Principle of Semiconductor Lasers

Modulation, Nonlinear & Ultra-fast Dynamics

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Textbooks

[1] J. M. Liu, *Photonic Devices*, Cambridge University Press

[2] L. A. Coldren & S. W. Corzine, *Diode Lasers and Photonic Integrated Circuits*, Wiley

[3] P. Blood, *Quantum Confined Laser Devices*, Oxford University Press

[4] J. Faist, *Quantum Cascade Lasers*, Oxford University Press

[5] J. Ohtsubo, *Semicondutor Lasers*, Springer

[6] T. Erneux & P. Glorieux, *Laser Dynamics*, Cambridge

[7] A. Uchida, *Optical Communications with Chaotic Lasers*, Wiley

Let there be light…

LASER: Light Amplification by Stimulated Emission of Radiation

Laser light is monochromatic, i.e., consisting of a single wavelength or color, and emitted in a narrow beam

T. H. Maiman was an American engineer and physicist credited with the invention of the first working laser. The laser was successfully fired on May 16, 1960

The laser disc player (1978), was the first successful consumer product to include a laser, but the compact disc player was the first laser-equipped device to become truly common in consumers' homes, beginning in 1982, followed shortly by laser printers

Theodore Maiman (1927-2007)

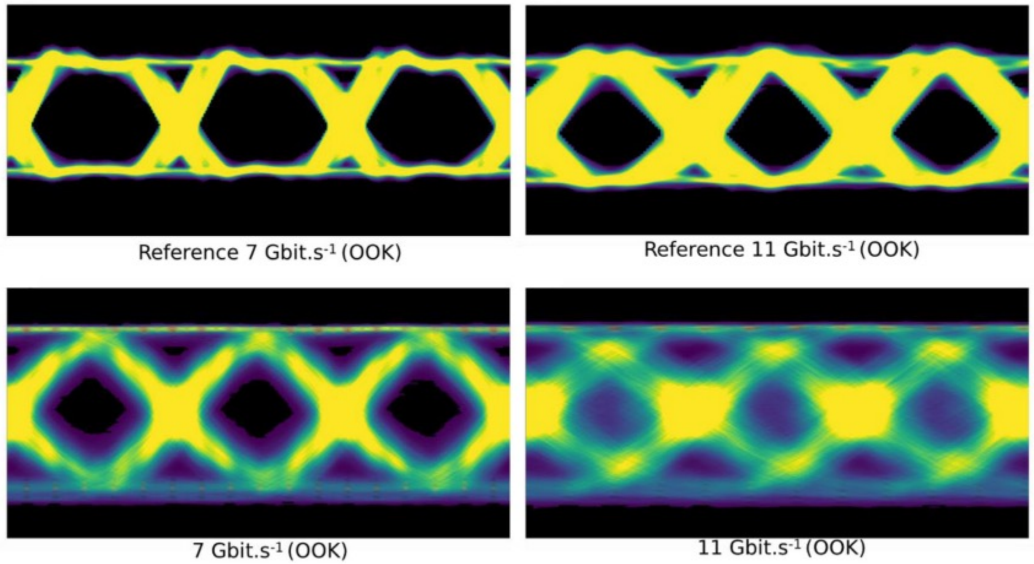
T. H. Maiman, Journal of the Optical Society of America (1960)

...and more!

Goldfinger (1964) was the first film to feature a laser

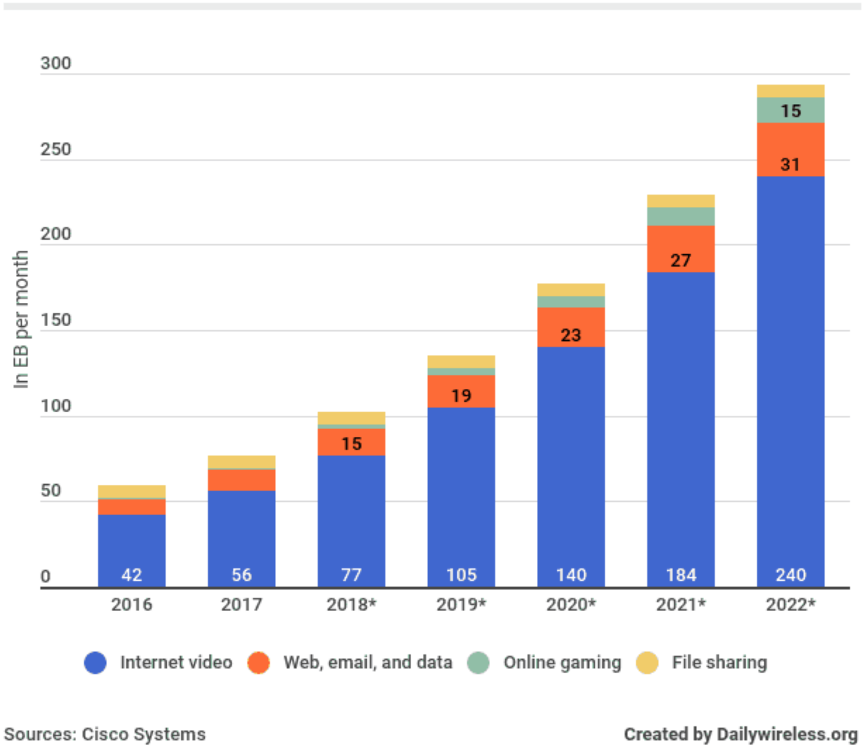
Lasers did not exist in 1959 when the book was written. In the novel, Goldfinger uses a circular saw to try to kill Bond, but the filmmakers changed it to a laser to make the film feel more fresh

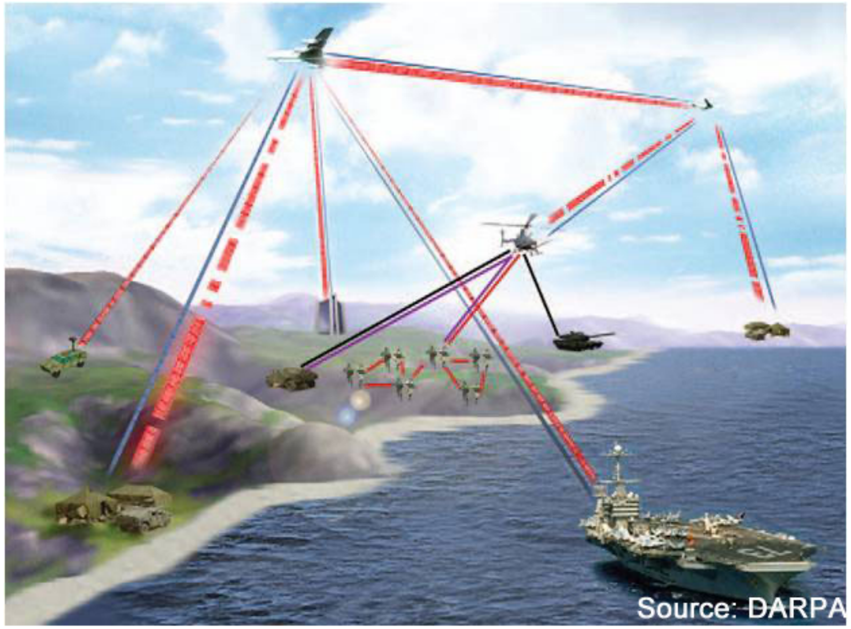
Fundamentals of Semiconductor Lasers
&
Modulation Dynamics



Optical communications

Predicted worldwide internet traffic (1EB = 10^{18})





Data transfer with light

→ 1000 x Larger bandwidth

→ 1000 x Lower loss

→ 100 x Larger distance

Scalability & power efficiency

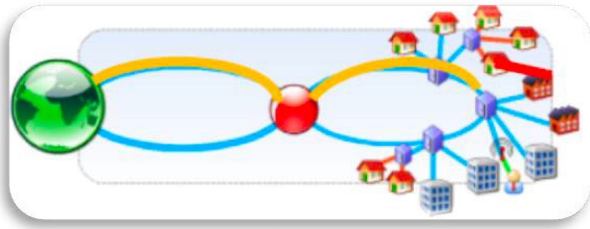
Growth of free-space optics

Optical communications

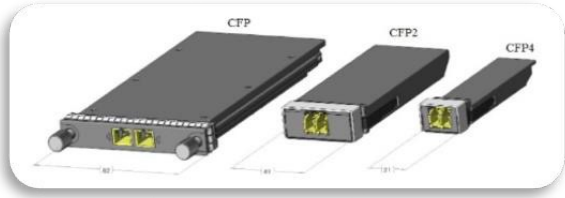
TELECOM

> 10 km

➢ Metro & Access



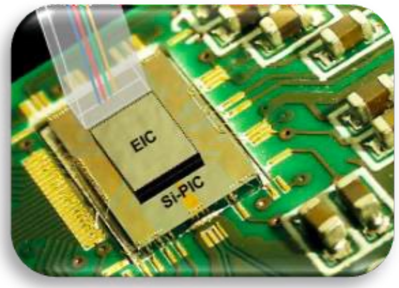
- **1.5 µm**
  - ➢ Transceivers modules



DATACOM

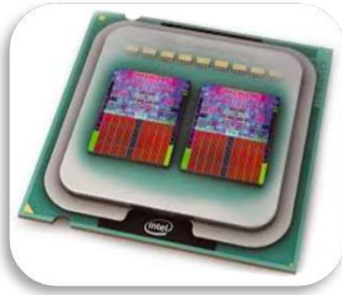
- **1m – 10 km**
  - ➢ Routers & switches

- **1.3 µm**
  - ➢ Rack-to-rack
  - ➢ Board-to-board

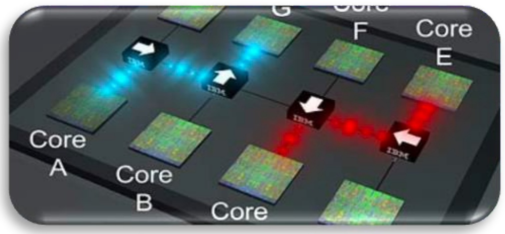


COMPUTERCOM

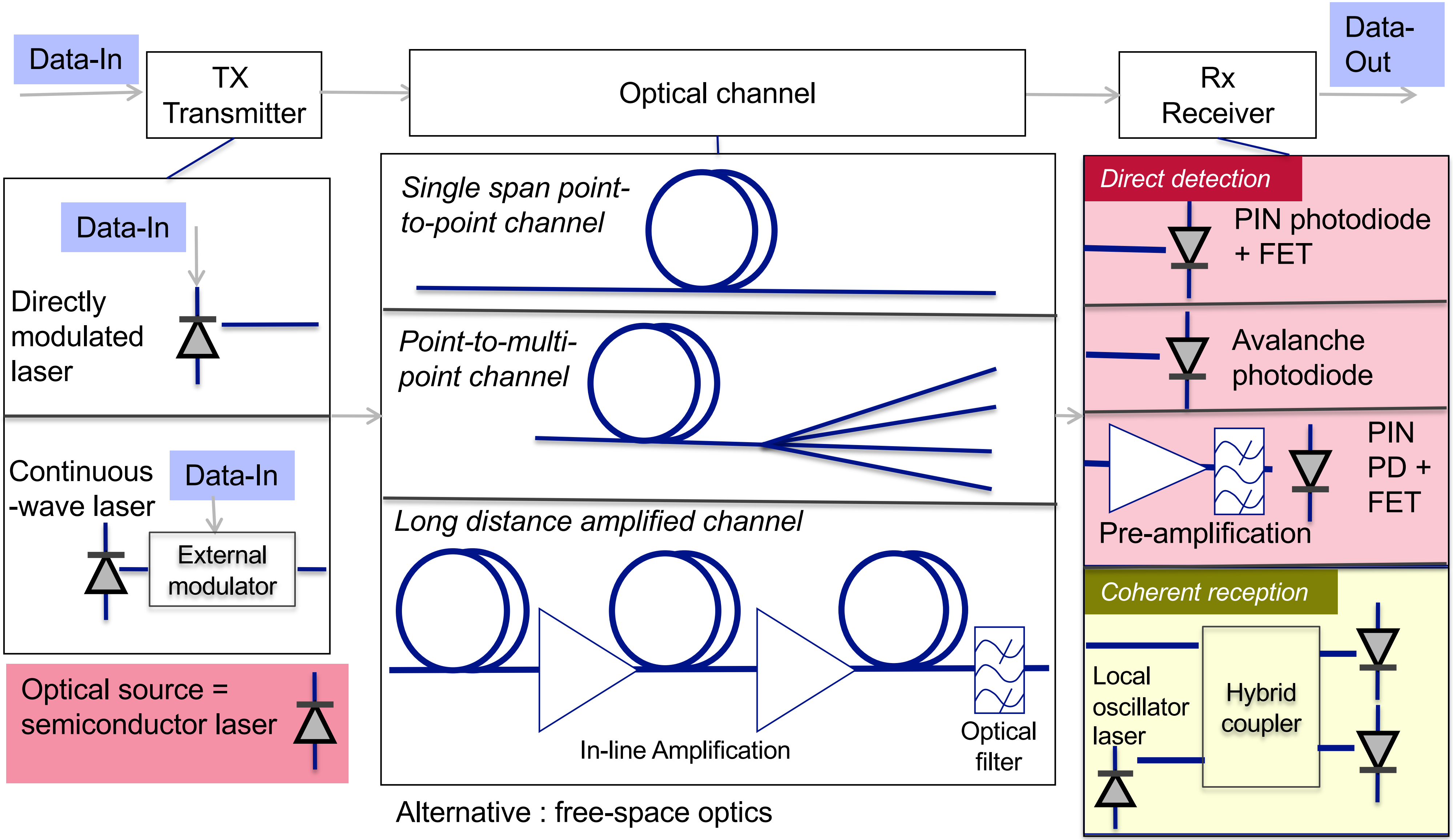
- **< 1m**
  - ➢ Photonics integrated circuits



- **1.3 µm**
  - ➢ Transceivers on chip



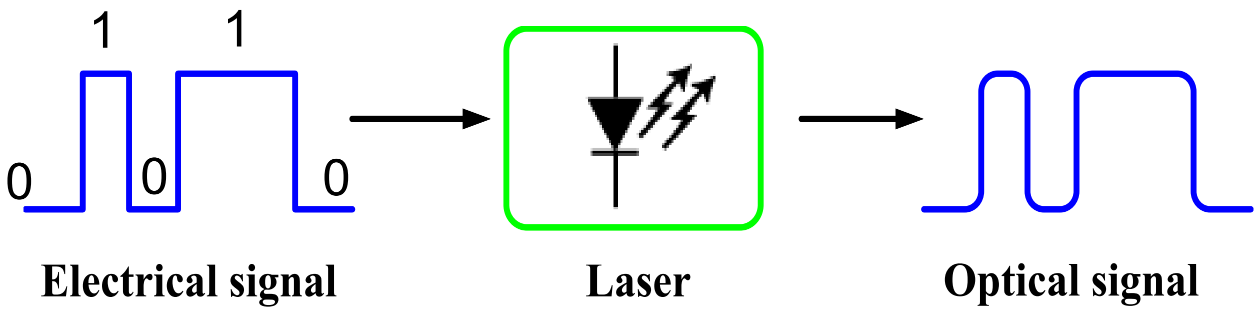
System architecture of an optical link



Alternative : free-space optics

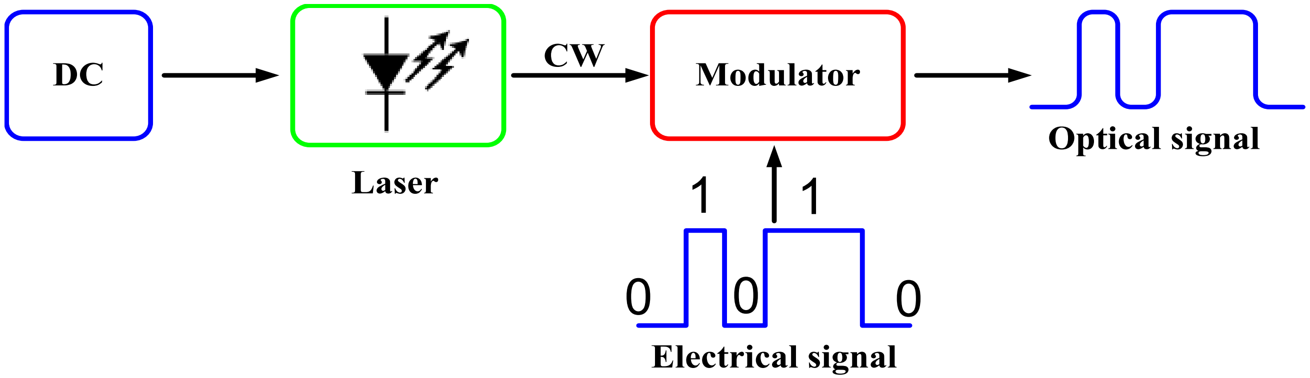
Modulation schemes

Direct modulation



- Chirp
- Simple
- Compactness
- Low cost
- Energy efficient

External modulation

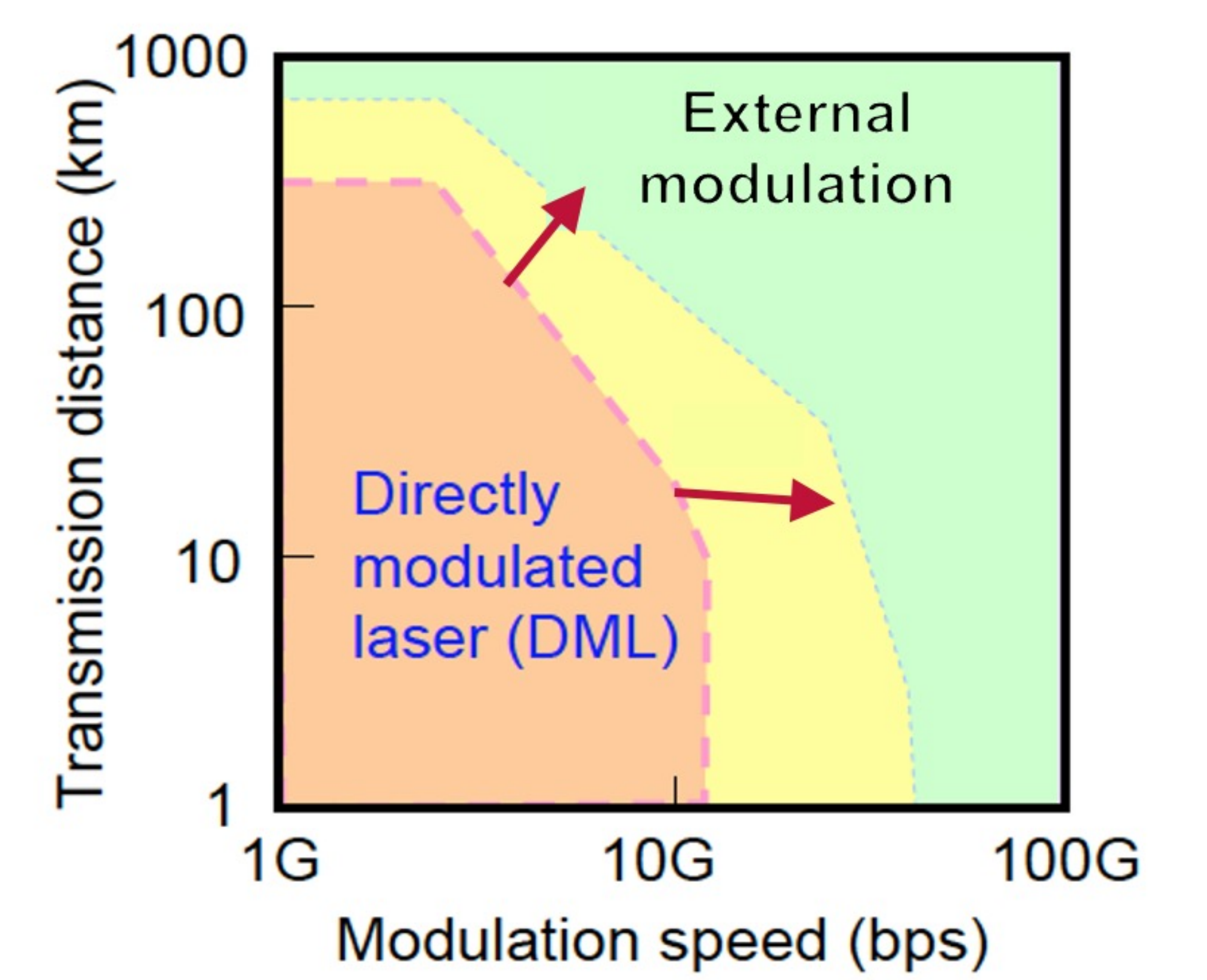


- Power consumption
- Cost
- High-speed
- Zéro chirp

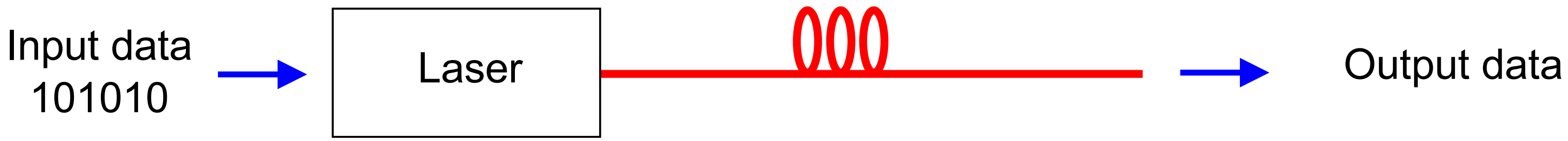
T. Anfray et al., Journal of Lightwave Technology (2012)

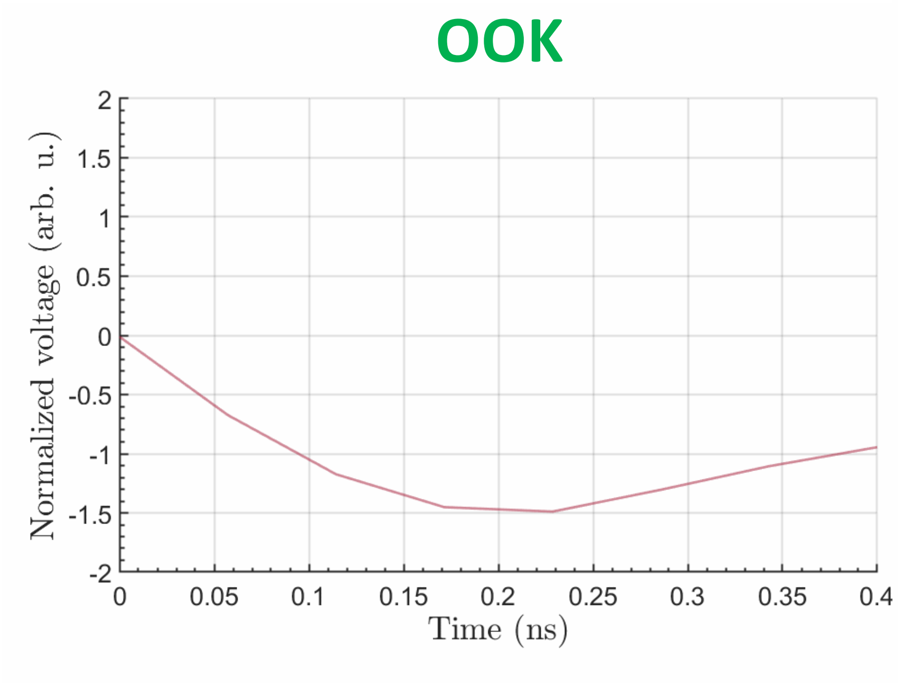
Speed & range limitations

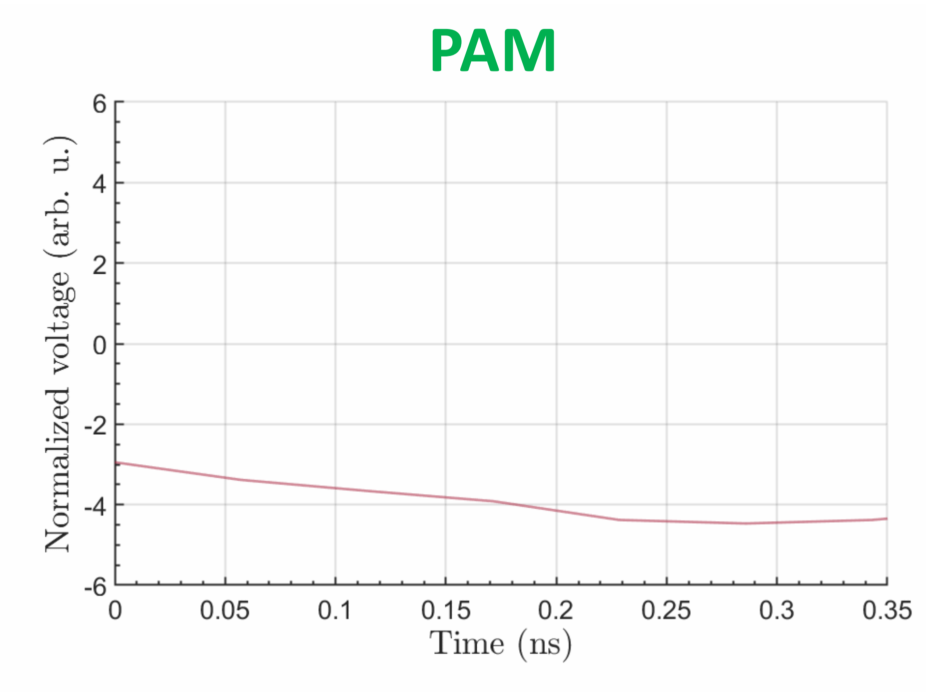
Tradeoff between the modulation speed and the range. For long-haul applications, external modulation is madatory



