Habilitation Exam

Communication, Compression, and Coordination over Networks: Benefits of Cooperation and Side-Information

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Distributed compression (source coding) with side-information

- Sensors observing sources
- Distributed storage system
- Nodes reconstructing sources
Communication networks with cooperating base stations or mobiles

BS-BS cooperation inside a cell

Mobile-BS cooperation inside a cell

BS-BS or mobile-mobile cooperation across cells
Coordinating partially-informed agents

Here: Coordinate hydro- and nuclear-plant to stabilize system and minimize productions

General cyber-physical system for transportation, rescue, health care, etc.
Information-theoretic setups

Asymptotically vanishing probability of error

- Distributed compression systems: how many bits to describe outcome of a source?

- Communication networks: how many bits can one transmit?

- Coordination of distributed agents: which actions-tuples are implementable?
Communication networks with cooperating users
BS-to-BS cooperation inside a cell: downlink

(AAlmost) lossless joint source-channel coding

Reliable communications for rates \((R_1, R_2)\) possible, if \(\exists\) encodings and decodings s.t.

\[
P \left[ \{ \hat{X}_1^n \neq X^n \} \cup \{ \hat{X}_2^n \neq X^n \} \right] \rightarrow 0
\]

Two scenarios for helper side-informations $V_1^n$ and $V_2^n$

Scenario 1: Scalar quantisation

$V_{k,t} = \phi_k(W_t)$

Scenario 2: Correlated sources

$X^n, Y_1^n, Y_2^n, V_1^n, V_2^n \text{ i.i.d.}$

$\sim P_{X_{Y_1 Y_2 V_1 V_2}}$

Background: No helper basestations, $R_1 = R_2 = 0$

Theorem [3]:
Reliable communications possible iff $\exists W \sim P_W$ s.t.

\[ H(X | Y_1) \leq I(W; U_1) \]

and

\[ H(X | Y_2) \leq I(W; U_2) \]

**Result for Scenario 1:** \( V_{k,t} = \phi_k(W_t) \)

**Theorem:**

Reliable communications possible with helper rates \((R_1, R_2)\) iff \( \exists W \sim P_W \) s.t.

\[
\begin{align*}
H(X|Y_1) &\leq I(W; U_1) + \min \left\{ R_1, \ I(W; V_1|U_1) \right\} \\
H(X|Y_2) &\leq I(W; U_2) + \min \left\{ R_2, \ I(W; V_2|U_2) \right\}
\end{align*}
\]

- Helper BS \( k \) randomly hashes \( V^n_k \) as in deterministic relay channels
Result for Scenario 2: \((X^n, Y^n_1, Y^n_2, V^n_1, V^n_2)\) i.i.d. \(\sim P_{X,Y,V_1, V_2}\)

Theorem:

Reliable communication possible with helper rates \((R_1, R_2)\) iff \(\exists W, A_1, A_2\) s.t.

\[
H(X|Y_1, A_1) \leq I(W; U_1) \quad R_1 \geq I(V_1; A_1|Y_1) \geq I(X; A_1|Y_1)
\]
\[
H(X|Y_2, A_2) \leq I(W; U_2) \quad R_2 \geq I(V_2; A_2|Y_2) \geq I(X; A_2|Y_2)
\]

and \((X, Y_1) \rightarrow V_1 \rightarrow A_1\) and \((X, Y_2) \rightarrow V_2 \rightarrow A_2\)

- Helper BS \(k\) uses Wyner’s helper source-code to compress \(V^n_k\) into \(A^n_k\)
Summary on: BS-to-BS cooperation inside a cell

- Capacity for two-scenarios of BSs-cooperation models
- Modularity/Duality of optimal solutions
- same operations at main BS
- Helper BSs: use Wyner’s helper source code or random hashing as for det. relay channels
- receiving mobiles: Tuncel’s decoding \textit{with improved}
  - channel outputs (Scenario 1); or
  - source side-information (Scenario 2)
BS-to-BS or mobile-to-mobile cooperation across cells

