Information-Theoretic Schemes for Integrated Sensing and Communication (ISAC)

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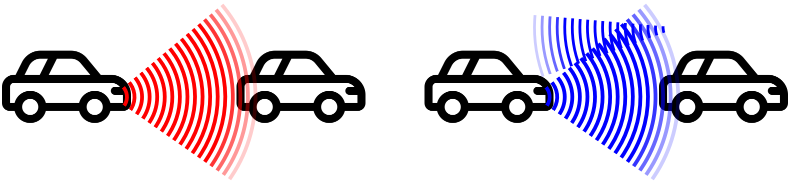
Traditional Sensing and Communications Separation

Communication

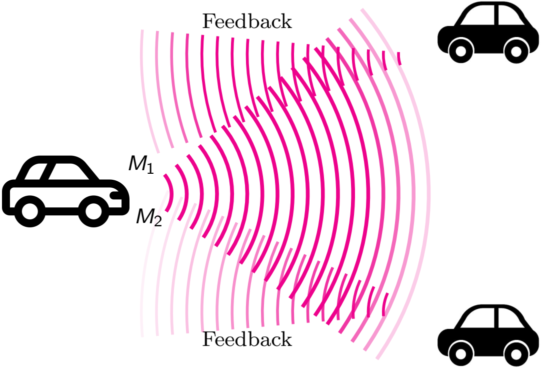
Sensing



Conventional approach: Resource splitting

Integrated Sensing and Communication (ISAC)

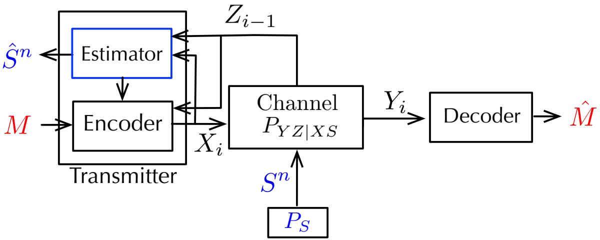
Sensing and Communication



- Synergistic Waveform for Sensing and Comm

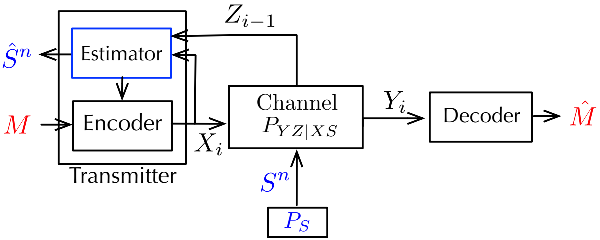
The Point-to-Point (P2P) Channel

Information Theoretic Model for P2P ISAC Kobayashi et al.



- i.i.d. state sequence $S^n = (S_1, \ldots, S_n)$
- Inputs with generalized feedback $X_i = f_i(M, Z^{i-1})$
- State estimation $\hat{S}^n = h(X^n, Z^n)$
- Arbitrary forward and backward channels $P_{Y|XS}$ and $P_{Z|YXS}$
  - Memoryless fading channels $Y = SX + N$
  - Receiver CSI: Y_i can include S_i or imperfect versions of S_i

Information Theoretic Model for P2P ISAC Kobayashi et al.



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Information-Theoretic Fundamental Limit

Definition

Capacity-distortion tradeoff $C(D)$ is largest rate R such that there exist encoder, decoder and estimator with

$$\Pr\left(\hat{M} \neq M\right) \to 0 \qquad \text{as} \qquad n \to \infty$$

and

$$\varlimsup_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[d(S_i, \hat{S}_i)] \leq D$$

Optimal Sensing under Distortion Constraints

- By memoryless assump.: Markov chain $(X^n, Z^n) \to (X_i, Z_i) \to S_i$

Lemma

The optimal estimator operates symbolwise on (X^n, Z^n)

$$\hat{s}^n(x^n, z^n) := (\hat{s}^*(x_1, z_1), \hat{s}^*(x_2, z_2), \ldots, \hat{s}^*(x_n, z_n)),$$

where the optimal per-symbol estimator is

$$\hat{s}^*(x, z) := \arg\min_{s' \in \hat{\mathcal{S}}} \sum_{s \in \mathcal{S}} P_{S|XZ}(s|x, z) d(s, s')$$

- Optimal estimator only depends on input sequence x^n but not on coding scheme $\to$ joint waveform design

Capacity-Distortion Tradeoff $C(D)$

Theorem (Kobayashi et al.)