Eye diagram



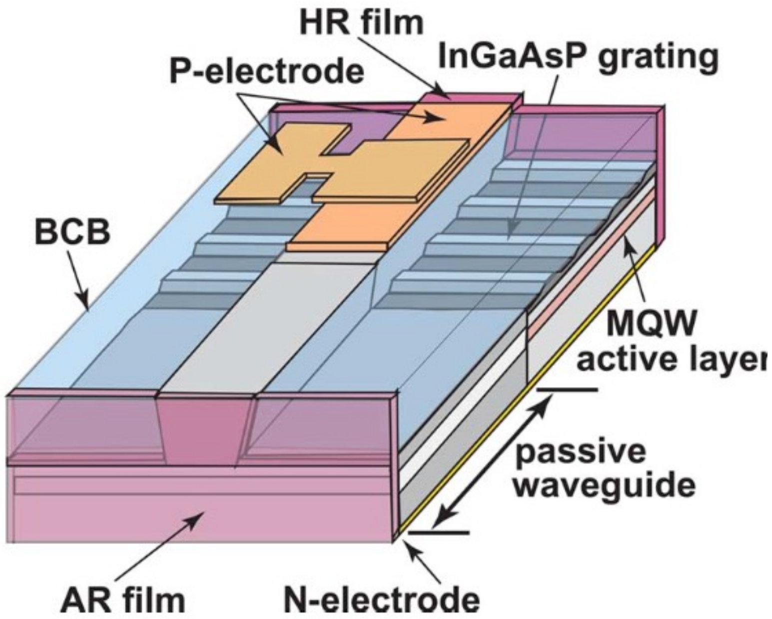




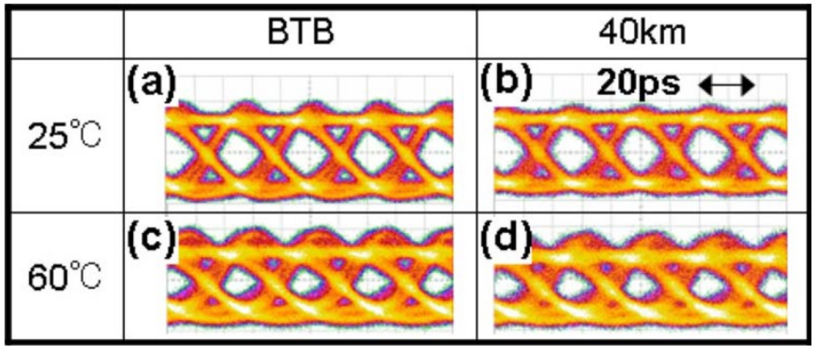
The eye diagram reflects the effect of noise, jitter, distortion, inter-symbol interference, and crosstalk, all contribute to close the eye and alter the data transmission

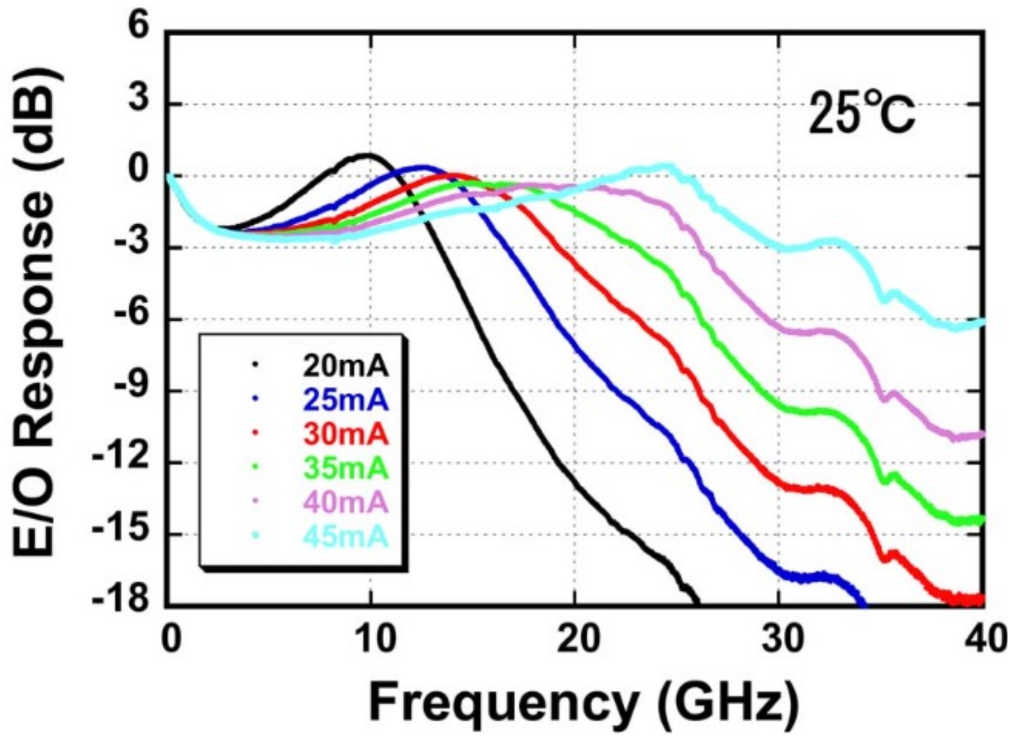
High-speed lasers

NTT Electronics



1.3-µm InGaAlAs DFB lasers
3-dB bandwidth of 30 GHz
Clear eye @43 Gb/s after 40 km transmission under direct modulation





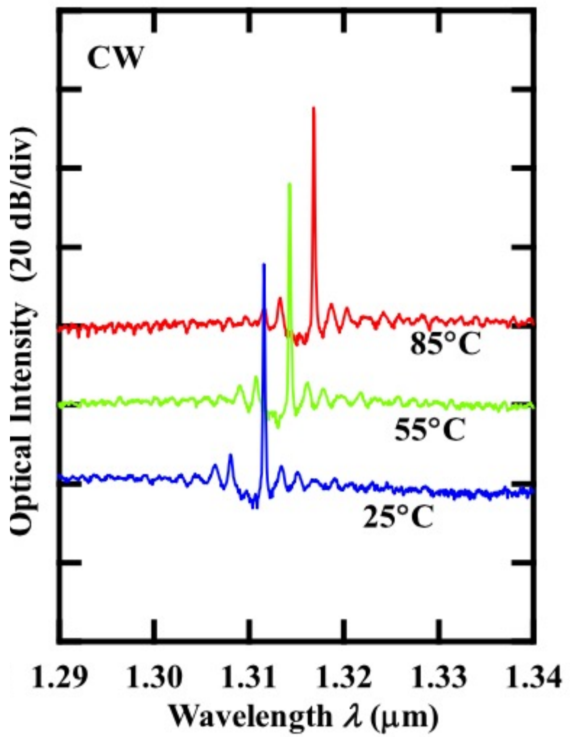
T. Tadokoro, IEEE J. of Lightwave Technology (2012)

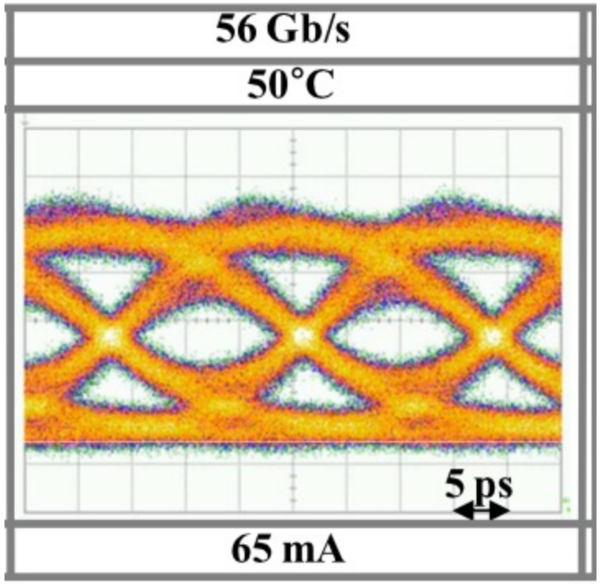
High-speed lasers

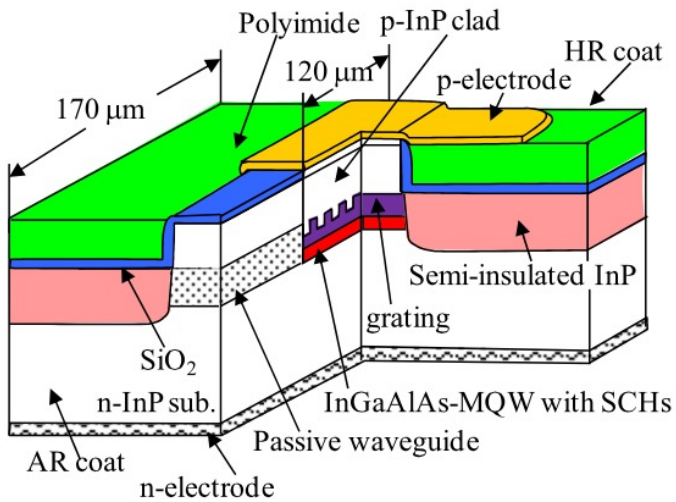
OCLARO

HITACHI

Direct modulation@56 and 50 Gb/s w/ 1.3-µm InGaAlAs DFB lasers
Transmission over 10 km



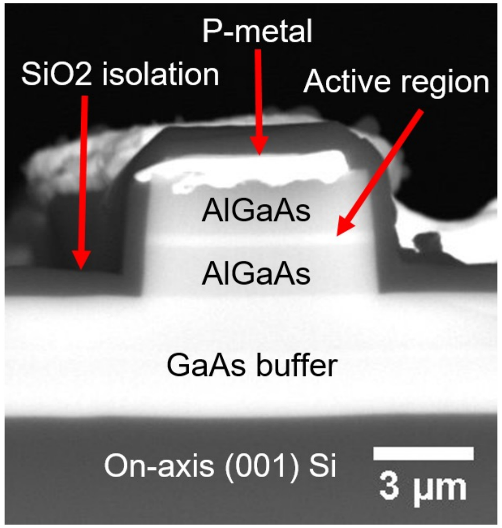


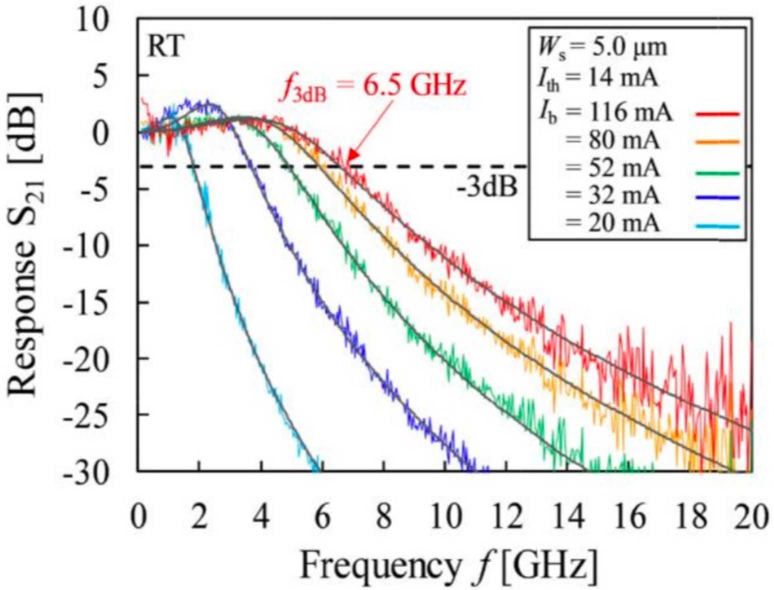


K. Nakahara, IEEE Photonics Technology Letters (2015)

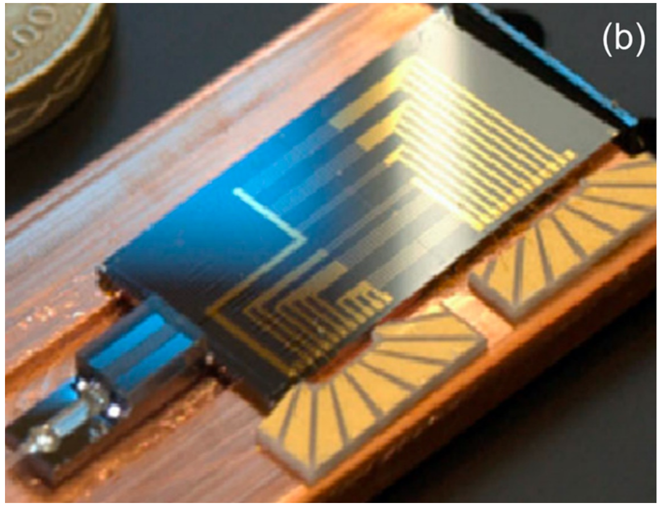
High-speed lasers

Directly modulated quantum dot (QD) lasers on silicon

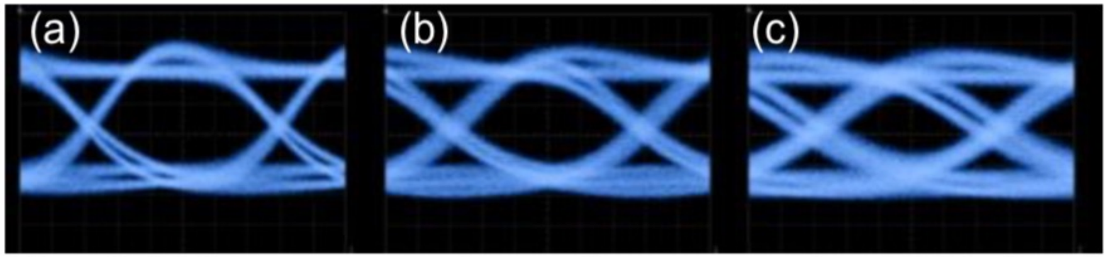




InAs QDs on a silicon oxynitride photonic chip



7.5 Gbps 10 Gbps 12.5 Gbps

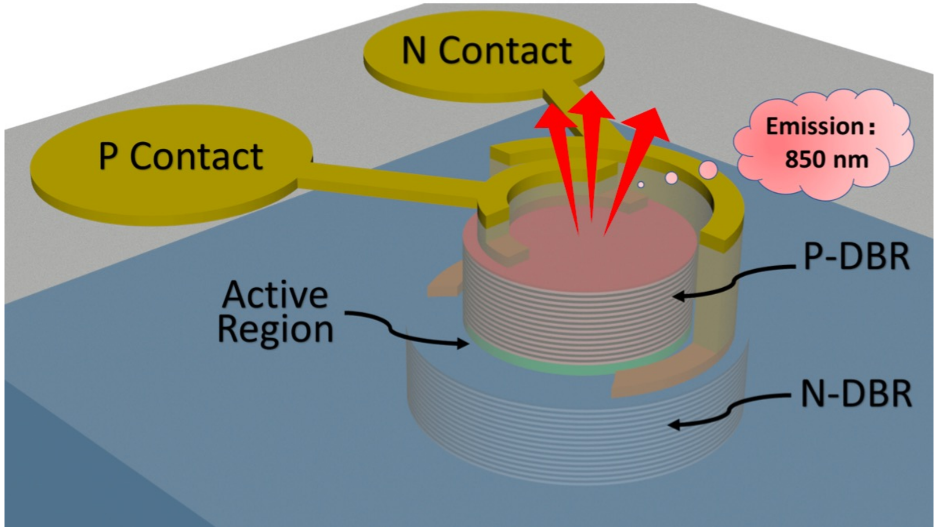


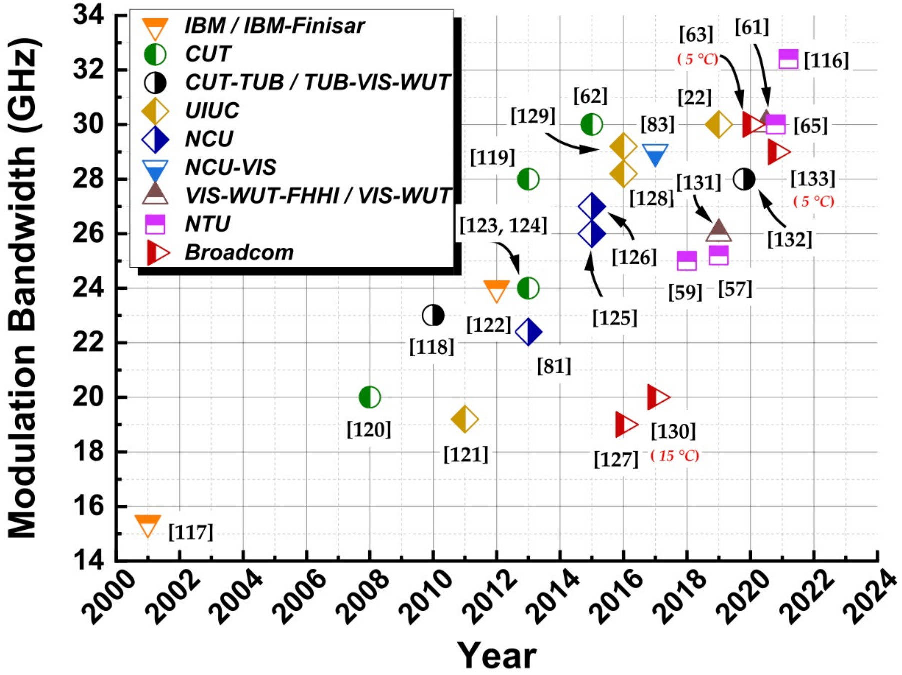
Direct modulation up to 12.5 Gbps

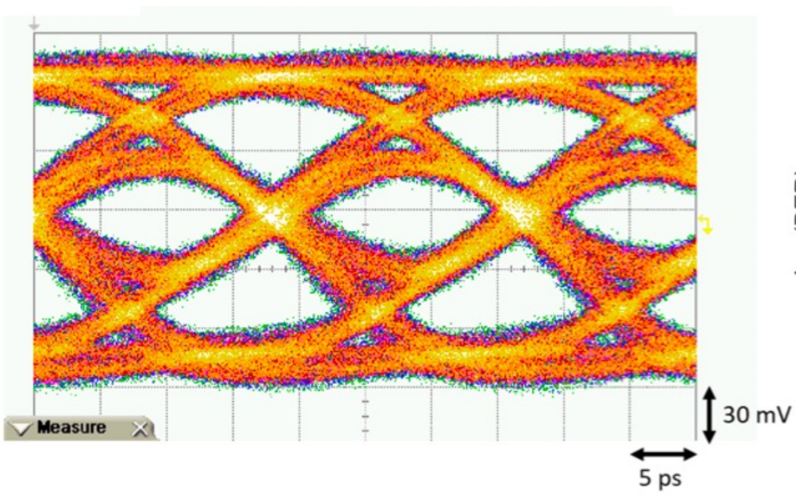
J. Norman et al., J. Vac. Sci. Technol. A (2021)

High-speed lasers

Vertical Cavity Surface Emitting Lasers (Optical Radars, Optical interconnects)





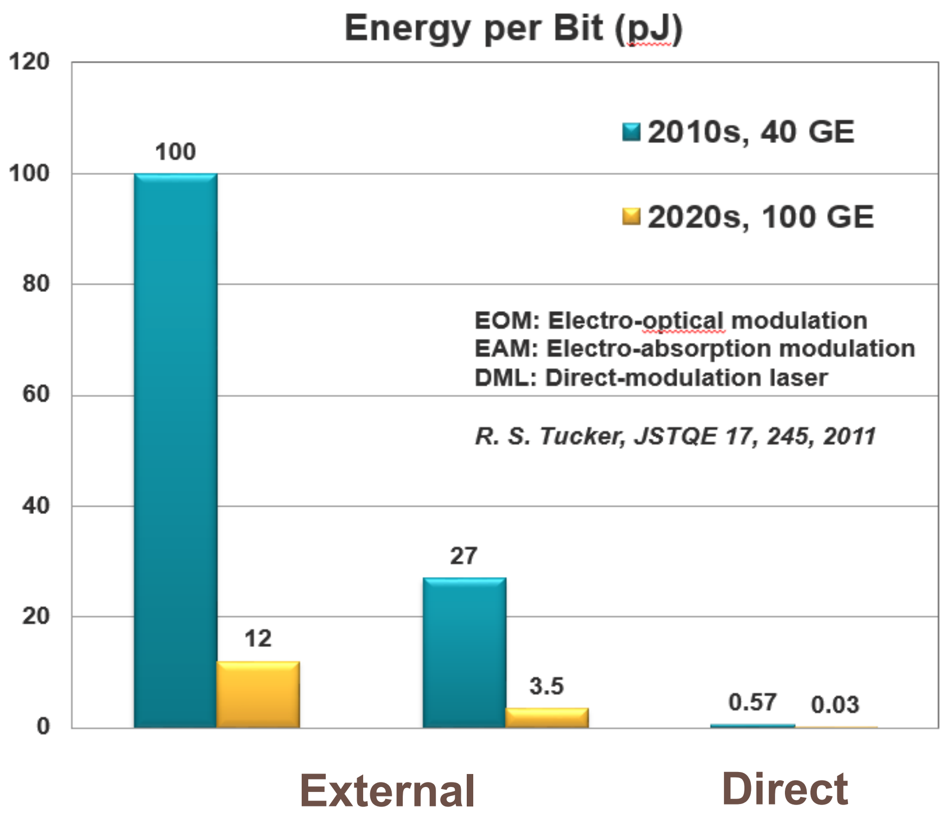


Direct modulation of VCSEL at 56 Gbps With forward error correction bit rates > 80 Gbps can be achieved*

H. T. Cheng et al., MDPI Applied Physics (2022)
*L. Chorchos et al., J. of Lightwave Technol. (2020)

Energy consumption

Transmitter Type	Component		Energy per Bit: 2010-era technology (40 Gb/s)	Energy per Bit: 2020-era Target (100 Gb/s)
	MUX		10 pJ	2 pJ
Directly Modulated	Laser $(\overline{V}_{laser}\overline{I}_{laser})$		358 fJ	10 fJ
	Driver (P_{driver} / B_r)		212 fJ	20 fJ
	Laser + Driver		570 fJ	30 fJ
Externally Modulated (Electro-Absorption)	Laser $(\overline{V}_{laser}\overline{I}_{laser} / B_r)$		1.5 pJ	500 fJ
	Lumped $V_{mod} = 3$ V	$\frac{1}{2} C_{mod}(V_{mod}^{p-p})^2$	800 fJ	50 fJ
		$2\widehat{V}_{mod}^2 / Z_0 B_r$	4 pJ	1.6 pJ
		$\overline{V_{mod} I_{mod}} / B_r$	1.5 pJ	600 fJ
	TW $V_{mod} = 1$ V	$2\widehat{V}_{mod}^2 / Z_0 B_r$	440 fJ	176 fJ
		$\overline{V_{mod} I_{mod}} / B_r$	500 fJ	200 fJ
	Driver (P_{driver} / B_r)		25 pJ	100 fJ (C_{mod}) 3 pJ (Z_0)
	Laser + Driver		27 pJ	600 fJ (C_{mod}) 3.5 pJ (Z_0)
Externally Modulated (Electro-Optic)	Laser $(\overline{V}_{laser}\overline{I}_{laser} / B_r)$		1.5 pJ	500 fJ
	$2\widehat{V}_{mod}^2 / Z_0 B_r$		4 pJ	1.6 pJ
	Driver (P_{driver} / B_r)		25 pJ	3 pJ
	Laser + 4 Drivers		~ 100 pJ	12 pJ



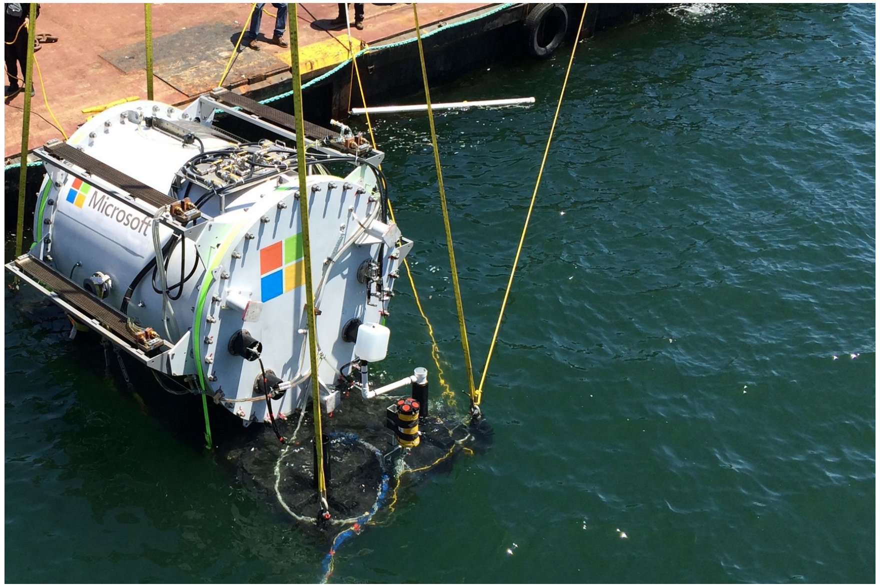
R. S. Tucker, IEEE J. Sel. Top. Quantum Electron. (2011)

Energy consumption

An optical datacenter burns about 5 MWh / m^2 / year

Air cooling 50% of the total energy consumption ie. 1kW used for running a server needs an additional 1kW toto cool to maintain its temperature to 22°C

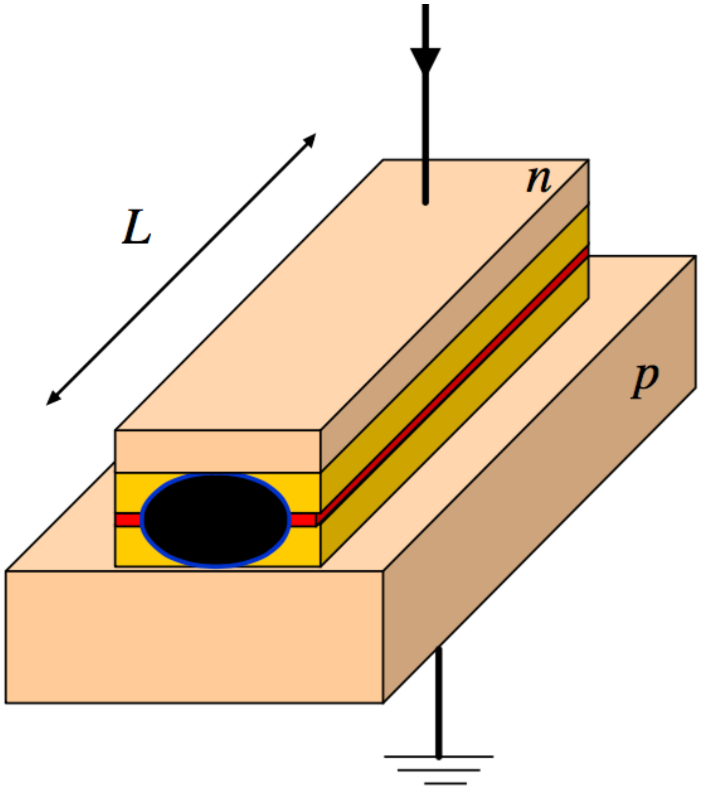
> ***→ Microsoft : under the sea data centers powered by renewable energy***
>
> ***→ Facebook: datacenters near the edge of the Artic circle***