- Cooperation over digital links of given capacities

# of conferencing rounds limited due to latency or complexity constraints

Wyner’s Asymmetric Soft-Handoff Model

- $K$ transmitter/receiver pairs

- Channel gains $\{\alpha_k\}$ fixed, constant, non-zero

- Memoryless Gaussian noises of variance $\sigma^2$ and equal power constraints $P$

**Goal**

Determine message rates $R_1, \ldots, R_K$ s.t. $\forall k: \Pr(\hat{M}_k \neq M_k) \to 0$ as $n \to \infty$
Communication takes place in 4 phases

- Conferencing over $\kappa$ rounds (here $\kappa = 2$):
  - Messages spread over $\kappa$ transmitters to left & right
  - Output signals spread over $\kappa$ receivers to left & right
Communication takes place in 4 phases

**Phase 1: Transmitter-conferencing**

- Conferencing over $\kappa$ rounds (here $\kappa = 2$):
  - Messages spread over $\kappa$ transmitters to left & right
  - Output signals spread over $\kappa$ receivers to left & right
Communication takes place in 4 phases

Phase 2: Cooperative communication over network

Conferencing over $\kappa$ rounds (here $\kappa = 2$):

→ Messages spread over $\kappa$ transmitters to left & right

→ Output signals spread over $\kappa$ receivers to left & right
Communication takes place in 4 phases

Phase 3: Receiver-conferencing

- Conferencing over $\kappa$ rounds (here $\kappa = 2$):
  - Messages spread over $\kappa$ transmitters to left & right
  - Output signals spread over $\kappa$ receivers to left & right
Communication takes place in 4 phases

Phase 4: Clustered decoding

- Conferencing over $\kappa$ rounds (here $\kappa = 2$):
  - Messages spread over $\kappa$ transmitters to left & right
  - Output signals spread over $\kappa$ receivers to left & right
High-SNR Performance: Multiplexing-Gain Per User

- **Sum-capacity:** $C_\Sigma$, maximum sum of rates $R_1 + R_2 + \cdots + R_K$ s.t. $p(\text{error}) \to 0$.

- **Asymptotic multiplexing gain per user $S$:**

  \[
  \text{Sum-capacity: } C_\Sigma \approx S \cdot \frac{K}{2} \log(1 + P/\sigma^2), \quad P\sigma^2 \gg 1
  \]

- **Conferencing prelogs $\mu_{Tx}$ and $\mu_{Rx}$:**

  \[
  R_{Tx} = \mu_{Tx} \cdot \frac{1}{2} \log(1+P/\sigma^2) \quad \text{and} \quad R_{Rx} = \mu_{Rx} \cdot \frac{1}{2} \log(1+P/\sigma^2)
  \]
Results:

Theorem (Achievability when number of conferencing rounds $\kappa < \infty$)

$$S \geq \begin{cases} 
\frac{1+2\mu_{Tx}+2\mu_{Rx}}{2} & 2\mu_{Tx} + 2\mu_{Rx} + \frac{2\max\{\mu_{Tx},\mu_{Rx}\}}{\kappa} < 1 \\
\frac{1+2\kappa+\max\{2\mu_{Tx},2\mu_{Rx}\}}{2\kappa+2} & \text{otherwise} \\
\frac{4\kappa+1}{4\kappa+2} & \min\{\mu_{Tx},\mu_{Rx}\} > \frac{\kappa}{4\kappa+2}
\end{cases}$$

Converse tight when $\kappa = 1$ and at the same time $\mu_{Tx} = 0$ or $\mu_{Rx} = 0$.

Theorem (Number of conferencing rounds $\kappa = \infty$)

$$S_{\kappa=\infty} = \min \left\{ 1, \frac{1 + 2\mu_{Tx} + 2\mu_{Rx}}{2} \right\}$$
Comparison

- For small conferencing prelogs $\kappa = 1$ suffices!

