Abstracts of the talks presented to

2011 EMPG

August 29-31, 2011

Telecom ParisTech

Editor: Olivier Hudry

2011 EMPG meeting

The *European Mathematical Psychology Group* (EMPG) was born in 1971, in Paris, through the impetus given by Jean-Claude Falmagne, as reported by Duncan Luce[1]: “By 1971, he was so involved in mathematical psychology that, after some discussions with others, he invited about 30 Europeans to participate in a meeting on mathematical psychology at the Maison des Etudiants Belges associated with the University of Paris. All came, leading to the formation of EMPG.”

Thus it is our pleasure to welcome the 2011 EMPG meeting and to celebrate the 40th birthday of the Group! Indeed, besides the annual meeting of the *Society for Mathematical Psychology* in the USA, there is also an annual meeting organized under the aegis of the EMPG, in different European countries: Finland (2010), Netherlands (2009), Austria (2008), Luxembourg (2007), France (2006), Italy (2005), Belgium (2004), Spain (2003), Germany (2002), Portugal (2001), just to quote the last ten meetings.

The 2011 meeting will be held on August 29-31, in the premises of Telecom ParisTech (http://www.telecom-paristech.fr/), in Paris, as 40 years ago! As usually, the main purpose of the meeting is to gather researchers and PhD students from different countries and to stimulate activities and discussions about the mathematical aspects of psychology. Indeed, all the communications dealing with any subject related to mathematical psychology are welcome: cognition, cognitive sciences, connectionism, decision-making, judgment, information processing, knowledge space, learning, memory, thinking, mathematical models, measurement, scaling, models for sensation and perception, neural modelling, neural network, perception and psychophysics, probabilistic models, psychological process models, psychometrics, statistical methods...

There will be 54 talks, including 6 plenary talks presented by Christine Choirat (Spain), Adele Diederich (Germany), Jean-Claude Falmagne (United States), Thierry Marchant (Belgium), Maartje Raijmakers (Netherlands), Luca Stefanutti (Italy), and an honorary speech dedicated to Jean-Claude Falmagne and his works presented by Dietrich Albert (Austria) and Cord Hockemeyer (Austria). The abstracts of these 54 talks can be found in this booklet, as well as the program (the bold names point the presumed speakers out) and the list of the participants to EMPG 2011: about 80 participants are expected, mainly from European countries (Belgium, France, Germany, Italy, Russia, Spain...), but also from Colombia or the USA. Among them, about 15 PhD students, which illustrates the attractiveness and the vitality of the field.

I seize this opportunity to thank all the people who helped me to organize this meeting:

- Telecom ParisTech and the staff of Telecom ParisTech provided the rooms, the necessary material, the help for logistics and communication, and so on;

[1] Duncan Luce, “Editoral”, *Journal of Mathematical Psychology* 49 (2005) 430–431.

- the sponsors of the meeting allowed us to cover the different expenses, since there were no fees: the ANR ComSoc (Computational Social Choice) research project, the French Society of Operations Research (Société française de recherche opérationnelle et d'aide à la décision, ROADEF; http://www.roadef.org/), the working group "Operations Research" (GdR RO; http://www-poleia.lip6.fr/~fouilhoux/gdrro/) of the Centre national de la recherche scientifique (CNRS, i.e. the French National Centre for Scientific Research);
- the Society for Mathematical Psychology (http://www.mathpsych.org/) offered "student travel awards" specifically provided in aid of the PhD students; they allowed us to invite the PhD students who attended the meeting to the conference dinner;
- Marc Barbut and Christiane Boghossian helped us in organizing a special issue of *Mathematics and Social Sciences* (http://msh.revues.org/3237?lang=en), as Vadim Lozin did for a special issue of the *Electronic Notes in Discrete Mathematics* (http://www.elsevier.com/wps/find/journaldescription.cws_home/681020/description);
- Dietrich Albert, Denis Bouyssou, Hans Colonius, Jean-Paul Doignon, Jean-Claude Falmagne, Janne Kujala entrusted me the organization of the 2011 EMPG meeting and gave me fruitful advices in order to prepare the meeting;
- the members of the scientific committee (see below) helped me in broadcasting information dealing with the meeting to interested people and to motivate some speakers;
- Irène Charon (Telecom ParisTech) and Antoine Lobstein (CNRS and Telecom ParisTech) constantly helped me throughout the preparation of the 2011 EMPG meeting; I thank them warmly.

Olivier Hudry

Scientific committee

- Dietrich Albert (Austria)
- Fuad Aleskerov (Russia)
- Helena Bacelar-Nicolau (Portugal)
- Raymond Bisdorff (Luxembourg)
- Denis Bouyssou (France)
- Irène Charon (France)
- Hans Colonius (Germany)
- Jean-Paul Doignon (Belgium)
- Jean-Claude Falmagne (United States)
- Miguel A. García-Pérez (Spain)
- Olivier Hudry (France)
- Janne Kujala (Finland)
- Jérôme Lang (France)
- Jean-François Laslier (France)
- Antoine Lobstein (France)
- Duncan Luce (United States)
- Boris Mirkin (Russia)
- Ákos Münnich (Hungary)
- Mike Regenwetter (United States)
- Roman Slowinski (Poland)
- Reinhard Suck (Germany)
- Alexis Tsioukias (France)

Program of EMPG 2011

Monday 29 August

8:30-9:00
* room B 310
  Welcome and opening session

9:00-10:00. Plenary session (M1)
* room B 310
  D. Bouyssou, **T. Marchant**: Measurement theory with unary relations

10:00-10:30.
* room E 200
  Coffee break

10:30-12:30. Parallel sessions (M2)
* room B 310
  H. Colonius, S. Rach: Towards a measure of visual-auditory integration efficiency
  K. Serrhini, G. Palka, S. Fuchs, W. Dorner, V. Meyer, S. Priest: Visual and Cognitive Perceptions to Improve Flood Risk Map
  Á. Münnich: Attitude and its measurement using subjective reference points
  G. Golovina, T. Savchenko: The approach to measurement of dynamic characteristics of the person
* room B 312
  D. Luce: Cross Modal Predictions for Binary and Unary Intensity Modalities
  J.-L. Dessalles: Algorithmic Simplicity May Control Some of our Emotions
  M. Cuevas: Effects of sample size, group sample size ratio, magnitude of DIF, and length of test in logistic regression as a method for identification of differential item functioning (DIF)
  V. Cervantes: Assessing the effects of the sample size, sample size ratio, magnitude of DIF and test length on the noncompensatory differential item functioning index (NCDIF)

12:30-14:00
lunch

14:00-15:00. Plenary session (M3)
* room B 310
  L. Stefanutti: When the correspondence between probabilistic and set representations of local independence becomes a requirement: constant odds models for probabilistic knowledge structures

15:00-15:30.
* room E 200
  Coffee break

15:30-17:30. Parallel sessions (M4)
* room B 310
- **R. Alcalá-Quintana**, M.A. García-Pérez: The effect of response format in synchrony judgments
- **M. García-Pérez**, R. Alcalá-Quintana: A model of indecision in 2AFC detection tasks
- **I. Fründ**, Felix A. Wichmann and Jakob Macke: Sequential dependencies in perceptual decisions
- **J. Shanteau**: Memory-Retrieval vs Decision-Making in Repetition Priming

* room B 312
- **Y. Guiard**: Is Your 'Ratio' a Quotient? Experimenters Need to Know
- **W. Vanpaemel**: Theory testing with the prior predictive
- **S. Noventa**, G. Vidotto: A variational approach to sensation and psychophysics
- **B. Mirkin**, S. Nascimento, T. Fenner: A Model of Interpretation of Concepts over a Taxonomy

17:30-17:45
short break

17:45-18:30. Plenary session (M5)
* room B 310
- **D. Albert, C. Hockemeyer**: JCF's impact is not limited to the foundation of the EMPG

19:30. **Conference dinner** at the Assanabel restaurant

Tuesday 30 August

9:00-10:00. Plenary session (T1)
* room B 310
- **M. Raijmakers**, V. D. Schmittmann, I. Visser: The application of latent Markov models in category learning

10:00-10:30
* room E 200

Coffee break and posters session (T2):
- **L. Burigana**, M. Vicovaro: Sets of transformations as criteria for rating the diversity within sets of objects
- **D. de Chiusole**, P. Anselmi, L. Stefanutti, E. Robusto: The Gain-Loss Model: bias and variance of the parameter estimates
- **N. Gauvrit**, H. Zenil, J.-P. Delahaye: Psychometrics of randomness. A Kolmogorov complexity approach
- **N. Gauvrit**: Mathematical consequences of the TBRS-model hypothesis
- **T. Savchenko**, G. M. Golovina: The use of content-analysis for reconstruction existential categories

10:30-12:00. Parallel sessions (T3)
* room B 310
  C. Pelta: Spatial prisoner´s dilemma and laws of imitation in Social Psychology
  O. Rioul, Y. Guiard: The Power Model of Fitts' Law Does Not Encompass the Logarithmic Model
  Ph. Robert-Demontrond, **D. Thiel**: Modeling the halo effect by using automata networks
* room B 312
  S. Barthelmé, N. Chopin: Approximate Bayesian Computation in Experimental Psychology
  R. Morey, J.N. Rouder, P.L. Speckman, J.M. Province: Default Bayes factors for inference in general ANOVA designs
  C. Zwilling, M. Regenwetter, D. Cavagnaro, A. Popova: Quantitative Testing of Decision Theories: Probabilistic Specification and Empirical Results from a Frequentist and Bayesian Framework

12:00-13:30
lunch

13:30-14:30. Plenary session (T4)
* room B 310
  J.-C. Falmagne: Learning Spaces in Real Life. How the large size of actual learning spaces guides the development of the theory

14:30-15:30. Parallel sessions (T5)
* room B 310
  E. Cosyn: Building a Learning Space from a Large Data Set
  P. Anselmi, E. Robusto and L. Stefanutti: Identifying the best skill multimap by constraining the problem error probabilities in the Gain-Loss Model
* room B 312
  M. Engelhart, J. Funke, S. Sager: Optimization-based Analysis and Training of Human Decision Making with a New Test-scenario
  A. Lambert-Mogiliansky, J. Busemeyer: Self-Control and Type Indeterminacy

15:30-16:00
* room E 200
  Coffee break and posters session (see above)

16:00-17:30. Parallel sessions (T6)
* room B 310
  D. de Chiusole, L. Stefanutti: Probabilistic skill multi-map models for skill dependence. An application to mathematical problem solving in children
  A. Spoto, L. Stefanutti, G. Vidotto: Skill map based knowledge structures: some considerations about their identifiability
  J. Heller: Parameter estimation and identifiability in probabilistic knowledge structures
* room B 312
  R. Bystricky: Comparison of various nonlinear distance based preferential voting systems
  E. Indurain: Means
  L. Wollschläger, A. Diederich: The random-walk-on-tree model for decisions between multiple alternatives

Wednesday 31 August

9:00-10:00. Plenary session (W1)
* room B 310
  C. Choirat: Separable representations in mathematical psychology and decision making

10:00-10:30
* room E 200
  Coffee break

10:30-12:00. Parallel sessions (W2)
* room B 310
  R. Suck: The knowledge space resulting from combining various skill orders
  J.-P. Doignon: Representations of interval orders
  S. Vautier, L. Hubert, M. Veldhuis: How mathematical psychologists could help test-score psychologists to perceive the empirical reality they have built?
* room B 312
  L. DeCarlo: When two heads are not better than one: A paradox in latent class signal detection theory
  S. Massoni, J.-C. Vergnaud: Direct and Indirect Elicitation Rules for Subjective Beliefs: Experimental Evidence
  H. C. Micko: A Multi-Threshold Model of Signal Detection

12:00-13:30
lunch

13:30-14:30. Plenary session (W3)
* room B 310
  A. Diederich: Optimal time windows: Modeling multisensory integration in saccadic reaction times

14:30-15:00
* room E 200
  Coffee break

15:00-17:00. (W4)
* room B 310
  M. Andreatta, E. Acotto: Mental and mathematical representations of music
  E. Golovina, V. Nosulenko: A new method of investigation of person's confidence expressed while comparing musical fragments
  D. Laming: Recalling the list-before-last: A cautionary tale
  V. Robinet, S. Portrat, **B. Lemaire**: Working memory capacity as a quantity of information

17:00
* room E 200
  Farewell cocktail

JCF's impact is not limited to the foundation of the EMPG

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Keywords: Mathematical Psychology in Europe.