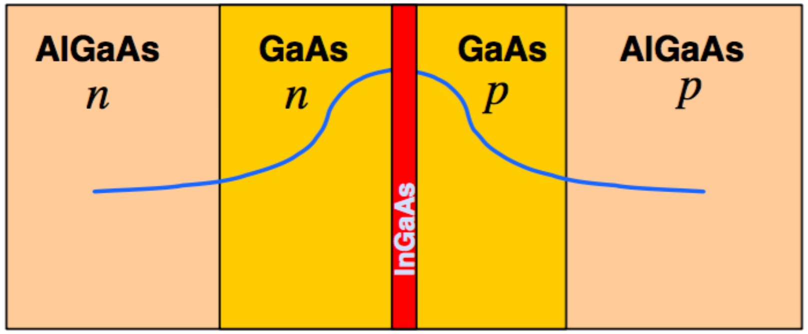
To reduce the global energy consumption, datacenters need faster optical and energy efficient optical devices

Semiconductor lasers

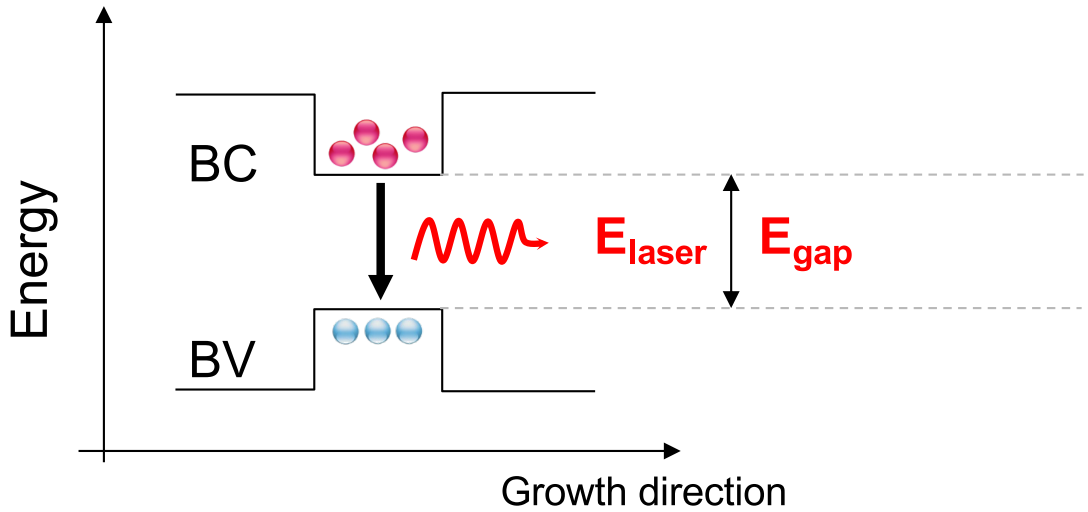
3 equivalent aspects



Technology

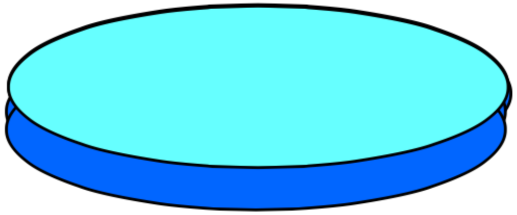


Electromagnetism

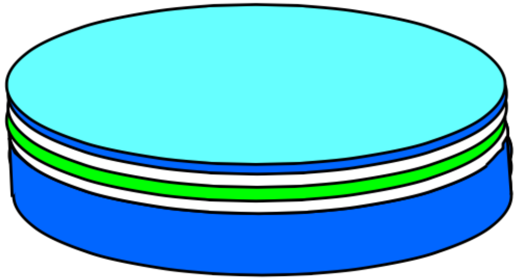


Interband structure

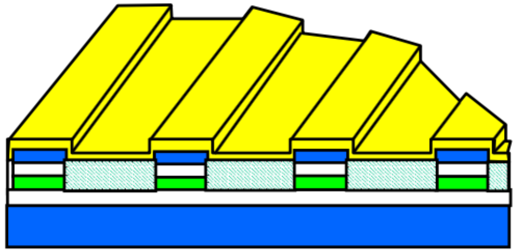
Technology & processing



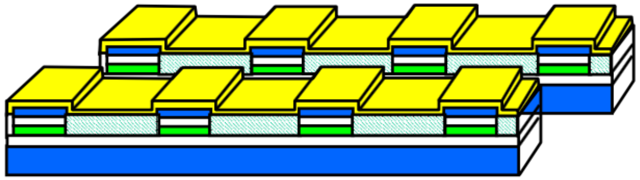
1- SUBSTRATE



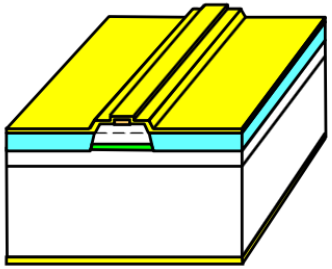
2- EPITAXIE



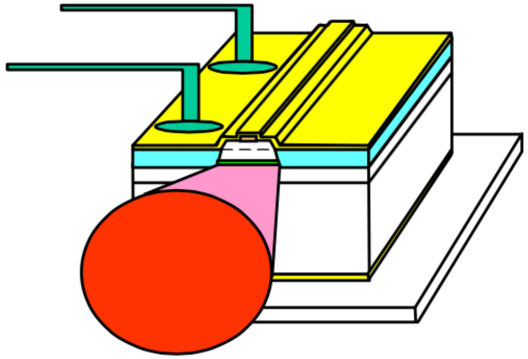
3- LASER PROCESSING



4- FACETS CLEAVING

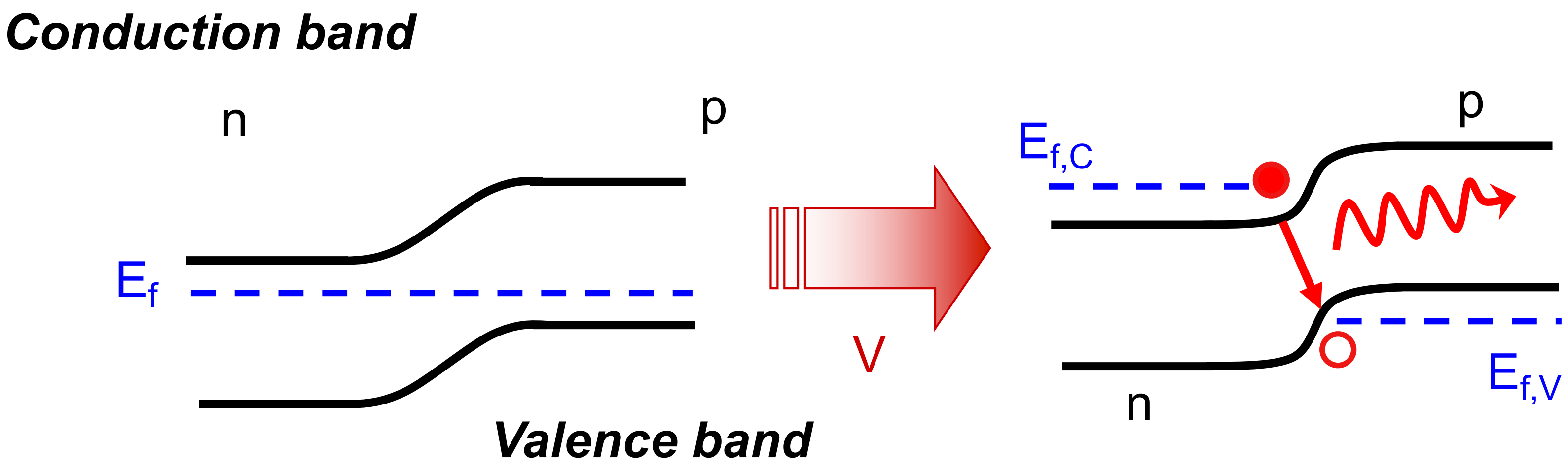


5- SINGLE CHIP PREPARATION



6- MOUNTING, BONDING

Building block: pn junction



Thermal equilibrium

Out of equilibrium

N = carrier density → optical gain

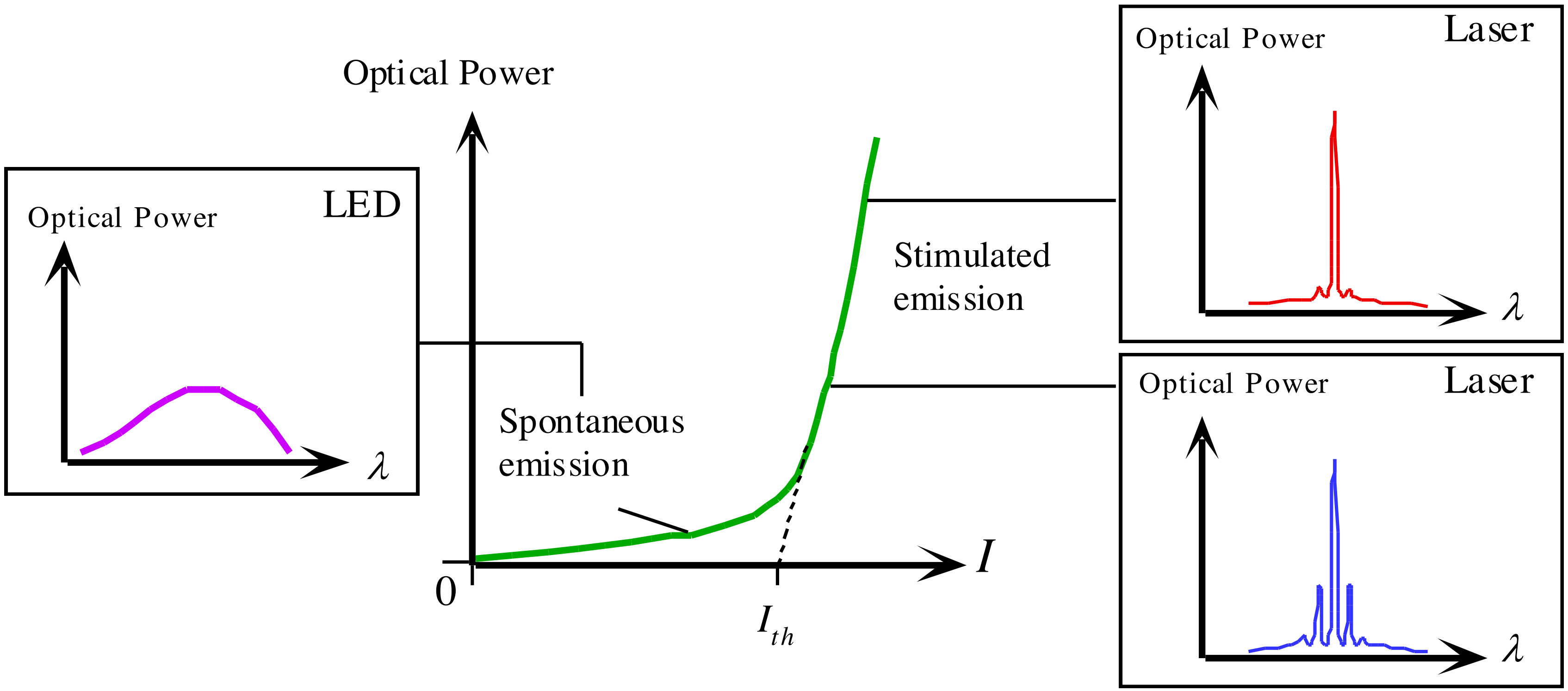
J = current density

τ_c = carrier recombination

d = width of the active zone

$$\boxed{N = \frac{J\tau_c}{qd}}$$

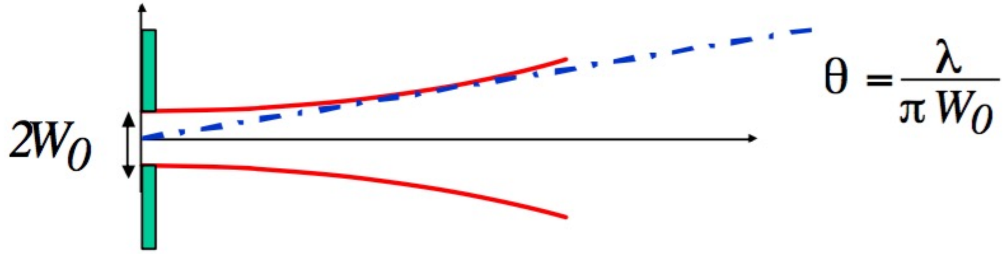
Emission properties

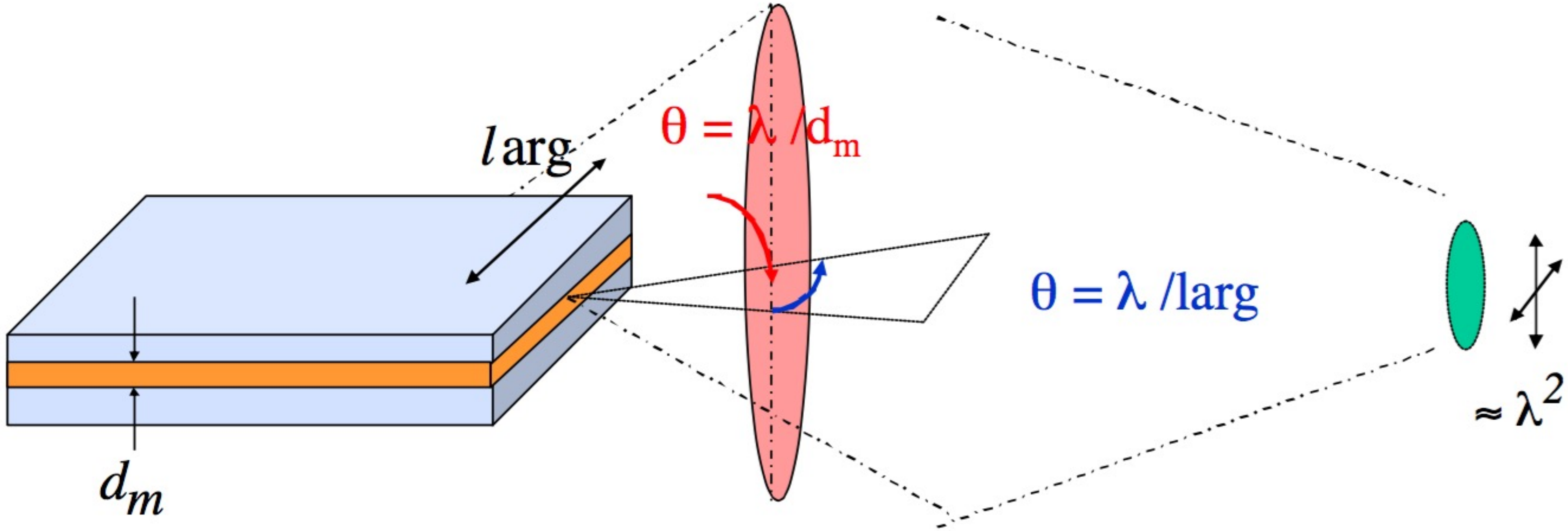


Typical output optical power vs. diode current (I) characteristics and the corresponding output spectrum of a laser diode.

Emission properties

Gaussian beam theory





Anisotropic divergence → anamorphosis optics

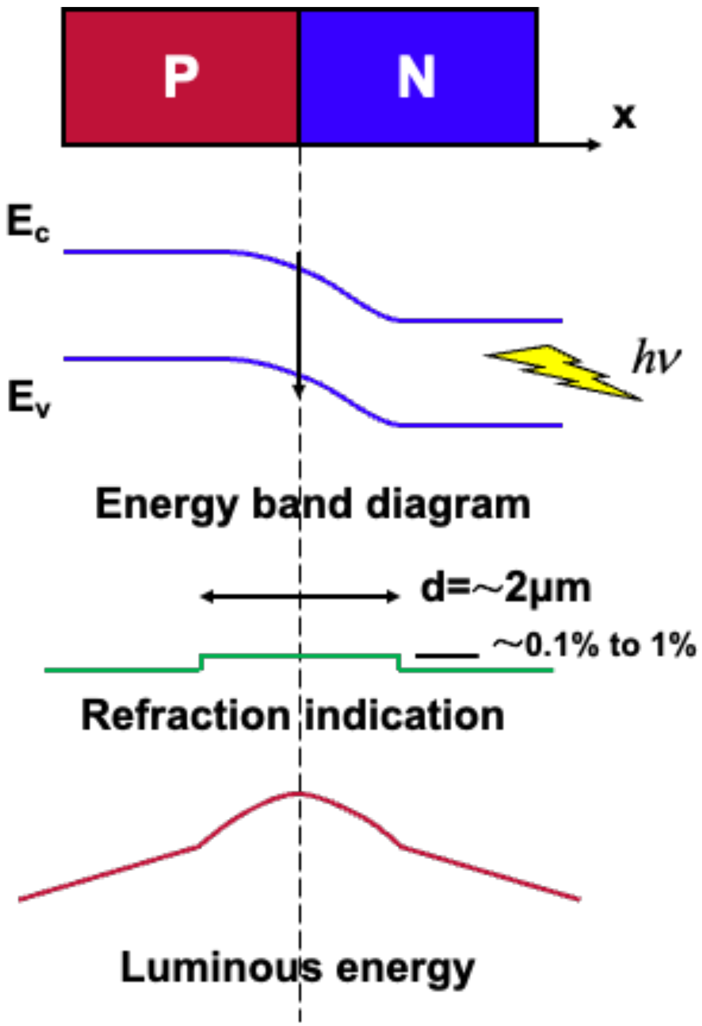
Homojunction laser

If J ↗ then N ↗
N must be large so that gain and loss balance each other

$$N = \frac{J\tau_c}{qd}$$

In practice

As J is very large ➔ Power dissipation critical ➔ Strong thermal issues



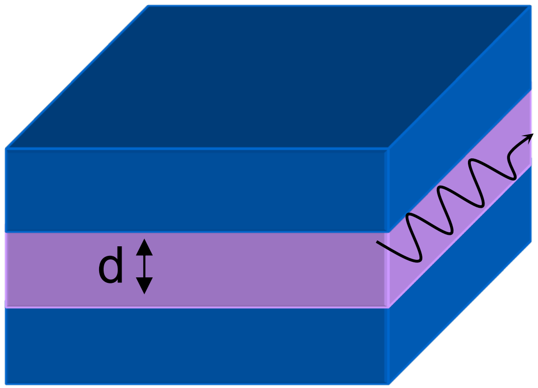
Strongly limited for applications because they only operates at cryogenic temperatures and under pulsed current

Double heterostructure

$$G(N) = G_N(N - N_t)$$

$$N = \frac{J\tau_c}{qd}$$

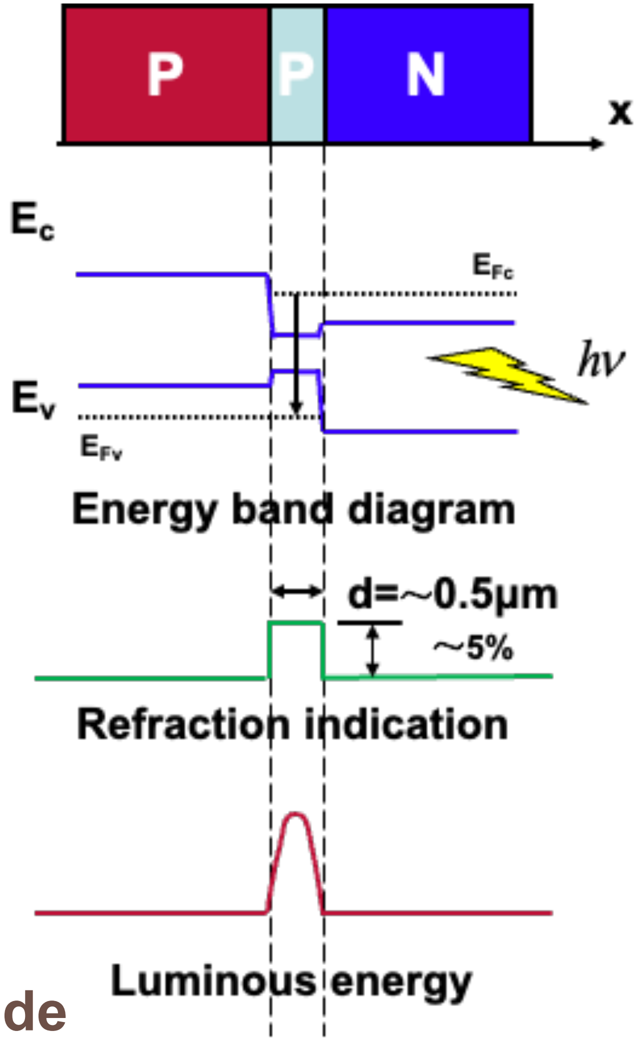
Gain ↗
means ↘ d



Wide gap
Small gap
Wide gap

$n_{small\ gap} > n_{wide\ gap}$

Photons are guided !



d < 500 nm against ~ 1 µm for a pn junction
Threshold decreased by one order of magnitude

1970: continuous-waves room temperature demonstrated!

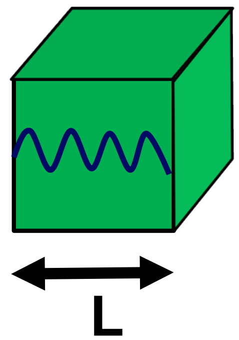
Quantization, what for?

Shape of density of states (gain spectral width)

Number of states (transparency current)

Carrier confinement

Energy tuning



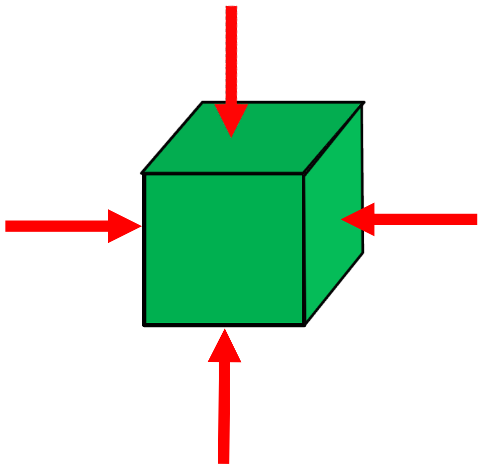
$$k_j L = n_j \pi$$

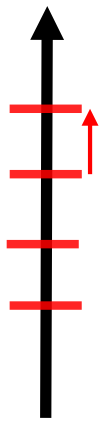
$$j = x, y, z$$

Energy quantization

$$E_n = \frac{h^2 \pi^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

Wavefunction confinement within the heterostructure potential





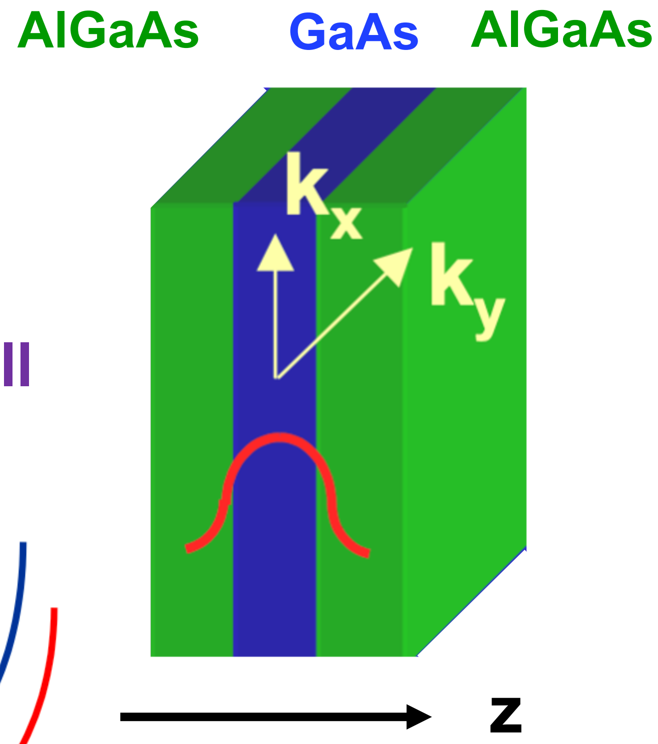
$$\Delta E \propto \frac{1}{L^2}$$

$$\Delta E > kT \Leftrightarrow L < \lambda \approx \frac{h}{\sqrt{2m^* kT}} \simeq 30 \text{ nm}$$

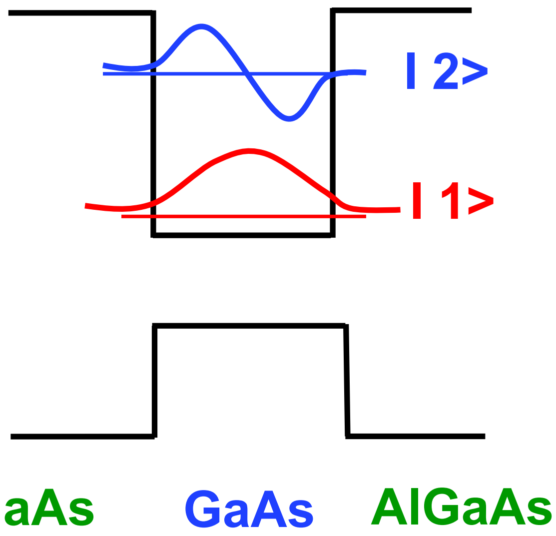
Electrons in GaAs at 300K

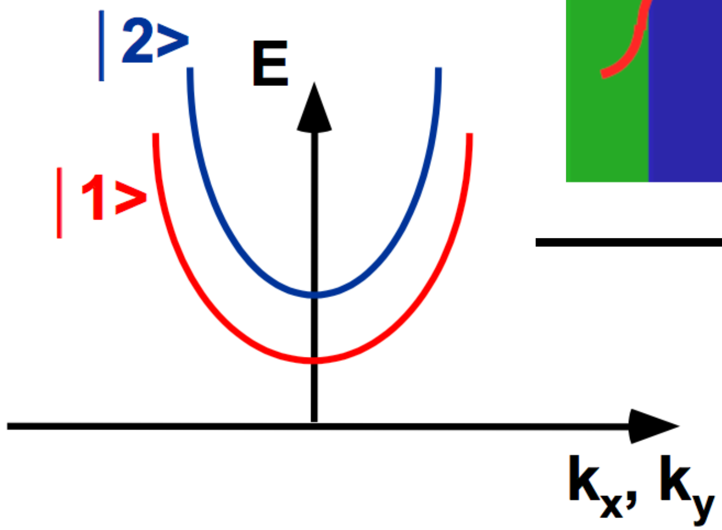
Quantization, what for?