Capacity-distortion tradeoff

$$C(D) := \max I(X; Y)$$

where maximum is over P_X *satisfying*

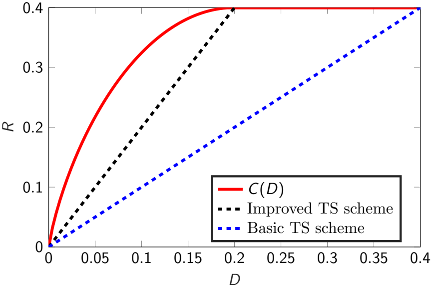
$$\mathbb{E}[d(S, \hat{s}^*(X, Z))] \leq D.$$

Here $(X, S, Y, Z) \sim P_X P_S P_{YZ|SX}$.

- Tradeoff between communication and sensing stems from P_X
- Generalized feedback not used for coding. Simple point-to-point codes are sufficient. It suffices to adjust input pmf P_X to desired sensing performance.

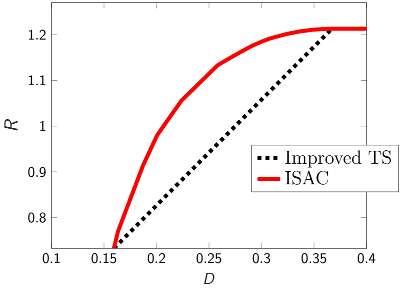
Ex. 1: Binary Multiplicative-State Channel

- $S \sim \mathcal{B}(q)$
- $Z = Y' = SX$ and $Y = (Y', S)$
- Hamming Distortion $d(s, \hat{s}) = s \oplus \hat{s}$.
- Minimize distortion: $X = 1 \;\rightarrow\; D = 0$ and $R = 0$
- Maximize rate: $X \sim \mathcal{B}(1/2) \;\rightarrow\; D = 1/2 \cdot \min\{q, 1-q\}$ and $R = q$



Ex. 2: Rayleigh Fading Channel

- Standard Gaussian state and noises $S_i, N_i, N_{\text{fb},i}$
- Rayleigh fading channel $Y_i' = S_i X_i + N_i$
- Rx observes $Y_i = (Y_i', S_i)$ and Tx $Z_i = Y_i' + N_{\text{fb},i}$
- Input power constraint $P = 10dB$
- Quadratic distortion $d(s, \hat{s}) = (s - \hat{s})^2$.



- $X \sim \mathcal{N}(0, P)$ achieves capacity
- $X \pm \sqrt{P}$ optimal for sensing

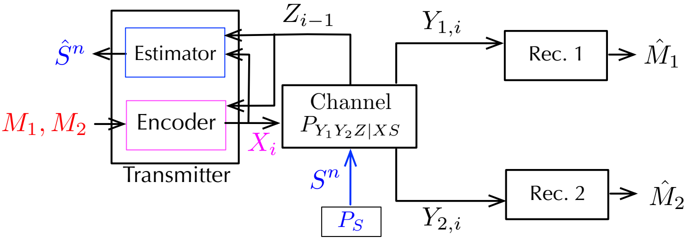
Take-Away Messages for P2P

- Information-theoretic model based on generalized feedback, memoryless state sequence, average distortion
- Symbol-by-symbol estimator optimal; sensing performance depends only on empirical statistics of X^n
- Use optimal data communication scheme under restriction on empirical statistics of x^n
  $\rightarrow$ generalized feedback not used for data communication
- Tradeoff between sensing and communication
- Resource-sharing schemes highly suboptimal

The One-to-Many Broadcast Channel

Information Theoretic Model for BCs

- State-dependent memoryless channels with generalized feedback



- Arbitrary forward and backward channels $P_{Y_1 Y_2|XS}$ and $P_{Z|Y_1 Y_2 XS}$
- Model includes as special cases Rx-CSI and two states $S = (S_1, S_2)$
- For most channels, feedback is helpful

Fundamental Capacity-Distortion Region for BC

Definition

Capacity-distortion region is the set of triples (R_1, R_2, D) so that there exist encoder, decoders, and estimator with

$$\lim_{n\to\infty} \Pr(\hat{M}_k \neq M_k) = 0, \quad k \in \{1,2\}, \qquad \overline{\lim_{n\to\infty}} \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[d(S_i, \hat{S}_i)] \leq D$$

Same per-symbol optimal estimator as for P2P!

