# Coding for Sensing

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June, 2022

# Traditional Radar and Communications Separation

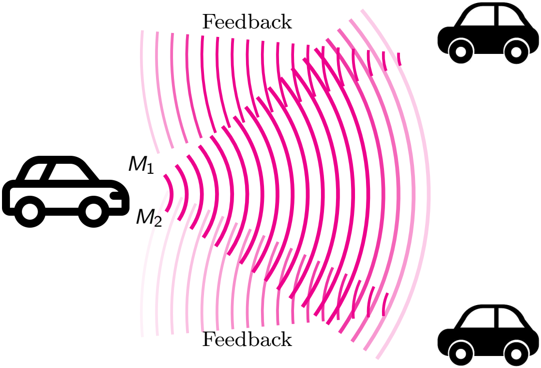
Communication

Sensing

Conventional approach: Resource splitting

# Integrated Communication and Sensing

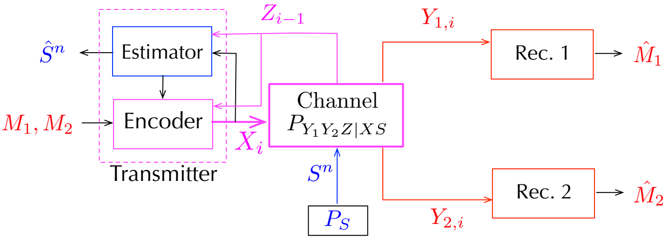
## Communication and Sensing




- Synergistic Waveform for Sensing and Comm
- Coding for Sensing

# Information Theoretic Model — The Channel

- State-dependent memoryless channels with generalized feedback




- Arbitrary forward and backward channels $P_{Y_1Y_2|XS}$ and $P_{Z|Y_1Y_2XS}$
- Model includes as special cases
  - Memoryless fading channels $Y_k = SX + N_k$ or $Y_k = \tilde{S}X + N_k$
  - Receiver CSI: $Y_k$ can include $S$ or imperfect versions of $S$
  - Generalized feedback can be $Z = SX + W_k$ or $Z = Y_k + W_k$ or ..

# Information Theoretic Model — Sensing

- Sensing Performance Measured by Average Block-Distortion:

$$\overline{\lim_{n\to\infty}} \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}[d(S_i, \hat{S}_i)] \le D.$$

- Arbitrary distortion functions $d(\cdot,\cdot)$, even functions of the state
- Can handle vector states $S = (S_1, S_2)$, e.g., 2 objects/environments
- Extends to multiple distortions:
  $\overline{\lim}_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}[d_k(S_i, \hat{S}_i)] \le D_k, \quad k = 1, 2, ...$

# Fundamental Capacity-Distortion Tradeoff for BC

## Definition

A rate-distortion tuple $(R_1, R_2, D)$ is achievable if there exist encoder, decoders and estimator s.t.

$$\Pr\left(\hat{M}_1 \neq M_1 \quad \text{or} \quad \hat{M}_2 \neq M_2\right) \to 0 \qquad \text{as} \qquad n \to \infty$$

and

$$\overline{\lim_{n\to\infty}} \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[d(S_i, \hat{S}_i)] \leq D$$

# Optimal Estimator for BC

**Lemma**

*The optimal estimator operates symbolwise on* $(X^n, Z^n)$

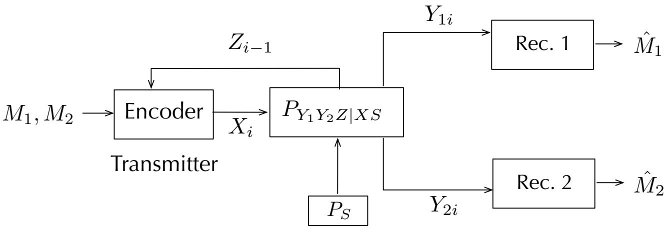
$$h^*(x^n, z^n) := (\hat{s}^*(x_1, z_1), \hat{s}^*(x_2, z_2), \dots, \hat{s}^*(x_n, z_n)),$$

*where the optimal per-symbol estimator is*

$$\hat{s}^*(x, z) := \arg\min_{s' \in \hat{\mathcal{S}}} \sum_{s \in \mathcal{S}} P_{S|XZ}(s|x, z) d(s, s').$$

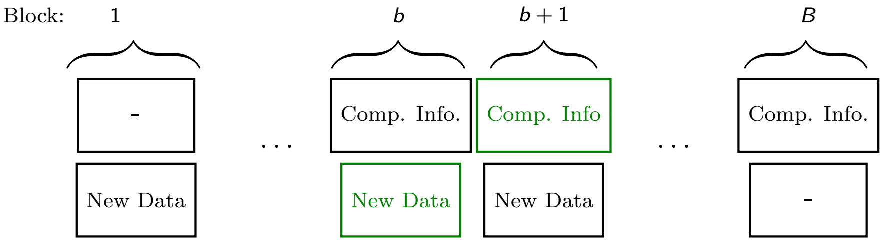
- Given $(X^n, Z^n)$, min. distortion $D_{\min} = \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}[d(S_i, \hat{s}^*(X_i, Z_i))]$
- Finding an optimal integrated sensing & comm. scheme reduces to finding optimal comm. scheme with desired $X^n$ statistics
- Tradeoff between comm and sensing stems from $X^n$ statistics

# State-Dependent BCs with Generalized Feedback




- Feedback does not increase capacity of degraded BCs (El Gamal,'79)
- Achievable scheme for general BCs
  (Shayevitz et al'12, Venkataramanan et al'13)
- Capacity of several BCs with full Receiver-CSI (Kim et al'16)

