



Rogue waves and extreme events in mid infrared quantum cascade lasers under external optical feedback

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Abstract— Quantum cascade lasers (QCLs) are optical sources exploiting radiative intersubband transitions within the conduction band of semiconductor heterostructures, which further guide the electron flow by tunneling and scattering to establish electronic inversion for a pair of quantum levels at a given electric field [1]. In order to increase the total gain, a module of several layers including the laser levels is repeated several times (Fig. 1 (left)), so that the electrons traverse the total structure like water in a cascade. Emission range of QCLs typically extends from the mid-infrared to the terahertz region hence making them candidates of choice for a wide range of applications such as free-space laser communications [2] or optical radars [3]. This work aims at exploring rogue waves and optical dragon-kings arising in QCLs under optical feedback. While it is obvious that disruptive events will affect a transmission link, the detection and suppression of rogue waves is important for improving free-space data transmission. Ultimately, the control of these extreme events to a level such that a QCL could be used as a rogue waves generator could even be utilized to disrupt a free-space transmission link.

1. Introduction

It is known that external optical feedback (EOF) can often produce unwanted instabilities into the laser [4], but they can also be well controlled to produce desirable laser properties, including improved modulation characteristics and spectral stability. Our initial analysis of distributed feedback QCL revealed five feedback regimes appearing successively when increasing the feedback ratio at a given external cavity length [5]. The first feedback regime is stable and single-mode, emitting at the free-running wavelength, but with an increased and phase-dependent output power. For slightly higher feedback ratios, a beating between two modes occurs, one of them being the free-running wavelength and the other one an external cavity mode. Moreover, in this second regime, the intensity of each mode also depends on the feedback phase. The third regime appears for intermediate feedback ratios, the laser is

stable and single-mode again, but it emits on the adjacent mode identified in the second regime. The fourth regime is unstable, mostly attributed to deterministic chaos essentially composed of low-frequency fluctuations [6] wherein the spectral pedestal and the intensity of the side-modes, that are otherwise well-suppressed, increases significantly. Finally, the fifth regime for very high feedback ratios is a stable single-mode regime on an external cavity mode with high output power. Under conventional optical feedback, QCLs have then been proven to display several nonlinear dynamical regimes, including undamped oscillations and low-frequency fluctuations, and adding a periodic forcing created an entrainment of the non-linear spiking at the largest coupling rates [7][8]. When injected into a response QCL, the deterministic chaos from the drive laser can be reproduced by the response laser and this corresponds to chaos synchronization [9]. It is also possible to further tune the non-linear dynamics emitted by a QCL when modifying the back-reflected light in the external cavity. A cross-polarization technique allows displaying square wave with a tunable duty cycle and period, while creating an asymmetric cavity gives raise to rogue waves [10][11]. Rogue waves are extreme events surging high above the average and occurring more often than expected from a Gaussian distribution. Contrary to rogue waves found in other semiconductor lasers, rogue waves in QCLs do not scale as time-delay phenomena because they are driven by optothermal effects. This phenomenon competes with optical dragon-kings, which are similar extreme events but with a huge probability for the most prominent bursts and which are not derived from a scale-free distribution [12]. In what follows, we describe some peculiar nonlinear dynamics arising in a distributed feedback QCL under EOF.

2. Optical feedback apparatus

Fig. 1 (right) shows the experimental setup used for the optical feedback analysis. It is made of two branches. On the one hand, there is a feedback path with a mid-infrared polarizer and a gold-plated mirror placed on an accurately-moving cart. This mirror, together with the emitting facet of the QCL, defines the external-cavity length. The polarizer is the cornerstone device for tuning the amount of opti-

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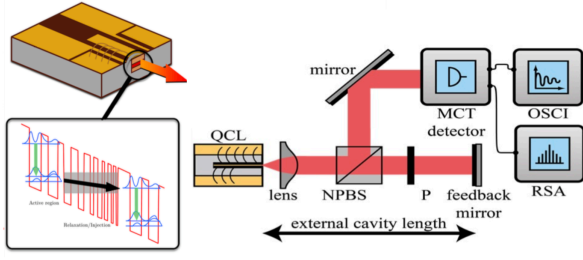
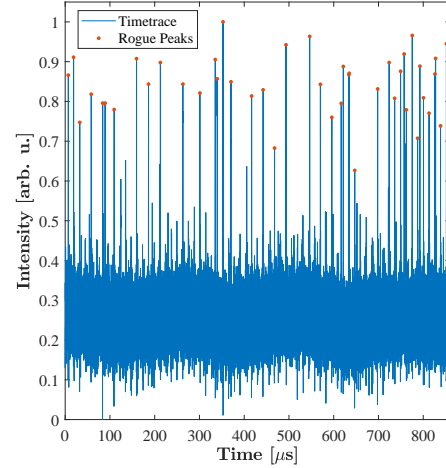


Figure 1: (left) Schematic representation of the QCL structure along with the cascade heterostructure; (right) Experimental apparatus used to study a QCL under EOF. P: polarizer to adjust the feedback strength, NPBS: non-polarizing beam-splitter, MCT: Mercury-Cadmium-Telluride, OSCI: digital oscilloscope, RSA: electric spectrum analyzer.

