Toward Optimal Destriping of MODIS Data Using a Unidirectional Variational Model

Marouan Bouali and Saïd Ladjal

Abstract—Images acquired by the Moderate Resolution Imaging Spectroradiometer (MODIS) aboard Terra and Aqua exhibit strong detector striping. This artifact is common to most pushbroom scanners and affects both visual interpretation and radiometric integrity of remotely sensed data. A considerable effort has been made to remove stripe noise and reduce its impact on high-level products. Despite the variety of destriping algorithms proposed in the literature, complete removal of stripes without signal distortion is yet to be overcome. In this paper, we tackle the striping issue from a variational angle. Basic statistical assumptions used in previous techniques are replaced by a much realistic geometrical consideration on the striping unidirectional variations. The resulting algorithm is tested on Aqua and Terra MODIS data contaminated with severe stripes and is shown to provide optimal qualitative and quantitative results.

Index Terms—Destriping, histogram matching, Moderate Resolution Imaging Spectroradiometer (MODIS), total variation, variational approach.

I. INTRODUCTION

THE MODERATE Resolution Imaging Spectroradiometer (MODIS) was first launched on December 18, 1999 aboard the Earth Observing System (EOS) Terra. Its remarkable design (Table I) intended to provide data acquired in a spectrum wide enough to improve our understanding of the Earth climate and the dynamic interactions involved between land, ocean, and atmosphere. The MODIS key component is a cross-track double-sided continuously rotating scan mirror that deflects the measured energy to photon detectors, allowing a monitoring of the Earth in 36 spectral bands ranging from the visible (0.4 µm) to the long-wave infrared (14.4 µm). Owing to the variety of products delivered by MODIS in the disciplines of ocean, land, and atmosphere, great advances have been made in Earth sciences and an increasing community of research scientists is actively involved in the improvement of MODIS data quality. Among the issues reported by the MODIS Characterization Science Team, the striping effect constitutes a persistent artifact that compromises the radiometric integrity of collected data. For instance, it is clearly visible in many of MODIS levels 2 and 3 products generated using emissive channels. In MODIS images, three different types of stripes have been identified [1]. Detector-to-detector stripes [Fig. 1(a)] are induced by a poor calibration of the relative gain and offset of each detector. These periodic stripes can be observed over an entire swath and are due to slight deviations between the input/output transfer function of adjacent detectors. Mirror-side stripes, also known as mirror banding [Fig. 1(b)], are due to a quasi-constant offset between forward and reverse scans and are dependent on the intensity level of the signal. It is often visible over bright targets displaying very high reflectance values and bringing the sensor close to its saturation mode. A typical case of mirror banding can be observed in homogeneous oceanographic images contaminated with sun glint (specular reflection of sun light on rough ocean surfaces) and/or strong atmospheric effects related to high concentrations of white aerosols. Because mirror-side stripes are related to the radiance level of the aforementioned phenomena, it is rarely present in the entire swath, and more importantly, it is often correlated with the scan angle. Noisy stripes [Fig. 1(c)] are random and only affect specific thermal bands. They appear as bright and dark stripes with random lengths within a scan line. In all cases, the presence of stripes cannot only be attributed to the imperfect relative calibration of the sensor detectors because other factors such as source spectral distribution and polarization or random noise in the internal calibration system can intervene [2], [3]. There exists an extensive literature related to the correction of stripes on satellite imagery. The first class of destriping approaches relies on digital filtering [4]–[9]. Because the striping effect can be viewed as a periodic noise, its frequency can be extracted