When Jean-Claude Falmagne invited to the first EMPG Meeting in Paris in 1971 he started what could be called an oddity in science: a series of annual conferences without any formal organisation behind it which has been successfully continuing for forty years until now. However, the impact of JCF's work is not limited to mathematical psychologists from all over the world meeting once a year in Europe. He has fostered research in the various areas of mathematical psychology in general and beyond this. This has to be recognised. Furthermore his work has still other long-lasting effects on European Research and Development. This we will exemplify by an area he has been (co-) founded, the theory of knowledge spaces.

The effect of response format in synchrony judgments

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Keywords: Probabilistic models in psychology, models for sensation and perception, temporal order judgment, simultaneity judgment, temporal order, time perception

Research on synchrony perception relies on two classic experimental tasks, temporal order judgment (TOJ) and simultaneity judgment (SJ). In both tasks a pair of stimuli (S_1 and S_2) is presented at various onset asynchronies, although observers are asked to judge their relative time of occurrence in a different way. In the TOJ task observers have to report which stimulus was presented first, whereas in the SJ task they have to report whether the two stimuli were presented simultaneously or successively. There is also a variant of the SJ task that allows for three response categories: "S_1 first", "S_2 first", or "simultaneous". The three tasks render an estimate of the point of subjective simultaneity (PSS) and a measure of sensitivity. In principle, these two values should agree (within sampling error) across tasks. However, agreement has been shown to be generally quite poor in empirical studies ([1]).

Here we use a probabilistic model of temporal-order perception to provide a common framework for performance under the three tasks. The core of the model is a unified representation of the decision space and the various experimental tasks differ as to how they require the observer to use this representation to produce a response. The perceived time of occurrence of any given event is modelled as an exponentially distributed random variable. Then, when two stimuli, S_1 and S_2, are presented the decision variable is defined as the difference between their perceived times of occurrence. The model includes two additional components: a *resolution* parameter that accounts for the lack of perfect temporal resolution of real observers, and a *response bias* parameter that comes into play when "simultaneous" responses are not allowed. Depending on the constrains imposed by the response format, this framework yields different psychometric functions, thus leading to discrepant estimates of the PSS and sensitivity. Implications for performance in the three types of task have been derived and tested against empirical data from a published within-subject study involving the three tasks ([1]). The model fit is satisfactory, explaining inconsistencies across tasks and opening a new way to analyze synchrony-judgment data.

[1] R.L.J. van Eijk, A. Kohlrausch, J.F. Juola, S. van de Par. "Audiovisual synchrony and temporal order judgments: Effects of experimental method and stimulus type", *Perception & Psychophysics, 70*, 2008, 955-968. doi:10.3758/PP.70.6.955.

Mental and mathematical representations of music

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Keywords: mental representations, mathematical models of music, musical cognition.

In musicological literature the concept of representation is quite widespread, even if in few cases it is completely analysed from a philosophical and psychological point of view [1]. We analyse here two different possible senses of the concept of musical representations: we distinguish between mental and mathematical representations of music [2]. Mental representations of music are the objects of the musical mind, the material of the musical cognition: they are *private representations* and they can be (meta)represented by public representations which are similar to the private mental representations [3]. On the other hand, mathematical representations of music are public representations which could be cognitively correlated with mental representations of music.

In many popular cognitive theories of music the mental representations of music are considered to be construed by the mind according with the musical flow [4], but in other models, like Generative Theory of Tonal Music [5] the mental representations of music are considered in the framework of a final-state theory. The computational approach to the musicology grounded on formal mathematical models uses written representations of music with a precise analytical function.

If mental representations of music are by definition a matter of cognitive psychology and philosophy, it can be argued that also mathematical representations of music have some cognitive correlates enabling the understanding of non-tonal music. Amongst the many typologies of mathematical representations of music we will analyse in details some examples about the so-called transformational analysis, which is a formalised subfield of computational musicology coming from the American Tradition [6]. The transformational paradigm in music also opens new questions about the cognitive and philosophical ramifications of algebraic approaches in music theory, analysis and composition, as we will discuss at the end of our talk by presenting some relationships between this approach in musicology and a category-oriented version of Piaget's genetic epistemology [7].

[1] C. Nussbaum, *The musical representation*, The Mit Press, Cambridge, Massachusetts, 2007.
[2] M. Chemillier, "Représentations musicales et représentations mathématiques", *Musique et Schème. Entre percept et concept*, Béatrice Ramaut-Chevassus ed., Publications de l'université de Saint-Etienne, 2007.
[3] D. Sperber, *Metarepresentations in an evolutionary perspective*, in Dan Sperber ed. *Metarepresentations: A Multidisciplinary Perspective* Oxford University Press, 2000, 117-137.
[4] E. Margulis, *A Model of Melodic Expectation*, Music Perception, 22, 4, Summer 2005, 663–714.
[5] R. Lerdahl, R. Jackendoff, A Generative Theory of Tonal Music, MIT press, 1983.
[6] M. Andreatta (dir.), *Around Set Theory*, Collection "Musique/Sciences", Ircam, 2008.
[7] M. Andreatta, « Calcul algébrique et calcul catégoriel en musique : aspects théoriques et informatiques », Le calcul de la musique, L. Pottier (éd.), Publications de l'université de Saint-Etienne, 2008, p. 429-477.

Identifying the best skill multimap by constraining the problem error probabilities in the Gain-Loss Model

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Keywords: knowledge space, learning process, skill multimap, constrained parameter estimation.

The Gain-Loss Model ([2], [3]) assesses the knowledge of students and the effectiveness of educational interventions in promoting specific learning. For the assessment to be accurate and trustworthy, information is required about which skills are measured by the assessment instrument and how they are related to the problems. In the Gain-Loss Model this information is translated into a skill multimap ([1]) associating each problem with a collection of subsets of skills that are necessary and sufficient to solve it.

In practical applications the skill multimap underlying the data is not known. As a consequence, more than one skill multimap could be plausible in theory, no one corresponding to the true skill multimap. In these cases, the skill multimap that best approximates the true one has to be identified. Comparing the fit to the data of alternative models by means of standard statistics (e.g., Pearson's Chi-square, likelihood ratio) does not guarantee that the best model is identified: An incorrectly specified model can obtain a good fit, even better than that of the correct model, by an ad hoc inflation of the error probabilities (careless error and lucky guess) of the problems ([4]).

An approach is proposed, which takes into account the information about model fit which derives from both the standard fit statistics and the estimates of the error probabilities. In particular, the approach compares the fit to the data of models which undergo a constrained estimation of their error probabilities. Error probabilities might inflate as a consequence of misspecifications in the skill multimap. Therefore, if we constrain these estimates, the fit of models which are furthest from the true one is expected to become worse than that of models which are closest.

Results of a simulation study and an empirical application are presented that highlight the usefulness of the proposed approach for identifying the skill multimap that best approximates the true one among a number of alternatives.

[1] J.-C. Falmagne, J.-P. Doignon, "Learning spaces: Interdisciplinary applied mathematics", Berlin: Springer-Verlag, 2011.
[2] E. Robusto, L. Stefanutti, P. Anselmi, "The Gain-Loss Model: A probabilistic skill multimap model for assessing learning processes", *Journal of Educational Measurement* 47, 2010, 373-394.
[3] L. Stefanutti, P. Anselmi, E. Robusto, "Assessing learning processes with the Gain-Loss Model", *Behavior Research Methods* 43, 2011, 66-76.
[4] L. Stefanutti, E. Robusto, "Recovering a probabilistic knowledge structure by constraining its parameter space", *Psychometrika* 74, 2009, 83-96.

Approximate Bayesian Computation in Experimental Psychology

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Keywords: Bayesian inference, likelihood-free inference, approximate Bayesian computation

How does one carry out statistical inference when the likelihood of the data under the model of interest is very difficult to compute, or even unavailable? Approximate Bayesian Computation (Pritchard et al., 1999) is an inference technique with roots in evolutionary biology, whose purpose is to deal with stochastic models with intractable likelihood functions. ABC enables “likelihood-free” inference in a Bayesian setting, and is applicable whenever the model is easy to simulate. In its most basic version, ABC requires that one defines a set of *summary statistics* s(y) for the dataset, and produces samples from an approximation to $p(\theta|s(y^*))$, the posterior distribution over parameters given the summarised data $s(y^*)$. Although the original algorithm has been much improved upon (e.g., Beaumont et al., 2002; Marjoram et al., 2003; Beaumont et al., 2009), reliance on summary statistics causes some practical and theoretical difficulties. The ABC-EP algorithm, introduced in Barthelmé and Chopin (2011), does not rely on summary statistics and can be much faster than other ABC methods, although not as generically applicable.

Many models in psychology and neuroscience can potentially benefit from ABC techniques, having often difficult or intractable likelihood functions but being relatively easy to sample from. In this work we will show how ABC-EP can be applied to hierarchical modelling of reaction time data. We will highlight some of the difficulties encountered as well as possible solutions.

Barthelmé, S. and Chopin, N. (2011). ABC-EP: Expectation propagation for likelihood- free bayesian computation. In *Proceedings of the 28th International Conference in Machine Learning*.
Beaumont, M., Zhang, W., and Balding, D. (2002). Approximate Bayesian computation in population genetics. *Genetics*, 162(4):2025.
Beaumont, M. A., Cornuet, J.-M., Marin, J.-M., and Robert, C. P. (2009). Adaptive approximate bayesian computation. *Biometrika*, 96(4):asp052-990.
Marjoram, P., Molitor, J., Plagnol, V., and Tavaré, S. (2003). Markov Chain Monte Carlo without Likelihoods. 100(26):15324-15328.
Pritchard, J., Seielstad, M., Perez-Lezaun, A., and Feldman, M. (1999). Population growth of human Y chromosomes: a study of Y chromosome microsatellites. *Molecular Biology and Evolution*, 16(12):1791.

Measurement theory with unary relations

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Keywords: measurement.

Measurement theory [4, for instance] has been proposed in the middle of the 20th century as an attempt to unify several disparate works published in economics, in psychology, in mathematics, in physics, ... but with a common interest: the numerical representation of empirically observable relations. Although the observable relations can in principle be of any order, most scholars have dealt with binary, ternary and quaternary relations and very few have considered the case of unary relations[2]. Some exceptions are [1, 2, 3, 5, 6, 7].

In this paper, we survey these results (obtained in the context of decision under risk, decision under uncertainty and conjoint measurement). We show that, in a sense, they are very similar to classic results obtained for binary relations (because they yield the same value function) although they use more parsimonious primitives (because a unary relation on a set *X* contains much less information than a binary relation on *X*).

We touch upon the problem of the simultaneous representation of several nested unary relations and the extension to other domains of measurement theory (extensive measurement, intensive measurement, ...).