- For finite $\kappa$, multiplexing gain per user saturates below 1

- Duality between transmitter-cooperation and receiver-cooperation
Coding scheme for $\kappa = \infty$

- $\mu_{Tx} = \mu_{Rx} = 1/2 \implies S_{\kappa=\infty} = 1$

- **Tx-conferencing:** $\hat{X}_k = f_k(M_1, \ldots, M_k)$

- **Rx-conferencing:** $\hat{M}_k = g_k(Y^n_1, \ldots, Y^n_k)$

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Coding scheme for $\kappa = \infty$

- $\mu_{T_x} = \mu_{R_x} = 1/2 \implies S_{\kappa=\infty} = 1$

- **Tx-conferencing:** $\hat{X}_k = f_k(M_1, \ldots, M_k)$

- **Rx-conferencing:** $\hat{M}_k = g_k(Y_1^n, \ldots, Y_k^n)$

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Coding Scheme for $\kappa = 1$

- $\mu_{Tx} = \mu_{Rx} = 1/2 \implies S_{\kappa=\infty} = 5/6$

- **Tx-conferencing:** $M_k$

- **Rx-conferencing:** $\hat{Y}_k^n = g_k(Y_k^n)$

Generally, interference mitigation causes interference to propagate
→ for finite $\kappa$ need to switch off txs!
Summary and Outlook on: Cooperation across cells

- For small conferencing prelogs $\kappa = 1$ suffices!

- For finite $\kappa$, multiplexing gain per user saturates below 1

- Duality between transmitter-cooperation and receiver-cooperation

- In future: analyze different cooperation constraints
  - oblivious codebooks
  - more accurate latency and complexity constraints? (mobiles!)
Mobiles-to-BS cooperation in a cell using feedback

- How and how much does feedback help on a memoryless BC
- Duality to the MAC


Rate-limited feedback on discrete memoryless BCs

Feedback rate constraint: $|F_{i,1}| \cdots |F_{i,n}| \leq nR_{fb,i}, \quad i = 1,2$

$X_t = f_t(M_1, M_2, F_{1}^{t-1}, F_{2}^{t-1})$
Dueck’s example provides first intuition how feedback helps on the BC

\[ X_t = \begin{pmatrix} B_{1,t} \\ B_{0,t} \\ B_{2,t} \end{pmatrix}, \quad Y_{1,t} = \begin{pmatrix} B_{1,t} + Z_{0,t} \\ B_{0,t} \end{pmatrix}, \quad Y_{2,t} = \begin{pmatrix} B_{2,t} + Z_{0,t} \\ B_{0,t} \end{pmatrix}, \quad Z_{0,t} \sim B(1/2) \]

- Capacity without feedback: \( 0 \leq R_1 + R_2 \leq 1 \)

- Capacity with feedback: \( 0 \leq R_1 \leq 1 \) and \( 0 \leq R_2 \leq 1 \)

\[ \rightarrow \text{send uncoded bits through} \ \{B_{1,t}\} \ \text{and} \ \{B_{2,t}\} \ \text{and send} \ B_{0,t} = Z_{0,t-1} \]

Feedback allows identifying update inform. that will be useful to both receivers

\[ \rightarrow \text{common update information allows to increase both private rates!} \]
Subsequent schemes building on this idea

- Wang’09: → Erasure BC
- Tassiulas&Georgiadis’10: → Erasure BC
- Shayevitz&W’10: → General channels, generalized feedback
- Maddah-Ali&Tse’10: → Fading channels and deterministic models; state-feedback
- Chen&Elia’13, Yang/Kobayashi/Gesbert/Yi’13

Feedback-gain remains unknown for most BCs
(Strictly) Less-Noise DMBCs

\[ Y_2 \succ Y_1 \]

For every auxiliary \( U \rightarrow X \rightarrow (Y_1, Y_2) \) with \( I(U; Y_1) > 0 \):

\[ I(U; Y_2) > I(U; Y_1) \]

- **Asymmetric** Binary Symmetric BC
- **Asymmetric** Binary Erasure BC
- Binary Symmetric/Erasure BC for certain parameters
Capacity of Less-Noisy DMBCs, \( Y_2 \succeq Y_1 \), i.e. \( I(U; Y_2) \geq I(U; Y_1) \)

- Capacity: all rate pairs \((R_1, R_2)\) where for some \( U \rightarrow X \rightarrow (Y_1, Y_2) \)

\[
R_1 \leq I(U; Y_1) \\
R_2 \leq I(X; Y_2 | U)
\]

