Institut Mines-Télécom
CNRS LTCI

On the Optimality of Mutual Information Analysis

Authors

Francois Bailly
Sylvain Guilley
Annelie Heuser
Olivier Rioul

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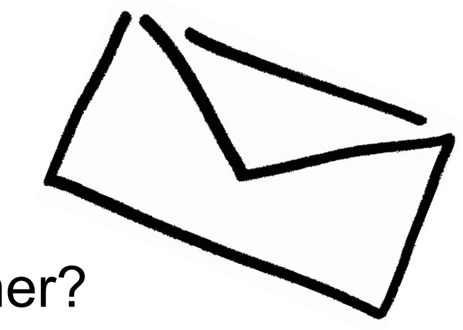
Motivation & State-of-the Art

- As the threat of side-channel attacks is well known, countermeasures are used for protection
- For example: Weakening the link between the measured leakage $\mathbf{x}$ and the sensitive variable $\mathbf{y}$
- Generic distinguisher cope with this scenario
  - Mutual Information Analysis (MIA) [1]
  - Kolmogorov-Smirnov distance (KSA) [2]
  - [3],…

Take home message!



- What is the *optimal* generic distinguisher?
- By applying the maximum likelihood principle, we derive the optimal generic distinguisher
- When the leakage has been quantified and probabilities are estimated from histograms, the optimal distinguisher's expression turns out to coincide with the mutual information analysis

[1] Benedikt Gierlichs, Lejla Batina, Pim Tuyls, and Bart Preneel. Mutual information analysis. In CHES, 10th International Workshop, volume 5154 of Lecture Notes in Computer Science, pages 426–442. Springer, August 10-13 2008. Washington, D.C., USA.
[2] Nicolas Veyrat-Charvillon and Fran¸cois-Xavier Standaert. Mutual Information Analysis: How, When and Why? In CHES, volume 5747 of LNCS, pages 429–443. Springer, September 6-9 2009. Lausanne, Switzerland.
[3] N. Veyrat-Charvillon and F.-X. Standaert. Generic side-channel distinguishers: Improvements and limitations. In P. Rogaway, editor, CRYPTO, volume 6841 of Lecture Notes in Computer Science, pages 354–372. Springer, 2011.
[4] Julien Doget, Emmanuel Prouff, Matthieu Rivain, François-Xavier Standaert: Univariate side channel attacks and leakage modeling. J. Cryptographic Engineering 1(2): 123-144 (2011)

Universal Maximum Likelihood Equivalent to MIA

Notations & Assumptions

- Values are quantized (discrete leakage)

$$\hat{\mathbb{P}}(x|y) = \frac{\sum_{i=1}^{m} \mathbb{1}_{x_i=x, y_i=y}}{\sum_{i=1}^{m} \mathbb{1}_{y_i=y}} = \frac{\hat{\mathbb{P}}(x,y)}{\hat{\mathbb{P}}(y)}$$

$$\hat{\mathbb{P}}(x,y) = \frac{1}{m}\sum_{i=1}^{m} \mathbb{1}_{x_i=x, y_i=y}$$

$$\hat{\mathbb{P}}(y) = \frac{1}{m}\sum_{i=1}^{m} \mathbb{1}_{y_i=y}.$$

$$\hat{\mathbb{P}}(x) = \frac{1}{m}\sum_{i=1}^{m} \mathbb{1}_{x_i=x}.$$

- Empirical Mutual Information

$$\hat{I}(\mathbf{x},\mathbf{y}) = \sum_{x\in\mathcal{X}, y\in\mathcal{Y}} \hat{\mathbb{P}}(x,y)\log_2 \frac{\hat{\mathbb{P}}(x,y)}{\hat{\mathbb{P}}(x)\hat{\mathbb{P}}(y)}$$

- From the Maximum Likelihood it is known that maximizing the success rate amounts to select the key guess $\hat{k}$ that maximizes

$$\mathbb{P}(\mathbf{x}|\mathbf{y})$$

- In practice if no profiling is possible the conditional distribution is unknown
- Therefore, we need a *universal* version (computed from the available data without prior information)

$$\hat{k} = \arg\max_k \hat{\mathbb{P}}(\mathbf{x}|\mathbf{y}) = \arg\max_k \prod_{i=1}^{m} \hat{\mathbb{P}}(x_i|y_i)$$

- Universal Maximum Likelihood is equivalent to mutual information analysis

$$\hat{k} = \arg\max_k \hat{I}(\mathbf{x},\mathbf{y})$$

MIA is the optimal tool for key recovery when the model is unknown.

Proof sketch

Denoting $n_{x,y} = \sum_{i=1}^{m} \mathbb{1}_{x_i=x, y_i=y} = m\,\hat{\mathbb{P}}(x,y)$

$$\hat{\mathbb{P}}(\mathbf{x}|\mathbf{y}) = \prod_{i=1}^{m} \hat{\mathbb{P}}(x_i|y_i) = \prod_{x\in\mathcal{X}, y\in\mathcal{Y}} \hat{\mathbb{P}}(x|y)^{n_{x,y}}$$

$$\hat{\mathbb{P}}(\mathbf{x}|\mathbf{y}) = \prod_{x\in\mathcal{X}, y\in\mathcal{Y}} \hat{\mathbb{P}}(x|y)^{m\,\hat{\mathbb{P}}(x,y)} = 2^{-m\hat{H}(\mathbf{x}|\mathbf{y})}$$

where $\hat{H}(\mathbf{x}|\mathbf{y}) = \sum_{x\in\mathcal{X}, y\in\mathcal{Y}} \hat{\mathbb{P}}(x,y)\log_2 \frac{1}{\hat{\mathbb{P}}(x|y)}.$

Empirical Future Work

- Experiments showing empirically the optimality of Mutual Information
- Especially, in comparison to Linear Regression Analysis [4]

Contact annelie.heuser@telecom-paristech.fr
Web http://perso.telecom-paristech.fr/~heuser/