![Fig. 1. (a) Detector-to-detector stripes. (b) Mirror-side stripes. (c) Random stripes.](image-url)
using spectral analysis and removed with an adequate low-pass or finite-impulse-response filter. Unfortunately, structural details with the same frequency components as stripes are inevitably filtered. Although digital filtering methods are sensor independent, easy and fast to implement, and provide a good visual improvement, they introduce blurring and ringing artifacts that compromise the radiometric accuracy of data. The second family of destriping algorithms focuses on the statistical properties of signals measured by individual detectors. The moment matching technique was introduced in [10] and removes stripes by assuming that the mean and standard deviation of the data acquired by each detector are identical. To overcome the limitations of first-order statistics, Horn and Woodham [10] and Weinreb et al. [11] suggested a refinement based on the histogram matching technique. The signal is considered as a random variable with a given empirical cumulative distribution function (ECDF). Measurements from each detector are subsequently corrected in order to match an imposed reference ECDF. Unlike digital filtering techniques, statistics-based methods do not interfere with the data radiometric accuracy, but their performance is limited by a criterion of homogeneity due to the strong assumption of subdetectors viewing the same scene. Wegener [12] proposed to overcome this issue by computing the ECDFs over homogeneous targets. Over the years, many improved algorithms combining statistical methods with advanced image-processing techniques have been suggested. Such hybrid approaches proved to be efficient in [13] where Rakwatin et al. combined histogram matching with an iterated weighted least squares filter to take into account noisy stripes on MODIS. In [14], the authors combined moment matching with the multiresolution analysis provided by wavelet transform. Radiometric equalization is the last class of destriping techniques, mainly used to eliminate nonperiodic stripes. In 2000, Corsini et al. [15] developed a destriping algorithm for the Modular Optoelectronic Scanner B-data (MOS-B). Assuming that stripes are constant over time, a data set of quasi-homogeneous images of sea targets acquired by MOS-B was used to determine the equalization curves for each detector prior to stripe removal. The destriping procedure proposed by Antonelli et al. [16] is extremely original. The methodology takes advantage of another artifact present in MODIS images, the bowtie effect. When the mirror scan angle is higher than ±25°, two consecutive detectors happen to observe the same instantaneous field of view (IFOV), and therefore, the redundant information in scan-to-scan overlaps can be exploited to establish an equalization curve used as an additional calibration stage. The destriping performances obtained with this approach are investigated in [17]. Only recently have variational methods been considered as a potential solution to the striping issue. In [18], the authors used a maximum-a-posteriori-based algorithm with a Huber–Markov regularization model for destriping and inpainting problems. This variational model differs from the one introduced in this paper on many points. First, in addition to the regularization coefficient λ, the authors introduce in the model two matrices related to the gain and offset of each pixel. The resulting optimization problem is then solved once the matrix values are estimated using the moment matching technique described in [19]. In terms of destriping performances, the algorithm proposed in [18] is equivalent to a Huber–Markov regularization where the data fidelity term is the original image processed with moment matching. In other words, this variational approach acts as an additional regularization aiming at reducing residual stripes that moment matching fails to remove. Aside from the limitations of moment matching discussed in [10], the Huber–Markov model is symmetrical and consequently provides results smoothed in both vertical and horizontal directions. Regardless of the choice of the Lagrange multiplier λ, reducing the effects of residual stripes with a similar symmetrical regularization model leads to solutions with considerable amount of blur, as shown in Fig. 2. The design of an adequate variational model is motivated by a simple question: Which part of the observed information is most reliable? When dealing with isotropic noise such as Gaussian noise, the Rudin, Osher, and Fatemi regularization model [20] (hereafter called ROF model) and its numerous derivatives provide reliable results. Directly using such models for destriping purposes leads to undesirable blurring due to the contradictive compromise between the fidelity term and the regularization term. Unlike other types of noises, striping has a clear directional signature that offers the possibility to choose a fidelity term much more reliable than the noisy image itself. For instance, as long as striping can be considered as an additive noise, the horizontal gradient of the image will not be affected with stripes. This simple observation makes it an optimal choice for the fidelity term of a destriping variational model, leaving the regularization term to process the noisy component, isolated in the vertical gradient. In this paper, we tackle the striping issue by introducing a unidirectional variational model. Our method is tested on MODIS imagery and compared qualitatively and quantitatively to other destriping techniques. This paper is organized as follows. Section II describes the variational model and the optimization method used for the destriping. Section III presents the data processing and defines several indexes used to evaluate the destriping quality. Section IV concludes this paper.