- Achieved by superposition coding

- If \( I(U; Y_2) > I(U; Y_1) \), Rx 2 could even decode an extra message in cloud center

- Problem: Rx 1 cannot decode, unless it knows this extra message...
Piggyback-coding

- Receiver 1 knows extra message $M_{\text{piggyback}}$
- Product codebook for messages $M_1$ and $M_{\text{piggyback}}$

\[ u^n(1,1) \quad u^n(1,2) \]
\[ u^n(2,1) \quad u^n(2,2) \]
\[ u^n(3,1) \quad u^n(3,2) \]

\[ u^n(2^nR_1,1) \quad u^n(2^nR_1,2) \]

\[ u^n(1,2^nR_{pg} - 1) \quad u^n(1,2^nR_{pg}) \]
\[ u^n(2,2^nR_{pg} - 1) \quad u^n(2,2^nR_{pg}) \]
\[ u^n(3,2^nR_{pg} - 1) \quad u^n(3,2^nR_{pg}) \]

Decoding possible if: $R_1 + R_{pg} < I(U; Y_2)$ and $R_1 < I(U; Y_1)$

$M_{\text{piggyback}}$ is not harming Receiver 1!
BC-scheme with feedback from the weaker Receiver 1

- Block-Markov coding with a piggyback superposition code in each block $b$:

Choose $M_{1,Fb,b-1}$ as a Wyner-Ziv message to compress $Y_{1,b-1}^n \rightarrow \tilde{Y}_{1,b}^n$

Rx 2 decodes $M_{2,b}$ based on $(\tilde{Y}_{1,b}^n, Y_{2,b}^n)$
A Simpler Achievable Region

**Theorem**

\((R_1, R_2)\) achievable, if for some \(P_U P_X U \hat{P}_Y 1 U Y_1:\)

\[
R_1 \leq I(U; Y_1) \\
R_2 \leq I(X; \hat{Y}_1, Y_2|U) = I(X; Y_2|U) + I(X; \hat{Y}_1|U, Y_2)
\]

and

\[
I(\hat{Y}_1; Y_1|U, Y_2) \leq \min\{R_{FB,1}, I(U; Y_2) - I(U; Y_1)\}.
\]

- Sending \(\hat{Y}_1\) is purely beneficial: not bothering Rx 1 and helping Rx 2
If $R_{FB,1} > 0$, Feedback Increases Entire Capacity Region

**Theorem:** For any DMBC $Y_2 \succ Y_1$, when $R_{FB,1} > 0$

Feedback improves all $(R_1 > 0, R_2 > 0)$ of the nofeedback capacity, unless $(R_1, R_2)$ lies on boundary of capacity of enhanced channel

- Ex.: Asymmetric Binary Symmetric, Binary Erasure, Gaussian BC
Extension to Two-Sided Feedback

- Marton-coding

- Send feedback messages $M_{FB,1,b-1}$ and $M_{FB,2,b-1}$ in cloud center of block $b$ using double piggyback-coding

- Feedback messages compress outputs $Y_{1,b}^n$ or $Y_{2,b}^n$

$M_{FB,1,b-1}$ “transparent” for Receiver 1, $M_{FB,2,b-1}$ for Receiver 2

→ like “double-booking” resources in cloud-center
Duality between Gaussian MIMO MAC and BC with feedback

Gaussian MIMO BC:

\[ Y_k = H_k X + Z_k, \quad k = 1, \ldots, K \]

power constraint \( P \)

Gaussian MIMO MAC:

\[ Y_k = \sum_{k=1}^{K} H_k^T X_k + Z \]

sum-power const.
\( P_1 + P_2 + \ldots + P_K = P \)

perfect feedback

Theorem

Rates achievable with linear-feedback schemes for dual BC and MAC coincide!


