Fault-Tolerant Computability in Anonymous Shared-Memory Model

Nayuta Yanagisawa

Department of Mathematics, Kyoto University

2017@France

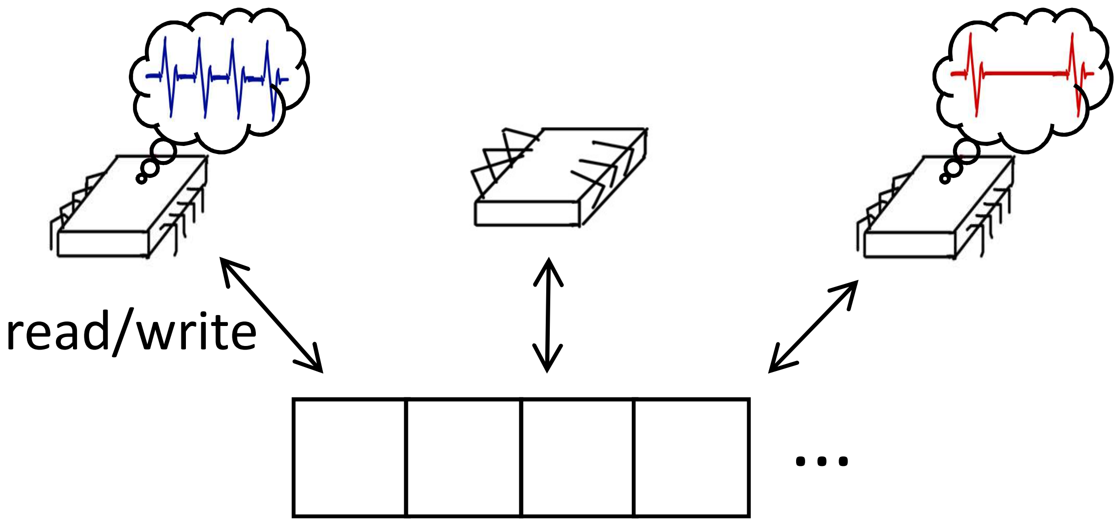
I’m here to talk about …

my recent results concerning anonymous shared-memory distributed computing

Anonymous Shared-Memory Model

A distributed system consists of

- A set of $n + 1$ *anonymous* and *asynchronous* processes, which are prone to *crash failures*;
- *multi-writer*/multi-reader atomic registers.



Why Difficult?

Anonymous processes execute an *identical program*, causing troubles:

- No single-writer shared object

  A value written by a process may be overwritten before other processes see it.

- Undetectability of multiplicity (clone)

  In the worst case, processes that have an identical local state cannot detect the activity of others.

Outline

I investigate distributed computability in the anonymous shared-memory model.

1. Infinitely-valued atomic weak set object
2. t-resilient $(t+1)$-set agreement protocol
3. Topological characterization of t-resilient solvable colorless tasks

1. Atomic Weak Set Object

Atomic Weak Set Object

An *atomic weak set object*, denoted by SET, is an atomic object used for storing values.

SET supports the two operations, *add* and *get*:

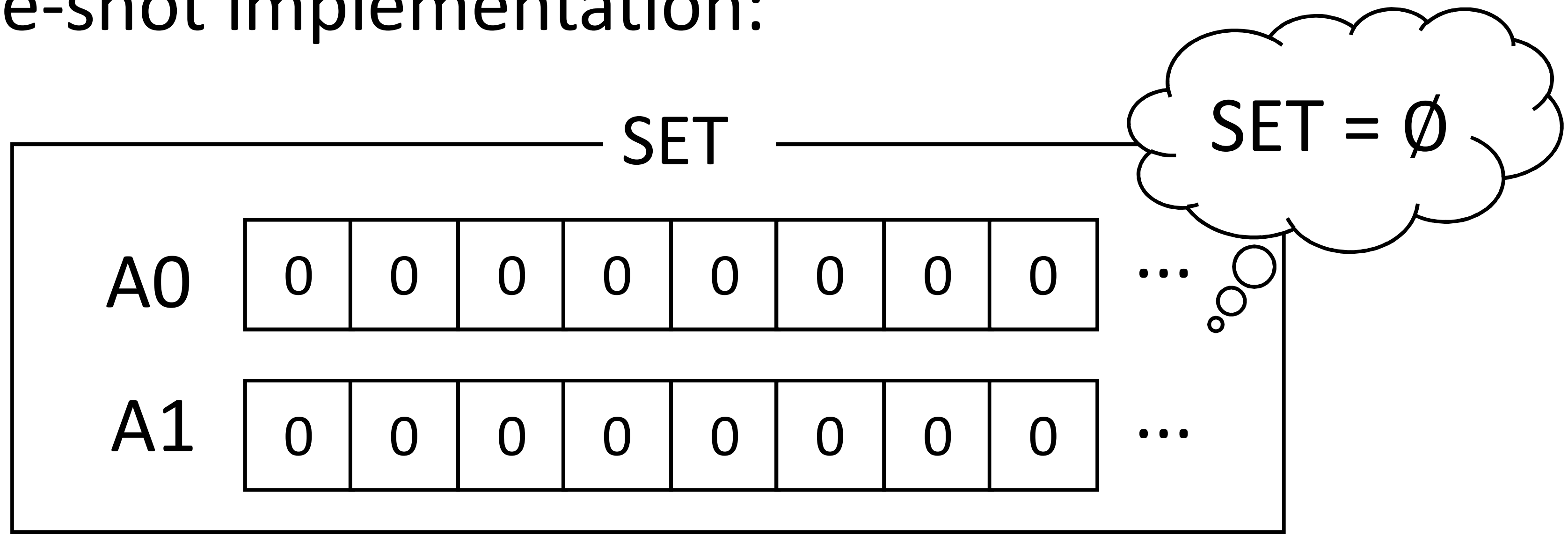
- A process can atomically add $v \in \{0,1,2 \ldots\}$ to SET by the add(v) operation.
- A process can atomically obtain the content of SET by the get() operation.