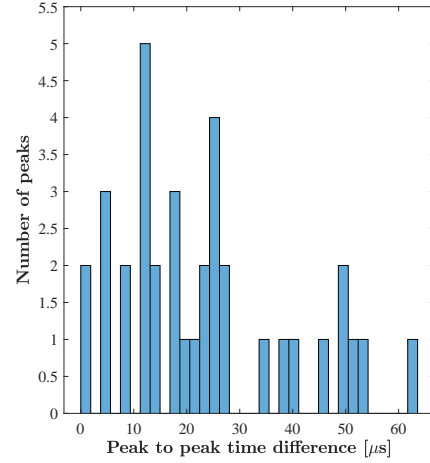
cal feedback knowing that the QCL wave is TM polarized: the angle of the polarizer defines the feedback strength. In the detection path, we use a high bandwidth mid-infrared detector (Vigo PEM Mercury-Cadmium-Telluride; MCT) operating at room temperature. The signal retrieved from the MCT detector is amplified with a wide-bandwidth low-noise amplifier, in order to overcome the background noise. The signal is then subsequently analyzed with a real-time digital oscilloscope and an electric spectrum analyzer. For an external-cavity length between 30-60 cm, the related external-cavity frequency is theoretically between 250-500 MHz. A 60/40 mid-infrared beam-splitter then splits the focused laser beam into both paths. Focusing is achieved with a lens in front of the laser. When the laser operates at room temperature, the QCL package is horizontally clamped over an indium foil and a copper mount with a Peltier module for temperature control. In that configuration, the wave hitting the beam-splitter is p -polarized and the transmission of the beam-splitter at this wavelength is about 60%. The current source is a low-noise source and the continuous bias delivered by the source can be modulated with an external signal from a waveform generator.

2.1. Rogues waves

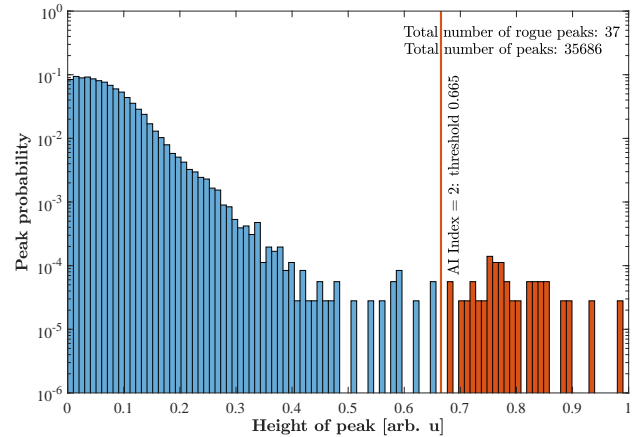
The term rogue waves describes random isolated events with amplitudes well above that of neighboring ones, and which occur more often than expected from the distribution of lower amplitude events. In order to detect rogue waves from experimental data, the probability density function (PDF) of an appropriate attribute of the data needs to be analyzed, and crest or wave heights are commonly used. Once extracted from the data, heights are compared to a threshold value above which events can be considered as extreme. This criterion is often defined in terms of the standard deviation of the whole time series frame : any event whose height is higher than the mean value (μ) plus height times the standard deviation (σ) is considered as an extreme event ($\mu \pm 8\sigma$). In the following, we focus on a



(a)



(b)



(c)

Figure 2: (a) Experimental time trace with events meeting the rogue wave criterion. (b) Histogram of the time-intervals between extreme events. (c) Histogram of the retrieved maxima and the related threshold corresponding to 2 times the significant height. Red bars gather the extreme events of the displayed time trace.