II. Destriping Algorithm

A. Problem Formulation

We consider an image to be a 2-D function defined in Ω, a bounded domain of \( \mathbb{R}^2 \). The striping effect is assumed to be an additive noise, so that the degradation process can be formulated as

\[
I_s(x, y) = u(x, y) + s(x, y)
\]

(1)

where \( I_s \) is the sensor output and \( I \) is the scene true radiance, both at the pixel \((x, y)\). The stripe noise \( s \) includes detector-to-detector stripes, mirror banding, and random stripes. Most destriping approaches tend to assume that \( s(x, y) \) can be considered as constant over a given scan line, which implies that striping could be properly removed by means of an additional calibration procedure that corrects relative gain and offset of noisy detectors. Although this assumption might apply to sensors that have shown stability over time (MOS-B), it has to be reconsidered for sensors like MODIS. In addition to predominant nonlinear effects, a progressive degradation of data quality in the form of random stripes has been observed in many emissive bands. In order to design an efficient destriping algorithm, able to overcome the residual stripes issue without introducing blurring, more realistic assumptions must be used.
For instance, it is clear that stripes can be viewed as a structured noise, of which variations are mainly concentrated along the $y$-axis. In mathematical words, most pixels of the stripe noise $s$ hold the following property:

$$\left| \frac{\partial s(x, y)}{\partial x} \right| \ll \left| \frac{\partial s(x, y)}{\partial y} \right|. \quad (2)$$

### B. Variational Model

Many ill-posed problems related to computer-vision applications such as image denoising and image restoration require the introduction of a regularizing constraint on the solution. Using prior information, an estimate of the true image can be computed by minimizing an energy functional that includes both a term that translates the fidelity of the estimated solution to the original image and a regularization term weighted by a positive parameter that regulates the smoothness of the solution. In [21], the author introduced an intuitive destriping methodology through a gradient-based iterative strategy. The proposed algorithm makes abstraction of the classical regularization coefficient but uses instead finite differences to minimize a quadratic energy functional of the following form:

$$E_k(u) = \int_\Omega \sum_{i+j=k, j \neq k} \left( \frac{k!}{i!(k-i)!} \left| \frac{\partial^k (u - I_s)}{\partial x^i \partial y^{k-i}} \right|^2 \right) dx dy + \left| \frac{\partial^k H_L \ast (u - I_s)}{\partial y} \right|^2 dx dy \quad (3)$$

where $H_L$ denotes a low-pass filter, $\ast$ is the convolution symbol, and $k$ is the number of iterations used in the destriping procedure. By exploiting the unidirectional signature of stripes in a variational framework, the method displayed excellent results on MODIS data and outperformed the histogram matching technique. However, one limitation of [21] is the introduction of local blurring artifacts over image sharp edges. This is a well-known drawback of quadratic variational models. Among nonlinear approaches, the total variation norm has become a major reference in image denoising applications. This model was first introduced by Rudin, Osher, and Fatemi (ROF) [20] and consists in minimizing the following energy

$$E(u) = \int_\Omega \| u - I \|^2 + \lambda TV(u) \quad (4)$$

where $TV(u)$ is the total variation of the estimated solution $u$ also expressed as

$$TV(u) = \int_\Omega |\nabla u| = \int_\Omega \sqrt{\left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2} \, dx \, dy. \quad (5)$$

Unlike quadratic restoration models, the ROF model is well known for its ability to preserve discontinuities. This feature is particularly important for remote-sensing applications where postprocessing artifacts are undesirable. Integration of (2) over the image domain $\Omega$ leads to the inequality

$$\int_\Omega |s(x, y)| \, dx \, dy \ll \int_\Omega \left| \frac{\partial s(x, y)}{\partial y} \right| \, dx \, dy. \quad (6)$$

The previous inequality translates a realistic characteristic of the stripe noise $s$

$$TV_x(s) \ll TV_y(s) \quad (7)$$

where $TV_x$ and $TV_y$ are horizontal and vertical variations, respectively. In order to obtain a robust stripe removal, we suggest the minimization of a functional that includes unidirectional variations

$$E(u) = TV_x(u - I_s) + \lambda TV_y(u) \quad (8)$$

where $\lambda$ is a regularization parameter that quantifies the degree of smoothness across the $y$-axis. This functional can also be
written as
\[
E(u) = \int_{\Omega} \left( \sqrt{\left( \frac{\partial(u - I_s)}{\partial x} \right)^2 + \lambda \sqrt{\left( \frac{\partial u}{\partial y} \right)^2}} \right) dx \, dy. \tag{9}
\]