Summary and outlook on: mobiles-to-BS cooperation using feedback

- Proposed two ways of exploiting feedback on memoryless BCs

- Even low-rate feedback increases capacity of large class of memoryless BCs
  \[\rightarrow \text{not only strictly less noisy!}\]

- Memoryless Gaussian MAC-BC duality when restricting to linear-feedback schemes (perfect feedback)

- Explore MAC-BC duality with feedback for discrete memoryless case
  \[\rightarrow \text{BC-dual to the Cover-Leung scheme for MAC with feedback?}\]
Coordinating partially-informed agents
Coordination over state-dependent networks: motivation

- Wish to coordinate inputs and state:

\[ \frac{1}{T} \sum_{t=1}^{T} P_{S_0,t, X_1,t, X_2,t} \rightarrow \bar{Q} \quad \text{as} \quad T \rightarrow \infty \]

Game-Theoretic Motivation: Infinitively Repeated Games

- Agents’ actions $X_{1,t}$ and $X_{2,t}$

- Payoff-functions $\omega_1(s_0,t, x_{1,t}, x_{2,t})$ and $\omega_2(s_0, x_{1,t}, x_{2,t})$

- Average expected payoff:

$$\overline{\omega}_k = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[\omega_k(S_0,t, X_{1,t}, X_{2,t})]$$

$$= \sum_{(s_0, x_1, x_2)} \omega_k(s_0, x_1, x_2) \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} P_{S_0,t}X_{1,t}X_{2,t}(s_0, x_1, x_2).$$
Setup and implementable distributions

\[ \{S_{1,t}\} \xrightarrow{\Gamma(y_1, y_2|s_0, x_1, x_2)} \{S_{2,t}\} \]

\[ S_{0,t} \sim \rho_0 \]

Agent 1 \hspace{1cm} Agent 2

\[ X_{1,t}, Y_{1,t} \hspace{1cm} X_{2,t}, Y_{2,t} \]

- Causal or non-causal SI:

\[ X_{k,t} = \sigma_{k,t}^{(c)}(S_k^t, Y_{k}^{t-1}) \quad \text{or} \quad X_{k,t} = \sigma_{k,t}^{(nc)}(S_k^T, Y_{k}^{t-1}) \]

Implementable distributions \( \overline{Q} \)

\[ \forall \epsilon > 0 \text{ there exist } T \text{ and encodings, s.t.} \]

\[ \left| \frac{1}{T} \sum_{t=1}^{T} P_{S_{0,t}X_{1,t}X_{2,t}}(s_0, x_1, x_2) - \overline{Q}(s_0, x_1, x_2) \right| \leq \epsilon. \]
Early related model and result: Gossner et al. 2006

\[
\begin{align*}
\{S_0,t\} & \quad \text{observed non-causally} \\
\text{Agent 1} & \xrightarrow{X_1,t} \Gamma(y_2|s_0,x_1) & Y_{2,t} = (X_{1,t}, S_{0,t}) & \xrightarrow{\text{Agent 2}} \text{Agent 2} & \xrightarrow{X_{2,t} = \sigma_2(Y_{2,t-1})}
\end{align*}
\]

- \(\{X_{1,t}\}\) communicates \(\{S_{0,t}\}\) to Agent 2

**Theorem[12]**

\[Q(s_0, x_1, x_2) \text{ implementable iff } I(S_0; X_2) \leq H(X_1|S_0, X_2)\]


New results under causal SI at both agents:

\[ X_{k,t} = \sigma_{k,t}^{(c)}(S^t_k, Y^{t-1}_k) \]

\[ \text{Agent 1} \quad \text{Agent 2} \]

\[ \{S_{1,t}\} \quad \{S_{2,t}\} \]

observed causally

\[ \text{observed causally} \]

\[ \Gamma(y_1, y_2|s_0, x_1, x_2) \]

\[ X_{1,t} \quad X_{2,t} \]

\[ Y_{1,t} \quad Y_{2,t} \]

Theorem

\[ \overline{Q} \) implementable iff it factorizes as

\[ \overline{Q}(s_0, x_1, x_2) = \sum_{u, s_1, s_2} \left[ \rho_0(s_0) \overline{\eta}(s_1, s_2|s_0) P_U(u) \prod_{k=1}^{2} P_{X_k|U,S_k}(x_k|u, s_k) \right] \]