[1] D. Bouyssou and T. Marchant, "An axiomatic approach to noncompensatory sorting methods in MCDM, II: the general case", *European Journal of Operational Research* 178, 2007, 246-276.
[2] D. Bouyssou and T. Marchant, "Ordered categories and additive conjoint measurement on connected sets", *Journal of Mathematical Psychology* 53, 2009, 92-105.
[3] D. Bouyssou and T. Marchant, "Additive conjoint measurement with ordered categories", *European Journal of Operational Research* 203, 2010, 195-204.
[4] D. H. Krantz, R. D. Luce, P. Suppes, and A. Tversky. *Foundations of measurement: Additive and polynomial representations*. Academic Press, New York, 1971.
[5] Y. Nakamura, "Threshold Models for Comparative Probability on Finite Sets", *Journal of Mathematical Psychology* 4, 2000, 353-382.
[6] Y. Nakamura, "Trichotomic preferences for gambles", *Journal of Mathematical Psychology* 48, 2004, 385-398.
[7] K. Vind. *Independence, Additivity, Uncertainty*. Springer, Berlin, 2003.

[2] A unary relation is simply a subset of the set of objects to be measured.

Sets of transformations as criteria for rating the diversity within sets of objects

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Keywords: Transformation, Diversity, Invariance, Spatial vision

Let X be a basic set (of objects) and T a set of transformations acting within X (to be interpreted as rules of permissible substitutions between elements of X). Let A be any subset of X. The diversity existing within set A is *no greater* (respectively, *no smaller*) than the diversity expressible by transformations in T if (i) for all $x \in A$ and all $x' \in A$ there is $t \in T$ such that $x' = t(x)$ (respectively, if (ii) for all $x \in A$ and all $t \in T$ there is $x' \in A$ such that $x' = t(x)$). [In particular, if T is a permutation group on domain X, then condition (i) means that A is the union of T-orbits (i.e., A is invariant to permutations in T), whereas condition (ii) means that A is a subset of one single T-orbit.] Presuming that a chain $T_1 \subset T_2 \subset ... \subset T_m$ of sets of transformations all acting within X is previously defined, then two (possibly identical) *diversity ranks* of a set A relative to that chain can be determined, one rank being the minimum $k \in \{1,...,m\}$ such that A satisfies condition (i) relative to T_k, and the other rank being the maximum $k \in \{1,...,m\}$ such that A satisfies condition (ii) relative to T_k. In this way a ranking of the diversity within any set of objects $A \subseteq X$ relative to the presumed chain of sets of transformations $(T_1,...,T_m)$ is made possible, which is somehow reminiscent of the Felix Klein's criterion for classifying geometrical properties ([1]-[4]). In our contribution we explore elementary formal properties (of algebraic type) implied by these definitions, and illustrate their possible use in the psychology of visual perception (in particular, spatial vision). Specifically, we link the concepts above to a way of characterizing the *conditional indeterminacy of percepts* (given the stimuli), a way that characterizes the conditional indeterminacy in terms a set of transformations suitable to cover the diversity within a set of alternative perceptual properties ([5]).

[1] Cassirer, E. (1944). The concept of group and the theory of perception. *Philosophy and Phenomenological Research*, *5*, 1-36.
[2] Klein, F. (1872/1893). Vergleichende Betrachtungen über neuere geometrische Forschungen ("Erlanger Programm"). *Mathematische Annalen*, *43*, 63-100.
[3] Narens, L. (1988). Meaningfulness and the Erlanger program of Felix Klein. *Mathématiques Informatique et Sciences Humaines*, 26 (101), 61-71.
[4] Suppes, P. (1991). The principle of invariance with special reference to perception. In J.P. Doignon, & J.C. Falmagne (Eds.), *Mathematical psychology. Current developments* (pp.35-53). New York: Springer Verlag.
[5] Todd, J.T., & Norman, J.F. (2003). The visual perception of 3-D shape from multiple cues: Are observers capable of perceiving metric structure? *Perception & Psychophysics*, *65*, 31-47.

Comparison of various nonlinear distance based preferential voting systems

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Keywords: preferential modeling, ordinal structures, distances, voting systems

It is not a mistake to say that voting systems are representing the aggregated collective result of individual preferences. Voting systems are many times reflecting much more their own characteristics than preferences of the voters, especially in the case of tight race. They are producing often in these cases unexpected or unwanted paradoxical results.

The preferential voting systems (as e.g. Borda scheme) are working most of the time with equidistant differences between the adjacent preferences of individual voter. These distances do not have to be the same, we can use different – mostly nonlinear functions to describe them. People usually do not have problems to distinguish between their first or second preference and many times to denote their worst choice – last preference, but the ability to make a decision between the "middle preferences" is far more doubtful. With the introduction of nonlinear distances to the voting systems, we can reflect these problems as well as at least partly prevent some unwanted paradoxical situations and produce modified voting systems according to the special needs. This approach we can insert to the different known scoring voting systems as well as to other systems as e.g. alternative vote system or STV. Comparison of selected cases is provided to describe impact of adopted method.

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Assessing the effects of the sample size, sample size ratio, magnitude of DIF and test length on the noncompensatory differential item functioning index (NCDIF)

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Keywords: DFIT framework, differential item functioning, noncompensatory DIF index, NCDIF, psychometric, statistical methods, sample size, ratio sample size, length of test, magnitude of DIF.

The noncompensatory differential item functioning index (NCDIF) has been proposed within the differential functioning of items and tests (DFIT) framework as an item response theory based alternative for DIF detection ([1]). Some studies have indicated that the NCDIF index may be affected by variables such as group sample size ratio, sample size, and seems related with specific items parameters like certain ranges of difficulty and discrimination ([2], [3]). Furthermore, it has also been noted that the proposed test statistic for testing the no-DIF hypothesis does not approximate the χ^2 distribution leading to inflated Type I error rates for large sample sizes under data simulated from the null hypothesis ([1], [3]); this has led to propose some cut-points (v.g. 0.006, [1]) as well as some Monte Carlo approaches ([2], [3]) for deciding whether an item should be considered as presenting DIF or not. Also, it is well known that for every null hypothesis test, larger sample sizes lead to sensitivity of smaller departures from the null hypothesis; thus, it is also recommended to assess the question regarding the practical importance of the observed effects (see e.g. [4]). Considering these aspects and some characteristics of large-scale assessments in our country, we conducted a simulation study aiming to assess if the NCDIF index was affected by the manipulated variables and if some cut-points could be obtained to be used as effect-size measures of varying DIF magnitude. In the simulation study I manipulated four partially crossed independent variables: sample size, group sample size ratio, DIF magnitude and length of test which resulted in 220 conditions and simulated 200 replications for each of these. Given the variables that affected the NCDIF index, we obtained cut-points for testing the null hypothesis in each resulting condition. We also found effect-size cut-points for classifying the DIF magnitude that seem to apply regardless of the levels of the manipulated variables. The results of this study are important because they allow knowing how the NCDIF procedure performs under different conditions providing cut-points for specific situations like extreme sample ratios between reference and focal group and also shed light on the applicability of the NCDIF statistic as an effect-size measure that might allow comparisons that are not tied up to the specific conditions under which the analyses are carried out.

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Separable representations in mathematical psychology and decision making

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Keywords: Stevens' model, separable representations, decision making.

Studying how individuals compare two given quantitative stimuli is a fundamental problem. One very common way to address it is through ratio estimation, that is to ask individuals to give their estimates of the ratio of two stimuli. In order to explain the results of this kind of experiments, Stevens formulated his famous psychophysical power law model, establishing that subjective value is a power function of physical value and that equal physical ratios produce equal psychological ratios. Stevens' approach has been criticized under various perspectives. This has led to the axiomatization, in particular by Narens and Luce, of various theories belonging to a class of so-called separable representations. During the last ten years, strong empirical evidence supporting this class of models has been collected. In this presentation, we review our work on the topic, especially regarding applications of separable representations to individual or group decision making. The presentation is based on joint work with Michele Bernasconi (Università Ca' Foscari di Venezia) and Raffaello Seri (Università degli Studi dell'Insubria).

Towards a measure of visual-auditory integration efficiency

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Keywords: measurement, visual-auditory integration, Fechnerian Scaling.

Visual-auditory integration refers to the process of combining information that has been extracted from the visual and auditory sensory channels It is important, for both theoretical and practical reasons, to distinguish between an individual‘s capability to extract auditory and visual information from the ability to integrate this information across the different modalities, i.e., an individual‘s integration efficiency (IE). Up to this date, there is no consensus about how to derive a quantitative measure of integration efficiency ([1]). Here, we suggest an approach based on the theory of Fechnerian Scaling ([2]). Fechnerian Scaling deals with the computation of subjective distances among stimuli from their pairwise discrimination probabilities. Different approaches of combining visual, auditory, and bimodal discrimination probabilities into an overall measure of IE will be discussed.

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Building a Learning Space from a Large Data Set

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Keywords: knowledge space, learning space, data mining.

Extracting a learning space on a large domain (in excess of 350 items) from extensive data presents challenges that require adaptation of and changes to traditional procedures. First, these procedures were tuned for tapping human expertise and not students data. Second, they were designed to build knowledge spaces without the assurance that the resulting spaces would be well-graded. The procedure that we propose here has a unique set of features.

- It relies on a computerized expert whose judgments are informed by querying a database of previous knowledge assessments.
- It maintains at all time and concurrently the entail relation and surmise system representations of the learning space under construction (by applying concepts exposed in Chapter 16 of [1]).
- It implements a bootstrapping process in which the building of the learning space at a given stage relies on the learning space built at the previous stage.

The procedure is illustrated with a learning space whose domain is the field of middle-school algebra.

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Effects of sample size, group sample size ratio, magnitude of DIF, and length of test in logistic regression as a method for identification of differential item functioning (DIF)

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Keywords: Logistic Regression, Differential item functioning, psychometrics, statistical methods, sample size, group sample size ratio, length of test, magnitude of DIF.

Logistic Regression (LR) has been used as a method to indentify differential item functioning (DIF) in different contexts like education and psychology. In this procedure, coefficients for group, total score and their interaction terms are calculated and tested for significance using the examinee's response (correct/incorrect) as dependent variable [1]. Some studies have shown DIF detection trough this procedure can be affected by variables such as group sample size ratio, sample size, and that it seems related with specific item parameters like certain ranges of difficulty and discrimination ([2], [3]). Considering these reasons and some characteristics of test development in our country we made a simulation study with four partially crossed independent variables: sample size, group sample size ratio, magnitude of DIF and length of test, which resulted in 270 conditions and simulated 200 replications for each of these. McFadden's R-squared Delta ($R^2\Delta$) was used as an effect size measure and as dependent variable. We used linear models to define which variables affected the effect size measure: $R^2\Delta$ when the group effect was introduced (Square R Delta for detecting items with uniform DIF – DRU) and $R^2\Delta$ when the interaction effect was introduced (Square R Delta for detecting items with non uniform DIF – DRN).

The results of this study show that all manipulated variables and the interactions between magnitude of DIF and both sample size and group sample ratio affect the magnitude of DRU for items with uniform DIF. These analyses with items without uniform DIF showed that this measure was affected by size, magnitude of DIF, length of test and the interaction between sample size ratio and sample size. For items with non uniform DIF, the analyses showed that DRN was affected by the same interactions than DRU for items with uniform DIF and also showed a significant interaction effect between magnitude of DIF and length of test. Finally, we found that DRN with items without non uniform DIF was affected by sample size, group sample size ratio, length of test and the interactions between size sample with magnitude of DIF and size sample ratio.

We also obtained cut-points, both for DRU and DRN, for different levels of the variables that affect the $R^2\Delta$ measures (test length was omitted because it has the smallest effect). The results of this study are important because they allow knowing how the RL DIF procedure and the $R^2\Delta$ perform under different conditions providing cut-points for specific situations like extreme sample ratios between reference and focal group and magnitude of DIF (small and big).

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The Gain-Loss Model: bias and variance of the parameter estimates

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Keywords: knowledge space theory, gain-loss model, maximum likelihood estimation, bias and variance

The Gain-Loss Model [3], [4] is a formal model developed within Knowledge Space Theory [1], [2]. It consists of five parameters: initial probabilities of the skills, probabilities of learning or losing skills (gain and loss) and careless error and lucky guess parameters of the items. The model parameters are estimated by maximum likelihood (MLE). The effect of high or low values of the initial and final item probabilities on the bias and variance of the lucky guess and careless error parameters is examined in four different simulation studies.