rogue wave threshold corresponding to 2 times the significant height of the dataset, which is equivalent to an abnormality index of 2. Figure 2 (a) shows a time trace with rogue waves when no external modulation is applied. This case differs from the usual configuration where EOF gives rise to deterministic chaos because the external cavity is asymmetric, or in other words, the feedback mirror is a bit tilted. The back-reflected light consequently hits the emission facet of the QCL in an uneven manner. Though this method is difficult to assess precisely, it seems to be mandatory in order to trigger events with a large amplitude without any external modulation. It is relevant to notice that variations of the non-linear dynamics in diode lasers due to a mirror tilt were already studied [13]. Figure 2 (b) shows that the time-interval between the events is not easily foreseen, which is one of the characteristics of rogue waves. Other studies have shown that the separation time between rogue events follows a Poissonian distribution when the time elapsed between these events. Fig. 2 (c) gathers the statistic of the spikes found in the time trace for the aforementioned criterion. The duration of the record was chosen as a trade-off between the number of displayed rogue waves and a sufficient resolution to ensure that the retrieved events were not artifacts. There is a clear long-tail distribution in the statistics which is incompatible with a regular distribution of maxima and this means that events with a large amplitude appear more frequently than expected. This configuration is very similar to what happens when studying deterministic chaos in QCLs. Indeed, EOF can lead to low-frequency fluctuations but they are mostly unpredictable, which is not the case when a periodic external modulation allows synchronizing the spiking dynamics. Consequently, applying a periodic modulation with a low amplitude are one of the levers to display huge pulses while maintaining the entrainment phenomenon [8].

2.2. Dragon-king events

Dragon-king can be observed in many different contexts, ranging from avalanches to chaotic electronic oscillators as far as to bursting neurons [14]. They are also characterized by giant events, but in this case, these events follow a power law except the occasional very large events which have a higher probability of occurrence. One main characteristic of rogue waves distributions is that they are scale free, which means that events of arbitrarily large sizes are caused by the same dynamical mechanisms governing the occurrence of small and giant events. In contrast, giant pulses related to dragon-king events cannot be explained by a scale free distribution. This means that dragon-king events possess a distinct formation mechanism which may ease the prediction of these giant pulses. As it is a sub-category of rogue waves, no study of the dragon-king events in optical systems have been reported so far. This does not mean that some of the aforementioned efforts on optical rogue waves do not partially contain dragon-kings among conventional

rogue waves. In our study of rogue waves in QCLs, we have observed a quite rare configuration where giant pulses do not follow the conventional PDF (like the one in Fig. 2). In this configuration, giant pulses all have more or less the same amplitude and consequently, there is a gap in the PDF. As this case rarely occurred, we were not able to compute enough data to draw a PDF showing a local minimum and then a local maximum for a very large value of the amplitude. Figure 3 shows the timetrace with the potential dragon-king events and the related peak-to-peak histogram and PDF.

3. Conclusions

This article experimentally investigates rogue waves and extreme events in the case of a QCL under EOF. The conditions that favor such events is described and confirmed the existence of rogue waves. As extreme events represent bursts of extra optical power, a method can also be used to trigger them precisely and that is compatible with sensing applications. Tools to predict such events can also be introduced with the view of deleting rogue waves in the cases where they could be hazardous. Peculiarities of extreme events such as dragon kings are also discussed. Even though it is possible to exhibit rogue waves in the case of a free-running QCL with EOF, it has to be noted that they are not easily predicted and extreme events are more likely to happen in a configuration with a periodic modulation of the pumping current and a small amplitude. As these extreme events do affect the performances of QCLs, it is also important to develop methods to get rid of them [15]. Overall, we believe that these results are of paramount importance for applications where extreme events need to be triggered on demand.

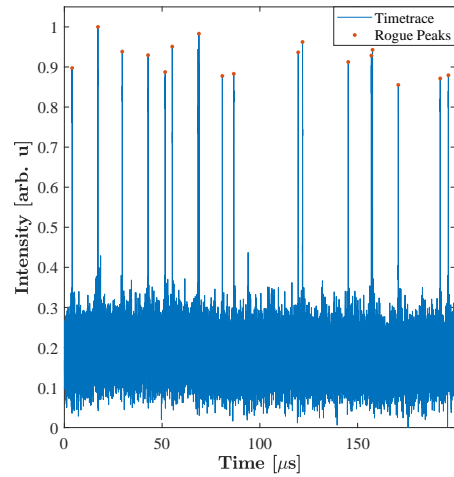
Acknowledgments

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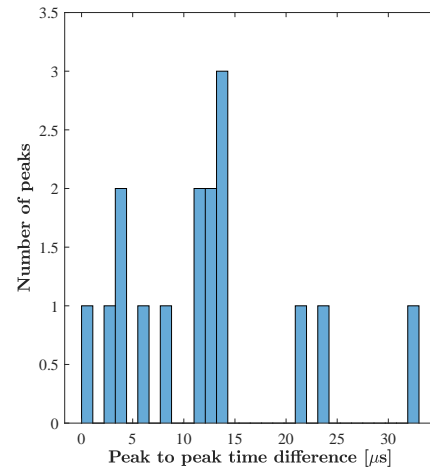
References