- No coding/communication required
- Extends to \( K \) agents
Non-causal state-info at both agents

\[ \{S_1, t\} \xrightarrow{\neg}(S_1, t, S_2, t | S_0, t) \xrightarrow{\neg} \{S_2, t\} \]

\[ S_{0,t} \sim \rho_0 \]

observed non-causally

\[ X_1, t \]

\[ \Gamma(y_1, y_2 | s_0, x_1, x_2) \]

\[ Y_1, t \]

\[ X_2, t \]

\[ Y_2, t \]

Agent 1  \quad \Gamma(y_1, y_2 | s_0, x_1, x_2) \quad \text{Agent 2}

Theorem: If \( S_{2,t} = f(S_{1,t}) \) or \( \Gamma(y_1, y_2 | s_0, x_1, x_2) = \tilde{\Gamma}(y_1, y_2 | s_0, x_1) \)

\[ Q(s_0, x_1, x_2) \text{ implementable iff it is marginal of some } \]

\[ Q(s_0, s_1, s_2, u, v, x_1, x_2, y_2) = \rho_0(s_0) \neg(s_1, s_2 | s_0) P_{UVX_1 | S_1}(u, v, x_1 | s_1) P_{X_2 | US_2}(x_2 | u, s_2) \Gamma(y_1, y_2 | s_0, x_1, x_2) \]

satisfying

\[ I(S_1; U | S_2) \leq I(V; Y_2, S_2 | U) - I(V; S_1 | U) \]

- Communication only in one direction: Agent 1 coordinates Agent 2
Summary and outlook on: coordinating partially-informed agents

- Proposed $K$-agent framework for coordination over state-dependent networks

- Only local coordination when state-information at agents local

- Implementable distributions under non-causal state-information when $K = 2$ and when communication only in one direction

- In future:
  - Distributed coordination, e.g., many-to-one
  - Benefits of coding strategies for real cyber-physical networks
More research plans

- Cache-aided communication networks (noisy channels!)
- Video streaming

- Interplays between information theory and statistics:
  - distributed hypothesis testing
  - distributed clustering (information bottleneck method)
Curriculum

- Master of Sciences ETH Zurich, March 2003
- PhD ETH Zurich, October 2008
- PostDoc University of California San Diego, May–November 2009
- Assistant Professor (Maître de Conférences) Telecom ParisTech, December 2009
- Visiting Professor at the Technion—Israel Institute of Technology June 2011
- Visiting Professor at ETH Zurich July/August 2010, August 2013, July/August 2015
Teaching

- Introduction to digital communications, Telecom ParisTech
- Information theory, Telecom ParisTech
- Iterative decoding methods, Telecom ParisTech
- Multi-user information theory, ETH Zurich
- Coding and cellular automata, ETH Zurich
Supervision of students

- **Post Docs:**
  - Roy Timo, November 2013–April 2014
  - Sadaf Salehkalaibar, February/March 2015

- **PhD students:**
  - Youlong Wu, November 2011–October 2014
  - Selma Belhadj Amor, October 2011–March 2015

- **Master Thesis students:**
  - Andreas Malär, June–December 2010
  - Thomas Laich, June–December 2012
Service to the society

- Associate Editor of IEEE Communication Letters, since December 2012

- TPC member of:
  - ITW 2015
  - ISWCS 2011

- Organization of conferences, workshops, and invited sessions:
  - Publicity chair ITW 2016, Cambridge, UK
  - Co-organizer of GdR ISIS workshop “Recent advances in information theory”, April 2012, Paris, France
  - Organizer of invited sessions at ISWCS 2011 and IZS 2012
Awards and grants

- ETH Medal for excellent PhD Thesis (8%)
- ETH Medal for excellent Master Thesis (2.5%)
- Diploma of Master of Science with Distinction

- "Emergences" Starting-Grant from the city of Paris, October 2012–March 2015
- CEFIPRA Project D2D, starting May 2015. Collaborative project with Alcatel-Lucent, INRIA, and IIT Mumbay
- Technion scholarship for visiting professors, June 2011
- IDEA League scholarships for visiting professors Jul/Aug. 2010 and Aug. 2013
- Google scholarship for research visit of PhD student Youlong Wu at the Technion
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