In each of the four studies, 500 data sets were simulated. The parameters of the model were estimated by MLE and bootstrapped bias and variance were obtained, for each parameter in the model.

The simulation studies show that high values of both the initial and final probabilities of an item produce a systematic overestimation and a considerably high variance of the lucky guess parameter of that item. On the other hand, low values of both the initial and final probabilities of an item produce a systematic overestimation and high variance of the careless error parameter of that item.

Both lucky guess and careless error are conditional probabilities. A re-parameterization of the model is proposed in which lucky guess and careless error parameters are replaced by joint probabilities of lucky guess and careless error. Simulation studies show, in fact, that these joint probabilities are much less affected, in terms of bias and variance, by the initial and final probabilities of the items.

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Probabilistic skill multi-map models for skill dependence. An application to mathematical problem solving in children

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Keywords: knowledge space theory, cognitive skill models, mathematical problem solving

A probabilistic model for skill multi-maps [1], which accounts for dependencies among skills, was applied for testing the empirical validity of a cognitive model of mathematical problem solving in children.

In knowledge space theory [2, 3] dependencies among skills belonging to some set *S* are represented by a *competence structure* [4], which is a collection of subsets of *S* including at least the empty set and *S* itself. Such representation is a deterministic model whose validation requires the specification of a probabilistic counterpart of it. In the proposed model, the two representations (probabilistic and deterministic) of skill dependence are in agreement.

Some interesting features of the model are: (a) the possibility to introduce ordinal skills with more than two (presence/absence) levels. Each level is represented by a sub-skill, and the set of levels of the skill is a linear order on the sub-skills; (b) dependencies among skills reduce the number of competence states associated to each knowledge state, reducing by this the uncertainty about the "true" competence state of a student.

We present an application of this probabilistic model to a group of Italian children from the primary school. The aim was to test the empirical validity of two alternative cognitive models of the main skills involved in the solution of mathematical problems, and of their interrelations. In the first model the skills were assumed to be independent, whereas the second model assumed specific dependencies among them. The results concerning a comparison of the two models are illustrated and discussed.

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When two heads are not better than one: A paradox in latent class signal detection theory

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Keywords: probabilistic models in psychology, signal detection theory, latent class models.

Suppose an observer attempts to detect the presence or absence of an unobserved (latent) event, such as the presence or absence of psychological or physical disease. It follows from a latent class extension of signal detection theory (DeCarlo, 2002) that the accuracy of classifications based on the observer's responses depends on his or her ability to detect, the observer's criterion location, and the probability of the event. In general, adding more observers and basing classifications on joint decisions increases the proportion of cases correctly classified, in a manner analogous to increasing reliability in classical test theory when items are added to a test. An interesting paradox arises, however, when there are two observers and the probability of an event is 0.5: if the second observer's detection is the same (or lower) than the first observer, and the criterion locations are at the optimal location for a single observer, then the proportion of cases correctly classified is exactly the same as when only the first observer is available, and so the second observer adds nothing. Thus, two heads are not better than one. An interesting result is that the paradox does not arise if the second observer has a different criterion location than the first observer; the optimal location for the second observer is derived. In this case, the proportion of cases correctly classified will be larger for two observers as compared to one observer, yet agreement and Kappa will be lower. Another interesting result is that the paradox does not arise for three observers, even if they all have the same (optimal) criterion locations; the paradox can generally arise, however, when there is an even number of observers, but under limited conditions (e.g., same detection across observers; event probability of 0.5); an example with four observers is given. It is noted that classification accuracy is of primary importance, and is really the 'reliability' of practical interest; Kappa and agreement are shown to only partially tap into classification accuracy.

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Algorithmic Simplicity May Control Some of our Emotions

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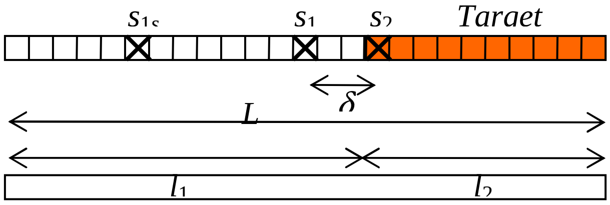
Keywords: complexity, simplicity, emotion

We studied emotional intensity in the context of near-miss situations. In a ‘near-miss’ (or ‘near-hit’) situation, an individual has almost won some substantial benefit, but gets nothing. In agreement with other studies [2-5], our experiment shows that the intensity of emotions in such cases depends on the ‘distance’ to the missed target. However, some additional effects that we could observe are only predicted by Simplicity Theory. We found for instance that missing a fragmented target is emotionally less intense than missing a connected one.

A basic, broadly used [3, 5] principle states that emotional intensity in a near-miss situation depends on $1/p$, where p is the *a priori* probability of the actual event. Unfortunately, objective probability is a bad predictor of emotional intensity here. It does not account for the influence of δ (see figure), nor for the influence of target fragmentation (which keeps winning probability constant), nor for the fact that emotion is extreme both for high and low winning probabilities.



Simplicity Theory (ST) [1] derives cognitive probability p from Kolmogorov complexity: $\boldsymbol{p = 2^{-U}}$. Unexpectedness U is the difference between *expected* complexity and *observed* complexity (see http://www.simplicitytheory.org). If we apply ST to the near-miss situation, we get the following prediction concerning the situation depicted above:

$$U(s_1) \geq \log(L/\delta) - 1$$

ST makes other predictions that we could check through several experiments. In these experiments, participants were asked to rank various near-miss situations depending on their emotional impact. The experiments were designed to show that alternative models are either silent or make wrong predictions. By contrast, ST accounts for most of the observed effects, including the role of target fragmentation.

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Optimal time windows: Modeling multisensory integration in saccadic reaction times

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Keywords: visual-auditory integration, optimal time window, TWIN model

The concept of a "time window of integration" holds that information from different sensory modalities must not be perceived too far apart in time in order to be integrated into a multisensory perceptual event. Empirical estimates of window width differ widely, however, ranging from 40 to 600 ms depending on context and experimental paradigm. In order to obtain a theoretically founded derivation of window width, we have recently proposed a decision-theoretic framework using a decision rule that is based on the prior probability of a common source, the likelihood of temporal disparities between the unimodal signals, and the payoff for making right or wrong decisions ([1]). Evoking the framework of the time-window-of-integration (TWIN) model ([2]), an explicit expression for optimal window width is obtained in the *focused attention* task where subjects are asked to respond to signals from a target modality only. Here we present several empirical predictions concerning the optimal time window that are derived from this decision framework. We discuss empirical studies designed to test these predictions in a number of different experimental paradigms and to probe for deviations from optimality. Moreover, we outline possible extensions of the TWIN model needed for paradigms with several response alternatives.

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Representations of interval orders

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Keywords : interval order, numerical representation, convex cone.

Assume a real, closed interval $[x_i, x'_i]$ is assigned to each alternative i from a finite set X, with $|X| = n$. A relation P on X is then defined by setting $i\,P\,j$ when $[x_i, x'_i]$ lies entirely before $[x_j, x'_j]$. Such a relation is an *interval order*. Equivalently, a relation P on X is an *interval order* if there exist some strictly positive real number ε and, for $i \in X$, real numbers x_i and x'_i, in such a way that

$$\left\{ \begin{array}{rcll} 0 & \leq & x_i, & \\ x_i & \leq & x'_i, & \\ x'_i + \varepsilon & \leq & x_j, & \text{when } i\,P\,j, \\ x'_i & \geq & x_j, & \text{when } i\,\overline{P}\,j. \end{array} \right. \tag{1}$$

By introducing ε above, we managed to keep only large inequalities in (1). As a consequence, for any choice of ε, the $2n$-tuples $(x_1, x'_1, x_2, x'_2, \ldots, x_n, x'_n)$ satisfying (1) form a closed, convex polyhedron $\mathcal{R}^P_\varepsilon$ in $\mathbb{R}^{2n}$. All such polyhedra $\mathcal{R}^P_\varepsilon$, for $\varepsilon \in \mathbb{R}^{+*}$, are homothetic; we call them collectively the *representation polyhedron* of the given interval order P.

The structure of the representation polyhedron $\mathcal{R}^P_\varepsilon$ is fairly well understood [2]. First, facets of $\mathcal{R}^P_\varepsilon$ are linked to 'noses' and 'hollows' of the interval order P. Second, $\mathcal{R}^P_\varepsilon$ happens to be a convex cone, with its unique vertex encoding the so-called 'minimum ε-representation' of P. Finally, the extreme rays of the cone $\mathcal{R}^P_\varepsilon$ are combinatorially characterized.

Our findings for interval orders are in strong contrast with the analogous ones for semiorders [1]. A *semiorder* on X is an interval order on X for which the interval length $x'_i - x_i$ can be made independent of i. In this case, the *representation polyhedron* $\mathcal{S}^P_\varepsilon$ consists of all $n+1$-tuples $(x_1, x_2, \ldots, x_n, r)$ in $\mathbb{R}^{n+1}$ that satisfy the inequalities

$$\left\{ \begin{array}{rcll} 0 & \leq & x_i, & \\ 0 & \leq & r, & \\ x_i + r + \varepsilon & \leq & x_j, & \text{when } i\,P\,j, \\ x_i + r & \geq & x_j, & \text{when } i\,\overline{P}\,j. \end{array} \right. \tag{2}$$

The structure of $\mathcal{S}^P_\varepsilon$ can be much involved, for instance there can be several vertices.

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Optimization-based Analysis and Training of Human Decision Making with a New Test-scenario

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Keywords: decision making, mixed-integer nonlinear programming, optimization, complex problem solving

Computer-based test scenarios have been used in complex problem solving for more than three decades. These scenarios have usually been defined on a trial-and-error basis, until certain characteristics became apparent. The more complex models become, the more likely it is that unforeseen and unwanted characteristics emerge in studies, e.g., the possibility to gain an infinite amount of money, [2].

To overcome this important problem, we propose to use mathematical optimization methodology on three levels: first, in the design stage of the complex problem scenario, second, as an analysis tool [2, 3], and third, to provide feedback in real time for learning purposes.

One of the most famous test scenarios in complex problem solving is the Tailorshop [1], developed by Dörner et al. in the 1980s. We present a novel test scenario, the *IWR Tailorshop*, with functional relations and model parameters that have been formulated based on optimization results. Furthermore we report on results that have been obtained by using optimal solutions as a basis for an objective and correct indicator function.

The implementation of the new model features a web-based interface and uses the widely spread AMPL interface, which allows, e.g., the use of a variety of powerful optimization algorithms.

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Learning Spaces in Real Life
How the large size of actual learning spaces guides the development of the theory

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Real life applications of learning space theory for assessing and teaching are rarely straightforward. The main reason is that the family of states may be on the order of billions. Maintaining and updating a probability distribution on such a large set is not feasible. Even the base of the family may be too large. A practical solution of this quandary is to perform a parallel search of the space.

This means:

- partitioning the domain of the spaces into a family of *n* subdomains;
- projecting the learning space on each subdomain, thereby producing *n* learning spaces, the bases of which are by several order of magnitude smaller than that of the original space;
- simultaneously assessing the subject on each subdomain, with concurrent updating, ultimately yielding *n* knowledge states;
- combining the *n* knowledge states into one final knowledge state.

The talk has three parts.

I. Recall of basic concepts of learning space theory.
   Learning spaces vs knowledge spaces.
II. Description of the parallel assessment.
   Construction the final knowledge state.
III. Exemplary application of the parallel procedure.