Wait-Free Implementation

Theorem 1

An atomic weak set object SET has a wait-free implementation in the anonymous model.

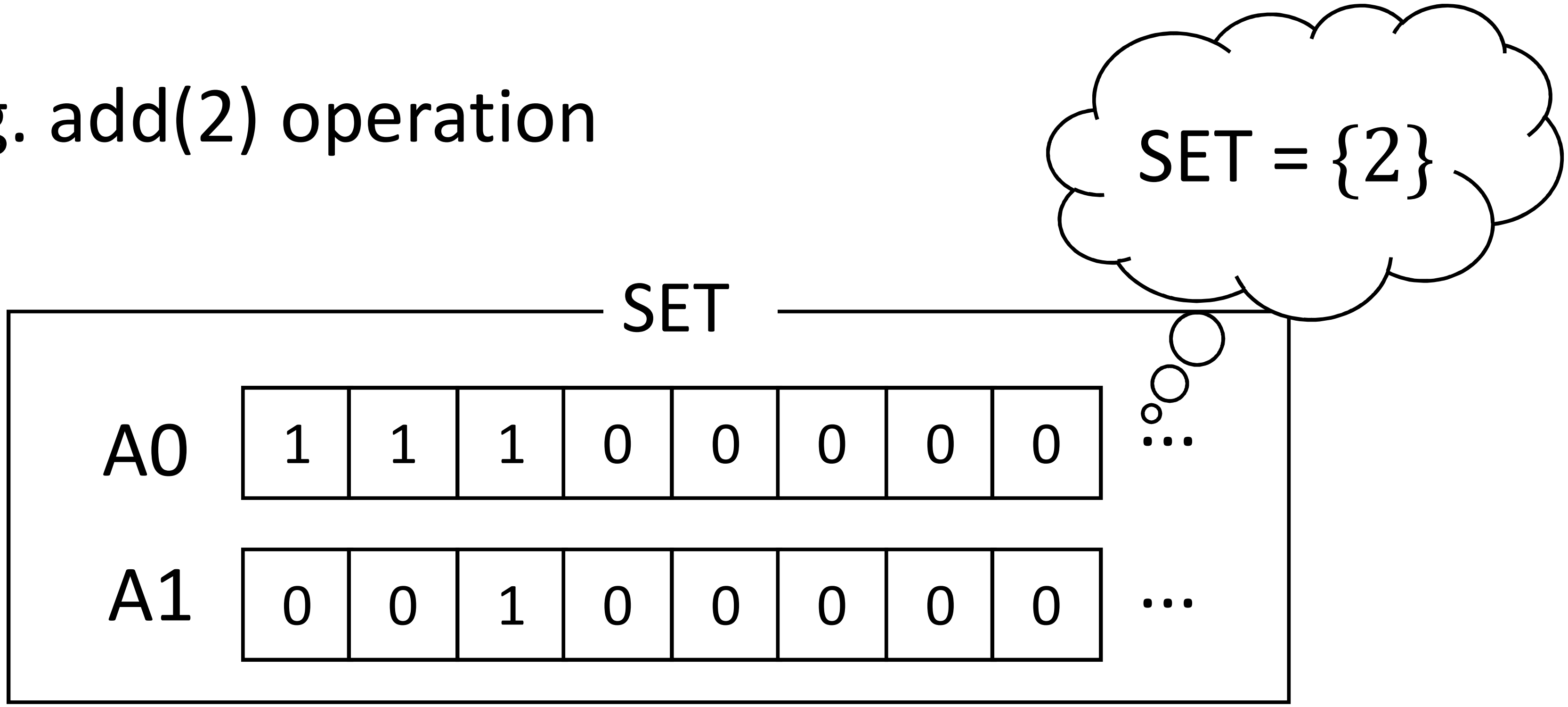
One-shot implementation:



Add Operation

To perform an add(k) operation, a process writes 1 to A0[i] ($i = 0, \dots, k$) and writes 1 to A1[k].

