Cellular Network Models with Less Connectivity

Wyner's symmetric network

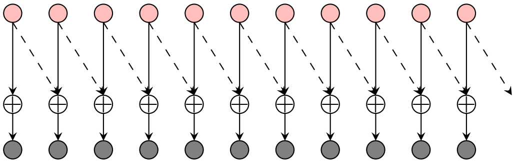
All pairs achievable satisfying

$$S^{(F)} \le \frac{\rho\rho_f}{2}, \qquad \underbrace{\rho(1-\rho_f)(2-\rho\rho_f)}_{\approx 2\rho(1-\rho_f) \text{ if } \rho_f \ll 1} S^{(F)} + S^{(S)} \le \rho(1-\rho_f).$$

Wyner's soft-handoff network

All pairs achievable satisfying

$$S^{(F)} \le \frac{\rho\rho_f}{2}, \qquad \underbrace{\rho(1-\rho_f)}S^{(F)} + S^{(S)} \le \rho(1-\rho_f). \quad \text{Exact}$$



Converse of Wyner's Soft-Handoff Model

Fix K and realizations of the sets $\mathcal{T}_{\text{slow}}$ and $\mathcal{T}_{\text{fast}}$. For each $k \in \mathcal{T}_{\text{slow}}$:

$$
\begin{aligned}
& R_k^{(F)} + R_k^{(S)} + R_{k+1}^{(F)} \\
& \qquad \le \frac{1}{2}\log(1 + (1 + |h_{k,k+1}|^2)\mathsf{P}) + \frac{1}{2}\log(1 + |h_{k,k+1}|^2) \\
& \qquad\quad + \max\{-\log|h_{k,k+1}|, 0\} + \frac{\epsilon_n}{n},
\end{aligned}
$$

- Sum up for all values of $k \in \mathcal{T}_{\text{slow}}$:

$$
\sum_{k \in \mathcal{T}_{\text{slow}}} \left(R_k^{(S)} + R_{k+1}^{(F)} \right) \le |\mathcal{T}_{\text{slow}}| \cdot \tilde{\Delta}.
$$

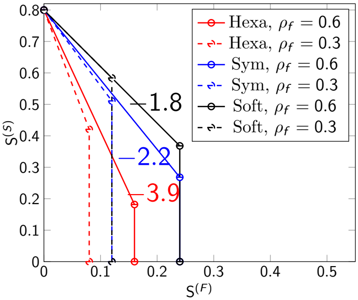
- Taking expectation and dividing by K, we obtain:

$$
\mathbb{E}[\bar{R}^{(S)}] + R^{(F)}(\rho\rho_f \cdot \rho(1-\rho_f)) \le \rho(1-\rho_f) \cdot \tilde{\Delta}.
$$

Model 2: An Active Tx Always Has a "Slow" Message

- With prob. ρ_f an active Tx sends "fast" data and always "slow" data $\rightarrow$ can schedule more "slow" Txs

$\rho = 0.8$



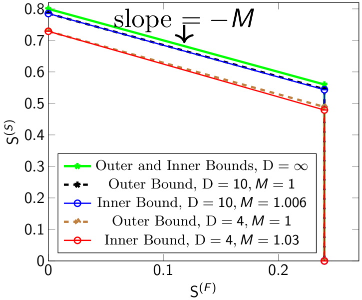
- Sum-MG is decreased with increasing "fast" MG.
- The penalty increases with ρ and the number of interfering links.

With Tx- and Rx-Cooperation

- No need to silence all Txs around "fast" Tx

  → Penalty for transmitting "fast" messages becomes vanishing (slope ≈ -1)

$\rho = 0.8$
$\rho_f = 0.6$



D $\to \infty$ or $\rho = 1$: matching inner and outer bounds

Conclusions

- Jointly designing mixed-delay systems can yield significant performance benefits in networks with cooperation
- Benefits are much larger when txs and rxs can cooperate
- For certain configurations, there is no loss in overall performance due to stringent delay constraints
- Similar observations extend to random arrivals and finite blocklengths (the latter needs more validation)

Related Information-Theoretic Literature

- M. Wigger, R. Timo, and S. Shamai (Shitz), "Conferencing in Wyner's Asymmetric Interference Network: Effect of Number of Rounds ," *IT-Trans*, Feb. 2017.
- W. Huleihel and Y. Steinberg,"Channels with cooperation links that may be absent," *IT-Trans*, Sep. 2017.
- T. Keresztfalvi and A. Lapidoth, " Semi-Robust communications over a broadcast channel", *T-IT*, 2019.
- K. Cohen, A. Steiner and S. Shamai (Shitz), "The broadcast approach under mixed delay constraints," *ISIT* 2012.
- Nadav Shulman and Meir Feder, "Static broadcasting", *IEEE International Symposium on Information Theory (ISIT)*, 2000.
- M. Langberg and M. Effros," Beyond capacity: The joint time-rate region", *arXiv:2101.12236v1*, Jan. 2021.
- Y. C. Liang, R. Zhang and J. Cioffi, "Transmit Optimization for MIMO-OFDM With Delay-Constrained and No-Delay-Constrained Traffic," *TSP* 2006.

Related IT-Literature in Finite Blocklength

- P. H. Lin, S. C. Lin and E. A. Jorswieck, "Early decoding for Gaussian BC with heterogeneous blocklength constraints," in *ISIT*, 2021.
- M. Mross, P. H. Lin, and E. A. Jorswieck, "New inner and outer bounds for 2-user Gaussian broadcast channels with heterogeneous blocklength constraints", Online: *arXiv:2202.02110v1*, Feb. 2022.
- H. Nikbakht, M. Egan, and J-M. Gorce, "Dirty paper coding for consecutive messages with heterogeneous decoding deadlines in the finite blocklength regime," *ISIT* , 2022.
- R. Kassab, O. Simeone, and P. Popovski, "Coexistence of URLLC and eMBB services in the C-RAN uplink: an information-theoretic study," in *Globecom*, 2018.

Our References

- H. Nikbakht, M. Wigger and S. Shamai (Shitz), "Random user activity with mixed delay traffic", *IEEE Information Theory workshop (ITW)*, 2020
- H. Nikbakht, M. Wigger and S. Shamai (Shitz), 'Multiplexing gains under mixed-delay constraints on Wyner's soft-handoff model," *Entropy*, vol. 22, no. 2, pp. 182, 2020.
- H. Nikbakht, M. Wigger and S. Shamai (Shitz), 'Coordinated multi point transmission and reception under mixed delay traffic," *IEEE Transaction on Wireless Communication*, under review.
- H. Nikbakht, M. Wigger, S. Shamai (Shitz), J.M. Gorce, H. V. Poor, "Joint Coding of URLLC and eMBB in Wyner's Soft-Handoff Network in the Finite Blocklength Regime," *IEEE GLOBECOM 2022*
- H. Nikbakht, M. Wigger, M. Egan, S. Shamai, J.-M. Gorce, H. V. Poor, "An information-theoretic view of mixed-delay traffic in 5G and 6G,", *Entropy*, vol. 24, no. 5.