Other variational models can be considered as an alternative for (9) using modified fidelity/regularization terms and different norms. However, given the results provided by previous variational methods [18], [21], we retain that an efficient destriping variational model requires the following: 1) the distribution of directional (vertical and horizontal) information separately into the fidelity term and the regularization term and 2) both fidelity and regularization terms to use an edge-preserving norm to avoid blurring artifacts. This justifies the uncommon use of the $L^1$ norm in the data fidelity term because the information related to image edges is ubiquitous in the energy functional (9).

C. Optimization Method

Using Euler–Lagrange equation, the differentiation of $E(u)$ with respect to $u$ is given by
\[
\frac{\partial E}{\partial u} = -\frac{\partial}{\partial x} \left( \frac{\partial(u - I_s)}{\partial x} \sqrt{\left( \frac{\partial(u - I_s)}{\partial x} \right)^2 + \lambda \left( \frac{\partial u}{\partial y} \right)^2} \right) - \lambda \frac{\partial}{\partial y} \left( \sqrt{\left( \frac{\partial u}{\partial y} \right)^2 + \lambda} \right). \tag{10}
\]
Let us point out the nondifferentiability of (9) at points where
\[
\begin{align*}
\frac{\partial u}{\partial x} &= \frac{\partial I_s}{\partial x} \\
\frac{\partial u}{\partial y} &= 0
\end{align*} \tag{11}
\]
and introduce two small positive parameters $\epsilon_1$ and $\epsilon_2$ to slightly perturb the energy functional (9) into
\[
E_{\epsilon_1, \epsilon_2}(u) = \int_{\Omega} \left( \sqrt{\left( \frac{\partial(u - I_s)}{\partial x} \right)^2 + \epsilon_1^2} \right) dx \, dy + \lambda \int_{\Omega} \left( \sqrt{\left( \frac{\partial u}{\partial y} \right)^2 + \epsilon_2^2} \right) dx \, dy. \tag{12}
\]
The Euler–Lagrange equation of (12) can then be written as
\[
\frac{\partial E_{\epsilon_1, \epsilon_2}}{\partial u} = -\frac{\partial}{\partial x} \left( \frac{\frac{\partial(u - I_s)}{\partial x}}{\sqrt{\left( \frac{\partial(u - I_s)}{\partial x} \right)^2 + \epsilon_1^2}} \right) - \lambda \frac{\partial}{\partial y} \left( \frac{\frac{\partial u}{\partial y}}{\sqrt{\left( \frac{\partial u}{\partial y} \right)^2 + \epsilon_2^2}} \right). \tag{13}
\]
It can be shown [22] that its solutions converge to the solutions of the initial problem (10) when $\epsilon_1, \epsilon_2 \to 0$. An estimation of the stripe-free scene is computed using a basic gradient descent optimization method
\[
\begin{cases}
\hat{u}_0 = I_s \\
\hat{u}_{n+1} = \hat{u}_n - \eta \frac{\partial E_{\epsilon_1, \epsilon_2}}{\partial u}(\hat{u}_n)
\end{cases} \tag{14}
\]
where $\eta$ is the time step. Several other techniques (not discussed in this paper) can be used to obtain a faster convergence to the solution.
D. Discretization

The implementation of the proposed algorithm requires a discrete version of the operators described in the previous section. We assume an image to be a 2-D vector of size $M \times N$. Its discrete gradient is a vector with vertical and horizontal components given by

$$
(\nabla_x u)_{i,j} = \begin{cases} 
  u_{i+1,j} - u_{i,j}, & \text{if } i < M \\
  0, & \text{if } i = M
\end{cases}
$$

$$
(\nabla_y u)_{i,j} = \begin{cases} 
  u_{i,j+1} - u_{i,j}, & \text{if } j < N \\
  0, & \text{if } j = N.
\end{cases}
$$