Shape of density of states (gain spectral width)
Number of states (transparency current)
Carrier confinement
Energy tuning



2D nanostructures: Quantum well



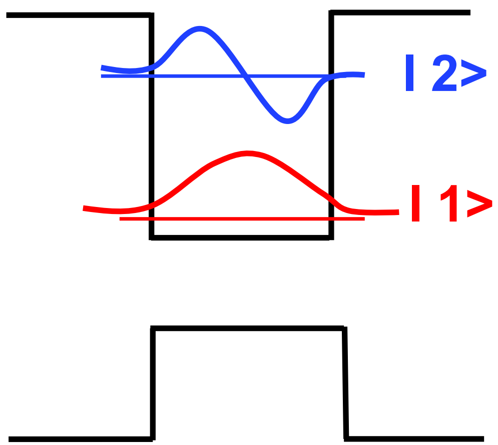


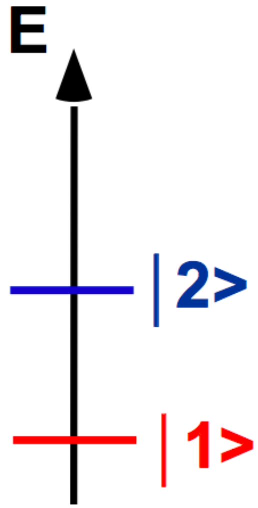
Continuum of energy states in two directions

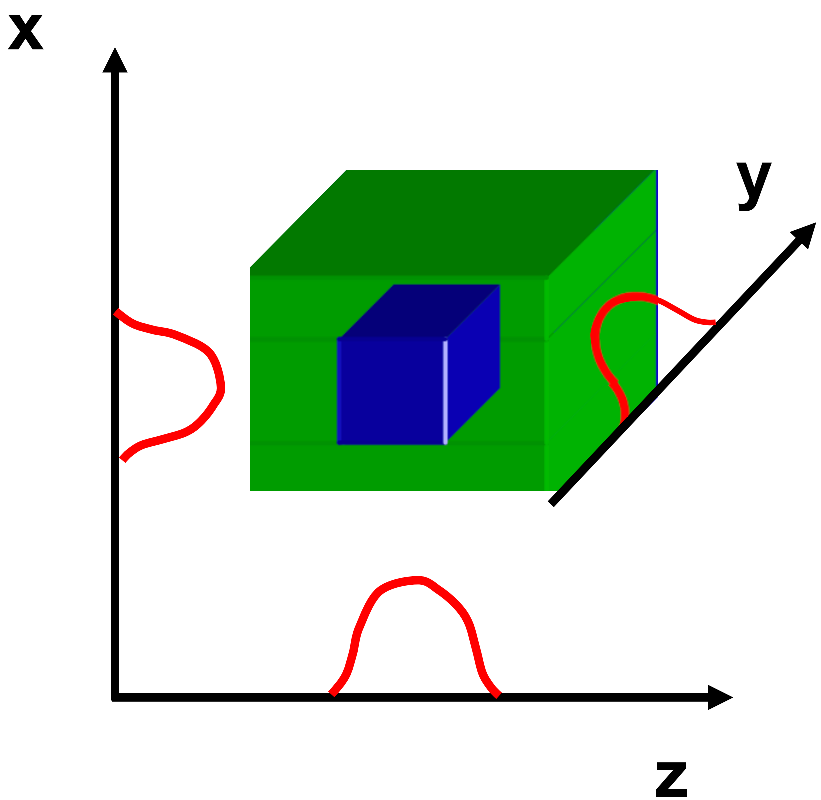
Quantization, what for?

Shape of density of states (gain spectral width)
Number of states (transparency current)
Carrier confinement
Energy tuning

0D nanostructure: Quantum dot



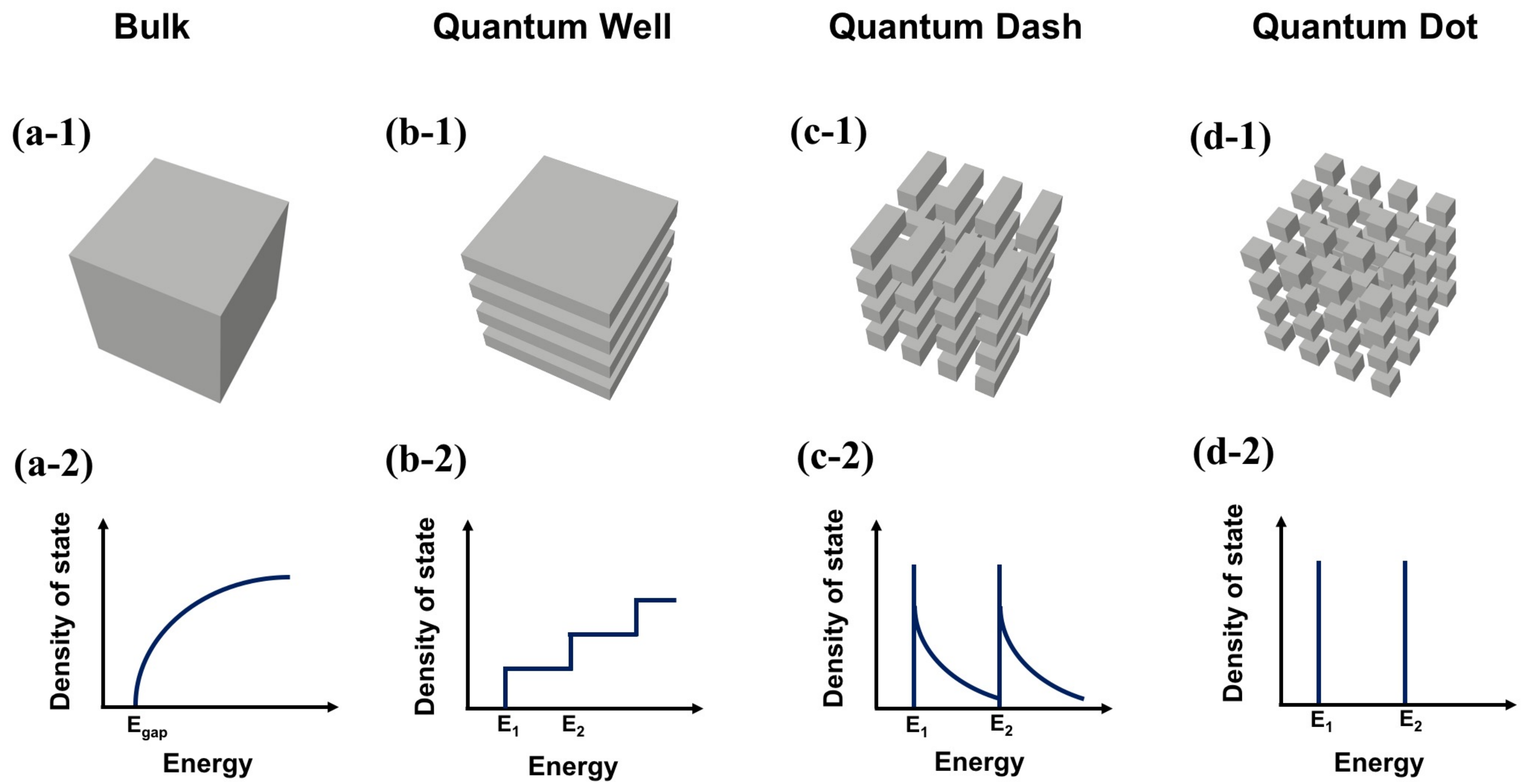




Only in 0D nanostructures, energy levels are completely discrete

→ semiconductor atoms

Quantization, what for?



$$\rho(E) \propto \sqrt{E - E_n} \longrightarrow \rho(E) \propto \delta(E - E_n)$$

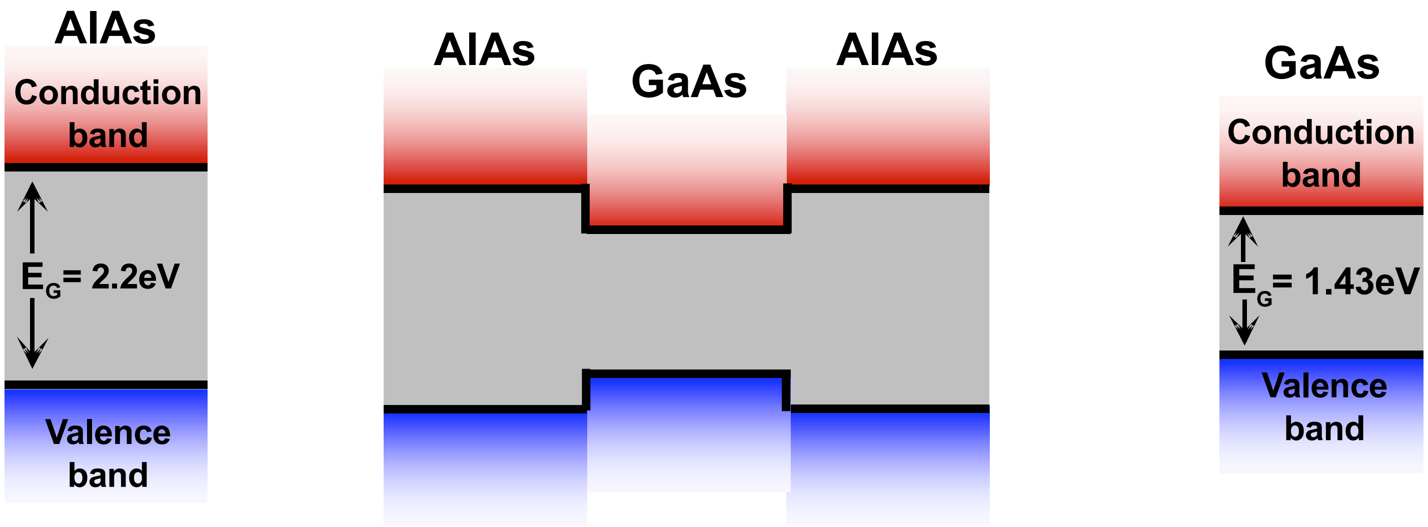
F. Grillot et. Al., Light: Sciences & Applications (2021)

Quantum well lasers

Principle: stick two different semiconductor materials

The lattice parameters must be compatible!

Example : GaAs = AlAs = 5.63Å



Quantum well
Small size double heterostructure (nm)

The quantum well is the building block of the quantum technologies

Quantum well lasers

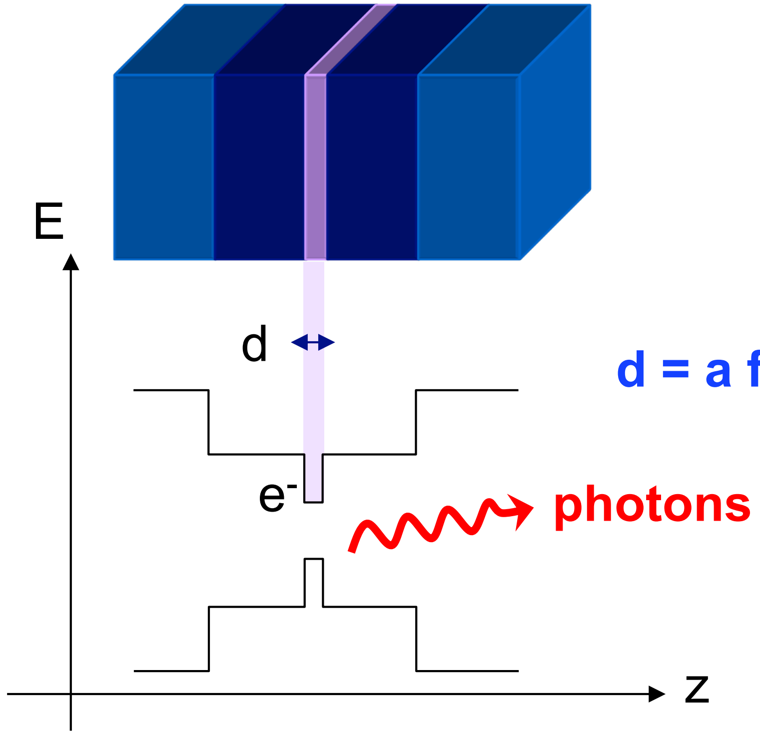
What is the optimal value for "d"?

Gain: the smaller "d", the better!

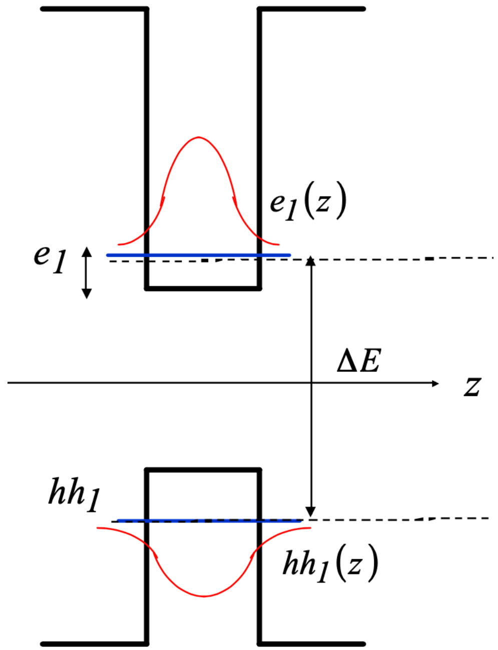
Guiding : If d ~ λ diffraction problem !

Schrödinger's equation

$$\frac{-\hbar^2}{2m}\frac{d^2}{dz^2}e_1(z) + V(z)e_1(z) = e_1\ e_1(z)$$

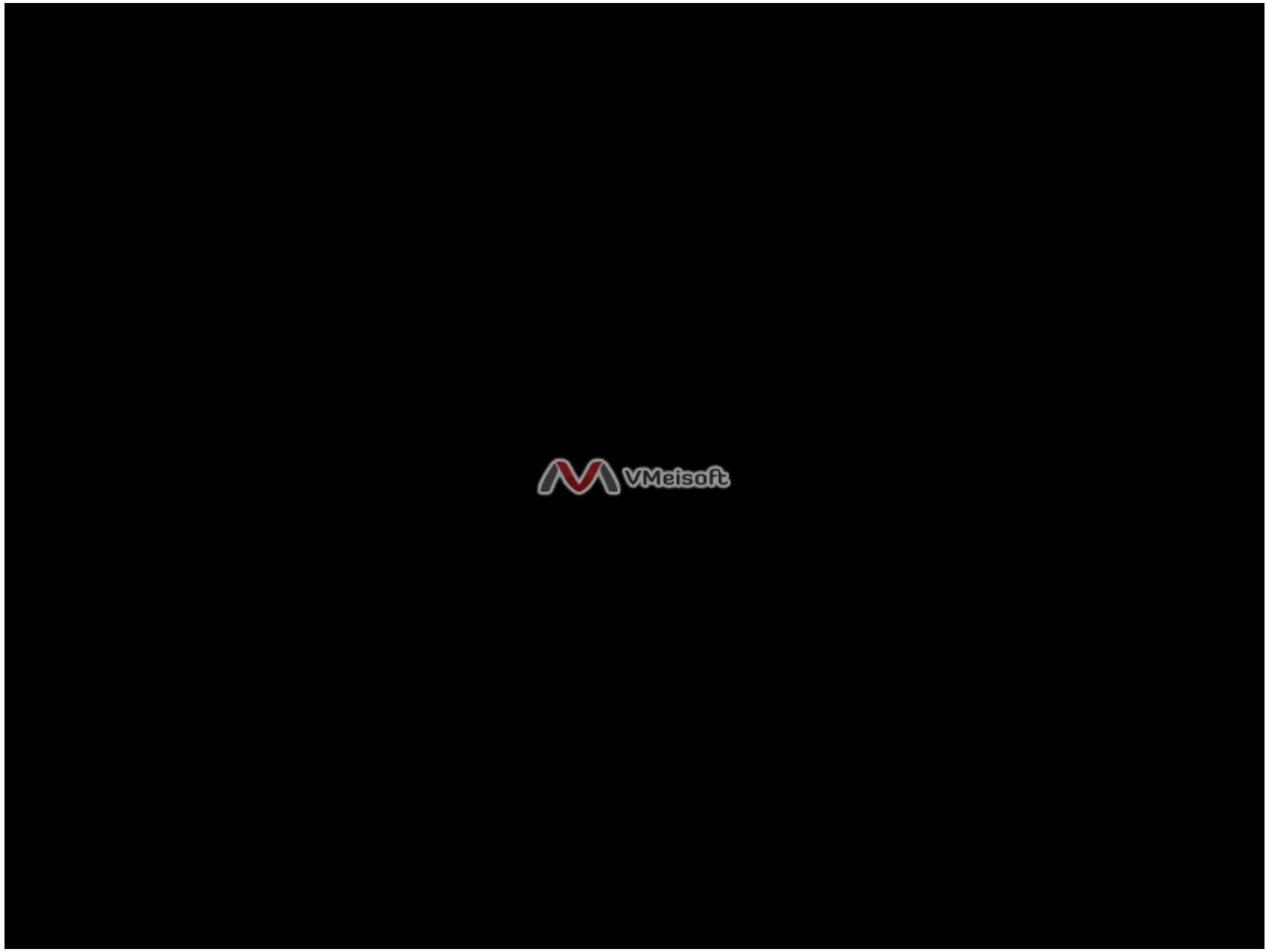


d = a few nm ≈ $\lambda_{\text{De Broglie}}$

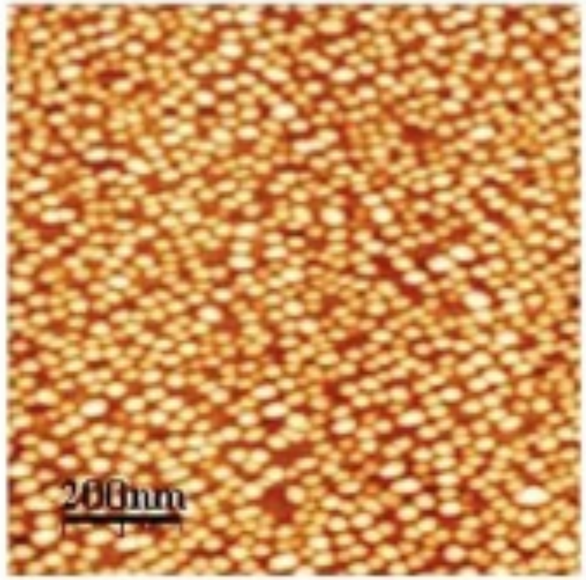


→ Enhanced performance e.g., large output power, high-temperature operation, very long lifetime,…

Quantum dot lasers

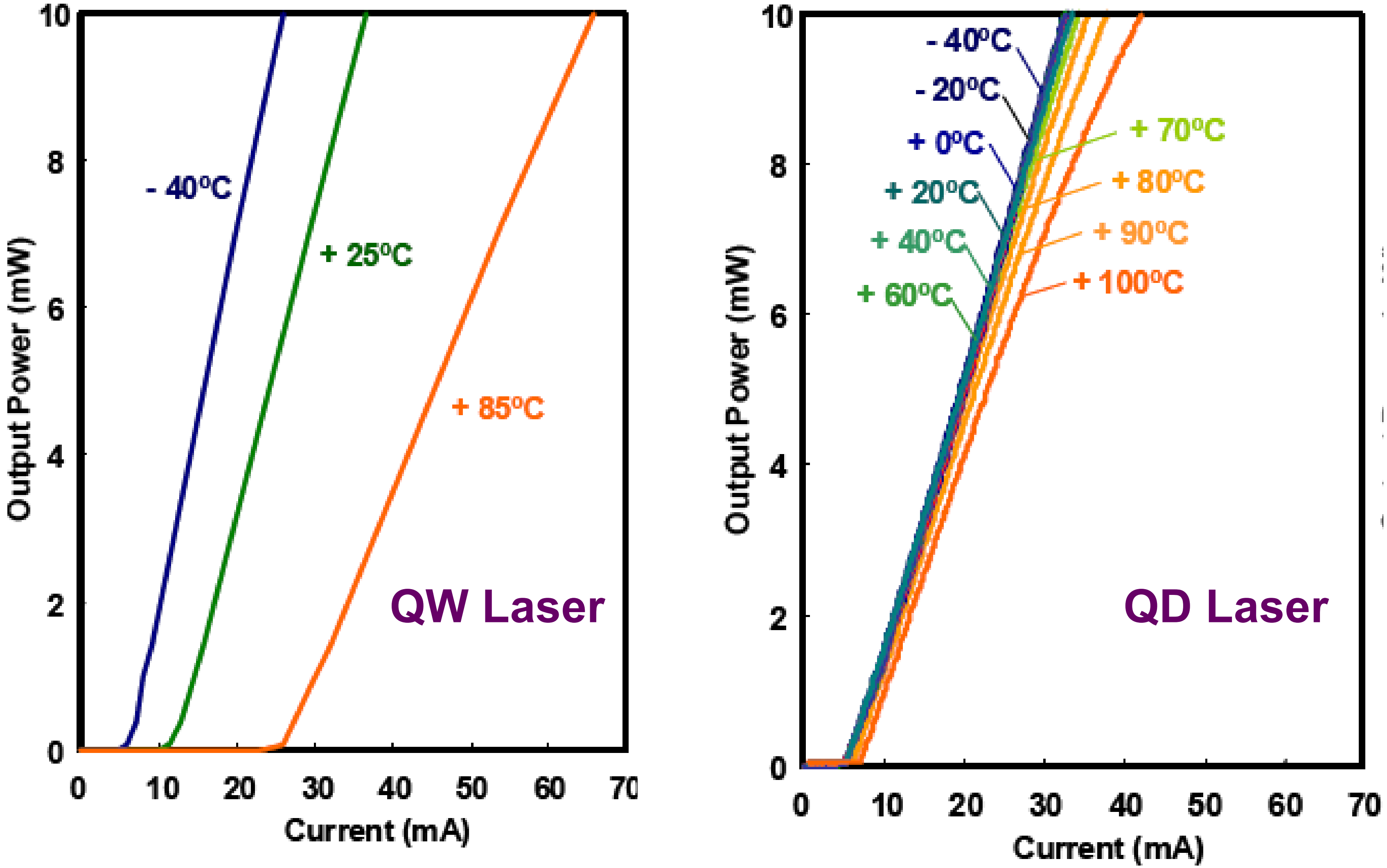


InAs QDs

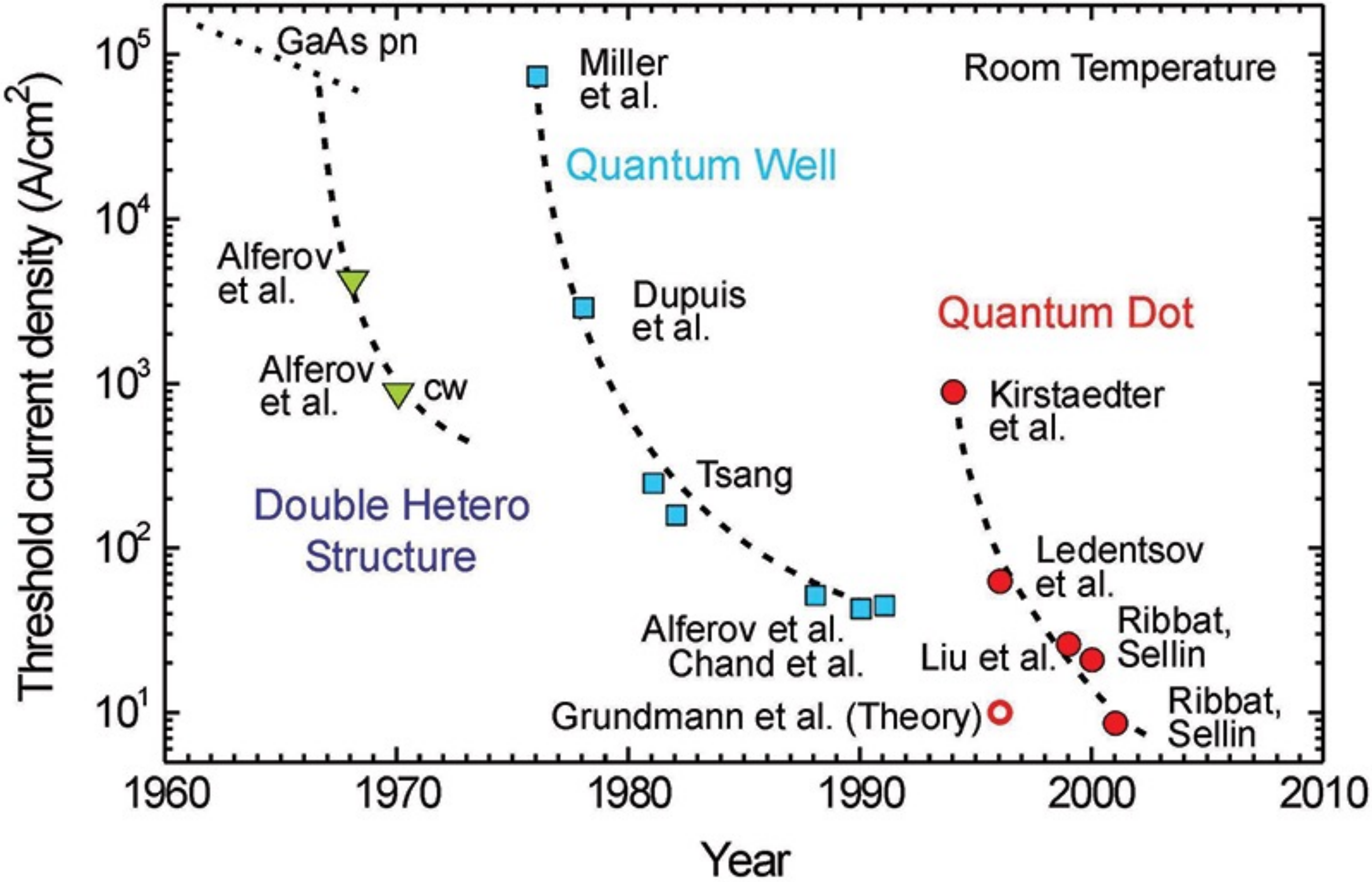


→ Electrons, holes are confined in a tiny structure, so the carriers can hardly move, even at elevated temperatures