Sequential dependencies in perceptual decisions

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Keywords: sequential dependencies, perception, mixture model

In most psychological experiments, observers respond to multiple trials that are presented in a sequence. In perceptual psychology, it is common to assume that these responses are independent of responses on previous trials, as well as of stimuli presented on previous trials. There are, however, multiple reasons to question the ubiquitous assumption of “independent trials” – for example, responses in cognitive experiments depend on previous stimuli and responses, and it is unclear why perceptual tasks should be unaffected by such serial dependencies. This observation raises two central questions: First, how strong are trial by trial dependencies in psychophysical experiments? Second, what are statistical methods that would allow us to detect these dependencies, and to deal with them appropriately?

Here, we present a model that allows for quantification of such trial by trial dependencies and apply it to psychophysical data-sets from perceptual decision tasks. Using multiple data-sets from one auditory and two visual experiments as well as simulated data, we show that our model successfully detects trial by trial dependencies if they are present and allows for a statistical assessment of the significance of these dependencies. Although the strength and direction of trial by trial dependencies varied considerably between observers, significant trial by trial dependencies were observed in 6 out of 7 observers. For those observers, model fits improved considerably if trial by trial history was incorporated into the model. The trial by trial dependencies we observed could be well captured by linear superposition of effects form multiple previous responses and stimuli.

We conclude that previous trials and responses influence responses in perceptual tasks, too.

A model of indecision in 2AFC detection tasks

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Topic: mathematical models in psychology, models for sensation and perception, perception and psychophysics, probabilistic models in psychology

Keywords: 2AFC, detection, psychophysical function, psychometric function, interval bias, order effects

Trials in a temporal two-alternative forced-choice (2AFC) detection experiment consist of two sequential intervals one of which presents a stimulus of some magnitude (the “test” stimulus) whereas the other presents a blank (the “null” stimulus). The observer is asked to report in which interval the test stimulus was presented. The magnitude of the test varies from trial to trial, and any given test magnitude is typically presented more than once. Data gathered in this way are either used (i) to fit a psychometric function to proportion data at each test magnitude or (ii) to adaptively track the detection threshold (i.e., the stimulus magnitude at which percentage correct in the 2AFC task is estimated to reach some arbitrary level). Interval bias has been occasionally reported to occur in 2AFC detection tasks, as manifested by the empirical fact that proportion correct differs systematically when the test stimulus is presented first or second in the trial (this interval bias is also known as “order effects”). A similar effect in 2AFC discrimination tasks was reported and thoroughly documented by Fechner more than 150 years ago, an effect that he referred to as a “constant error.”

We present a re-analysis of published data from visual detection experiments whose results reveal strong order effects that vary in sign and magnitude in a systematic manner across observers. These order effects appear to have gone unnoticed in the original papers because data were pooled across presentation orders before they were analyzed. We also discuss alternative signal detection models and show that only two of them are consistent with the empirical characteristics of order effects as revealed by the preceding re-analysis of data. In one of these models (the so-called model with bias), order effects arise as a result of *decisional bias* in observers who are never undecided about which interval presented the test; in the other (the so-called model with indecision) order effects arise because observers are occasionally undecided and guess with some *response bias*. Because the two models differ only in characteristics that do not render identifiably different outcomes in conventional 2AFC data, we also re-analyze data from unforced-choice detection tasks that allow testing which of these two causes of order effects is more tenable. The results of these re-analyses support the model with indecision and refute the model with bias.

Mathematical consequences of the TBRS-model hypothesis

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The Time Based Resource Sharing (TBRS) model of working memory developed by Camos and Barouillet has not yet been expressed in a mathematical way. It is, however, based on strong hypothesis that may be mathematized. Here we will investigate some mathematical foundations, and consequences of TBRS' assumptions. These assumption impose a link between the decay and re-activation functions (evolution of the activation with time). Special cases of TBRS are studied (with exponential decay / exponential reactivation), and implications in terms of subspan and supraspan recall discussed.

Psychometrics of randomness.
A Kolmogorov complexity approach

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Since human randomness production have been studied and widely used to assess executive function (especially inhibition), many measures have been suggested. However, each of them focuses on one feature of randomness, leading authors to use multiple measures at a time. Here, we provide a new measure of randomness based on “Kolmogorov complexity for short sequences”. This measure does not rely on a specific property of sequences, but formalizes the idea that a sequence is random if a random computer program is unlikely to produce it. We also propose a re-analyze of the classical Radio Zenith data based on this measure.

A new method of investigation of person's confidence expressed while comparing musical fragments

E. Golovina, V. Nosulenko

Keywords: person's confidence in task of music fragments comparison, confidence mathematical measurement, "perceived quality" of an acoustic event, verbalizations' analysis.

The present investigation is an elaboration and approbation of a new method of person's confidence estimation in psychophysical task of music fragments' comparison. This method is based on the free verbalizations data analysis and is aimed at revealing confidence indexes, used to characterize some aspects of the "perceived quality" of an acoustic event.

We used the verbal data obtained by V. Nosulenko and I. Starikova (2009, 2010) in experimental study of comparison of sounds that differ in the way of encoding. In this experiment, 9 musical fragments recorded in two digital sound formats (WAVE and MP3) were presented to the auditors in the framework of a paired comparison procedure. Participants had to choose a preferred sound in a pair and to evaluate the difference between the two sound formats by means of the 8-point scale. In addition, participants verbally described the differences they perceived between the sounds. Confidence in the auditors' answers was indirectly estimated on the base of the corresponded verbalizations' data and compared with the data obtained by a traditional direct method of confidence analysis (questions like "Are you sure or no in your answer?").

Analyzing the auditors' verbalizations the following confidence components were revealed: number of requests to repeat the musical fragment; verbal units, describing confidence state (for example: "exactly, sure, of course, no matter" and so on) and unconfidence state (for example: "it seems to me, I doubt" and so on); number of answers' changes. Sum index of confidence was suggested:

$CON = 1- \sum\sum (Wj\ N\ i\ j\ /\ J)$,

N i j – i-construct frequency in estimation of j-pair of musical fragment, Wj – weight coefficients, determined empirically.

$j = 1, \ldots, J$; $i = 1, \ldots, I$. J – number of fragments pairs, I – number of confidence components.

Sum index of confidence is considered to be a formalization of verbalizations of musical fragments' perceived quality. Confidence mathematical measurement gives an opportunity to get its numeral meaning and then to correlate it with personal features meanings. It will allow making psychological portraits of people, preferring one or those music records' formats (for example, MP3 or WAVE).

The approach to measurement of dynamic characteristics of the person

G. Golovina, T. Savchenko

Keywords: method of dynamics modeling, dynamic psychodiagnostics, typical cycle, automatic models

Any state of a person is a result of mental process and method of dynamics modeling at macro level (style of behaviour level) on the basis of results of micro dynamic processes modeling is considered to be an adequate method of its studying. Thus, static states are considered to be a result of micro dynamic modeling and construction of limiting cycles. The described problem is possible to be solved investigating internal dynamics of mental states (in psychological time).

The empirical fact that a person could usually endure cycles of states (in everyday life they are simple cycles) lies in a heart of psychodynamic diagnostics. It means that if the anger is followed by fear there is a high probability that the fear will be followed by anger. Within time a cyclic change of two states becomes a typical psychodynamics, a typical cycle of a person. The dynamic psychodiagnostics is aimed at revealing a typical cycle of one or another states of a person.

Use of the dynamic approach in personal psychodiagnostics has allowed to realize a new way of construction the express methods (questionnaires). The difference of dynamic diagnostics from traditional one is in the fact that not only measured feature is taken in dynamics, but the process itself is taken in dynamics also.

Each question of a technique represents a description of one of four states and an offer to choose one variant of an exit from the given situation. There are three variants. Thus, there are six possible combinations from four presented states. Passing mentally from one condition in another, the person, at last, reverts to the original state; the cycle comes to the end. Last transition is called a limiting cycle. The psychological analysis of unstable balance between the first and last states allows to give the name of each dynamic type of the person.

Two techniques constructed by means of the given method are described.

In summary it is necessary to notice that the given approach help to develop modelling experiment for the description of dynamics of satisfaction a life on the basis of microdynamic process of cyclic transitions of a condition life satisfaction in a condition a life dissatisfaction. The automatic models wear applied to modelling of such dynamics.

Is Your 'Ratio' a Quotient? Experimenters Need to Know

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Keywords: mathematical modeling, substantive theory, experimentation.

Modeling equations make up an interface that improves the testability of substantive theories [2]. The numerical quantities that appear in equations map onto both abstract quantities of the theory and concrete variables to be measured and manipulated in the laboratory. Focusing on the numerical/physical mapping, the paper warns that fractional expressions, which abound in modeling equations, are ambiguous from the standpoint of experimenters [1].

Take the example of Fitts' law, a simple empirical rule of thumb of the form $T = f(W/D)$, where f denotes some strictly decreasing function linking average movement time T (the dependent measure) to the ratio of target width W and target distance D (which experimenters manipulate systematically). Depending on whether or not the division of W by D is assumed to have been done, the fractional expression or 'ratio' W/D exhibits one quantity or two quantities—either the quotient $Q_{W/D}$ of the division of W by D or its two operands W and D. Thus it is unclear whether one should view Fitts' law as a relation of the form $T = f(Q_{W/D})$, meaning that movement time depends on a single dimensionless variable (relative target tolerance), or of the form $T = f(W, D)$, meaning that movement time depends on two lengths (target width W and target distance D).

It seems that most Fitts' law students have tacitly taken the writing W/D to denote a quotient. However, they have apparently not realized that (*i*) given their option, scale is the other important independent variable of Fitts' aimed-movement paradigm, (*ii*) the alternative understanding is potentially useful too, and (*iii*) the two understandings are mutually exclusive.

I will argue that one can enquire either into the influences of relative target tolerance (the dimensionless quotient $Q_{W/D}$) and scale (of which D is a possible estimate) or, alternatively, into the influences of target width W and target distance D. But one cannot ask about the respective contributions of lengths W and D to the effect of W/D on movement time, as for example in [3], because no more than two variables can be singled out on the right-hand side of a Fitts' law equation.

The main purpose of this communication is to submit an analysis that has appeared in [1] to the discussion of mathematicians.

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Parameter estimation and identifiability in probabilistic knowledge structures

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Keywords: theory of knowledge structures, parameter estimation, local identifiability

Practical applications of the theory of knowledge structures often rely on a probabilistic version, known as the basic local independence model [1]. The presentation discusses various procedures for estimating its parameters, including maximum likelihood (ML) via the EM algorithm [2], minimum discrepancy (MD) estimation as well as MDML, a hybrid method combining the two approaches. For the first time it presents results on the identifiability of the parameters by characterizing classes of probabilistic knowledge structures that are not (locally) identifiable.

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Means

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Keywords: Means; functional equations; binary operations; aggregation; social choice

In this communication, we analyze different approaches to the mathematical concept of a *mean*. The possible alternative settings are related to a large set of branches of Mathematics as well as to multidisciplinary applications.

To fix our ideas, let us consider as starting point the arithmetic mean defined in the set of real numbers. Obviously, it can be interpreted under different mathematical points of view, namely: 1) as an algebraic binary *operation* defined on the additive group of reals, 2) as a *choice* of a number that is located between (in fact just in the middle, in the case of the arithmetic mean) two given real numbers, 3) as a *bivariate map* that satisfies several *axioms* as, e.g., symmetry or idempotence. 4) as a solution of a set of *functional equations* (e.g.: symmetry, idempotence, etc.) on real-valued functions on two real variables... It is also interesting to take into account that these points of view are closely related: To put an example, a bivariate real-valued map defined in the set of real numbers can obviously be interpreted as an algebraic binary operation, and conversely.

Having these ideas in mind, we may ask ourselves which could be the best definition of the concept of a mean (or, perhaps better, of a *generalized mean*) on an abstract set X. Needless to say that we will consider means defined for any finite number n of elements (i.e.: n-variate means), not necessarily two as in the aforementioned example.