In its discrete form, the functional defined in (12) becomes

$$
E(u) = \sum_{i=1}^{M} \sum_{j=1}^{N} \sqrt{((\nabla_x u)_{i,j} - (\nabla_x I_s)_{i,j})^2 + \epsilon_1^2} 
+ \lambda \sum_{i=1}^{M} \sum_{j=1}^{N} \sqrt{((\nabla_y u)_{i,j})^2 + \epsilon_2^2}
$$

(17)

and its Euler–Lagrange equation leads to

$$
\nabla_u E(u) = -\nabla_x \left( \frac{\nabla_x u - \nabla_x I_s}{\sqrt{(\nabla_x u - \nabla_x I_s)^2 + \epsilon_1^2}} \right) 
- \lambda \nabla_y \left( \frac{\nabla_y u}{\sqrt{(\nabla_y u)^2 + \epsilon_2^2}} \right). 
$$

With the discrete settings, the gradient descent method translates to

$$
\begin{align*}
\hat{u}_{k+1} &= \hat{u}_k + \eta \left[ \nabla_x \left( \frac{\nabla_x \hat{u}_k - \nabla_x I_s}{\sqrt{(\nabla_x \hat{u}_k - \nabla_x I_s)^2 + \epsilon_1^2}} \right) 
+ \lambda \nabla_y \left( \frac{\nabla_y \hat{u}_k}{\sqrt{(\nabla_y \hat{u}_k)^2 + \epsilon_2^2}} \right) \right]
\end{align*}
$$

(19)

The iterative solving procedure is stopped when

$$
|\hat{u}_{k+1} - \hat{u}_k| \leq \epsilon \tag{20}
$$

where $\epsilon$ is a tolerance parameter.

E. Choice of the Lagrange Multiplier $\lambda$

The choice of an optimal coefficient $\lambda$ plays a key role in the regularization. A large value of $\lambda$ will induce oversmoothing. If $\lambda$ is too small, the destriped results will contain residual stripes (when $\lambda \to 0$, the minimization of (17) converges to $I_s + A$, where $TV_x(A) = 0$). An intuitive way to tackle the issue of the Lagrange multiplier’s choice is to rely on iterative procedures such as the multiscale hierarchical decomposition proposed by Tadmor et al. in [23] or, more recently, the iterative regularization methodology introduced in [24]. The basic idea behind such techniques is not to evaluate the optimal $\lambda$ itself but to converge to the solution provided by its value. In this paper, we propose a direct adaptation of the methodology.
BOUALI LADJAL: TOWARD DESTRI珀NG OF MODIS DATA

Fig. 8. (a) Original subimage from MODIS Aqua band 36 (14.085–14.385 µm) captured on November 10, 2009, north of Baja California peninsula. (b) Destriped with a low-pass filter. (c) Destriped with histogram matching (IMAPP). (d) Destriped image with the proposed method.

It consists in recursively solving the following optimization problem:

$$\hat{V}_{k+1} = \arg\min_u TV_x(u - (I_s - \hat{U}_k)) + \frac{\lambda_0}{2^k} TV_y(u) \quad (21)$$

where \(\lambda_0\) is the initial Lagrange multiplier and \(\hat{U}_k\) is the destriped image at the iteration \(k\), computed as

$$\hat{U}_k = \sum_{i=0}^{k} \hat{V}_i. \quad (22)$$

Since the primary objective of the hierarchical decomposition proposed in [23] is to provide a multiscale representation of images, the stopping rules proposed by the authors are not meant to be used for denoising purposes. The stopping criterion suggested in [24] and based on the discrepancy principle cannot be applied in our case because it requires an approximative estimate of the noise level in the observed image. An alternative option would consist in using a measure that quantifies the degree of smoothing in a direction that should not be disturbed by the destriping algorithm, namely, the image distortion (ID) index defined in the next section. We suggest stopping the iterative procedure as soon as the ID index of the estimated solution \(\hat{U}_k\) reaches a value higher than 0.95. This threshold was determined experimentally with the simultaneous requirements of nonresidual stripes and minimum distortion in mind. Depending on the user’s final application of the algorithm, this threshold value can be revised. In Fig. 25, we illustrate the destriping results obtained through the successive iterations of the methodology of Tadmor et al. In Fig. 26, we show the ID index as a function of the number of iterations where \(\lambda_0\) is fixed to 5 and updated with \(\lambda_{k+1} = \lambda_0 / 2^k\). As shown in Fig. 26(b), the number of iterations can be considerably reduced by using a first guess of \(\lambda_0\) equal to \(TV_x(I_s) / TV_y(I_s)\). The destriping procedure described in this section can be summarized with the following.