# Intuition about the Shayevitz-Wigger BC Scheme




1. Block-Markov strategy:
   - Compression info sent in block $b+1$: info about channel in block $b$ learned via feedback
   - Block-$b$ outputs improved with compression info sent in block $b+1$
2. New data and compression info sent with Marton's BC scheme (without feedback)

# Results on Capacity-Distortion Region of BCs

## Degraded Broadcast Channels $X \to Y_1 \to Y_2$

Capacity-distortion region $\mathcal{CD}$: set of triples $(R_1, R_2, D)$ satisfying

$$\begin{aligned} R_1 &\leq I(X; Y_1 \mid U) \\ R_2 &\leq I(U; Y_2), \\ \mathbb{E}[d(S, \hat{s}^*(X, Z))] &\leq D \end{aligned}$$

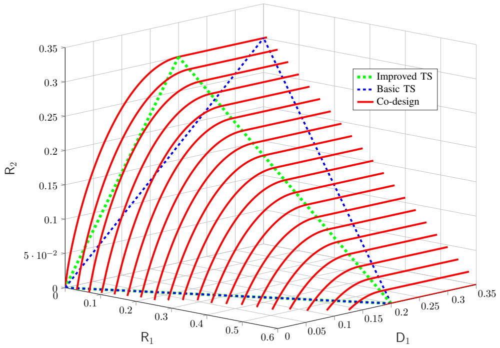
for some pmf $P_{UX} P_S P_{Y_1 Y_2 Z|XS}$.

## General Broadcast Channels

Inner and outer bounds (feasible and infeasible regions) based on Shayevitz-W. scheme and genie-aided bound

# Binary Fading Example: Capacity-Distortion Region

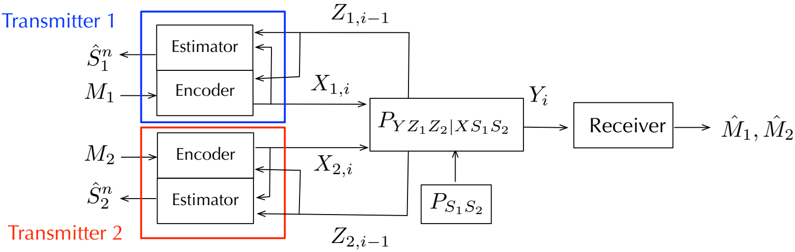
- Double-State $S = (S_1, S_2)$ with corr. components, known at Rxs!
- Fading outputs $Y_k = S_k X$, for $k = 1, 2$ (without noise)
- Perfect Rx CSI and both outputs fed back $Z = (Y_1, Y_2)$
- When $X = 1$ Tx learns $S_1, S_2$;  when $X = 0$ it learns nothing




- Resource/time-sharing approaches sub-optimal
- Tradeoff betwen optimal sensing and comm. performances

# Multiple-Access Channels

- Two Txs sense (correlated) states and also send data messages
- Feedback allows for communication/coordination between Txs




- Sensing at Tx $k$ based only on $(X_k^n, Z_k^n)$ is suboptimal!
- Use coding not only for communication but also for sensing!

# Coding for Sensing – A Toy Example

- Via feedback, Tx 1 can get info about $S_2$ that Tx 2 does not get

  $\rightarrow$ use coding and send it to Tx 2 through the feedback link

- Example: $Y = S_2 X_2 \qquad Z_1 = S_2 \qquad Z_2 = X_1$

  Can achieve distortion 0 at Tx 2 if Tx 1 repeats its feedback

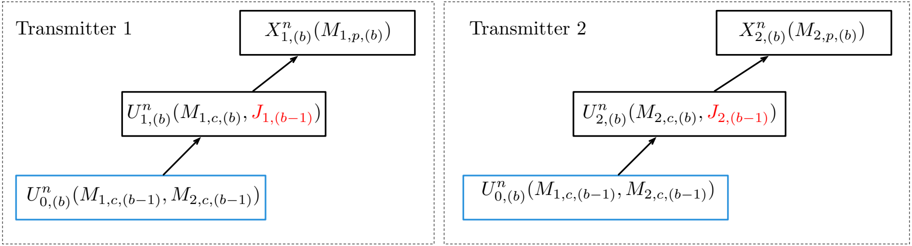
  $$X_{1,i} = Z_{1,i-1} = S_{2,i-1} \quad \Longrightarrow \quad Z_{2,i} = S_{2,i-1}$$

  Without Tx 1 repeating the feedback this is not possible!

Coding for sensing can help if Tx 1 has info. on $S_2$ that is not available at Tx 2 (or vice versa)

# Coding Scheme

- Extends [Willems'83] scheme (MAC with generalized feedback)
- Block-Markov coding and backwards decoding
- Feedback allows to create cooperative $U_0$-codeword
- Txs exchange message parts and compression info for sensing




- $J_{k,(b)}$: compression info about $(Z^n_{k,(b)}, X^n_{k,(b)}, U^n_{k,(b)})$

# Results on MAC with Integrated Sensing and Communications

- New idea of coding-for-sensing
- Improved achievable region (better sensing performance) over state of the art
- Capacity-distortion region for the MAC remains an open problem

# Summary

- Presented information-theoretic framework for integrated sensing and communication (Kobayashi,Caire,Kramer'18)
- Single Tx: sensing performance depends only on $X^n$ statistics, i.e., on the chosen waveform
- Tradeoff between rates and distortion(s)
- Multiple Txs: *fully integrated coding for sensing and comm.* $\rightarrow$ we included coding for sensing in Willems'83 scheme

- Interesting future research direction: channels with memory