The search for a suitable definition of a mean is indeed an old question, on which we want to analyze new trends and possibilities, coming from Pure Mathematics as well as from a miscellaneous wide set of interdisciplinary applications (e.g.: aggregation of individual preferences into a social one, in Mathematical Social Choice). Several examples of contexts where some kind of a mean plays a crucial role are: 1) topological spaces where a continuous topological mean can be defined, 2) divisible groups, where an algebraic mean makes sense, 3) totally ordered sets, 4) aggregation operators arising in fuzzy set theory (e.g.: triangular norms and conorms), 5) some particular spaces of continuous functions, 6) sets of profiles of individual preferences, arising in Social Choice, 7) extensions of orderings from a set to its power set following different criteria...

Through the communication, we shall consider a variety of examples of these kinds, *trying to establish a common theory* that relates the different mathematical properties that a mean can have.

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Self-Control and Type Indeterminacy

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The last decade has seen the rapid development of a literature that extends the use of the mathematical formalism of quantum mechanics to psychology and social sciences. The objective is to capture contextual features of individual preferences and explain a variety of so-called behavioural anomalies. The basic idea is that behaviour presumably generated by preferences (and beliefs) exhibits non-classical features akin those encountered in subatomic physics. Of particular interest are instances of non-commutativity in revealed preferences. The Type Indeterminacy (TI)-model of decision-making has been developed to explain those phenomena relying on two main ideas: 1. choice-making is similar to a measurement of an individual preferences, and 2. preferences are psychological objects that are characterized by an irreducible, intrinsic uncertainty. Together they imply that choice-making modify preferences so the individual characteristics of an agent are not given once for all but change with the experiences (choices) she makes.

In this paper we study the implications of quantum type indeterminacy in a dynamic decision problem i.e., for optimal decision-making over a series of consecutive decisions. We assume that the agent is aware that her decision today affects the preferences that will be relevant for her decisions tomorrow. We show that dynamic optimization in a TI-model translates into a game with multiple selves and provides a suitable framework to address issues of self-control. The TI-model delivers a theory of self-management in terms of decentralized Bayes-Nash equilibrium among the potential eigentypes (selves). In a numerical example we show how the predictions of the TI-model differ from that of a classical model. We argue that the predictions of the TI-model provide a description of human behaviour that makes a lot of sense. In particular, what may be perceived as a feature of dynamic inconsistency, may instead reflect rational optimization by a type indeterminate agent.

Recalling the list-before-last: A cautionary tale

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Keywords: free recall, memory, retrieval.

An unresolved question in memory research is: What cues free recall? An experiment by Shiffrin (1970), in which participants were instructed to recall the list-before-last, has been cited as evidence that retrieval in free recall experiments is (somehow) cued. The alternative is that retrieval is spontaneous and the words recalled are selected only in retrospect, after they have been retrieved. This talk illustrates the difference between these two hypotheses and then compares Shiffrin's published data with similar data from Murdock and Okada (1970), and with two other experiments, to argue that the rate of recall from the list-before-last is no greater than one should expect from a spontaneous retrieval that (potentially) addresses all preceding lists. The number of intrusions, when recall is requested from the last list, corrected by an estimate of the proportion of unwanted retrievals that are suppressed, is sufficient to account for the number of words recalled from the list-before-last.

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Cross Modal Predictions for Binary and Unary Intensity Modalities.

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Keywords: binary and unary intensity modalities, cross modal matching, *p*-additive representations

A substantial empirical literature on cross modal matching of subjective intensities exists, but there is very little by way of theory. For attributes with two relatively independent sense organs, such as ears, eyes, arms, etc., called binary modalities, [2] formulated behavioral assumptions leading to numerical representations. Empirical work of R. Steingrimsson & Luce for loudness and that of Steingrimsson for visual brightness has provided support for the theory. Moreover, [3] established that the strong empirical evidence for bisymmetry and against commutativity in both domains forces a simple additive representation over the two sense organs which also have also been shown to be power functions.

However, many intensity modalities, such as vibration, odor, money, etc., seem to be unary rather that binary. A theory of extensive measurement [1] seems suited to the unary cases. In addition to the additive representation, as in the binary theory, two others arise when the representation is into the additive and multiplicative real numbers. These have gone unnoticed for over 100 years. Unlike the binary case, no behavioral properties seem to rule them out. A criterion is given for deciding which of the 3 a respondent satisfies. This complexity must be taken into account when considering cross-model matches [4]. Quite a few predictions result: for the binary/binary cases (1), for the unary/unary cases (6), and for the binary/unary cases (3). Thus, an extensive experimental effort needs to be undertaken: Testing whether the unary theory is sustained as the binary one has been, and, if so, whether or not the complex of cross-modal predictions seems to be correct. With support from the Air Force Office of Scientific Research to the U.C. Irvine, my collaborator Steingrimsson and I plan to make serious inroads on this empirical work during the coming 18 months.

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Direct and Indirect Elicitation Rules for Subjective Beliefs: Experimental Evidence

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Keywords: metacognition, signal detection theory, perception and psychophysics, scoring rules.

In this communication, we describe a *Signal Detection Theory* model to predict the level of subjective beliefs of subjects facing a Type 2 task [4], [2]. In an experimental setting we measure the ability of individuals in the metacognitive process according to their level of ROC area. We use three different elicitation rules [3] to reveal their level of confidence in their own decision facing a perceptual task and a cognitive one. The scoring rules are different according to their process of revelation (direct by choice of probabilities or indirect by choice of stakes) and their incentives (monetary rewards or free elicitation). The model of Signal Detection fits well with our data and allows to predict the behaviors of subjects. Our results show that direct rules perform better in terms of discrimination than indirect ones. Furthermore we obtain inter-task correlations only for direct rules. Our data support the evidence of a high inter-individual heterogeneity in metacognitive performance [1].

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A Multi-Threshold Model of Signal Detection

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Keywords: Models for Sensation and Perception, Signal Detection.

Multi-threshold theory assumes several or many detectors to operate independently by high-threshold theory with guessing probability reinterpreted as probability of spontaneous activation. Some criterion number or more of them must be activated for a signal report to occur. Due to the flexibility of its binomial signal and noise distributions, it generalizes most existing detection models or mimics them for special parameter values. It provides a theoretical foundation for an observed empirical relationship among signal and noise distribution means and standard deviations. A modification assuming sequential activation of detectors mimics exponential distribution detection models as well as a random walk model and, after replacing criterion number by first activation of a detector under time pressure, supports Rasch-separability of simple reaction time distributions…

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A Model of Interpretation of Concepts over a Taxonomy

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Our method for interpretation of concepts involves a pre-specified ontology represented by a hierarchy of subjects (taxonomy) and includes the following stages: (a) representation of a the query concept as a fuzzy leaf set of the taxonomy, and (b) mapping the set's topics to higher ranks of the taxonomy tree. The former method involves matching the query concept to unstructured texts found by a search engine. The latter method involves a penalty function summing penalties for the chosen "head subjects" together with penalties for emerging "gaps" and "offshoots". The method finds a mapping minimizing the penalty function in recursive steps involving two different scenarios, that of 'gaining a head subject' and that of 'not gaining a head subject'. The method is illustrated by applying it to illustrative and real-world data.

Default Bayes factors for inference in general ANOVA designs

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Keywords: Bayes factor, statistical inference, ANOVA.

Bayes factor has long been advocated as superior to *p* values for assessing statistical evidence from data (Edwards, Lindman, & Savage, 1963; Wagenmakers, 2007). Despite the advantages of the former and the drawbacks of the later, inference by *p* values is still nearly ubiquitous, and inference by Bayes factor is rare. One impediment to adoption of Bayes factor is a lack of practical development, especially a lack of ready-to-use formulas and algorithms. Building on the previous work of Liang et al. (2008) and Rouder et al. (2009), we discuss a set of default Bayes factor tests for common ANOVA and regression designs. These tests are based on multivariate generalizations of Cauchy priors on standardized effects, and have the desirable properties of being invariant to the location and scale of measurement units, and of leading to consistent inference in the large-sample limit. Moreover, these Bayes factors are computationally convenient, with straightforward sampling algorithms. We cover models with fixed, random, and mixed effects, including random interactions, for within-subject, between-subject, and mixed designs. We also generalize the discussion to regression models; that is, those with continuous covariates. Our development makes the computation of Bayes factors straightforward for the vast majority of designs in experimental psychology.

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Attitude and its measurement using subjective reference points

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Keywords: attitude measurement, subjective evaluation, reference point, functional equation

It is widely accepted, that attitude is best to define as an overall evaluation of an object, and recently it has also become accepted that evaluations are comparative in nature, and relative to a norm or standard (reference point). We present an axiomatic characterization of value judgments relative to a reference point. The derived models present closed formulas for the reference comparison function, and include the reference point. By these functional forms of the possible reference comparison functions, the role of the reference point in the relative judgment is characterized. Summary, a new model of subjective evaluations (which includes reference points), and technique for measuring attitude will be presented (its applicability will be illustrated by survey examples).

A variational approach to sensation and psychophysics

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Keywords: variational calculus, psychophysical law, neuroelectrical law.

Variational methods play a fundamental and unifying role in several fields of physics, chemistry, engineering, economics and biology, since they allow one to derive the behavior of a system as a consequence of an optimality principle [1]. An important example of their application to psychological studies concerns the description of motor control [2].

We suggest a possible application of these methods to a theoretical model of sensation that leads one to consider a psychophysical law as the solution of an Euler-Lagrange equation and to connect the Hamiltonian, namely the energy of the process, with neurophysiological features. As an example of the suggested methodology, a free particle Lagrangian, with a varying mass depending both on time and stimulus intensity, is used as a sufficient condition to derive the fundamental psychophysical laws while accounting for time-varying features [3] and the measurability of prothetic continua on interval scales [4]. In addition, the choice of a Hamiltonian based on the Naka-Rushton model [5] allows one to define a possible relation between a behavioral law and a neuroelectrical response of primary afferent units. In particular, the adaptation phenomenon appears to be based on the minimization of the total number of action potentials. Preliminary results are investigated on data obtained on the sense of touch [6].

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Spatial prisoner´s dilemma and laws of imitation in Social Psychology

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Keywords: game theory, spatial prisoner´s dilemma, imitation, Tarde, differential association, Sutherland.

In this communication it is designed a game based on the spatial prisoner ([1]) introducing the three laws of imitation defined by Tarde ([2]). The French author described (1) the law of close contact (individuals in close intimate contact with one another imitate each other´s behavior), (2) the law of imitation of superiors by inferiors (people follow the model of high status in hopes their behaviour will procure the rewards associated with the "superior" class) and (3) the law of insertion (new behaviors reinforce or discourage previous customs). Our computational simulation presents the following characteristics: (a) a lattice of nxn cells, occupied by an N initial population of 1,000 individuals (n and N are adjustable parameters). The payoffs for the prisoner´s dilemma matrix are the values R=1, T=b(b>1), S=P=0; (b) 1.2<b<2.5. The initial distribution of cooperators takes their values from the set {0.1, 0.3, 0.5, 0.7, 0.9}; (c) the neighborhood of each individual is composed of the 8 adjacent cells plus the individual´s own cell; (d) the law of close contact (LCC) is defined like a Conf(ormist) rule: if your behavior is different from that of the neighboring agent, copy its behavior; the law of imitation of superiors (LIS) is defined like a Maxi(mization) rule: if the neighbor agent gets higher payoffs, it copies its behavior and the law of insertion (LOI) is defined like a Fashion rule (copy the behaviour with the highest frequency of appearance in your neighborhood-in case of equal frequency, copy at random-) or a Snob rule (copy the behavior with a lower frequency appearance in your neighborhood-copy at random in case of equal frequency-); (e) the agents have no memory. We have conducted a comprehensive tour of all the possibilities (see Fig. 1, for instance), combining all possible values of b between 1.2 and 2.5, with an initial distribution of cooperators (c) between 0.1 to 0.9. We have also been changing the population of agents and we have increased the number of rounds to 100, yielding results qualitatively and quantitatively very similar: the formation of little "clusters" of cooperators supporting Tarde´s second law of imitation and also the interpretation of Sutherland based on the idea of "differential association" which explains the imitation of deviance or criminal behavior as a process of communication within intimate personal groups ([3], [4]).