- [1] J. Faist, "Quantum cascade lasers," Oxford, 2013.
- [2] X. Pang, O. Ozolins, R. Schatz, J. Storck, A. Udalcovs, J. R. Navarro, A. Kakkar, G. Maisons, M. Carras, G. Jacobsen, S. Popov, and S. Lourdudoss, "Gigabit free-space multi-level signal transmission with a mid-infrared QCL operating at room temperature", *Optics Letters*, 2017.
- [3] F.-Y. Lin and J.-M. Liu, "Chaotic LIDAR", *IEEE Journal of Selected Topics in Quantum Electronics*. vol. 10, p. 991, 2004.

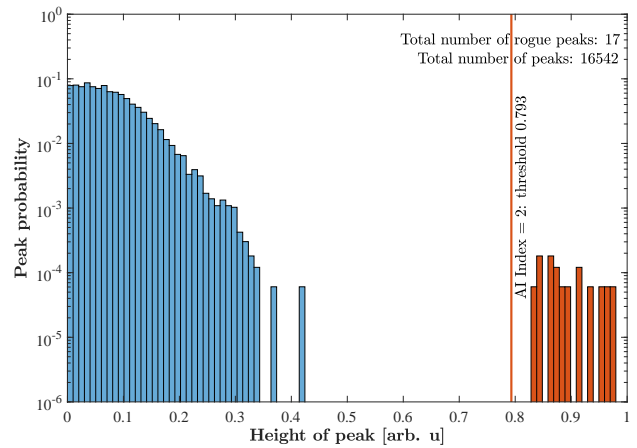
- [4] D. M. Kane and K. A. Shore, "Unlocking dynamical diversity: optical feedback effects on semiconductor lasers", John Wiley & Sons, 2005.
- [5] L. Jumpertz, M. Carras, K. Schires, and F. Grillot, "Regimes of external optical feedback in 5.6 micron distributed feedback mid-infrared quantum cascade lasers", *Applied Physics Letters*, vol. 105, p. 131112, 2014.
- [6] L. Jumpertz, K. Schires, M. Carras, M. Sciamanna, and F. Grillot, "Chaotic light at mid-infrared wavelength", *Light Science and Applications*, Vol. 5, e16088, 2016.
- [7] O. Spitz and F. Grillot, "A review of recent results of mid-infrared quantum cascade photonic devices operating under external optical control", *Journal of Physics: Photonics*, vol. 4, p. 022001, 2022.
- [8] O. Spitz, L. Durupt, and F. Grillot, "Competition between entrainment phenomenon and chaos in a quantum cascade laser under strong optical reinjection", *MDPI Applied Science*, vol. 9, p. 29, 2022.
- [9] O. Spitz, A. Herdt, J. Wu, G. Maisons, M. Carras, C.-W. Wong, W. Elsaesser, and F. Grillot, "Private communication with quantum cascade laser photonic chaos", *Nature Communications*, vol. 12, p. 3327, 2021.
- [10] O. Spitz, J. Wu, A. Herdt, G. Maisons, M. Carras, W. E. Elsaesser, C.-W. Wong, and F. Grillot, "Extreme events in quantum cascade lasers", *Advanced Photonics*, vol. 2, p. 066001, 2020.
- [11] O. Spitz, A. Herdt, W. Elsaesser, and F. Grillot, "Stimulating polarization switching dynamics in mid-infrared quantum cascade lasers", *Journal of the Optical Society of America B*, vol. 38, p. 35, 2021.
- [12] A. Mishra, S. Saha, M. Vigneshwaran, P. Pal, T. Kapitaniak, and S. K. Dana, "Dragon-king-like extreme events in coupled bursting neurons", *Physical Review E*, vol. 97, p. 062311, 2018.
- [13] P. Besnard; B. Meziare; G.M. Stephan, "Feedback phenomena in a semiconductor laser induced by distant reflectors", *IEEE Journal of Quantum Electronics*, Vol. 29, p. 1271, 1993.
- [14] T. Jin, C. Siyu, and C. Masoller, "Generation of extreme pulses on demand in semiconductor lasers with optical injection", *Opt. Express*, vol. 25, p. 3126, 2012.
- [15] N. M. Granese, A. Lacapmeasure, M. B. Agüero, M. G. Kovalsky, A. A. Hnilo, and J. R. Tredicce, "Extreme events and crises observed in an all-solid-state laser with modulation of losses", *Optics Letters*, vol. 41, p. 3010, 2016.



(a) a



(b) b



(c) c

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