Optimal estimator: $\quad s^n = (\hat{s}^*(x_1, z_1), \hat{s}^*(x_2, z_2), \ldots, \hat{s}^*(x_n, z_n)),$

with

$$\hat{s}^*(x,z) := \arg\min_{s' \in \hat{\mathcal{S}}} \sum_{s \in \mathcal{S}} P_{S|XZ}(s|x,z) d(s,s').$$

- Sensing performance depends only on statistics of x^n
- Find optimal generalized-fb BC code and adapt X^n statistics.

Capacity-Distortion Region of Degraded BCs

Degraded Broadcast Channels $X \to Y_1 \to Y_2$

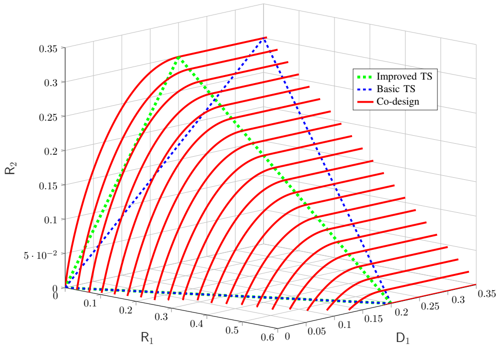
Capacity-distortion region: all (R_1, R_2, D) that for some P_{UX} satisfy

$$\begin{aligned} R_1 &\leq I(X; Y_1 \mid U) \\ R_2 &\leq I(U; Y_2), \\ \mathbb{E}[d(S, \hat{s}^*(X, Z))] &\leq D. \end{aligned}$$

- Tradeoff between communication and sensing from P_X.
- No-feedback codes with appropriate P_X.

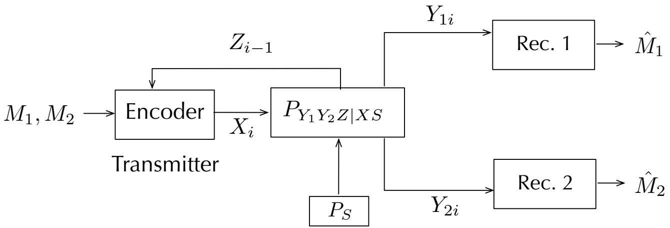
Binary Fading Example: Capacity-Distortion Region

- Double-State $S = (S_1, S_2)$ with corr. components, known at Rxs
- Fading outputs $Y_k = S_k X$, for $k = 1, 2$ (without noise)
- Perfect Rx CSI and both outputs fed back $Z = (Y_1, Y_2)$
- When $X = 1$ Tx learns S_1, S_2; when $X = 0$ it learns nothing



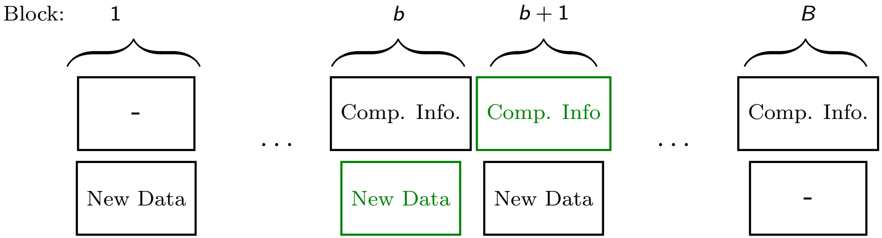
- Resource/time-sharing approaches sub-optimal
- Tradeoff betwen optimal sensing and comm. performances

State-Dependent BCs with Generalized Feedback



- Feedback does not increase capacity of degraded BCs (El Gamal,'79)
- Achievable scheme for general BCs (Shayevitz et al'12, Venkataramanan et al'13)
- Capacity of several BCs with full Receiver-CSI (Kim et al'16)

Intuition about the Shayevitz-Wigger BC Scheme



1. Block-Markov strategy:
   - Compression info sent in block $b+1$: info about channel in block b learned via feedback
   - Block-b outputs improved with compression info sent in block $b+1$
2. New data and compression info sent with Marton's BC scheme (without feedback)

Bounds on Capacity-Distortion Region of General BCs

General Broadcast Channels

Inner and outer bounds (feasible and infeasible regions) based on Shayevitz-W. scheme and genie-aided bound

- Bounds in general case tight only in special cases.