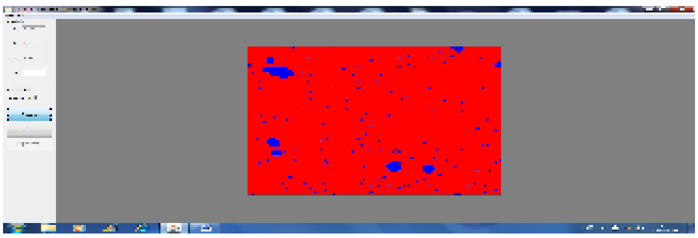

Fig. 1. Computational simulation for b=1.2 and c= 0.5 ("clusters" of cooperators in blue color).

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The application of latent Markov models in category learning

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Keywords: Latent Markov Models, Category Learning, Automaticity

Learning ill-defined categories ([1]) involves multiple categorization learning systems, which gives rise to individual differences in generalization performance that are of categorical nature. Statistical modeling of category-learning data should take into account this source of *inter-individual* variation. Johansen and Palmeri [2] proposed that categorical differences in learned representations may also occur as a function of experience. Hence, they also expected *intra-individual* differences in generalization patterns during the course of learning. In addition, a third source of variation is expected that is due to *erroneous* generalization of the learned category structure.

Latent Markov analysis (LMA) of multivariate nominal time series can account for these three sources of variation ([3]). We modeled the *inter-individual* variation by defining a mixture of latent categorization strategies at each time point. *Inter-individual* differences were also modeled as different trajectories of shifts between strategies during the course of learning. A transition matrix that describes the conditional probability of transitions between categorization strategies models also the *intra-individual* variation. The third source of variation, due to making *errors* (i.e., inconsistencies) in generalization, is modeled by the parameters that describe the probability of a response given a categorization strategy. All three sources of variance are modeled in parallel, which results in a robust statistical description of the categorization data.

The presentation will show how the application of latent Markov models allows for analyzing multiple latent categorization strategies separately in a statistically robust way, without using a probe procedure. The analysis of accuracy and reaction time data from a study by Johansen and Palmeri [2] provides detailed information about rule- and exemplar-based categorizations and the interaction between learning systems during the course of learning. The interaction between the learning systems appears to be at the output level. A shift towards exemplar-based categorization has a benefit in consistency (for the transfer items) and accuracy (for the training items), but a general cost in speed, which vanished after 32 learning blocks. As predicted by automaticity theories [4], exemplar-based categorization follows a power law of learning and the speed of responding to individual items depends on the number of presentations.

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The Power Model of Fitts' Law Does Not Encompass the Logarithmic Model

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Keywords : mathematical models in psychology, functional equations, simple aimed movement.

Fitts' law is a well-known empirical rule of thumb which predicts the average time T it takes people, under time pressure, to reach with some pointer a target of width W located at distance D. Within the classic experimental paradigm settled by Fitts [1], the law is a relation of the form $T = f(D/W)$, where f stands for some strictly increasing function. Two formulations are well-known:

$$T = a + b\cdot\log_2\left(2\frac{D}{W}\right) \qquad \text{(Fitts [1])} \qquad (1)$$

$$T = a + b\cdot\left(\frac{D}{W}\right)^{1/n} \qquad \text{(Meyer } et\ al. \text{ [2, 4])} \qquad (2)$$

Whether Fitts' law is a logarithmic (1) or a power law (2) has remained unclear so far. The curves look similar over the rather narrow range of D/W that can be actually investigated in the laboratory.

In two widely cited papers [2, 4], Meyer *et al.* have suggested there is no real log vs. power issue about Fitts' law. Arguing that $a + b\cdot(D/W)^{1/n} \to a' + b'\cdot\ln(D/W)$ as the maximum number of submovements $n \to +\infty$, they claimed that the power model of Fitts' law they derived from their substantive theory—the celebrated stochastic optimized submovement theory—encompasses the logarithmic model as a limiting case.

We review the submovement theory [2, 3]: Consider the recursive functional equation predicted by the theory after n submovements, assuming uniformly distributed endpoints:

$$f_n\left(\frac{D}{W}\right) = \min_s\left\{\frac{D/W - 1/2}{s} + \frac{2}{s}\int_{1/2}^{s/2} f_{n-1}(x)\,\mathrm{d}x\right\} \qquad (n > 1) \qquad (3)$$

We derive an easy proof that the solution $T = f_n(D/W)$ is given by the positive root T of the nth order equation

$$2\frac{D}{W} = 1 + 2T + \frac{(2T)^2}{2} + \cdots + \frac{(2T)^n}{n!}. \qquad (4)$$

The resulting model does indeed tend to the logarithmic $T = \ln(2D/W)/2$ as $n \to +\infty$, while for $n = 2$ we do recover the square-root model derived by Meyer *et al.* in [2]. However, our analysis makes it clear why the solution cannot be, even to a rough approximation as n grows large, identified with a power law of the form (2).

Even if one takes (2) for granted, we demonstrate that Meyer *et al.*'s claim is false: there do not even exist sequences a_n, b_n such that the model $a_n + b_n(D/W)^{1/n}$ tends to a logarithmic model as $n \to +\infty$, as was suggested in [4, Fig. 6.13].

Meyer *et al.* [2, 4] have convinced the community of Fitts' law students that their submovement theory leads to a power model that encompasses the logarithmic models. But it appears that (i) their theory does not lead to a genuine power model, and (ii) their supposedly power model does not encompass the logarithmic one. At any rate, awareness that in fact the two classes of candidate mathematical descriptions of Fitts' law are not equivalent should stimulate experimental research.

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Modeling the halo effect by using automata networks

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Keywords: Social Sciences, human behaviour representation techniques, discrete automata networks, technological changes, attitudes forming, attitudes transformation

We will try to show how automata network models can conceptualize and simulate the halo effect in human attitudes faced with a new “offer” which can be weight up according to different formal attributes. From a first presentation of the classical representations of the attitude concept using a cognitivist inspiration, we will propose a connectionist approach based on automata networks. Then, we will show how this kind of models can represent in term of conceptual and formal unification - by integration in the same movement based on the *Gestalt* and cognitive perspectives, the human attitudes forming process and the halo effect. We based our connexionnist model on the Beckwith & Lehman multiattributes theory and apply it to consumer food product choice.

Working memory capacity as a quantity of information

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Keywords: working memory, information theory, computational models.

Assessing the capacity of human working memory (WM) has been a constant concern for researchers since the seminal paper from Miller [1] which estimated that capacity to be about 7 chunks. The dominant trend is still that all chunks take the same place in WM.

However, recent papers question this view and suggest that compression mechanisms may occur in WM [2,3]. This paper therefore study the idea that the capacity of WM is not a fixed amount of chunks but rather a fixed quantity of information, in the information theory sense.

This idea is in line with the fact that a principle of simplicity guide cognition [4]: our WM would store the simplest representation, that is the shortest one in a information coding paradigm. We discuss that issue in terms of Kolmogorov complexity but use a more tractable technique to compute the shortest representation, namely the minimal description length two-part coding [5]. A representation of a sequence of data is a set of rewriting rules (chunks) plus the data rewritten given the chunks. The best representation is the one that minimizes the total codelength, computed using the Shannon-Fano coding. Given two ways of representing some data, for instance, (∅; abc,abcde,cde,abcd) and (X=abc; X,Xde,cde,Xd), the one that is assumed to be chosen is the one having the shortest total codelength. We also present a generative method for exploring the different representations and creating chunks.

Our basic experiment is the following: 64 sequences of items are presented to the participant who should recall as much items as possible after each presentation. To study compression in WM, sequences contain a specific pair of items (a chunk). There are 3 different chunks appearing in the sequences according to 3 frequency values (½, ¼ and 1/8). Behavioural results (30 human participants) reveal that mean recall performance are significantly better on sequences containing high-frequency chunks than on sequences containing low-frequency chunks.

Crucially, we show that a model whose capacity is based on a fixed number of chunks converge to a situation with identical performance whatever the chunk presented in the sequence: the chunk with 50% frequency is learned earlier, but when all 3 chunks have been learned, there is no difference at all. However, a model based on a fixed quantity of information always makes a difference between the 3 sequences: those containing the most frequent chunk are better recalled. Indeed, given the high frequency of the chunk, their codelengths are shorter. WM capacity could then be a quantity of information.

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The use of content-analysis for reconstruction existential categories

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The issues of choosing and making decisions, realising one's own mortality often appear to be the reasons for deep suffering (Maddi, 2006). There is often a conflict that underlies these feelings, which is caused by misunderstanding or refusing to accept such essential aspects of our life like sense, death, freedom and existential isolation (***Seligman*****,** Csikszentmihalyi, 2000). These very factors are the basic existential categories. According to the existential approach a person's whole existence is predetermined by 4 characteristics: self-distance, self-transcendence, freedom and responsibility. Having the basic existential categories and being able to operate with them determines the existential fullness of a personality, which in our opinion is a significant characteristic of a psychologist (Längle, 1995). The aim of this work is studying the attitude to the basic aspects of human life of the Psychological Faculty graduates and analysing the phenomenon of existential fullness.

In this work we applied the method of the so-called "cascade" content-analysis. A group of experts had to single out semantic constructs in the essays (Stubbs, 1983). At the next step we analysed their frequency in the texts. Finally we used other methods like the "Existential scale" survey, the Meaning-of-Life Orientations Test, locus of control, the content-analysis of the essays.

The semantic content-analysis let us single out 4 main motives for studying psychology as a second higher education. Those are: acquiring extra knowledge to improve the quality of existing professional activity, changing one's profession, using psychological knowledge to improve one's personal life as well as being able to help relatives, and the altruistic motive – to be able to help and support people who need it, to bring harmony to the world.

Primary analysis of the creative works let us single out 10 semantic constructs for each of the categories of death, sense and freedom, and four constructs for isolation. Some of the constructs were similar enough to be united together and eventually there were 6 constructs for the category of **death**: avoiding, accepting, negative feelings, close people, higher mind, the accelerator of life; 5 constructs for **freedom**: freedom, sacrifice, responsibility, acceptance of desires, experience: 5 constructs for **sense**: super-sense, me and close people, life, altruism void of sense: and finally, 4 constructs for the category of **isolation**: loneliness, solitude, existential isolation, existential seclusion.

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Visual and Cognitive Perceptions to Improve Flood Risk Map

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Keywords: flood risk map, visual and cognitive perceptions, eye tracking, static and dynamic analyses, preferences graphs.

Designing a real map involves knowledge of the rules of graphic semiology [**1**] (visual variables of shape, size, colour, etc.). Yet when making a cartographic document, authors do not usually take the users into account (their culture, how cartographically literate they are): the communication pattern is therefore <u>top down</u> (from the specialist or sender to the user or sendee of the message).

In order to increase public involvement, we suggest that the realisation process in map-making be reversed going from the reader to the specialist then back to the reader (<u>loop</u> system) using the method known as *Experimental Graphic Semiology* (**S**émiologie **G**raphique **E**xpérimentale **-SGE**). This means that features of the users' visual perception are considered first [**2**].

To achieve this goal, the SGE approach [**3**] studies the eye movements (fixations, saccades [**4**]) recorded from approximately 40 test subjects (German, Austrian, English and French [**5**]) when faced with different European flood risk maps (*Flood Directive* 2007).

The SGE approach is complemented with a <u>cognitive survey</u> that defines the users' needs (local politicians, general public, etc.) regarding flood risk management and the different subjects' semiological preferences (preferences graphs).