Impact of quantization



Impact of quantization



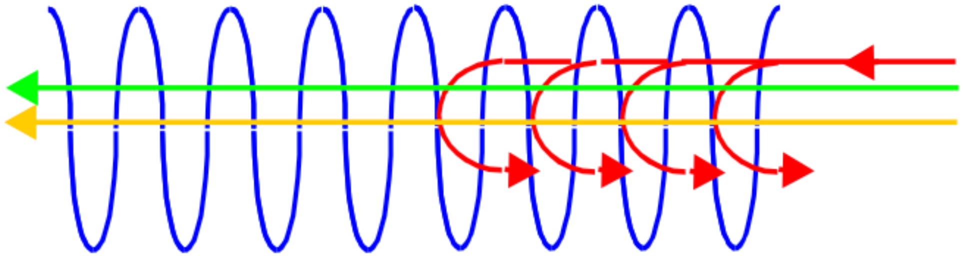
Single-mode laser (DFB)

DFB lasers are designed to experience internal feedback all along the gain ridge → no end mirrors are needed

Periodic perturbations of the refractive index along the laser cavity provide the frequency-selective feedback

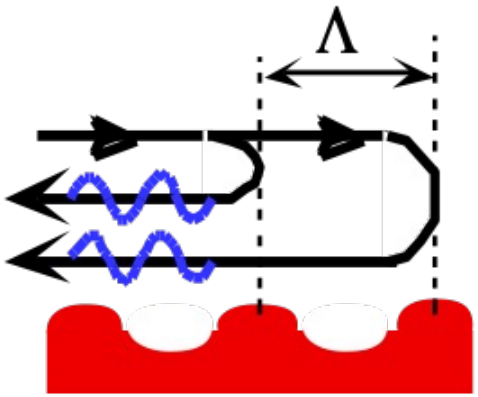
Fabry-Perot
Localized reflection
λ-independent
Many modes

Distributed feedback
Distributed reflection
λ-dependent - Phase π/2
1 or 2 modes

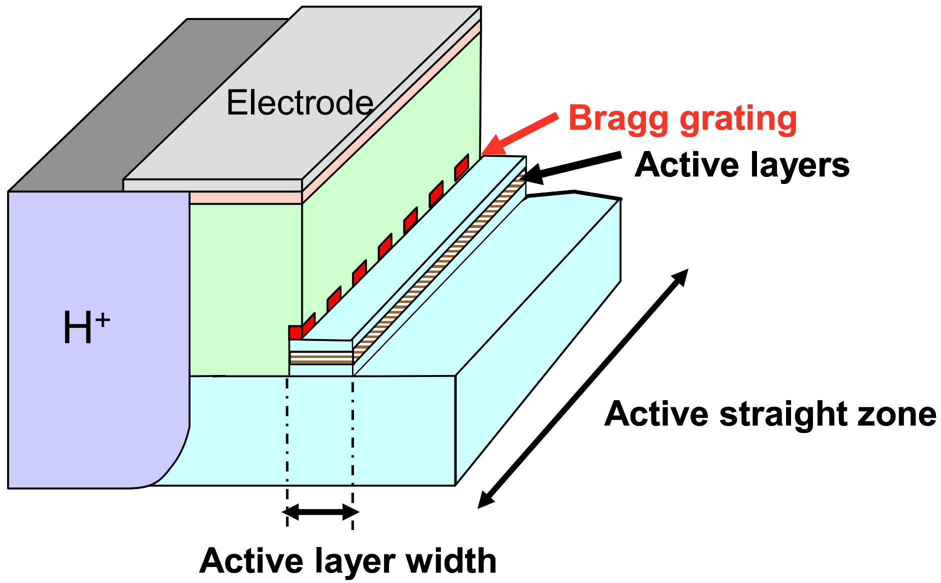


H. Kogelnik and C. V. Shank, J. Appl. Phys. (1972)

Single-mode laser (DFB)

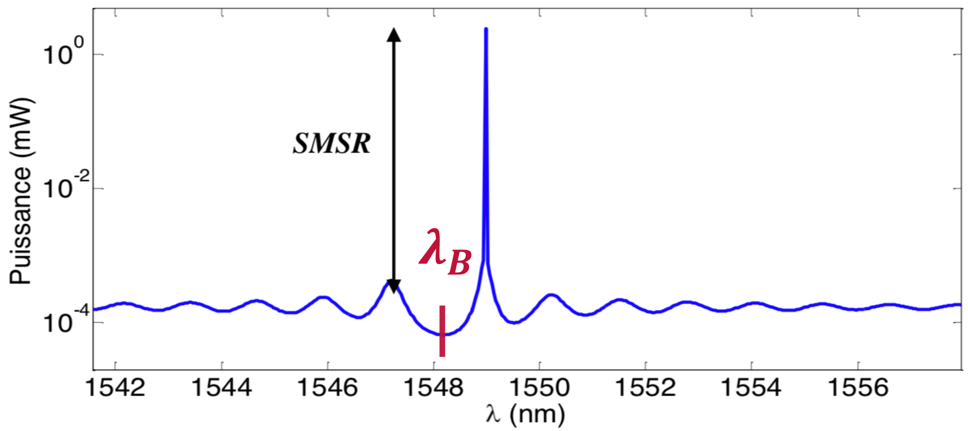


$$q(\lambda_B/2n) = \Lambda$$



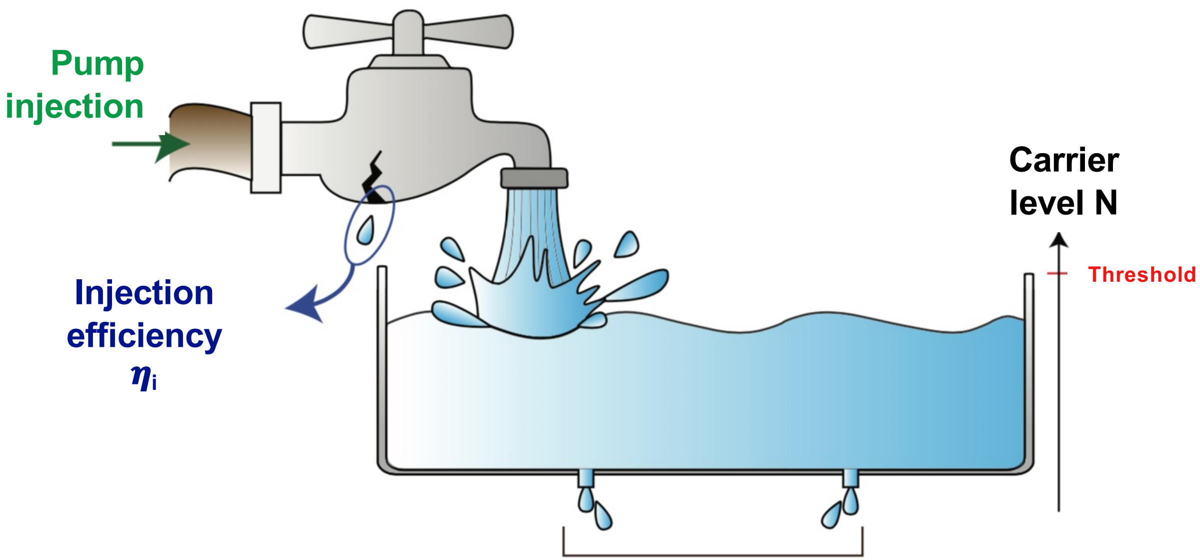
Active zone

InAsP QWs	QW Width	Barrier Width	QW Strain	Barrier strain
9	8 nm	8 nm	+1%	-0.6%



Corpuscular equations

Electric field $E(t) = \sqrt{S(t)}\, e^{-j\phi(t)}$ $S = |E|^2$



Loss : non-radiative recombination and spontaneous emission R_{sp}

N : carrier number
S : photon number

$$\frac{dN}{dt} = \frac{\eta_i I}{q} - \frac{N}{\tau_c} - G_N(N - N_t)S$$

$$\frac{dS}{dt} = G_N(N - N_t)S - \frac{S}{\tau_p} + R_{sp}$$

$$\frac{d\phi}{dt} = \frac{\alpha_H}{2} G_N\left(N - N_t - \frac{1}{\tau_p}\right) = \frac{\alpha_H}{2} G_N(N - N_{th})$$

* Here we consider η_i=1

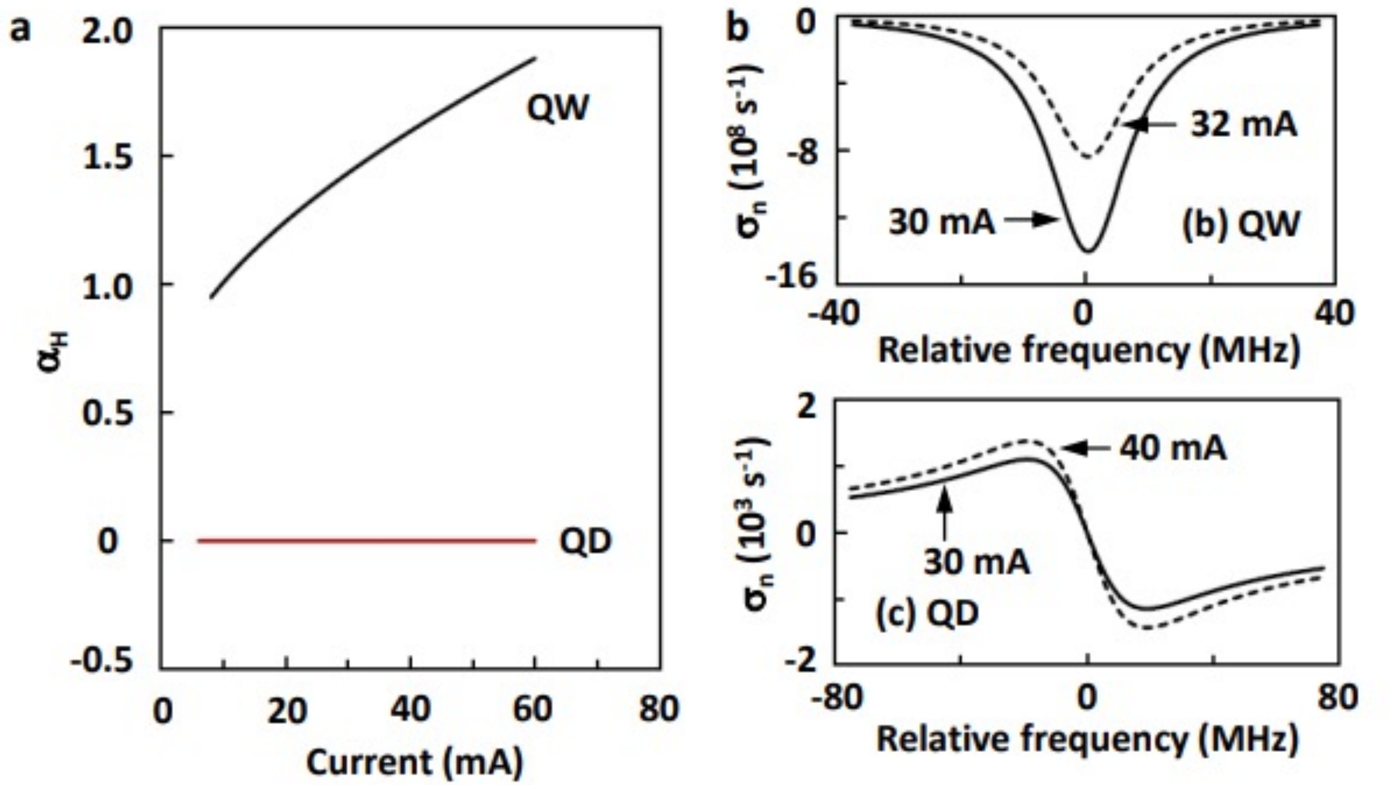
** Linewidth broadening factor α_H

L. A Coldren, S. W Corzine. Diode lasers and photonic integrated circuits, John Wiley & Sons (2012)

Linewidth broadening factor

Carrier density induced gain (g_n) and refractive index changes (σ_n)

$$\alpha_H = \frac{2\,\delta\sigma_n/dN}{\delta g_n/dN}$$



Impact

Spectral linewidth

Phase noise

Nonlinear properties

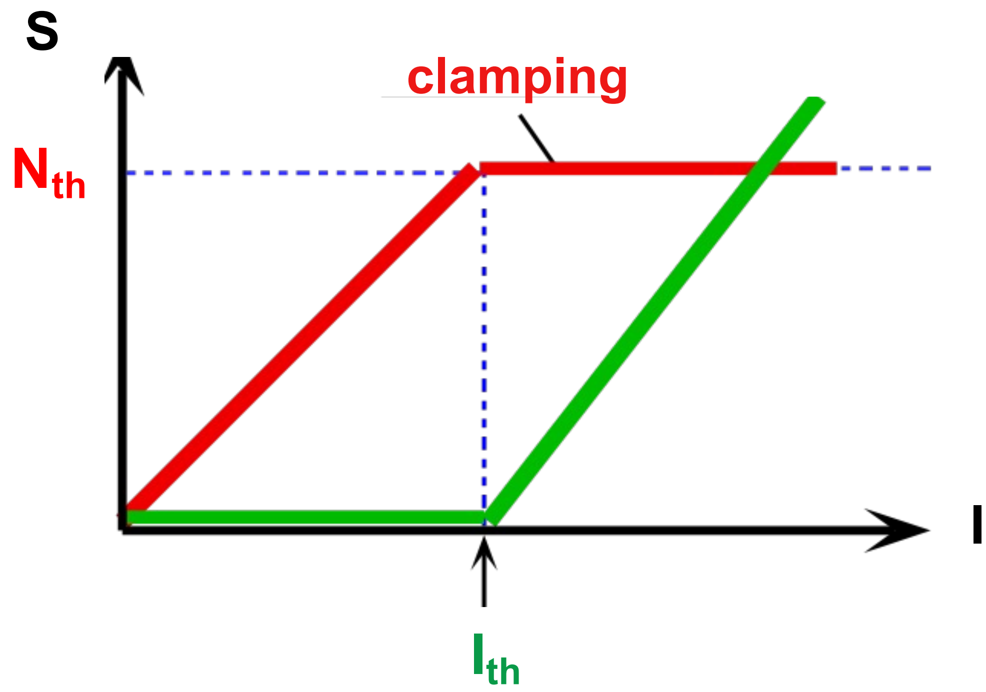
Modulation capabilities

F. Grillot et al. , IEEE Journal of Quantum Electronics (2008)

Clamping & saturation

$$\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_c} - G_N(N - N_t)S$$

$$\frac{dS}{dt} = G_N(N - N_t)S - \frac{S}{\tau_p} + R_{sp}$$



Gain (linear approximation)

$$G(N) = G_N(N - N_t)$$

Cavity frequency

$$\omega(N) = \omega_{th} + \omega_N(N - N_{th})$$

Carrier density at transparency

→ N > N_t: luminescence (G>0)

→ N < N_t: absorption (G<0)

Threshold ($N=N_{th}$): carrier clamping

→ Gain = $\sum \text{Loss}$

→ $G(N_{th})=G_{th}$

Relaxation oscillations

Relaxation oscillation: beating between carrier and photon populations

Semiconductor lasers display very large relaxation frequencies (>GHz) allowing very high-speed modulation

	Population lifetime (carrier lifetime) [s]	Photon lifetime [s]	Relaxation oscillation frequency [Hz]
Semiconductor lasers	10^{-9}	10^{-12}	$\sim 10^{9}$
Solid-state lasers	10^{-3}	10^{-9}	$\sim 10^{5}$
Gas lasers	10^{-8}	10^{-7}	$\sim 10^{6}$

Relaxation oscillations

$$\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_c} - G_N(N - N_t)S$$

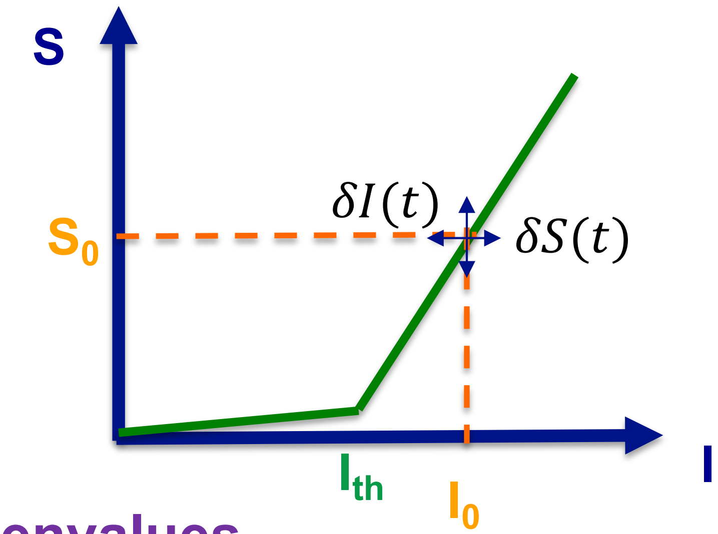
$$\frac{dS}{dt} = G_N(N - N_t)S - \frac{S}{\tau_p} + \cancel{R_{sp}}$$

Small perturbation of the steady-state

$$N(t) = N_0 + \delta N(t)$$

$$I(t) = I_o + \delta I(t)$$

$$S(t) = S_0 + \delta S(t)$$

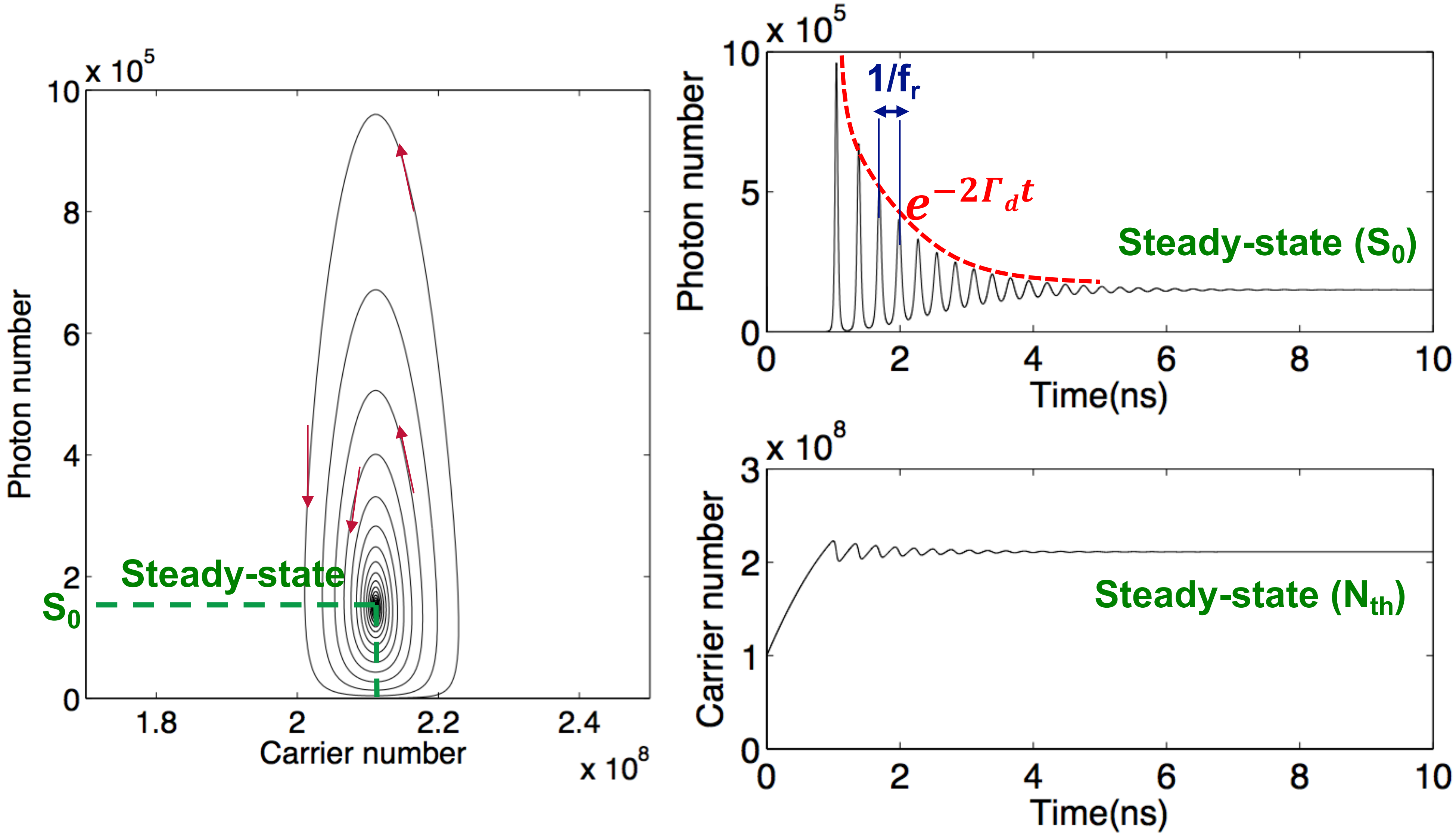


$$\begin{pmatrix} \delta\dot{N} \\ \delta\dot{S} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\tau_c} - G_N S_0 & -\frac{1}{\tau_p} \\ G_N S_0 & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta S \end{pmatrix}$$

The evolution matrix gives access to the eigenvalues

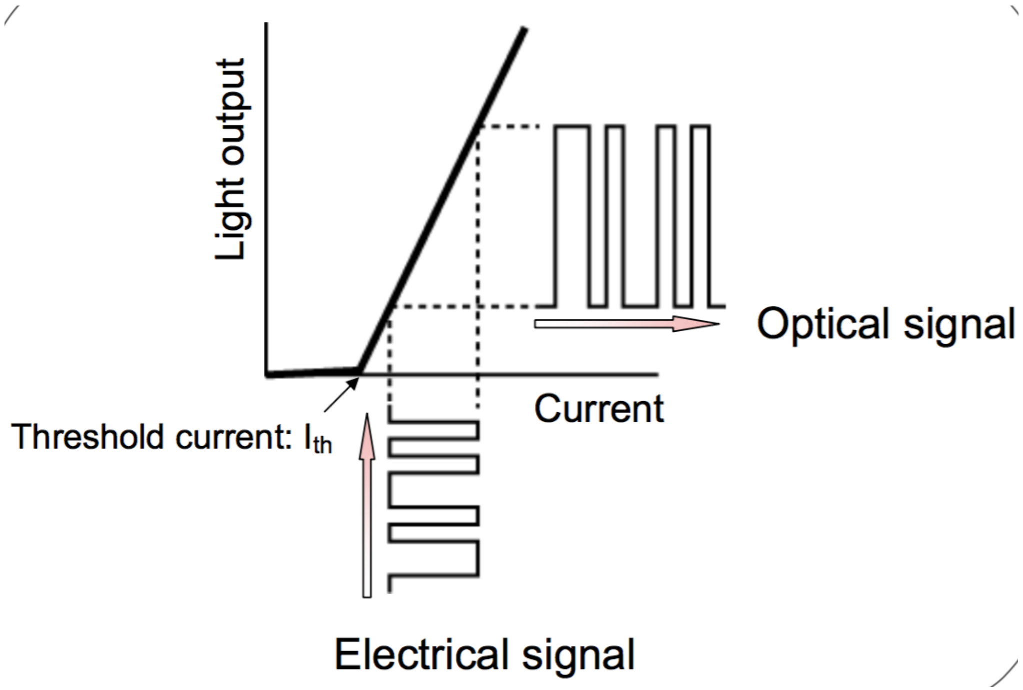
$$f_{\pm} = -\Gamma_d \pm \sqrt{\Gamma_d^2 - f_r^2}$$

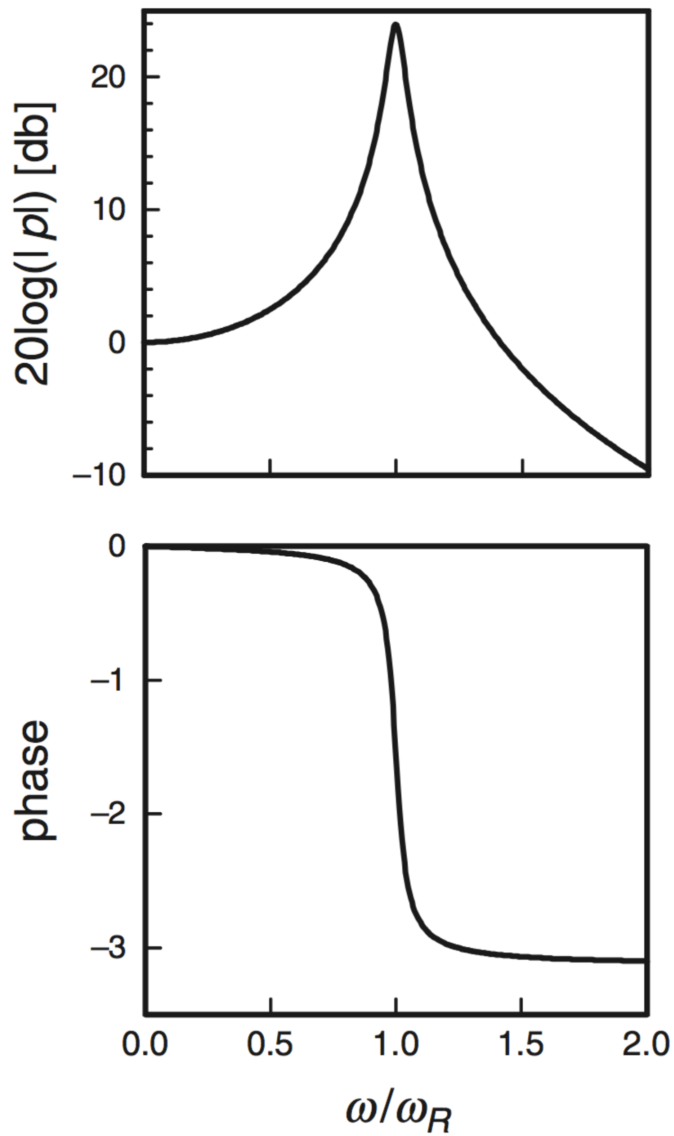
Relaxation oscillations



T. Erneux and P. Glorieux, Laser Dynamics, Cambridge

Modulation dynamics





Laser's modulation response

$$H(j\omega)=\frac{\delta S}{\delta I}=\frac{1}{1-\omega^2/\omega_r^2+2j\omega\Gamma_d/\omega_r^2}$$

$$\phi(j\omega)=\frac{\delta\phi}{\delta I}=tan^{-1}\left(-\frac{2\omega\Gamma_d}{\omega_r^2-\omega^2}\right)$$