E.g. add(2) operation



Get Operation

To perform a get operation, a process read A1[i] until it sees A0[i] $= 0$. (first collect)

Then, the process read A1 again in the same manner. (second collect)

If the two collects are identical, return $\{i \mid \text{first\_collect}[i] \neq 0\}$; otherwise repeat.

SET

A0	1	1	1	1	1	0	0	0	⋯
A1	0	0	1	0	1	0	0	0	⋯

Application

- Set agreement protocol (next section)
- Simple approximate agreement protocol based on [Moran 95]

[Moran 95] Shlomo Moran. Using approximate agreement to obtain complete disagreement: the output structure of input-free asynchronous computations. (ISTCS 1995)

2. Set Agreement Protocol

k-Set Agreement

- *Termination*: Every non-faulty process eventually decide;
- *k-agreement*: The set of outputs holds at most k distinct values;
- *Validity*: Every output value is equal to some process's input value.

$(t+1)$-Set Agreement

Theorem 2

There exists an anonymous t-resilient protocol for the $(t+1)$-set agreement problem.

Note: I assume that every value is encoded into a non-negative integer.

Set Agreement Protocol

See manuscript

Correctness of Protocol

Termination: A process waits only when it sees more than $t + 1$ values and its value is the minimum among them. This ensure that some $t + 1$ set of processes never jump to Line 13 in each execution.

k-agreement: see manuscript.

Remark

Our protocol can be seen as an extension of the anonymous consensus protocol proposed by Attiya et al. [Attiya02]

[Attiya02] Hagit Attiya, Alla Gorbach, and Shlomo Moran. Computing in totally anonymous asynchronous shared memory systems. Information and Computation, 2002.

3. Topological Characterization

Colorless Task

A colorless task is a triple $T = (I, O, \Delta)$, where

- I and O are finite simplicial complexes;
- $\Delta: I \to 2^{O}$ is a carrier map.

Topological Characterization

Theorem 3

A colorless task $T = (I, O, \Delta)$ is t-resilient solvable in the anonymous model iff

$(*)$ there is a continuous map $f\colon |\mathrm{skel}^t I| \to |O|$ s.t. $f(|s|) \subseteq |\Delta(s)|$ for all $s \in \mathrm{skel}^t I$.

Theorem 4 [Herlihy & Rajsbaum '10]

A colorless task $T = (I, O, \Delta)$ is wait-free solvable in the *non-anonymous* model iff $(*)$.

Computational Equality

Corollary 5

A colorless task $T = (I, O, \Delta)$ is t-resilient solvable in the anonymous model iff it is t-resilient solvable in the non-anonymous model.

Anonymous shared-memory computing
= Non-anonymous shared-memory computing
(… as long as colorless tasks are concerned)

Proof of Thm 3: Only If Part

T is solvable in the anonymous model

$\Rightarrow T$ is solvable in the non-anonymous model

$\Rightarrow$ A continuous map exists (Theorem 4)

Proof of If Part

$\exists f : |\mathrm{skel}^t I| \to |O|$ s.t.
$f(|s|) \subseteq |\Delta(s)|$ for all $s \in \mathrm{skel}^t I$.

$\Rightarrow \exists \delta : \mathrm{Bary}^k \mathrm{skel}^t I \to O$ s.t.
$\delta\left(\mathrm{Bary}^k \mathrm{skel}^t s\right) \subseteq \Delta(s)$ for every $s \in I$.
(finite approximate agreement theorem)

There is an anonymous protocol that solves
$T = \left(I, \mathrm{Bary}^k \mathrm{skel}^t I, \mathrm{Bary}^k \mathrm{skel}^t\right)$.

Summary

I have investigate distributed computability in the anonymous shared-memory model.

1. Infinitely-valued atomic weak set object
2. t-resilient $(t+1)$-set agreement protocol
3. Topological characterization of t-resilient solvable colorless tasks

Further Research

Uniform solvability of colorless tasks

- Infinite simplicial complex
- Generalized simplicial approximation
- Reducing to Gafni & Koutsoupias 2002

Computability for general decision tasks

- Full-information protocol is not known.
- Just started.

Thank you!

Why Important?

why important