**Input** Image \(I_s\) with stripe noise

1: Select \(\lambda_0 = TV_x(I_s) / TV_y(I_s)\)
2: Initialize \(\hat{U}_0 = \hat{V}_0 = 0\)
3: WHILE \(\{\text{ID}(\hat{U}_k) < 0.95\}\)
   Update \(\lambda_{k+1} = \lambda_0 / 2^k\)
   Solve \(\hat{V}_{k+1} = \arg\min_u \{TV_x(u - (I_s - \hat{U}_k)) + \lambda_{k+1} TV_y(u)\}\)
   Compute \(\hat{U}_{k+1} = \sum_{i=0}^{k+1} \hat{V}_i\)
4: END

**Output** Destriped image = \(\hat{U}_{k+1}\)

III. EXPERIMENTAL RESULTS

In this paper, we used Terra and Aqua MODIS level 1 B data downloaded from the Level 1 and Atmosphere Archive and Distribution System (http://ladsweb.nascom.nasa.gov/). MODIS level 1 B data corresponds to top-of-atmosphere calibrated radiances coded to a 16-b scale and are in hierarchical data format–EOS format, the standard format for Terra and Aqua sensors. To illustrate the visual quality improvement, we selected two swaths captured by Terra MODIS on July 1, 2009 over the Mediterranean Sea and by Aqua MODIS on November 10, 2009 in the north of Baja California peninsula. The proposed algorithm is applied to entire swaths (2030 × 1354 pixels) and compared with low-pass filtering and the histogram matching technique [11], as implemented in the
International MODIS/AIRS Processing Package (IMAPP). The IMAPP destriping software was downloaded from http://cimss.ssec.wisc.edu/imapp/. To improve the visual analysis of destriping performances, the results are illustrated on 512 × 512 pixel images extracted from entire swaths. We specifically selected emissive bands 27 (6.535–6.895 µm), 30 (9.580–9.880 µm), 33 (13.185–13.485 µm), and 36 (14.085–14.385 µm) because they are contaminated with severe striping effect.

Figs. 3–8 show the visual quality improvement provided by low-pass filtering, histogram matching (IMAPP), and the proposed algorithm. Visual analysis of destriped images clearly shows that low-pass filtering blurs the images. With the histogram matching technique, many residual stripes related to detector nonlinearities can be observed. Our approach effectively removes all types of stripes, and the details related to the original signal are left intact. In order to highlight the visual improvement offered by the new algorithm, a zoomed version of Fig. 6 images is shown in Fig. 9.

Figs. 10–15 show the mean cross-track profiles before and after destriping. As expected, cross-track profiles of the original striped images display strong periodic and random variations across the swath. These fluctuations are expected to be properly filtered according to the efficiency of the adopted destriping approach. For instance, Figs. 10–15(c) clearly show that the histogram matching technique only corrects for detector-to-detector stripes and mirror-side stripes. On the contrary, for all tested images, our variational technique provides a very smooth curve. Obviously, the analysis of cross-track profiles is not sufficient to assess the quality of destriping. In fact, low-pass filtering also provides a good smoothing of cross-track profiles but at the cost of a significant loss of details from the original images.

Figs. 16–21 show the mean column power spectrum as a function of normalized frequency before and after destriping. To improve visibility, the spectral magnitudes are plotted with a logarithmic scale. Detector-to-detector stripes and mirror-side banding translate in these graphs as clear pulses in the frequency domain. In the case of detector-to-detector stripe noise, the pulses are clearly located at frequencies of 1/10, 2/10, 3/10, 4/10, and 5/10 cycles per pixel. In the selected images, mirror-side stripes do not display clear pulses but ex-
Fig. 14. Mean cross-track profiles of Fig. 7 images. (a) Original Aqua MODIS band 30. (b) Low-pass filtering. (c) Histogram matching (IMAPP). (d) Proposed method.