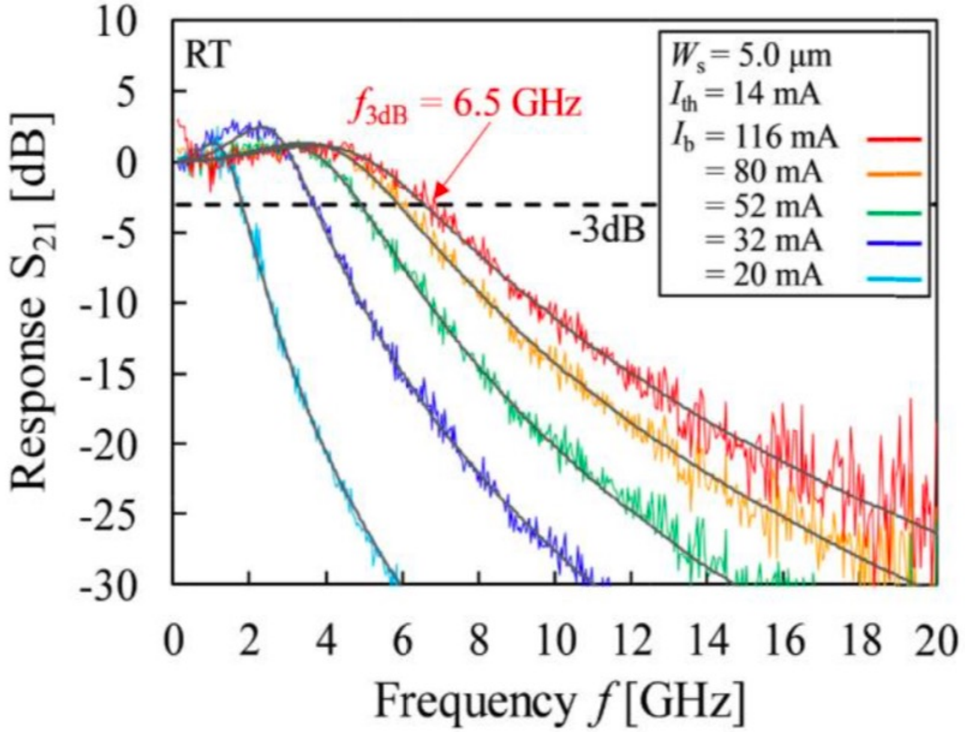
Modulation dynamics

The modulation response peaks at the relaxation oscillation frequency

$$f_r^2 = \frac{1}{4\pi^2}\frac{G_N S_0}{\tau_p}$$

The 3-dB modulation bandwidth scales with the oscillation frequency

$$f_{3dB} = f_r\sqrt{1+\sqrt{2}}$$



At high bias & high modulation frequencies, the modulation response falls down (damping effect, carrier transport across the junction)

Modulation dynamics

How to increase the modulation speed ? Need to enhance the relaxation oscillation frequency & the 3dB modulation bandwidth

1) Increasing the photon density via a better confinement of the optical field in the active layer

2) Biasing the laser at a higher pump current

3) Increasing the differential gain G_N coefficient by cooling the device, doping active areas or by using quantum confined structures (wells, dots)

4) Reducing the photon lifetime by decreasing the laser cavity length

Frequency chirp

Input data 101010 → Laser → Output data

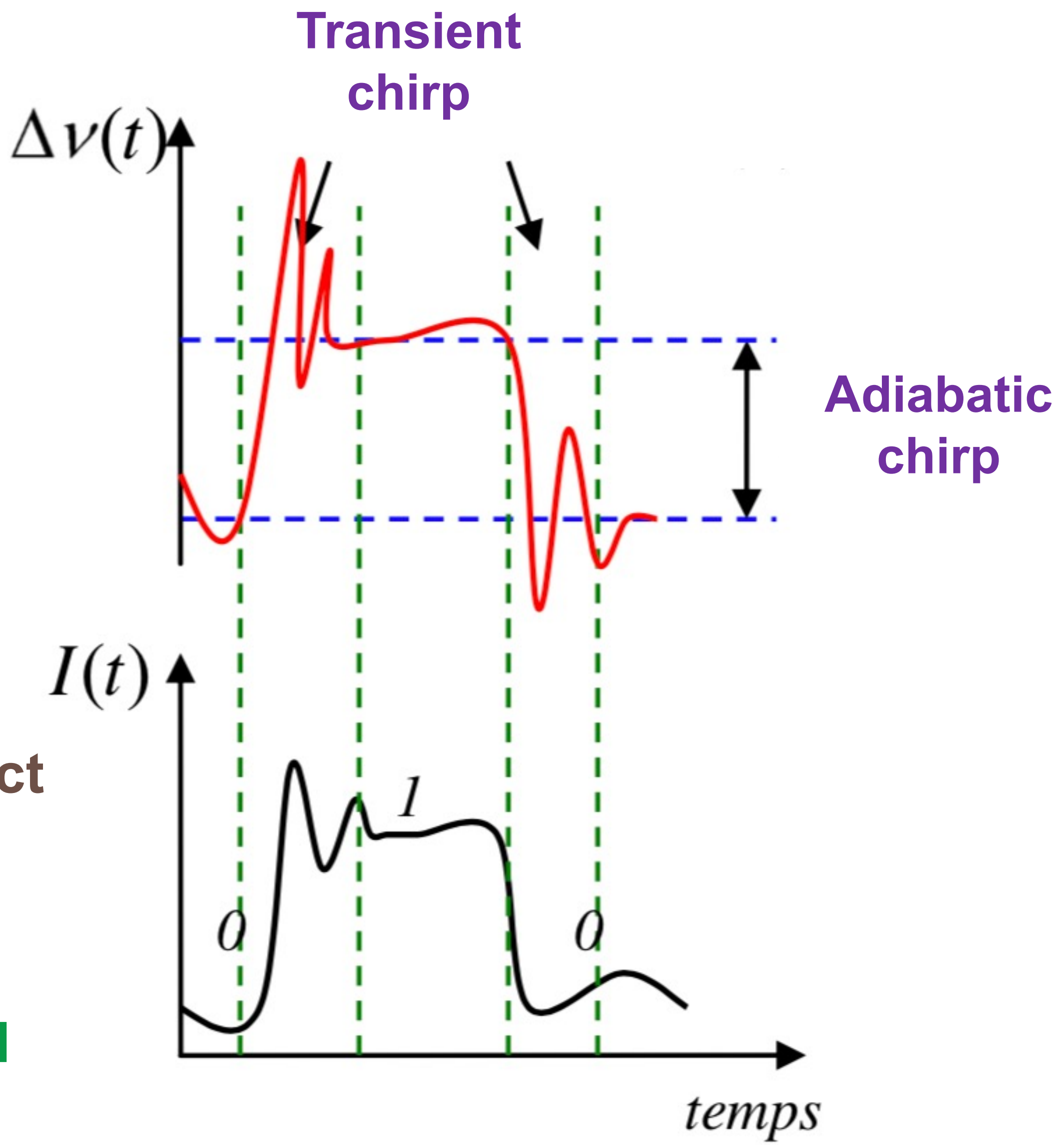
Transient | Adiabatic

$$\Delta\nu = \frac{\alpha_H}{4\pi}\left(\frac{d}{dt}\ln S(t) + \kappa S(t)\right)$$

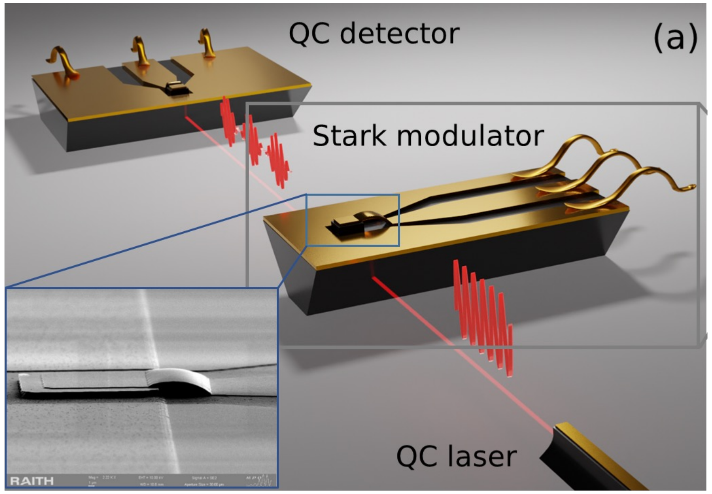
Chirp only occurs under direct modulation

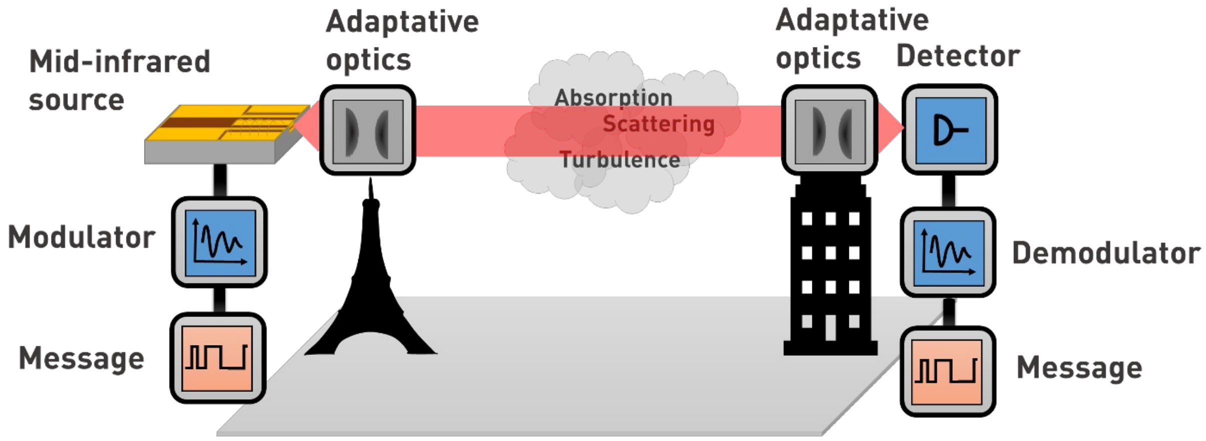
$\alpha_H = 0$ → chirpfree lasers

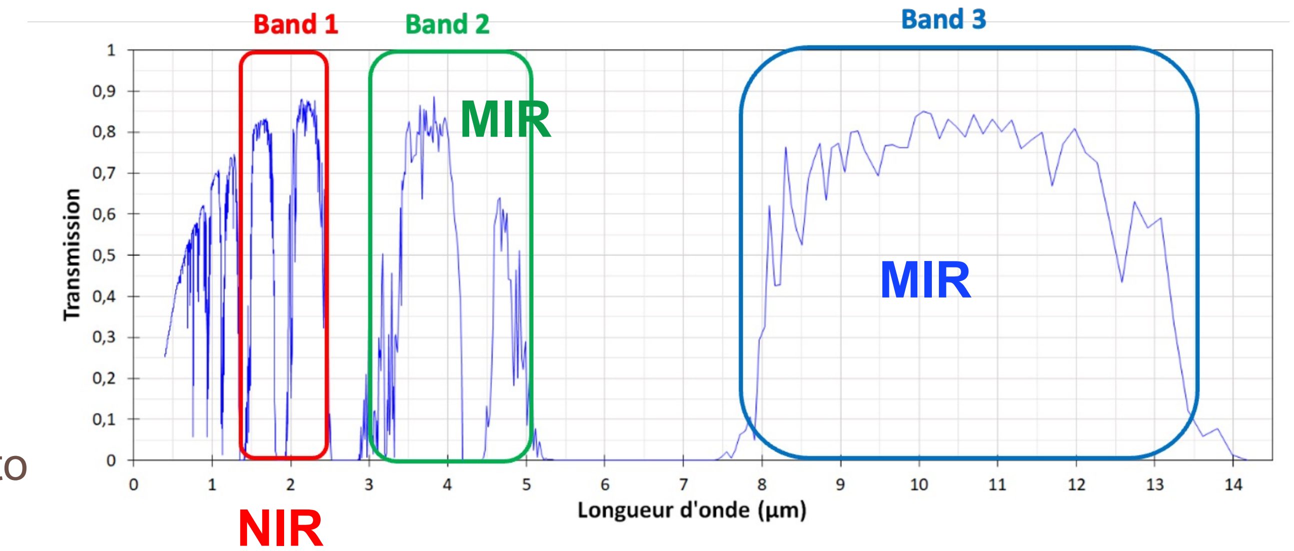
Data transmission not degraded



Unipolar quantum optoelectronics







Mid infrared (MIR) free-space laser communications

Low absorption

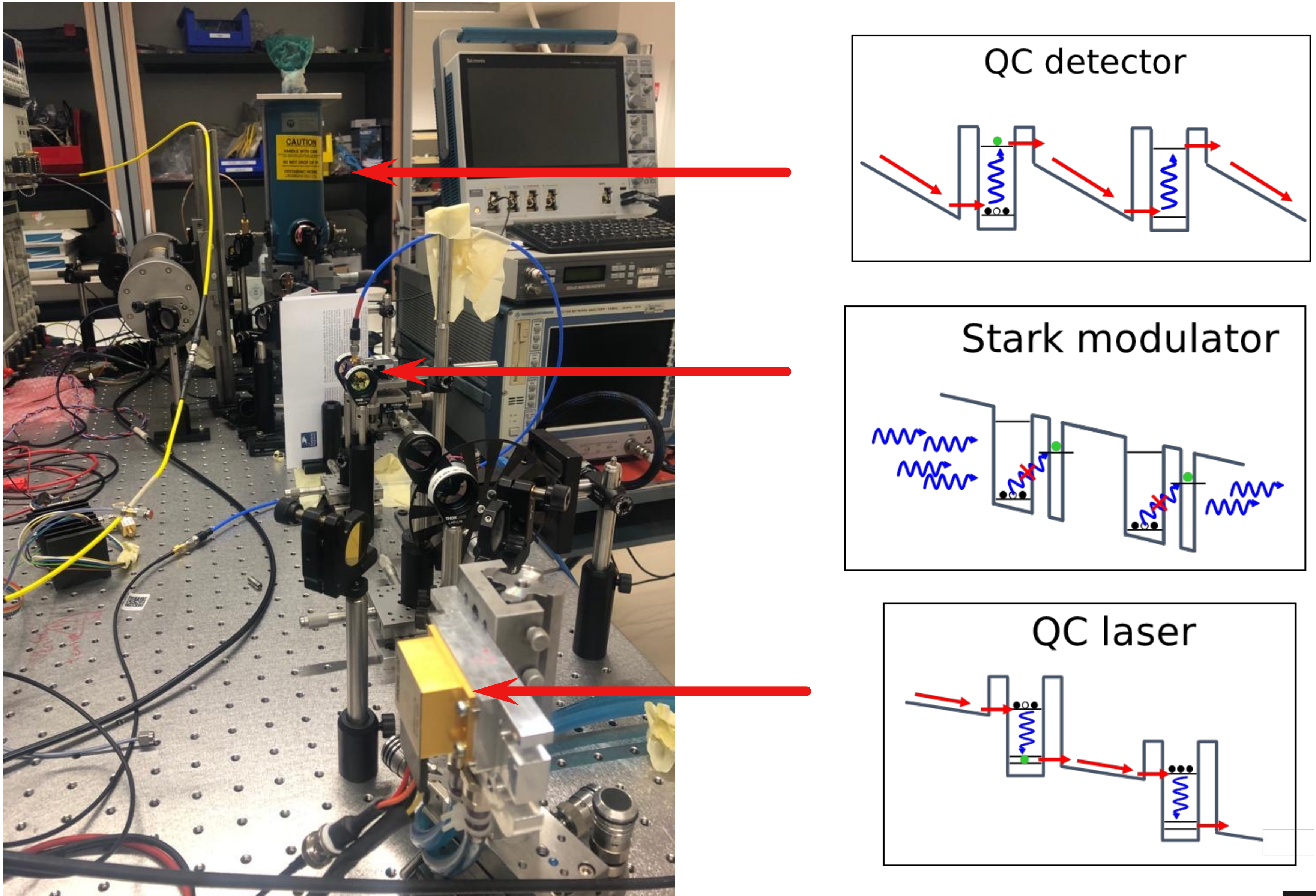
Resistance to turbulence

Low scattering

Stealth

Enhanced performance compared to near infrared (NIR)

Unipolar quantum optoelectronics

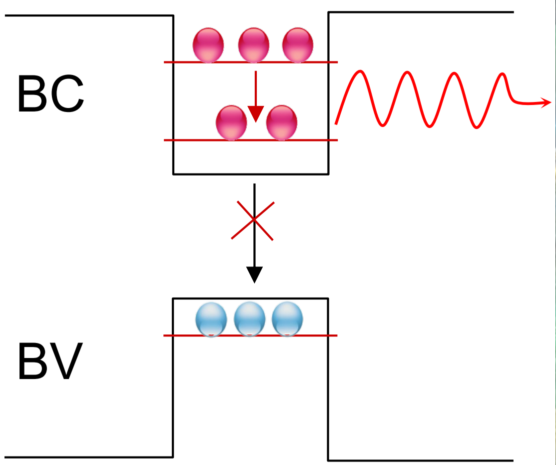


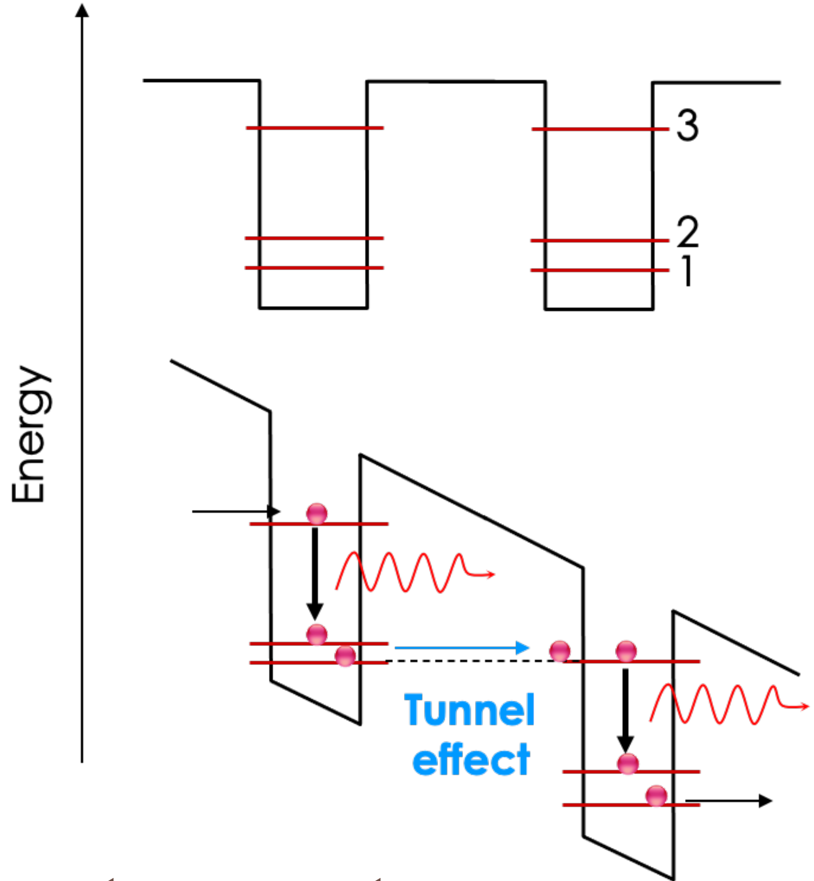
Quantum cascade lasers

Greatly tunable from 3 to 300 microns optical wavelength

Short carrier lifetime (intersubband process with picosecond transitions)

Unipolar optoelectronics: only electrons are involved



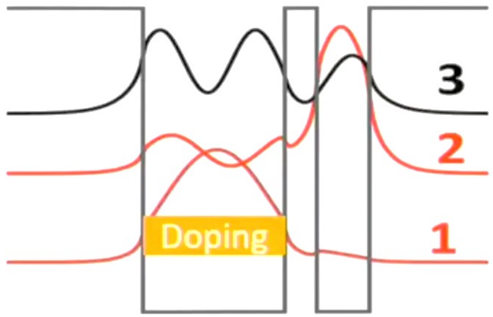


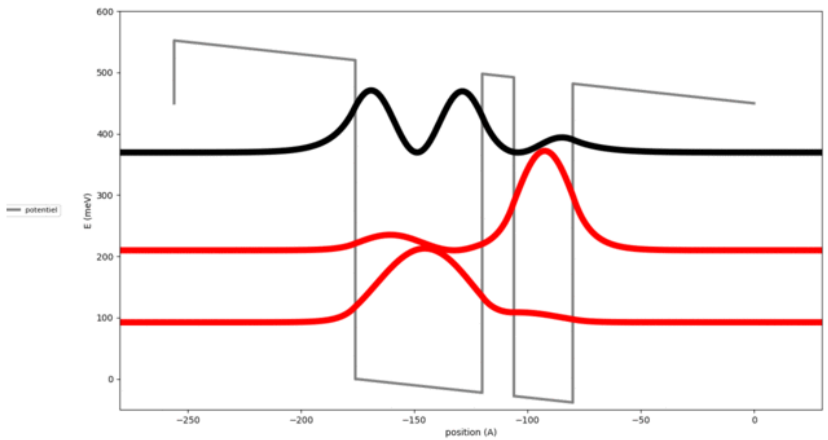
Applications : free-space communications, spectroscopy, etc.

