Markov Point Process for Multiple Object Detection

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Outline

- Motivation
- Some results using MPP
- The different issues to solve
- First example: Trees detection
- A new optimization algorithm based on a multiple births and deaths dynamics (coll. IITP Moscow: ÉCONET / Associated Team ODESSA)
- Second example: Flamingos counting
- Parameter Estimation (CNES contract)
- Third example: Boats detection
- Perspectives: numerous applications
MPP and (very) high resolution data

- Take into account the geometric information
- Modeling the scene structure
- Using and extending a huge algorithmic corpus
Some examples of results

- First applications: cartography
  - Road network
  - Building reconstruction
The different issues

- **Modeling:**
  - A configuration space (localization + marks)
  
  ![Configuration space diagram]

  \[ x = \{(x_1, m_1), \ldots, (x_i, m_i), \ldots (x_n, m_n)\} \]

  - A density: a priori and a data term

  \[
  h(x) = f(x)g_Y(x) \\
  = f(x)\prod_i g_Y^i(x_i, m_i) \\
  \propto \exp\left[ U_1(x) + \sum_i U_Y(x_i, m_i) \right]
  \]
The different issues

- Optimization:
  - A reference measure
  - Some perturbation kernels (proposal)
    - Adding an object?
    - Removing object?
    - Modifying object?
    - Merging objects?
    - …

\[(h(x))^\frac{1}{T} d\pi(x)\]

- Choose a proposition kernel \(Q_m(x, \cdot)\) with probability \(p_m(x)\), or let the configuration unchanged probability \(1 - \sum_m p_m(x)\).
- Sample \(x'\) according to the chosen
- Compute the acceptance ratio:
  \[R_m(x, x') = \frac{D_m(x', x)}{D_m(x, x')} = \frac{(h(x'))^{\frac{1}{T}} \pi(dx') Q_m(x', dx)}{(h(x))^{\frac{1}{T}} \pi(dx) Q_m(x, dx)}\]
- With probability \(\alpha = \min(1, R_m)\) set \(x_{t+1} = x'\), else reject the proposition: \(x_{t+1} = x\).
Objects counting: trees example
Objects counting : example of trees

- Objects : Ellipses
  \[ \chi = \mathcal{P} \times \mathcal{M} \]

- Prior
  \[ U_p(x) = \sum_i \sum_{u \in x} U_{r_i}(u) + \sum_{-i} \sum_{u \in v} U_{-i}(u, v) \]
  \[ U_{r_{ab}}(x) = \gamma_{ab} \sum_{u \in x} \frac{b_u}{a_u} - 1 \]
  \[ U_{-r}(x) = \gamma_r \sum_{u \sim_r v} \frac{\Lambda(S_u \cap S_v)}{\min(\Lambda(S_u), \Lambda(S_v))} \]
  \[ U_{-\rho}(x) = \gamma_{\rho} \sum_{u \sim_{\rho} v} \frac{\Lambda(S_u^{\rho} \cap S_v^{\rho})}{\min(\Lambda(S_u^{\rho}), \Lambda(S_v^{\rho}))} \]

\[ \mathcal{P} = [0, X_M] \times [0, Y_M] \]

\[ (a, b, \theta) \in \mathcal{M} = [a_m, a_M] \times [b_m, b_M] \times [0, \pi], a \geq b \]
Likelihood

- 2 Gaussian distributions (trees $C_A$, background $C_F$)
- Parameter estimation using k-means:

$$\mathcal{L}(I / x) = \prod_{p \in C_A} \frac{1}{\sqrt{2\pi \sigma_A}} \exp \left[ - \frac{(y_p - \mu_A)^2}{2\sigma_A^2} \right] \prod_{p \in C_F} \frac{1}{\sqrt{2\pi \sigma_F}} \exp \left[ - \frac{(y_p - \mu_F)^2}{2\sigma_F^2} \right]$$
More sophisticated likelihood

\[ \mu^I(p,t) = A \left( 1 - \frac{d(p,t)}{d(p,r_t)} \right) + B \]

\[ \mu^s(p,t) = A \cos \left( \frac{\pi}{2} \frac{d(p,t)}{d(p,r_t)} \right) + B \]

\[ \int_{t \in S(u)} \mu(t) \frac{dt}{\pi a_u b_u} = \mu_A \]

\[ \int_{t \in S(u)} (\mu(t) - \mu_A)^2 \frac{dt}{\pi a_u b_u} = \sigma_A^2 \]

\[ \mathcal{L}(I|x) = \prod_{p \in \mathcal{A}} \max_{u \in S(u)} \left\{ \frac{1}{\sqrt{2\pi} \sigma_A(u,p)} \exp \left[ -\frac{(y_p - \mu_A(u,p))^2}{2\sigma_A^2(u,p)} \right] \right\} \prod_{p \in \mathcal{F}} \frac{1}{\sqrt{2\pi} \sigma_F} \exp \left[ -\frac{(y_p - \mu_F)^2}{2\sigma_F^2} \right] \]
Reconstructions
**Objects counting : example of trees**