Fig. 15. Mean cross-track profiles of Fig. 8 images. (a) Original Aqua MODIS band 36. (b) Low-pass filtering. (c) Histogram matching (IMAPP). (d) Proposed method.

Fig. 16. Mean column power spectrum of Fig. 3 images. (a) Original image from Terra MODIS band 27. (b) Low-pass filtering. (c) Histogram matching (IMAPP). (d) Proposed method.

Fig. 17. Mean column power spectrum of Fig. 4 images. (a) Original image from MODIS Terra band 30. (b) Low-pass filtering. (c) Histogram Matching (IMAPP). (d) Proposed method.

Experiments conducted on images affected with sun glint (visible on the reflective bands) indicate that they can be located at frequencies of 1/20, 3/20, 5/20, 7/20, and 9/20 cycles per pixel. Histogram matching does reduce these pulses but it does not process random stripes. Although low-pass filtering provides a visual improvement, Figs. 16–21(b) show a significant bias in the resulting power spectrum. With our variational approach, frequency components related to periodic stripes are extremely attenuated, and the power spectrums show no evidence of noise in the high-frequency range. In Figs. 23 and 24, a realistic stripe noise extracted from Terra MODIS band 27 using the proposed technique was added to stripe-free images [Figs. 23(a) and 24(a)] extracted from Terra MODIS band 31 (10.780–11.280 µm) and Aqua MODIS band 31. The figures show the destriping results obtained with low-pass filtering, moment matching, histogram matching, and the new approach. For these experiments, we implemented the histogram matching technique as described in [10] instead of using the IMAPP destriping software. This is because the IMAPP software processes random stripes (mainly present on Terra MODIS) by replacing noisy detectors with neighbors. Although such procedure improves visual quality, it tends to decrease the ID index (cf. Table II) and therefore impacts the radiometric integrity of the denoised data. This experiment emphasizes the limitations of statistic-based techniques in the presence of nonlinear stripes.

The original images from band 31 are used as reference to compute the peak signal-to-noise ratio (PSNR) obtained with different destriping techniques. The PSNR results are reported in Table IV. It is also important to point out that the proposed technique is able to remove stripes without interfering with the bowtie effect, visible on MODIS images in areas with high scan angle. This is shown in Fig. 22. To go further and provide a quantitative assessment of the new destriping technique, we make use of the qualitative indexes defined in the following sections.

A. Noise Reduction (NR) Ratio and Image Distortion (ID)

Let us define by \( P_0 \) and \( Q_0 \) the ensemble averaged power spectrums down the columns and lines of the original image and...
by $P_1$ and $Q_1$ the averaged power spectrums down the columns and lines of the destriped images. Assuming

$$
N_0 = \sum_{BW} P_0(D)
$$
$$
N_1 = \sum_{BW} P_1(D)
$$

where $D$ is the distance from the origin in the Fourier space and $BW$ is the stripe noise region of the averaged power spectrum down the columns, $D \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. Noise reduction (NR) and ID are then defined as

$$
NR = \frac{N_0}{N_1}
$$
$$
ID = 1 - \frac{1}{\text{card}(BW)} \sum_{BW} \left| Q_0(D) - Q_1(D) \right|
$$

where $BW$ is the entire spectrum of frequencies. A different ID index was used in [2], but its definition makes use of averaged power spectrums down the columns (instead of lines) and, thus, does not take into account noisy stripes. As a consequence, the histogram matching technique will provide illusive ID values equal to 1. NR and ID value results are reported in Table II. The NR ratio was not computed for Terra MODIS band 33, because this band is mainly affected with random stripes.