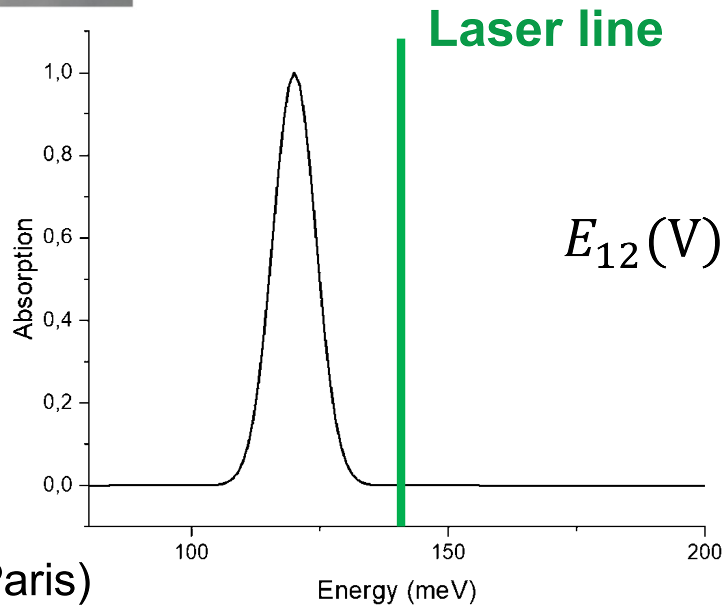
Stark Modulator

Asymmetric coupled quantum wells
An external electric field affects the absorption energy of the 1-2 transition
Applied AC voltage provides amplitude modulation of the transmitted light

$$\boldsymbol{E_{12}(V) \neq E_{12}(0)}$$







Animation made by Thomas Bonazzi (ENS Paris)

Stark Modulator

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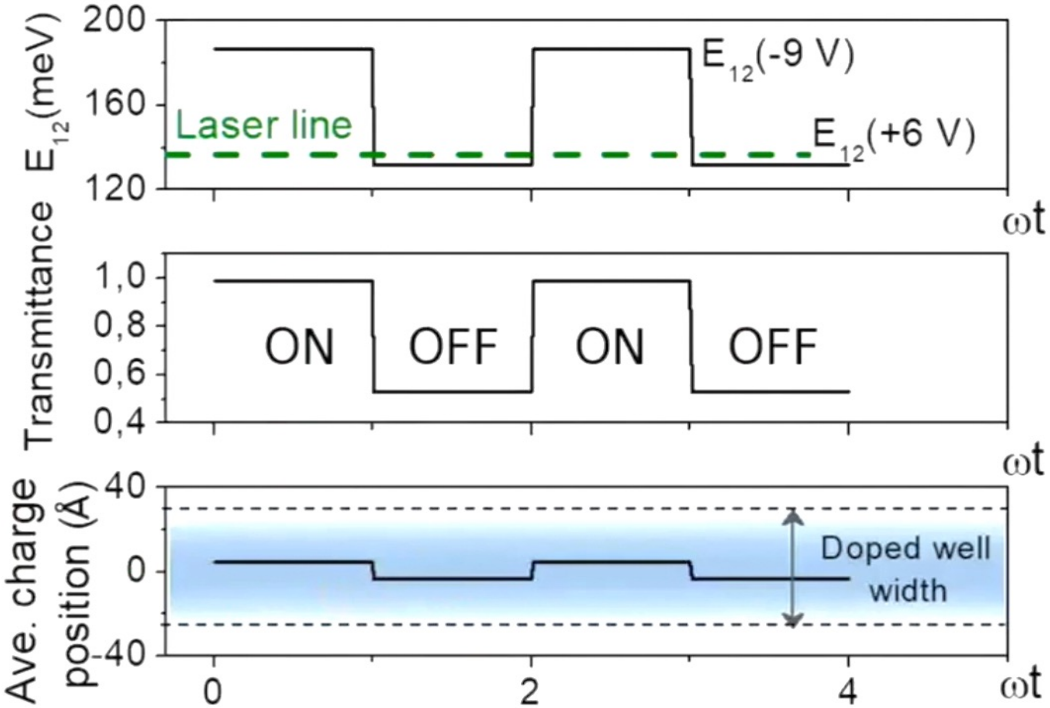
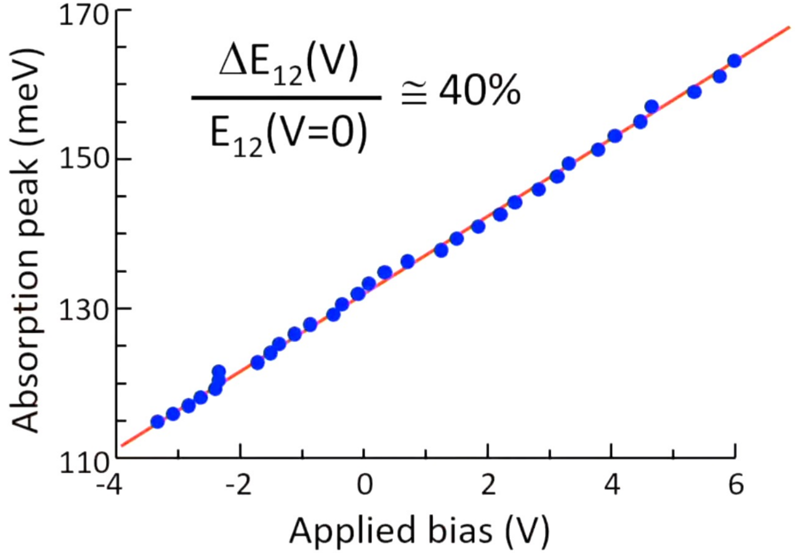
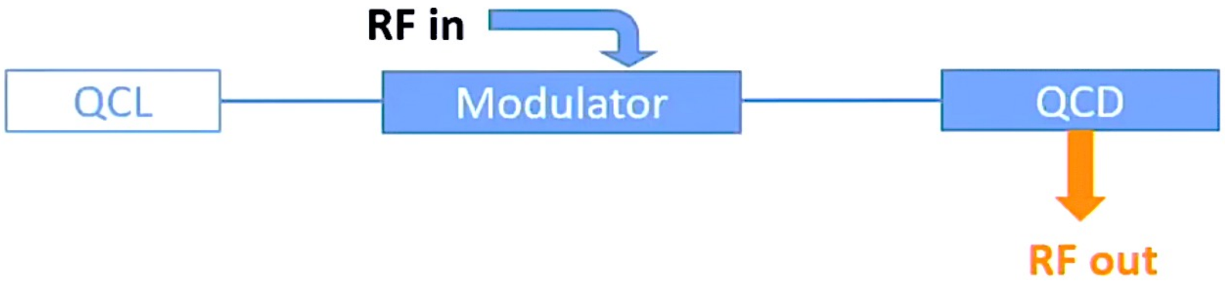


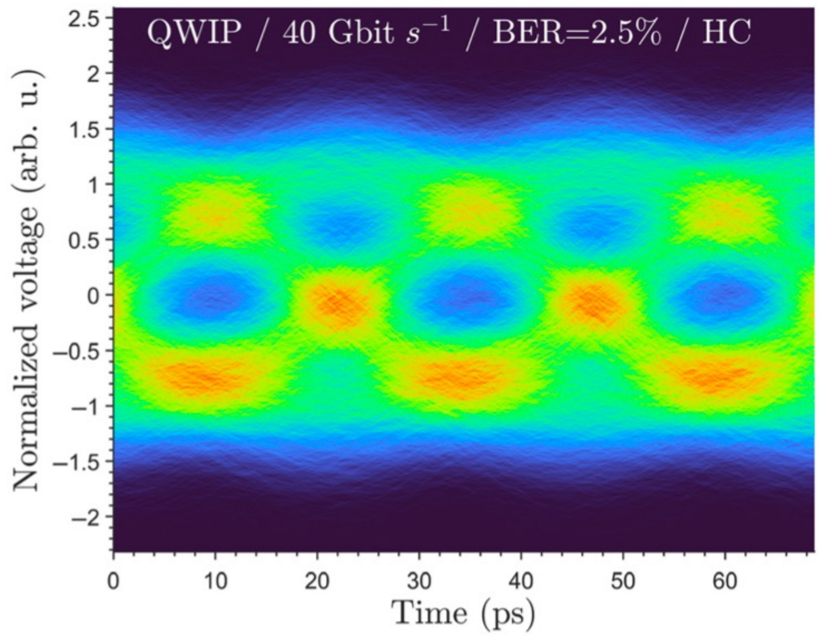


H. Dely et al., Laser & Photonics Reviews (2021)

High-speed data transfer



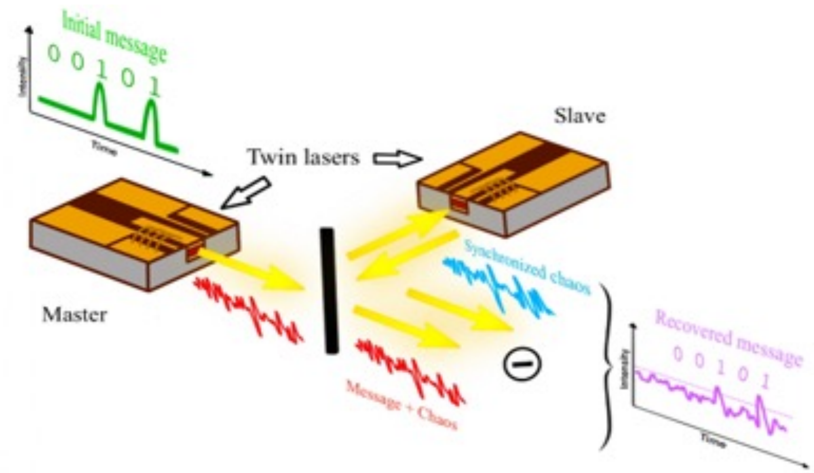
	Attenuation (dB/km)		
Visibility (km)	1550 nm	4000 nm	9000 nm
20 (Clear)	0.2	0.06	0.02
5 (Haze)	1.2	0.47	0.2
2 (Mist)	4	2	1.1
1 (Fog)	9.3	5.4	3.3



Budget link: L=1 km, P=30 mW output power, 4 mW received power

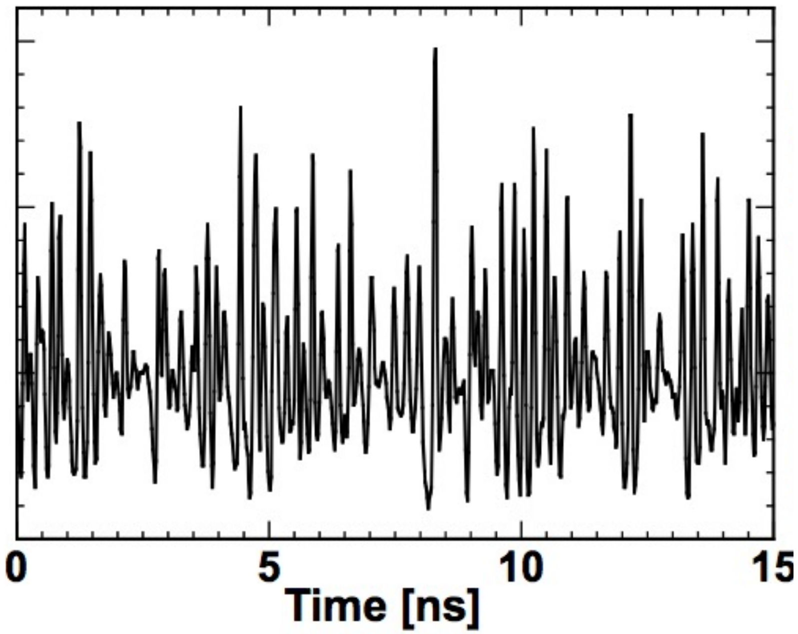
P. Didier et al., Advanced Photonics (2022)

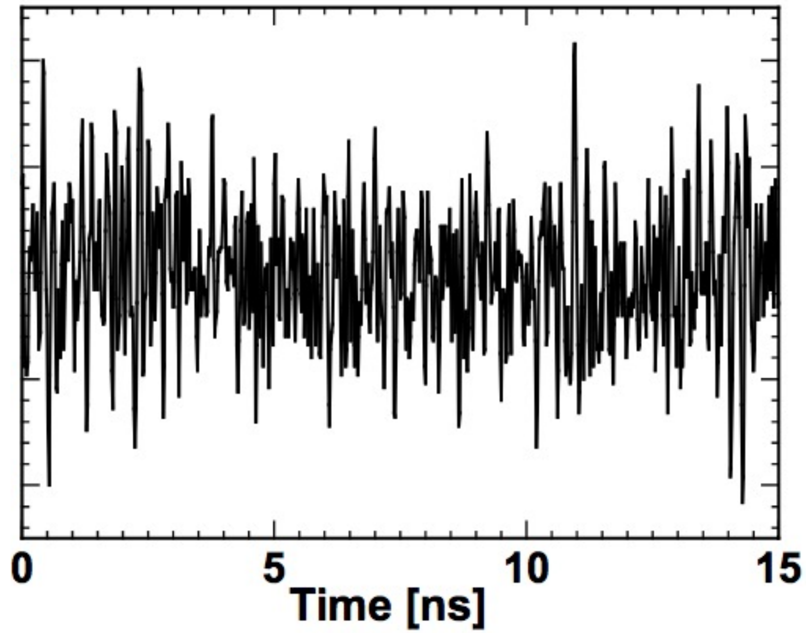
Non linear Dynamics & Chaos-based Applications of Semiconductor Lasers

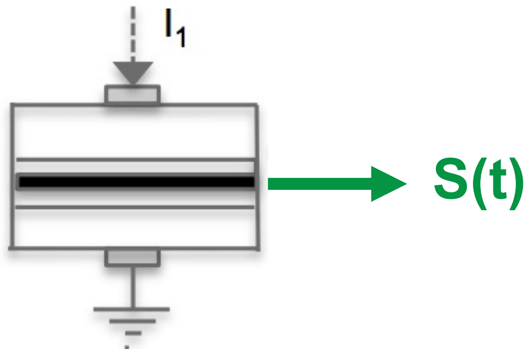


Chaos versus noise?

Temporal waveforms of laser intensity output in the diode laser

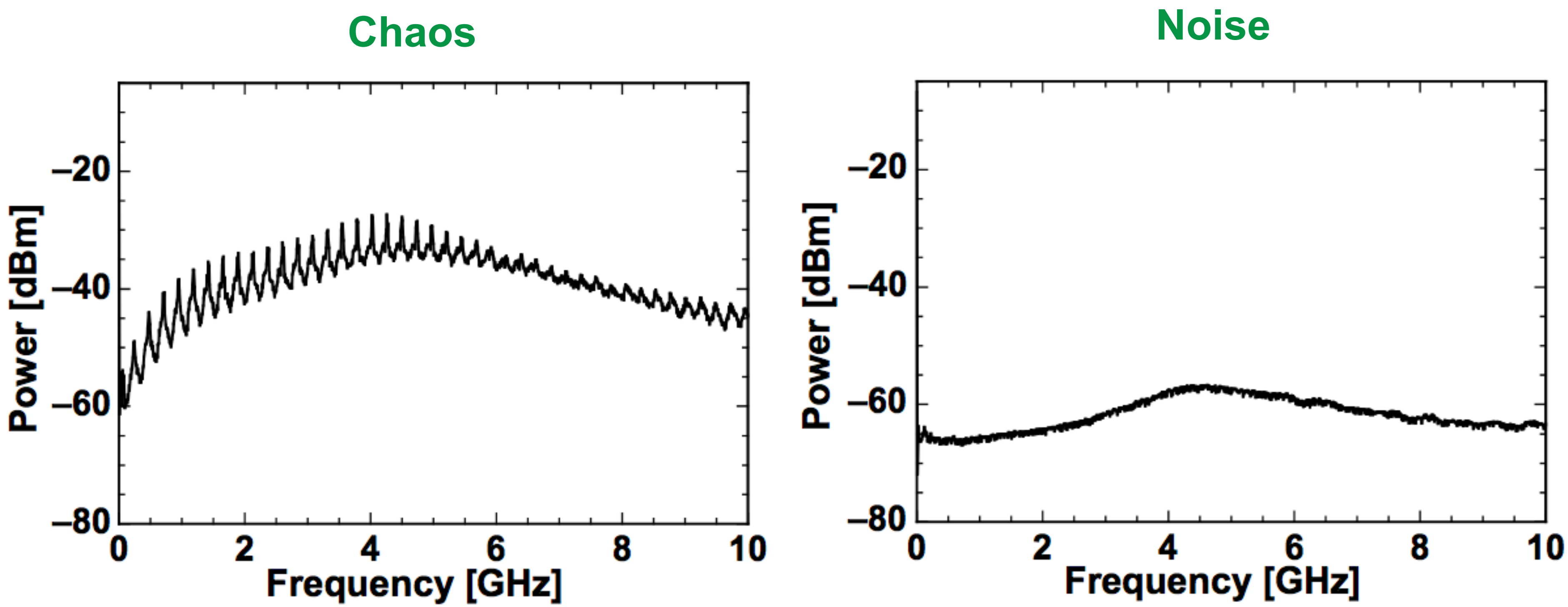






Chaos versus noise?

Fourier transform of the temporal waveforms



Chaos is fundamentally driven by a characteristic frequency of the nonlinear system that is to say for a diode laser, the so-called relaxation frequency as seen before

The three revolutions in Physics

Relativity theory

Quantum mechanics

Chaos theory

Chaos is not an invention but a real discovery!

Navier-Stokes equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \quad (1)$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho u v)}{\partial y} + \frac{\partial (\rho u w)}{\partial z} = -\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2)$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial (\rho u v)}{\partial x} + \frac{\partial (\rho v^2)}{\partial y} + \frac{\partial (\rho v w)}{\partial z} = -\frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (3)$$

$$\frac{\partial \rho w}{\partial t} + \frac{\partial (\rho u w)}{\partial x} + \frac{\partial (\rho v w)}{\partial y} + \frac{\partial (\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (4)$$

$$\frac{\partial \rho E}{\partial t} + \frac{\partial (\rho u E)}{\partial x} + \frac{\partial (\rho v E)}{\partial y} + \frac{\partial (\rho w E)}{\partial z} = -\frac{\partial p u}{\partial x} - \frac{\partial p v}{\partial y} - \frac{\partial p w}{\partial z} + S \quad (5)$$

where ρ is the air density, u, v, w are the components of the air's velocity, E is measure of the air's internal energy (which allows us to compute its temperature) and p is the air pressure.

These equations used for weather prediction can not be solved analytically and even numerically it requires the use of a super computer!

Brian J. Cantwell, Fundamentals of Compressible Flows, Course from Stanford University

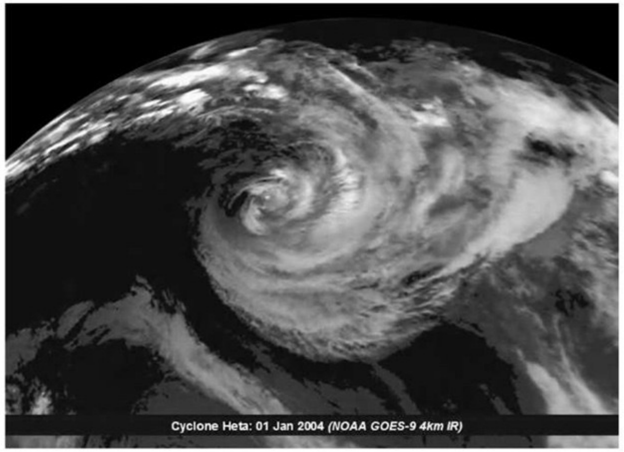
The Lorenz model

Lorenz gave rise to the modern field of chaos theory. His model consists on a simplified approach made with three coupled ordinary differential equations, describing the fluid convection and temperature

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = -xz + rx - y$$

$$\frac{dz}{dt} = xy - bz$$



Edward Lorenz
(1917-2008)

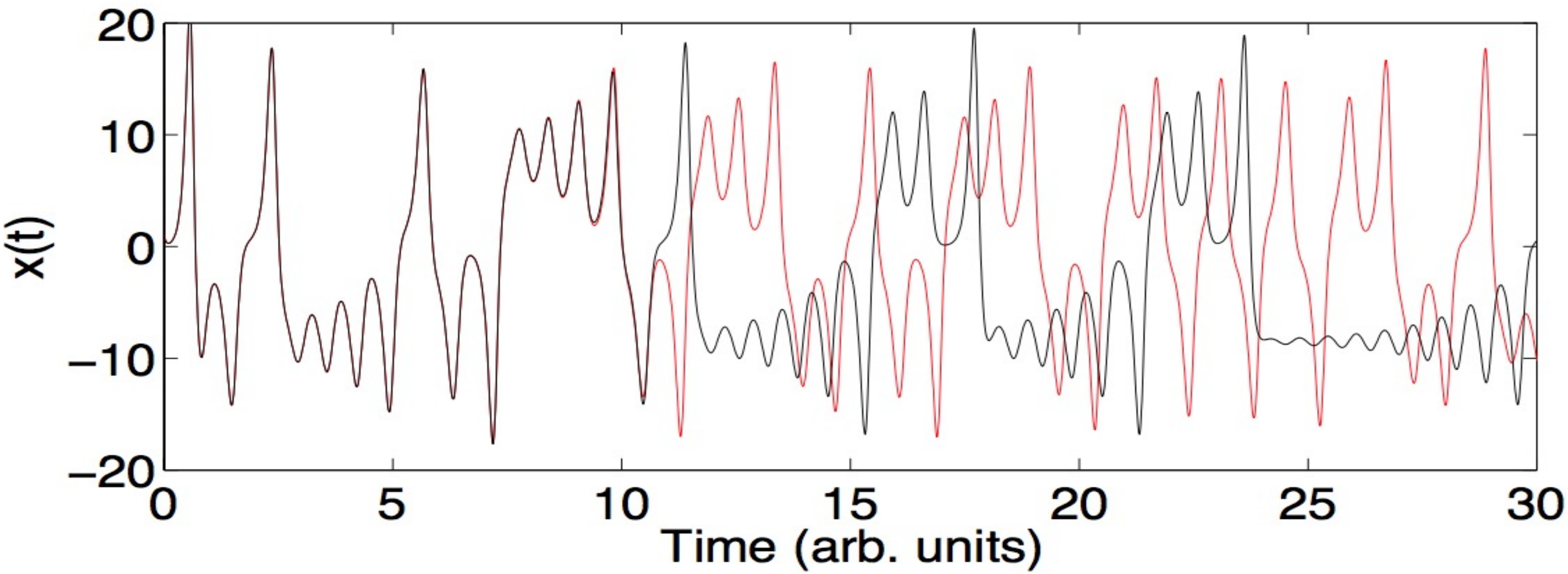
where x(t), y(t), and z(t) symbolizes a state of the atmosphere

The Lorenz's model has three degrees of freedom and enough nonlinearity to generate chaotic dynamical behavior

E. Lorenz, Deterministic nonperiodic flow, J of the Atmospheric Science (1963)

The Lorenz model

The Lorenz's model shows a remarkable feature, the sensitive dependence on initial conditions that result in the trajectories in phase space diverging exponentially fast away from each other



Two initial conditions	$[X_0,Y_0,Z_0] = [0,1,1.05]$ $[X_0,Y_0,Z_0] = [0.000001,1.000001,1.05000001]$

E. Lorenz, Deterministic nonperiodic flow, J of the Atmospheric Science (1963)

The Lorenz model

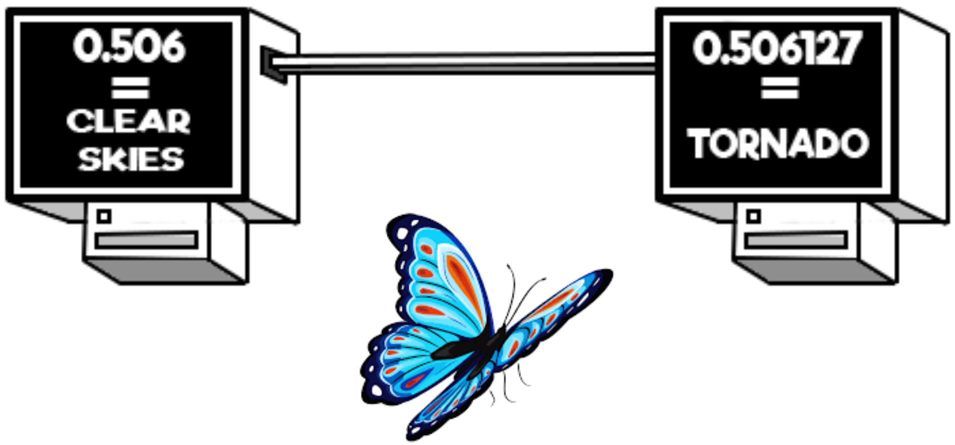
A sad ending? In view of the inevitable inaccuracy and incompleteness of weather observations, precise very long-range forecasting would seem to be non-existent

Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?

Edward Lorentz

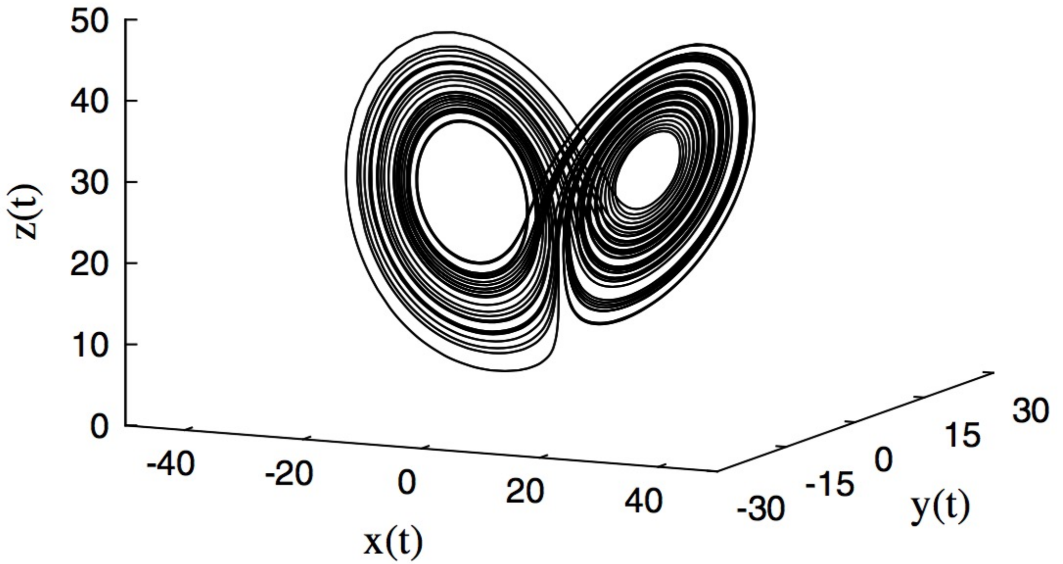
Paper presented at a conference

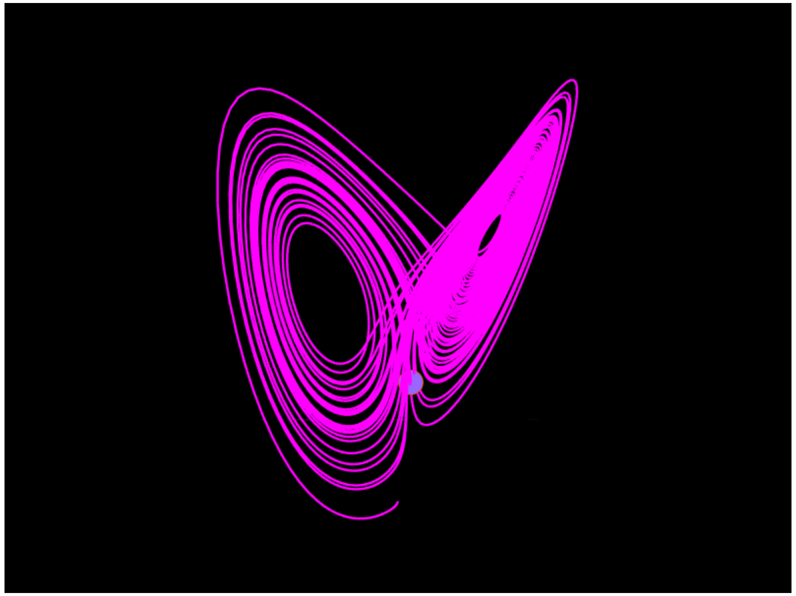
Washington D.C. 1972



The Lorenz attractor

A strange attractor consists on a fractal structure, where multiscale similarity of the density of trajectories exists. Here the attractor looks like a butterfly, and it is called the butterfly attractor





Two points on the attractor that are near each other at one time will be arbitrarily far apart at later times. Strange attractors are unique in that the motion of the system never repeats (non-periodic)

E. Lorenz, Deterministic nonperiodic flow, J of the Atmospheric Science (1963)

The three fingerprints of chaos

1) A trajectory is never periodic. It coasts along an imaginary surface called a strange attractor

2) Extreme sensitivity to initial conditions. This means that nearby trajectories will diverge exponentially e.g. continuous systems in a 2-dimensional phase space cannot experience such a divergence

3) Poincaré-Bendixson theorem: Chaotic behaviors can only be observed in deterministic continuous systems with a phase space of dimension 3, at least