- **Data term**

\[
U_d(x) = \gamma_d \sum_{u \in x} U_d(u)
\]

\[
d_B(u, F(u)) = 100 \left[ \frac{(\mu_1 - \mu_2)^2}{4(\sigma_1 + \sigma_2)} - \frac{1}{2} \log \left( \frac{2\sqrt{\sigma_1 \sigma_2}}{\sigma_1 + \sigma_2} \right) \right]
\]

\[
Q_B(d_B) = \begin{cases} 
1 - \left( \frac{d_B}{d_0} \right) . & \text{if } d_B < d_0 \\
\exp \left( -\frac{d_B - d_0}{3d_0} \right) - 1 . & \text{if } d_B \geq d_0
\end{cases}
\]

\[
U_d(u) = Q_B(d_B(u, F(u)))
\]
Objects detection: example of trees

- Reference measure:
  - Non homogeneous Poisson

\[ \lambda(x) = \beta \lambda_{NDVI}(x) \]

- Alignment term (for plantations)

\[ u \sim_a v \iff p_u p_v = \pm V_k + \tilde{\epsilon}, k = 1, 2 \]

\[ U_a(x) = \gamma_a \sum_{u \sim_a v} \sigma \left( \min_{k=1,2} \left\| p_u p_v \pm V_k \right\| \right) \]

\[ \sigma(t) = \frac{1}{\varepsilon^2} \left( \frac{1 + \epsilon^2}{1 + t^2} \right) - 1 \]
Objects counting: example of trees

- Optimization: RJMCMC
  - Perturbations kernel:
    - Uniform birth and death
    - Object modification:
      - Merge / Split
      - Birth and death in a neighborhood:
Likelihood vs data term
Objects counting: example of trees

Results:
Multiple births and deaths dynamics

- Continuous dynamics in time (Stochastic Differential Equation)
- After discretization:
  - **Precomputing of the data term / birth map**
  - **Birth:**
    - For each pixel, add an object with probability \( \delta \cdot B(Ud(x)) \)
  - **Sort the objects with respect to their data term value**
  - **Death:**
    - For each object \( u \) taken in the list order, remove it with probability \( d(U(u), \delta, \beta) \)

\[
d(u_c) = \frac{\delta a_\beta(u_c)}{1 + \delta a_\beta(u_c)} \quad \text{with} \quad a_\beta(u_c) = \exp(-\beta U(u_c))
\]

**Theorem:** When \( N \to \infty, \beta \to \infty, \delta \to 0 \), convergence toward the configuration minimizing the energy
Application to flamingos counting
Application to flamingos counting

- Objects: ellipses
- Prior: non overlapping
- Data term: distance between internal and external radiometry
- Optimization: multiple birth and death dynamics
Application to flamingos counting

**Estimation of the size of a colony in Camargue:**

- 557 detected flamingos
Application to flamingos counting

Estimation of the size of a colony in Turkey (2004):

3682 detected flamingos (Tour du Valat: 3684 flamingos)
Application to flamingos counting

Estimation of the size of a colony in Mauritania:

♣ Very high density

©Tour du Valat
Application to flamingos detection

Estimation of the size of a colony in Mauritania:

- 14595 detected flamingos (Tour du Valat: 13650 flamingos)
Parameter Estimation

- Incomplete data problem: EM Algorithm
- E-step: evaluate the conditional expectation:
  \[ E_{\theta^k} \left[ \log f_{\theta}(x, y) | y \right] \]
- M-step: maximize the expectation
  \[ \theta^{k+1} = \arg\max_{\theta \in \Theta} E_{\theta^k} \left[ \log f_{\theta}(x, y) | y \right] \]
- No explicit expression for the expectation
- Stochastic version of the EM algorithm (SEM)
SEM algorithm

- S-step: simulate a configuration
  \[ x^{(k)} \approx f_{\theta^k}(X \mid y) \]

- E-step: Approximate the expectation
  \[ Q(\theta, \theta^k; y) = \log f_{\theta}(x^{(k)}, y) \]

- M-step: Maximise the expectation
  \[ \theta^{k+1} = \arg \max_{\theta \in \Theta} Q(\theta, \theta^k; y) \]
Pseudo-likelihood

- No analytical nor numerical expression of the extended likelihood

\[ f_\theta(x, y) = \frac{h_\theta(x, y)}{c(\theta)} \quad c(\theta) = \iint h_\theta(x, y) \mu(dx) \nu(dy) \]

- Approximation by the pseudo-likelihood

\[ \lambda_\theta(u; x, y) = \beta \exp \left( -\gamma_d U_d(u) - \sum_{x_i \in x / x_i \neq u} t_s(u, x_i) \right) \]

- Papangelou intensity

\[ PL_w(\theta; x, y) = \left[ \prod_{x_i \in x} \lambda_\theta(x_i; x, y) \right] \exp \left( -\int W \lambda_\theta(u; x, y) \Lambda(du) \right) \]
Automatic detection
Automatic detection
Automatic detection
Perspectives

- Numerous applications:
  - Population counting (animals, …)
  - Numeration/Classification (cells, …)
  - …

- GPU programming