Different analytical methods are implemented: **statistical analysis** of the eye movements (number, direction, amplitude, multiple correlations ACP); **static** (or spatial) and **dynamic analyses** to study in which order the different components (title, key…) are visually accessed while the map is being viewed.

The ultimate goal is to put forward specific model(s) for mapping flood risk on the basis of a pluridisciplinary approach that unites planning issues (flood management), medecine ("eye-tracking"), statistical data and cognitive perception.

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Memory-Retrieval vs Decision-Making in Repetition Priming

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Keywords: decision-making, memory, mathematical models, statistical methods.

As described by Meyer and Schvaneveldt [1], repetition priming (RP) has two properties: (a) *Improved performance*, as seen by shorter Reaction Time (RT) or higher accuracy on repeated exposure to words or images; and (b) *Lack of awareness* of the earlier exposure. Following initial research on word recognition, RP has been extended to a variety of domains, ranging from reading and learning to perceptual identification and face recognition [2]. Dependent variables used to study RP include RT, percent correct, and fMRI [2].

Although there is much research of *when* RP effects are observed, there is less understanding of *why* RP occurs. Most accounts focus on *implicit memory* and *spreading activation* of memory traces. In contrast, the purpose here is to evaluate relative contributions of memory-retrieval and decision-making to RP. This research uses a question-answering task suggested by Anderson [3] and initially studied by Shanteau and McClelland [4]. Question sets were devised to all have the same answers. This allows for estimation of memory/decision times.

A 4-component model was proposed consisting of Read-In (I), Memory-Retrieval (R), Decision- Making (D), and Response-Output (O) times. Thus, if the first word is an answer, the total time to respond is: $RT_{q1} = I_q + (R_1 + D_q) + O_1$. If the first two words are answers, then cumulative RT can be described as: $Cum\ RT_{q2} = I_q + (R_1 + D_q) + O_1 + (R_2 + D_q) + O_2$.

There are two assumptions: (1) *S* only needs to read the question once, i.e., just one I_q value; (2) Retrieval time, R_i and question difficult, D_q are independent. This implies *Cum RTs* for different questions = serial position x *f* (decision difficulty). To analyze cumulative RTs, Functional Measurement (developed in [5] to analyze rating data) was extended to RTs in [6].

In Study 1, 16 *Ss* memorized a list of words to perfect recall. They then answered sets of questions with identical answers; randomization was used throughout. The results were recorded to .01 sec accuracy. Using program FM#1 [7], tests were conducted on interaction trend components. Both graphical and statistical analyses revealed support for the model. Estimation of parameters showed larger differences in decision times than in retrieval times.

Study 2 examined RP with questions repeated at the end of the session. Results showed a 15% reduction in RTs due to RP, with decreases in both memory retrieval and decision making times. Study 3 extended these results by having Ss do a visual search of the list, so that there was no memory retrieval. Comparable results were obtained, thus highlighting the role of decision-making above and beyond memory-retrieval in RP.

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Skill map based knowledge structures: some considerations about their identifiability

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Keywords: Identifiability, Knowledge Structures, Skill Maps.

The high values of the error rates, i.e. careless error (α) and lucky guess (β), resulting from the fitting procedure of a skill multi map based knowledge structure ([1], [2]) to the data might indicate some misspecification of the model. When fitting these structures the standard goodness of fit statistics could provide only a partial description of the actual fit of the model to the data [3]. Furthermore, it seems critical, in this situation, to interpret the estimated α and β as error parameters. A more convincing way to look at those estimates is considering them as a clear symptom of some problems in the built theoretical model.

A reasonable solution to cope with this challenge is to introduce some modifications in the skill map. In this paper we show how some possible modifications to the skill map may lead the knowledge structure to be not identifiable. More specifically, we refer to particular modifications such as adding skills (either included or not in the initial set of skills), or adding competencies for an item *q*. We demonstrate how these changes in the skill map lead the derived knowledge structure to be *backward-graded* or *forward-graded* respectively, and how these two particular kinds of structures are not identifiable. We show how in a backward-graded knowledge structure the α parameter can be brought down to zero while preserving the initial fit of the model. The same procedure can be applied to the β parameter of a forward-graded knowledge structure. Of course, by using this solution the opportunity to measure the true value of the error parameters when they are not zero is totally abandoned. Solutions both allowing the modification of the skill map and avoiding the renounce of the measurement of the actual error rates are explored.

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When the correspondence between probabilistic and set representations of local independence becomes a requirement: constant odds models for probabilistic knowledge structures

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Keywords: probabilistic knowledge structures; local independence; set representations of independence

Knowledge structures are set representations of the dependence among items belonging to a certain domain of knowledge. The formal definition of a knowledge structure [1] is given in terms of a pair $(\mathcal{Q}, \mathcal{K})$, where $\mathcal{Q}$ is a set of items and $\mathcal{K}$ is a collection of subsets of $\mathcal{Q}$, containing at least the empty set and the full set $\mathcal{Q}$. Each subset in this collection is called a *knowledge state* and represents the specific subset of items that an individual is capable of solving.

A knowledge structure is a discrete deterministic model whose empirical validation requires the specification of some suitable probabilistic representation. A rather general formulation of this probabilistic representation is the so called *probabilistic knowledge structure* [2, 3], a triple $(\mathcal{Q}, \mathcal{K}, \pi)$, where $\pi : \mathcal{K} \to [0, 1]$ is a probability distribution on the knowledge states.

Set representations of global independence of the items in a knowledge structure have been investigated by [4]. Such representations are here extended to describe independence among items that arises locally, in certain parts of the global structure, and conditionally to specific outcomes of a partial knowledge assessment.

On a parallel direction, the probabilistic framework of (globally) independent substructures introduced by [3] is extended to represent local independence among items in a probabilistic knowledge structure.

Assume that a correspondence between the set representation and the probabilistic representation of local independence does matter. Then a probabilistic knowledge structure does not respect this type of correspondence in general.

We study the implications on the form of the probability distribution π, of the introduction of a *requirement of correspondence* between the probabilistic and the set representations of local independence. We show that somewhat restrictive results are obtained for the probability distribution π when the knowledge structure is well graded. In particular, for this class of knowledge structures, the odds between the probabilities of pairs of knowledge states that differ by exactly one item only depends on the item, and is constant across such pairs.

Some empirical applications are presented where such restrictive model is compared to the unrestricted basic local independence model (BLIM) by Doignon and Falmagne (1988). In particular, cases where the more restrictive model outperforms the BLIM, and cases where only the BLIM displays an adequate fit to the data are examined and discussed.

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The knowledge space resulting from combining various skill orders

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Keywords: partial orders, operations with partial orders, knowledge spaces, set representations.

In this paper knowledge spaces are regarded as a partial order of skills and a set representation of this partial order where the range of such a representing function is the power set of a set of items, problems, questions etc. In this view the (set representation of the) skill order corresponds to the basis of the knowledge space. Given this approach several methods of combining (small) knowledge spaces into a big one can be constructed. They correspond to four operations which produce new orders from (two or more) given orders (addition, dominated addition, product order, and lexicographical order on the Cartesian product). These are applied to skill orders and the properties of the ensuing big knowledge space are investigated.

For the above mentioned operations the intersection of the sets on which the orders are defined are usually assumed disjoint. This assumption may be too strong for some of the intended applications. (For example, the skills for reading and for writing may partially overlap). We investigate how this requirement can be incorporated in the present theory.

Theory testing with the prior predictive

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Keywords: model selection, cognition, mathematical models in psychology, probabilistic models in psychology, psychological process models.

Many existing model selection methods do not consider whether a good fit is meaningful and are insensitive to the prior. In this communication, I propose the prior predictive test, which is based on the prior predictive distribution. Upon observing data, three situations can occur. First, if the observed data are not among the central predictions of the model, the model is invalidated. Second, if the observed data are among the central predictions of the model, and a good fit is meaningful, the model is supported. Third, if the observed data are among the central predictions of the model, and a good fit is not meaningful, the model is neither supported nor invalidated. A good fit is considered meaningful and impressive if plausible outcomes exist that are not among the central predictions of the model. The prior predictive test is sensitive to the prior and takes the plausibility of data into account. An application example focusing on category learning demonstrates the potential of the prior predictive test for testing psychological models.

How mathematical psychologists could help test-score psychologists to perceive the empirical reality they have built?

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Keywords: nominal confusion, partial orders, ordinal forcing, test scores.

Test scores x used by empirical psychologists are aggregate functions of multivariate qualitative data, usually mere sums of numerical labels. We analyse such functions as a function composition of a descriptive function f and a metaphorical function g. Let $\Omega_n = \{\omega_i;\ i = 1, ..., n\}$ the set of objects to be described, $M^m = \prod_{i=1}^{m} M_i$, $|M_i| = k_i$, the set of the possible m-tuples, and (x) the series of the possible numbers. Thus,

$$f : \Omega \to M^m$$
$$\omega \to \mathbf{x} = f(\omega) \quad (1)$$

and

$$g : M^m \to (x)$$
$$\mathbf{x} \to x = g(\mathbf{x}). \quad (2)$$

Empirical confusion results from the fact that while (x) is totally ordered by the usual relations $\leq$ and $\geq$, M^m is partially ordered by the direct product order relations $\prec$ and $\succ$ [1]. Thus, a statement of type $g \circ f(\omega_1) \leq g \circ f(\omega_2)$ may have no meaning with respect to how $f(\omega_1)$ and $f(\omega_2)$ can be related.

The present communication details this framework, identifies two kinds of empirico-logical confusions, namely nominal confusion and ordinal forcing, and introduces the issue of how mathematical psychologists could help test-score psychologists become aware of the need to take into account the logical consequences of their observational techniques.

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The random-walk-on-tree model for decisions between multiple alternatives

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Keywords: multi-alternative preferential choice, information sampling, random walk on trees, Decision Field Theory, Leaky Competing Accumulator Model.

In order to simulate reaction times and choice probabilities in multi-alternative preferential choice between more than three alternatives, we define a random walk on a tree (i.e. a cycle-free graph). Transition probabilities depend on the evaluation of the alternatives on multiple attributes, as in Diederich (1997). Leakage, inhibition and noise constrain the information sampling process. Inhibition can be implemented either locally as in Decision Field Theory (Roe, Busemeyer and Townsend, 2001) or globally as in the Leaky Competing Accumulator Model (Usher and McClelland, 2004). For choices between three alternatives, the model can simulate similarity, compromise and attraction effects. Simulation of choices with optional or fixed stopping times between four or more alternatives is also possible. Furthermore, numerical estimation of expected reaction times (in the optional stopping time case) and choice probabilities is computationally feasible.

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Quantitative Testing of Decision Theories: Probabilistic Specification and Empirical Results from a Frequentist and Bayesian Framework

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We provide a new quantitative framework for testing algebraic theories of pairwise preference on binary choice data. This framework bridges the conceptual, mathematical, and statistical gap between algebraic decision theory in the deterministic realm and highly variable empirical data that originate from sampling processes in the laboratory.

Previous research has shown that specifying an appropriate probabilistic structure is a crucial step in testing algebraic decision theories against real data. Here, we discuss a general methodology that accommodates “distance-based,” “aggregation-based,” and “mixture-based” probabilistic specifications. All of these probabilistic specifications require state-of-the-art order-constrained inference.

We formally implement these specifications, in both a Frequentist and Bayesian framework, for leading theories of decision making, including Expected Utility theory, Cumulative Prospect theory and the Transfer-of-Attention-Exchange model. The analysis uses a particular functional form for each theory. Using new lab data, our method avoids aggregation paradoxes by analyzing within respondent only. These tests are extraordinarily powerful. We also discuss differences and commonalities between the frequentist and Bayesian analyses. We reinforce other authors’ warning that probabilistic specification is not an add on to algebraic theory.

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