### B. Improvement Factors (IF) of Radiometric Quality

In [15], two improvement factors (IFs) have been defined and used. Let us evaluate the sequences

\[
\begin{align*}
    d_R[j] &= m_{IR}[j] - m_I[j] \\
    d_E[j] &= m_{IE}[j] - m_I[j] \\
    \Delta_{IR}[j] &= m_{IR}[j] - m_{IR}[j-1] \\
    \Delta_{IE}[j] &= m_{IE}[j] - m_{IE}[j-1]
\end{align*}
\]

where $m_{IR}[j]$ and $m_{IE}[j]$ are the mean radiance values of the $j$th line in the raw and destriped images, respectively. $m_I[j]$ is
the mean value of the \( j \)th line in the image destriped with a low-pass filter. We define the IFs as

\[
\text{IF}_1 = 10 \log_{10} \left( \frac{\sum_j d_R^2[j]}{\sum_j d_E^2[j]} \right)
\]

\[
\text{IF}_2 = 10 \log_{10} \left( \frac{\sum_j \Delta I_R^2[j]}{\sum_j \Delta I_E^2[j]} \right). \tag{26}
\]

The values of \( \text{IF}_1 \) and \( \text{IF}_2 \) are reported in Table II.

C. Inverse Coefficient of Variation (ICV)

The inverse coefficient of variation (ICV) used in [25] and [26] is defined as

\[
\text{ICV} = \frac{R_m}{R_s} \tag{27}
\]

where \( R_m \) and \( R_s \) are the mean and standard deviation of pixel values. The ICV index is computed in homogeneous regions within a window of \( 10 \times 10 \) pixels. The results of ICV are reported in Table III.

D. Peak Signal-to-Noise Ratio (PSNR)

We define the PSNR as

\[
\text{PSNR} = 10 \log_{10} \left( \frac{d^2 \times MN}{\|I_d - I_{\text{ref}}\|^2} \right) \tag{28}
\]

where \( d \) is the image dynamic range, \( I_d \) is the denoised image, and \( I_{\text{ref}} \) is the reference image.

It is very important to use all the indexes defined earlier in order to evaluate the performances of a given destriping algorithm. For instance, when compared with other techniques, a basic low-pass filter will often lead to better values in the NR ratio, in the radiometric IFs, and in the ICV. However, these indexes are improved at the expense of an extremely poor ID index which automatically discards low-pass filtering for applications requiring radiometric accuracy.

IV. DISCUSSION AND CONCLUSION

In this paper, a new destriping algorithm based on a variational framework is described and applied to MODIS images heavily affected with detector-to-detector stripes, mirror-side stripes, and random stripes. Aside from the great qualitative improvement shown in cross-track profiles and ensemble aver-
Fig. 23. (a) Original stripe-free image from MODIS Aqua band 31 (10.780–11.280 µm). (b) Image (a) with additional stripe. (c) Destriped with low-pass filtering. (d) Destriped with moment matching. (e) Destriped with histogram matching. (f) Destriped with the proposed method.

Table IV

<table>
<thead>
<tr>
<th>Band</th>
<th>Low-pass filter</th>
<th>Moment</th>
<th>Histogram</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terra Band 31</td>
<td>24.07 dB</td>
<td>29.54 dB</td>
<td>29.71 dB</td>
<td>45.77 dB</td>
</tr>
<tr>
<td>Aqua Band 31</td>
<td>27.78 dB</td>
<td>28.09 dB</td>
<td>28.11 dB</td>
<td>40.08 dB</td>
</tr>
</tbody>
</table>

Fig. 24. (a) Original stripe-free image from MODIS Terra band 31 (10.780–11.280 µm). (b) Image (a) with additional stripe. (c) Destriped with low-pass filtering. (d) Destriped with moment matching. (e) Destriped with histogram matching. (f) Destriped with the proposed method.

From level 1 B data affected with stripe noise can benefit from this technique. The new destriping procedure is able to remove periodic and random stripes while preserving the radiometric integrity of the data, a feature of major importance for ocean-color products. Although only MODIS data have been investigated in this paper, the methodology can be applied to a variety of satellite/airborne sensors contaminated with a similar artifact.

ACKNOWLEDGMENT

The authors would like to thank the four anonymous reviewers for their valuable comments.

REFERENCES