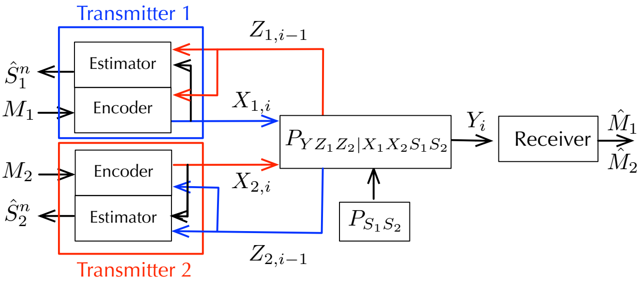
Take-Away Messages for BC

- Information-theoretic model based on generalized feedback, memoryless state sequence, average distortion
- Symbol-by-symbol estimator optimal; sensing performance depends only on empirical statistics of X^n
- Use optimal data communication scheme under restriction on empirical statistics of x^n
  $\rightarrow$ generalized feedback used for data communication
- 3-dimensional tradeoff between 2 rates and distortion
- Resource-sharing schemes highly suboptimal

The Many-to-One Multiaccess Channel

ISAC over Multiaccess Channels

- Two Txs sense (correlated) states and also send data messages



- Symbol-wise estimator at Tx k based on $(X_{k,i}, Z_{k,i})$ is suboptimal!
- Collaborative coding and sensing through Tx-Tx- paths!

Coding for Sensing – A Toy Example

- Via feedback, Tx 1 can get info about S_2 that Tx 2 does not get

  $\rightarrow$ use coding and send it to Tx 2 through the feedback link

- Example: $Y = S_2 X_2 \qquad Z_1 = S_2 \qquad Z_2 = X_1$

  Can achieve distortion 0 at Tx 2 if Tx 1 repeats its feedback

$$X_{1,i} = Z_{1,i-1} = S_{2,i-1} \quad \Longrightarrow \quad Z_{2,i} = S_{2,i-1}$$

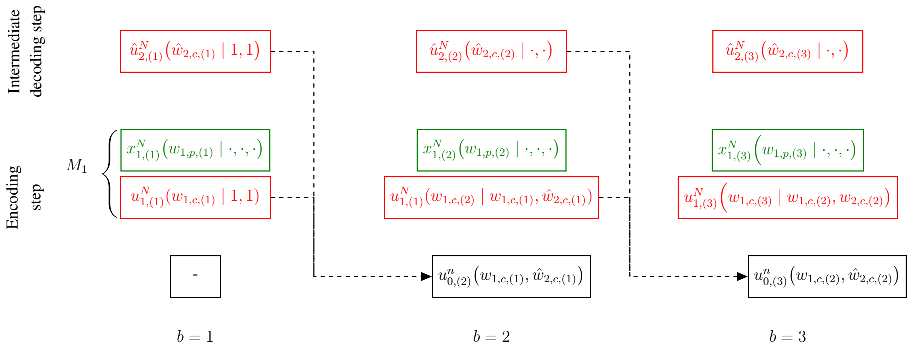
  Without Tx 1 repeating the feedback this is not possible!

Coding for sensing can help if Tx 1 has info. on S_2 that is not available at Tx 2 (or vice versa)

Idea of Collaborative Coding and Sensing Scheme

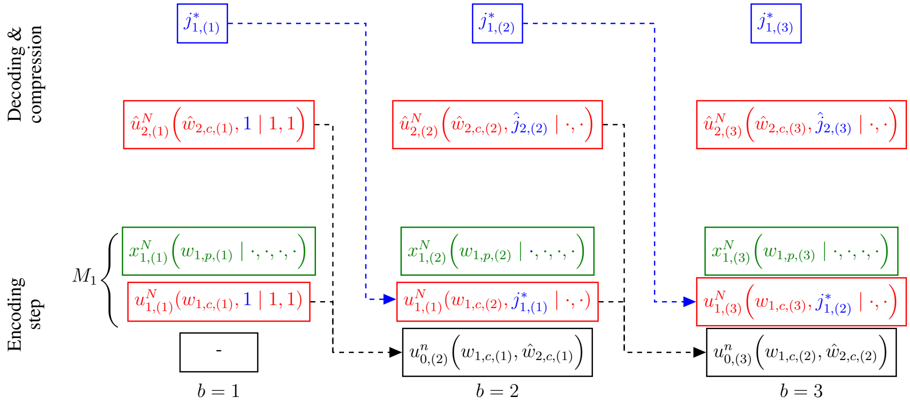
- [Willems'83] scheme (MAC with generalized feedback)
  - Block-Markov coding and backwards decoding
  - Txs exchange message parts over Tx-Tx paths
  - Exchanged message parts are collaboratively re-transmitted in the next block
- Extension of [Willems'83] to ISAC with collaborative sensing
  - After each block, each Tx extracts sensing info of interest to the other Rx
  - Sends this sensing info in the next block in the codeword decoded at other Tx
  - Each Tx estimates state based on inputs/outputs, decoded codewords, and sensing info from other Tx.

Willems' Scheme for MAC With Generalized Feedback



- Backward decoding at the Rx: In block b decode $M_{1,c,(b-1)}, M_{2,c,(b-1)}, M_{1,p,b}, M_{2,p,b}$

Scheme for ISAC MAC with Collaborative Encoding



- Sensing index $J_{k,(b)}$: compression info about $(Z^n_{k,(b)}, X^n_{k,(b)}, U^n_{\bar{k},(b)})$
- $\hat{S}^n_{k,(b)}$ based on $(Z^n_{k,(b)}, X^n_{k,(b)}, U^n_{\bar{k},(b)}, V^n_{\bar{k},(b)}(J_{\bar{k},(b)}))$

Binary MAC Example

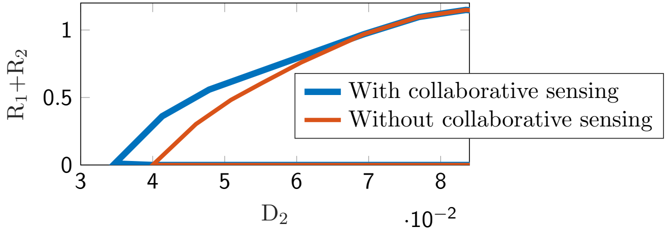
- S_1, S_2 i.i.d. Bernouilli-0.9, noises B_0, B_1, B_2 ind. Bernoulli, and

$$\begin{aligned} Y' &= S_1X_1 + S_2X_2 + B_0, \qquad & Y = (Y', S_1, S_2), \\ Z_k &= S_1X_1 + S_2X_2 + B_k, \qquad & \forall k \in \{1, 2\}. \end{aligned}$$

- Hamming distortion $d(s, \hat{s}) = s \oplus \hat{s}$

- Choose auxiliaries U_0, U_1, U_2 binary and

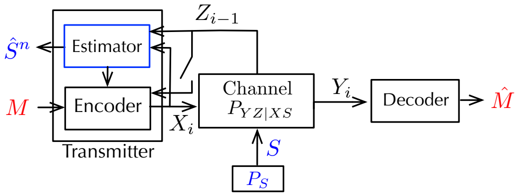
$$V_k = \begin{cases} \mathbb{1}\{Z_k = 1\} + 2 \cdot \mathbb{1}\{Z_k = 2\} & \text{if } E_k = 0 \\ \text{“?”} & \text{if } E_k = 1 \end{cases} \qquad \forall k = \{1, 2\}$$



Take-Aways for the MAC and Related Scenarios

- Symbol-by-symbol estimator based on inputs/outputs suboptimal
- Base estimator also on decoded codewords
- Sensing performance improved through collaborative sensing $\rightarrow$ Use the Tx-to-Tx path already used for feedback communication!
- Improved schemes are possible using interactive two-way schemes (Han) and joint source-channel coding
- Tradeoff between sensing and communication
- Capacity-distortion region for the MAC remains an open problem

Interesting Related Models (Joudeh et al., Wu et al., Chang et al.)



- Single sensing parameter $S \in \{0, 1\}$ constant for all times
- Sensing performance measured in detection-error exponent
  $E := -\frac{1}{n} \log \Big(\max_{s\in\{0,1\}} \frac{1}{|\mathcal{W}|} \sum_{w\in\mathcal{W}} \mathbb{P}[\hat{S} \neq s \mid W = w, S = s] \Big).$

Theorem (Chang et al.)

Without feedback coding, (R, D) pairs are achievable iff for some P_X:

$$R \leq \min_{s\in\mathcal{S}} I(X; Y),$$
$$D \leq \min_{s\in\{0,1\}} \max_{l\in[0,1]} -\sum_x P_X(x) \log \Big(\sum_z P_{Z|XS}(z|x, 0)^l P_{Z|XS}(z|x, 1)^{1-l} \Big).$$

With feedback for coding the set is larger.

Summary and Outlook

- Presented information-theoretic framework for integrated sensing and communication (Kobayashi,Caire,Kramer'18)
- Single Tx: sensing performance depends only on X^n statistics, i.e., on the chosen waveform.
- Tradeoff between rates and distortion(s).
- Multiple Txs: *Fully integrate coding for collaborative sensing and comm.*
- Interesting future research directions:
  - Channels with memory
  - Other sensing performance criteria
  - Secrecy constraints

References on Information-Theoretic ISAC

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