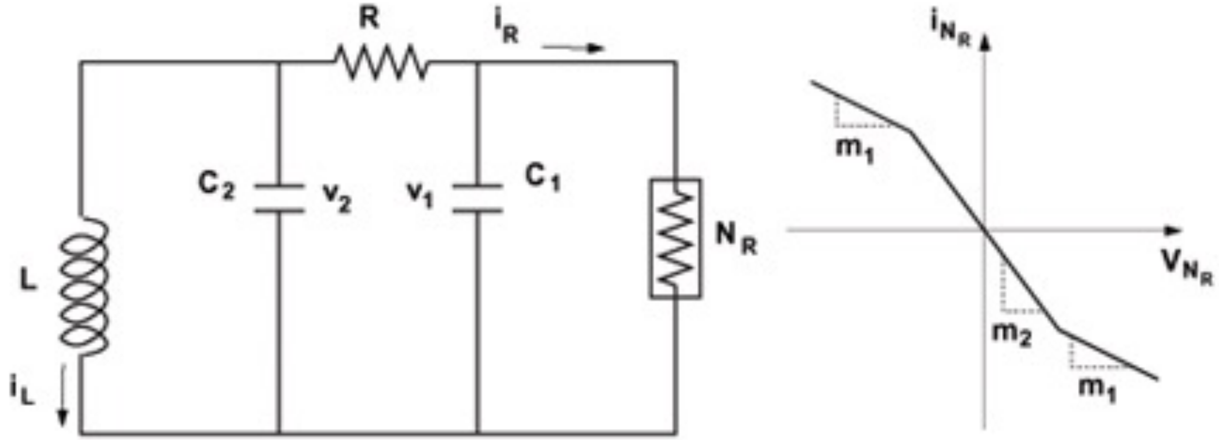
S. Strogatz, "Nonlinear Dynamics and Chaos with application to physics, biology, chemistry and engineering", Perseus Book (1994)

Henri Poincaré (1854-1912)

Ivan Bendixson (1861-1935)

Birth of chaos-based applications

In 1983, L. Chua, proposed an electronic circuit exhibiting rich complex nonlinear dynamics including chaos



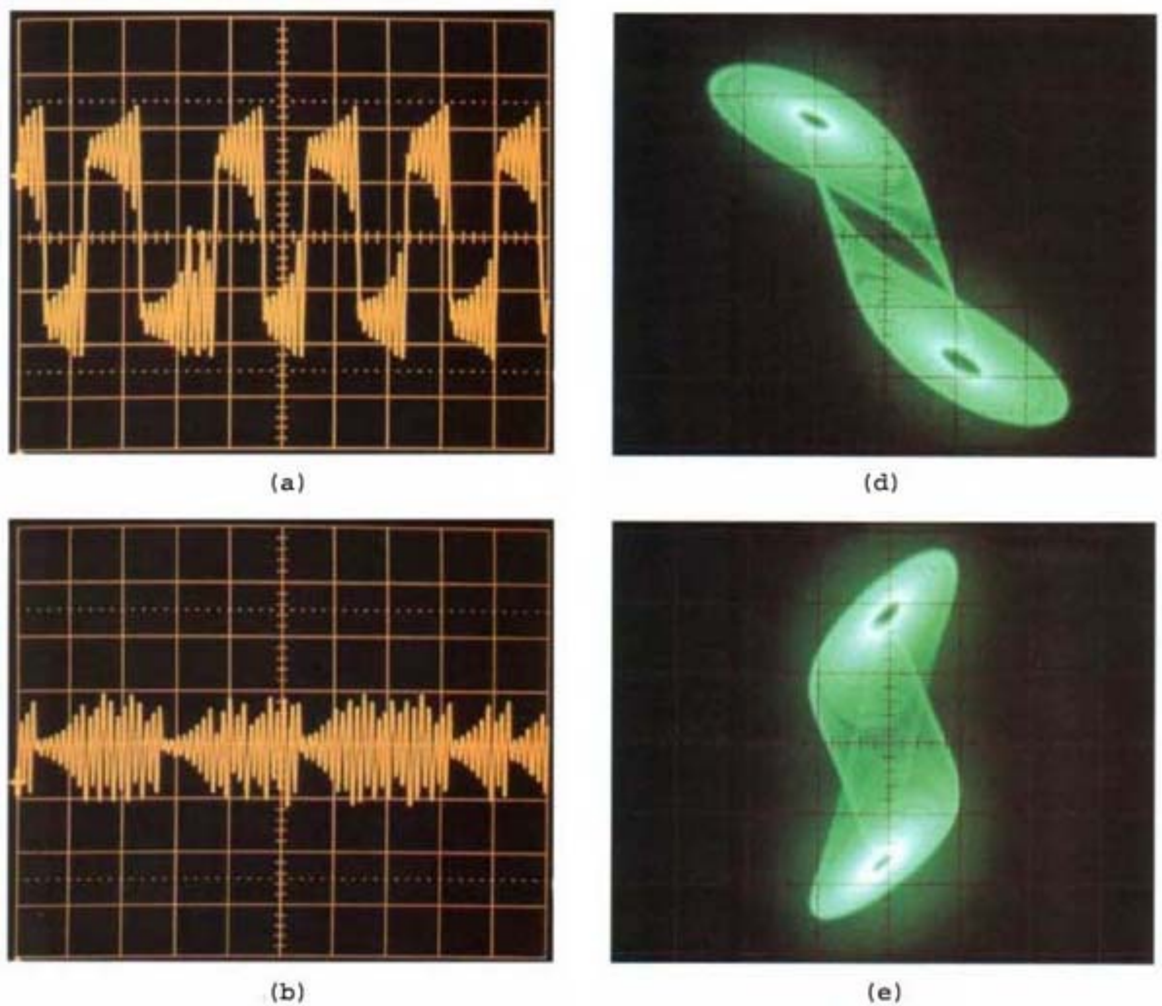
Leon Chua (1936-)

$$
\begin{aligned}
C_1 \frac{dv_1}{dt} &= \frac{(v_2 - v_1)}{R} - h(v_1)\ , \\
C_2 \frac{dv_2}{dt} &= \frac{(v_1 - v_2)}{R} + i_L\ , \\
L \frac{di_L}{dt} &= -v_2\ ,
\end{aligned}
$$

The nonlinearity is provided with the Chua's diode (N_R). Here, the dimension of the phase space is 3

L. Chua et al., IEEE Transactions on Circuits and Systems (1986)

Birth of chaos-based applications



The Chua's circuit exhibiting a double-scroll attractor named as the Chua's attractor is the first experimental proof of chaos (1986)

The first laser instabilities (chaos)

PHYSICAL REVIEW VOLUME 123, NUMBER 4 AUGUST 15, 1961

Stimulated Optical Emission in Fluorescent Solids. II. Spectroscopy and Stimulated Emission in Ruby

T. H. Maiman,* R. H. Hoskins,* I. J. D'Haenens, C. K. Asawa, and V. Evtuhov
Hughes Research Laboratories, A Division of Hughes Aircraft Company, Malibu, California
(Received January 27, 1961)

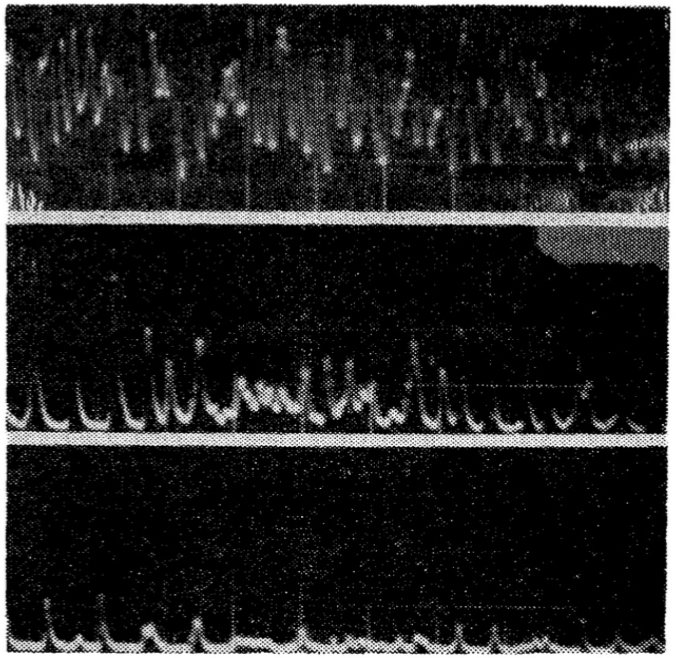


Fig. 13. Output pulse from ruby on an expanded time scale (10 μsec/division). (a), (b), and (c) represent the output approximately 600, 1000, and 1200 μsec after the onset of oscillation. The vertical sensitivity and the base line are the same in each case.

These initial observations were either left unexplained or wrongly attributed to noise!

T. H. Maiman et al., Phys. Rev. (1961)

Analogy between fluids and lasers

Lorenz

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = -xz + rx - y$$

$$\frac{dz}{dt} = xy - bz$$

Lorenz-Haken

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = -xz + rx - (1 - i\delta)y$$

$$\frac{dz}{dt} = Re[x^* y] - bz$$

Lorentz: x(t), y(t), and z(t) represent properties of the convecting fluid flow, and temperature differences between the left- and right-hand sides of the cell of fluid and between its top and bottom, respectively

Lorenz-Haken: x(t), y(t), and z(t) represent the time evolution of the electric field, the atomic polarization, and the population inversion

H. Haken, Physics Letters A (1975)

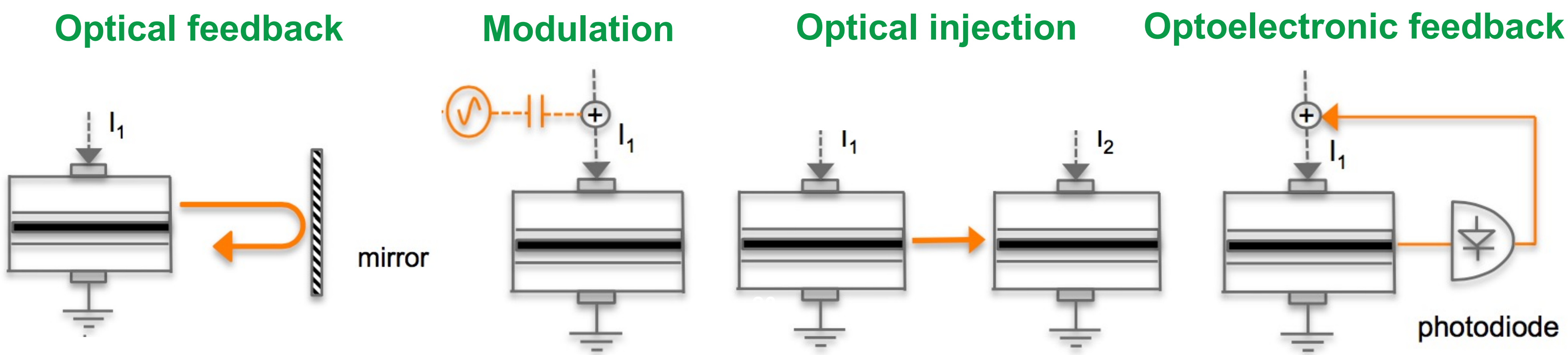
Chaos generation in semiconductor lasers

$$\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_c} - G_N(N - N_t)S$$

$$\frac{dS}{dt} = G_N(N - N_t)S - \frac{S}{\tau_p} + R_{sp}$$

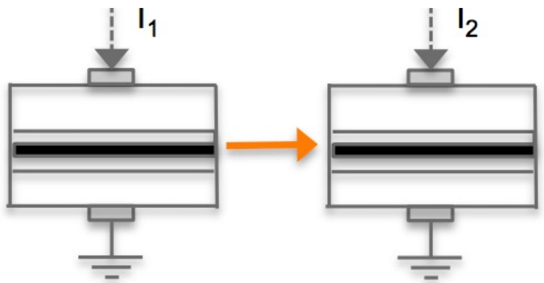
Solitary semiconductor lasers do not exhibit chaotic waveforms because the dimension of the phase space is 2

Adding an external perturbation allow to fulfill the Poincaré-Bendixson theorem and a minimum of 3 degrees of freedom is at least obtained



M. Sciamanna and K. A. Shore, Nature Photonics (2015)

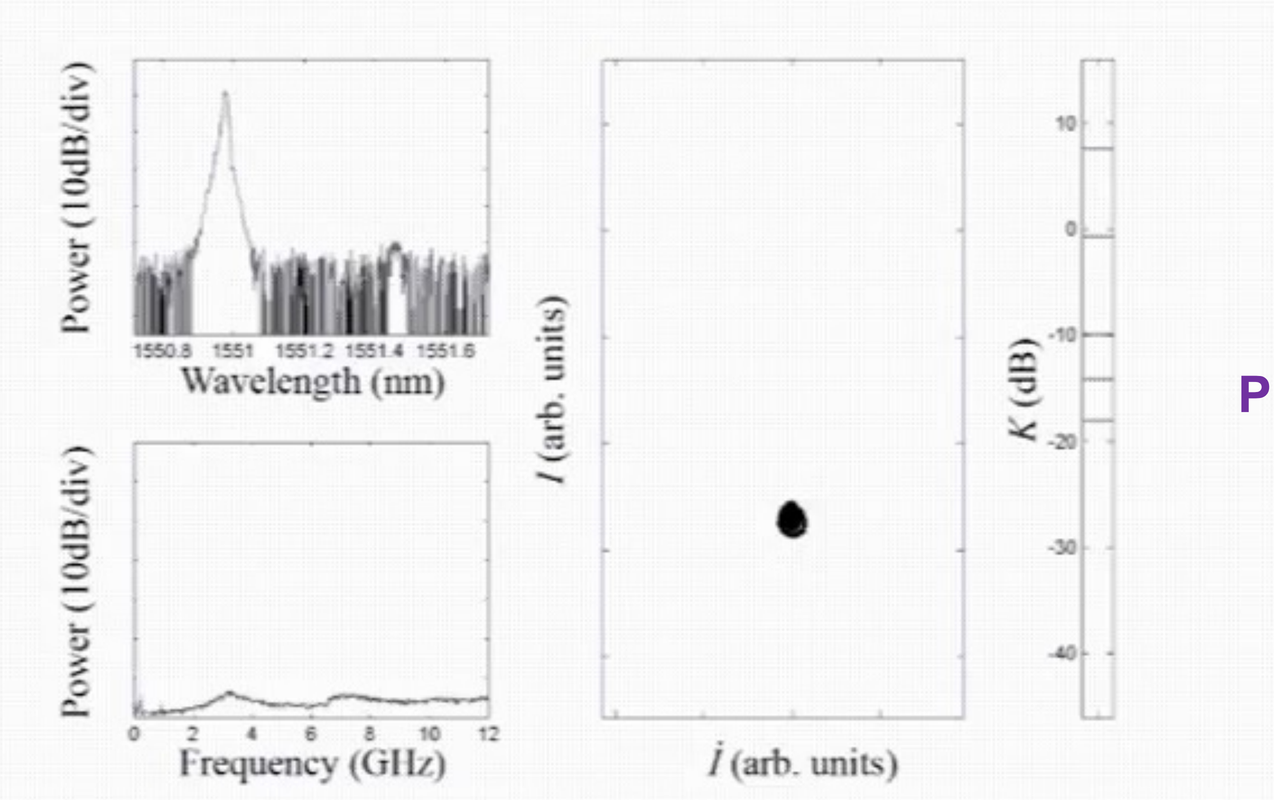
Chaos in semiconductor lasers



A slave laser is optically injected by a master laser. The strange attractor occurs for a certain level of injection strength

Optical spectrum

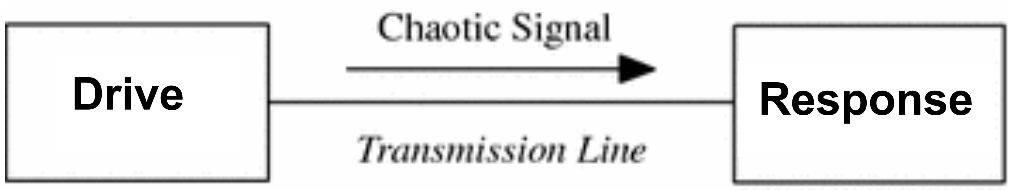
Electrical spectrum



Phase Space

Chaos synchronization

Drive & response lasers synchronize themselves



Awase-Gai

VOLUME 64, NUMBER 8 PHYSICAL REVIEW LETTERS 19 FEBRUARY 1990

Synchronization in Chaotic Systems

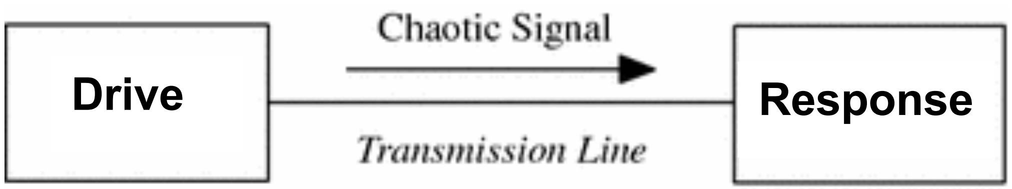
Louis M. Pecora and Thomas L. Carroll
Code 6341, Naval Research Laboratory, Washington, D.C. 20375
(Received 20 December 1989)

Assuming $x=x_r$ on the driver, a very elegant mathematical proof of the synchronization can be conducted by extracting the Lyapunov function associated to the error differences $y-y_r$ and $z-z_r$ (see my notes)

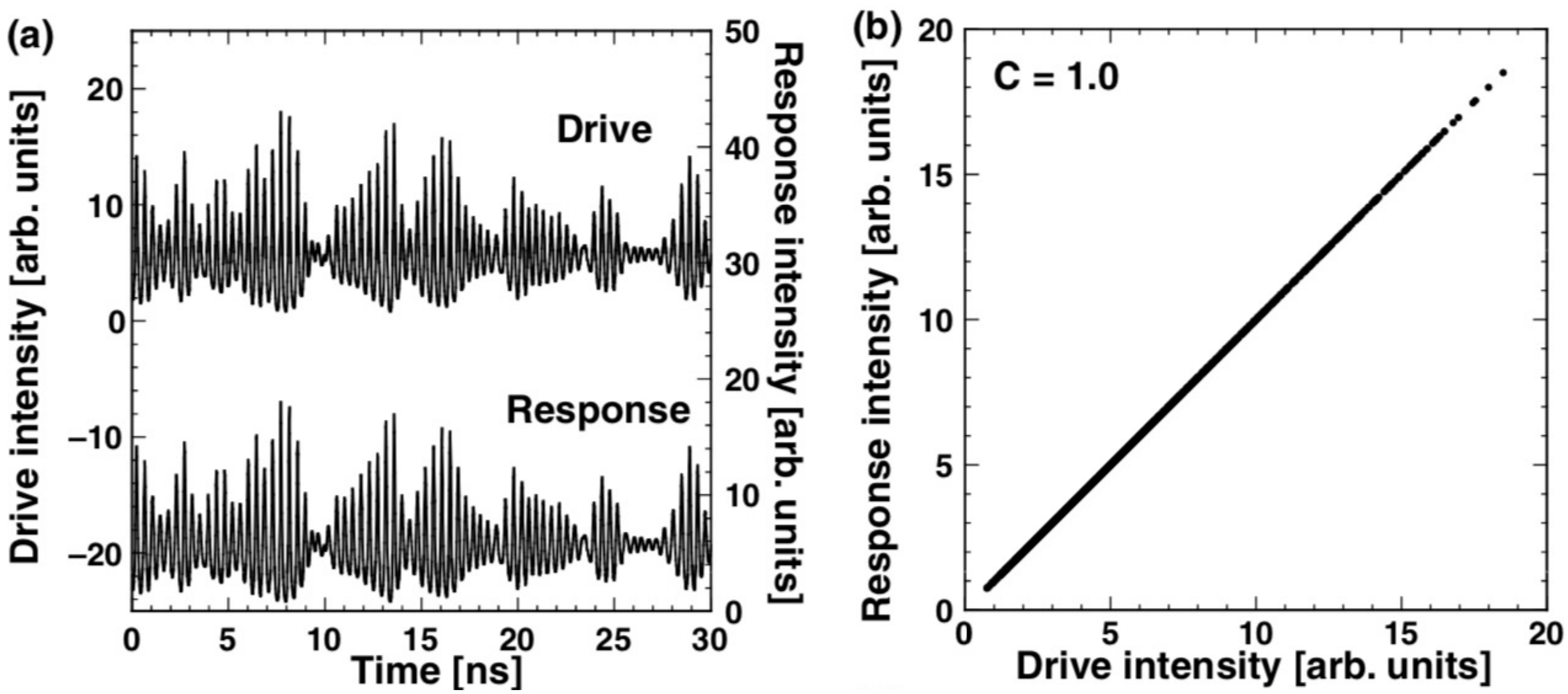
R. He and P. G. Vaidya, Phys. Rev. E. (1998)

Chaos synchronization

Drive & response lasers synchronize themselves



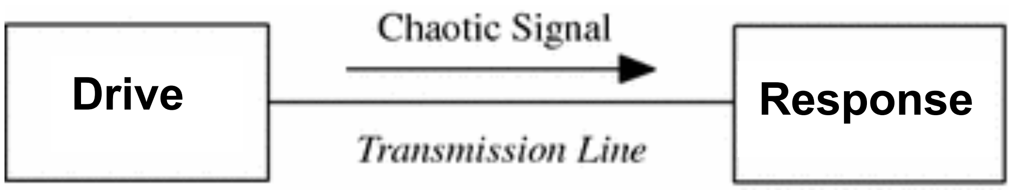
Awase-Gai



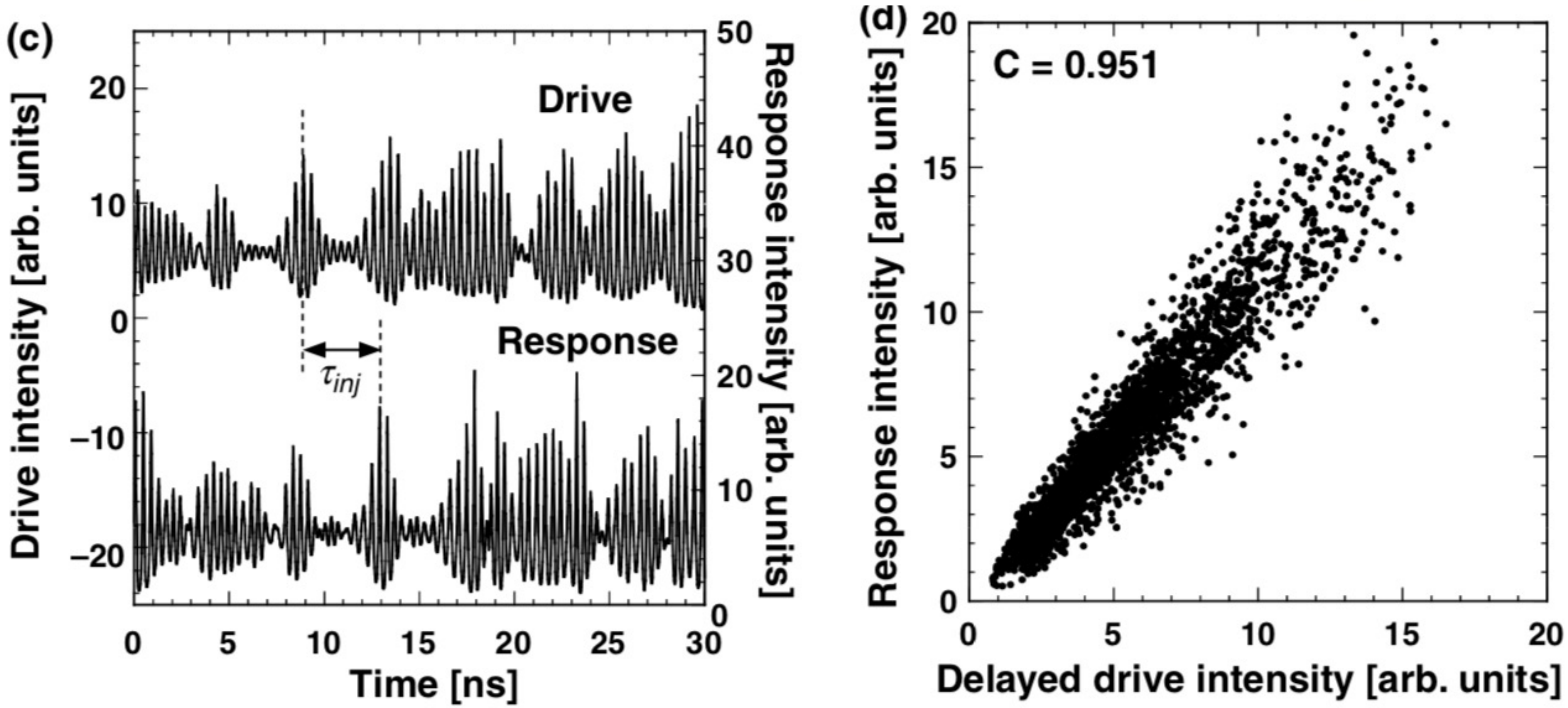
A. Uchida, Optical Communication with Chaotic Lasers, Wiley (2012)

Chaos synchronization

Drive & response lasers synchronize themselves



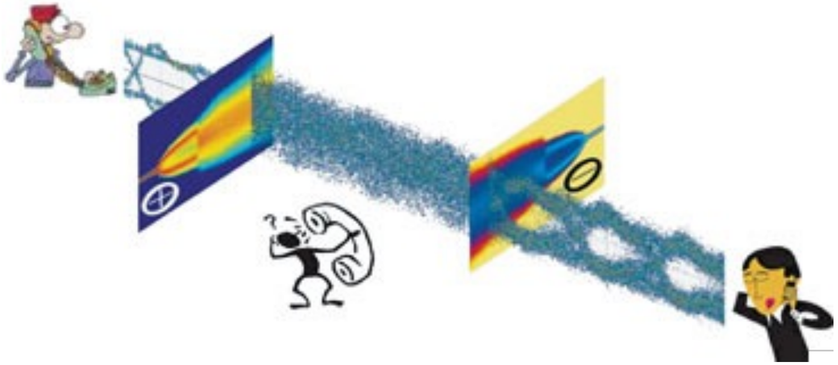
Awase-Gai



A. Uchida, Optical Communication with Chaotic Lasers, Wiley (2012)

Private communications

Photonic chaos can be used for communications with increased privacy



Communication with Chaotic Lasers

Gregory D. VanWiggeren and Rajarshi Roy*

Recent experiments with chaotic electronic circuits have shown the possibility of communication with chaos. The experimental demonstration of chaotic communication with an optical system is described. An erbium-doped fiber ring laser (EDFRL) was used to produce chaotic light with a wavelength of 1.53 micrometers. A small 10-megahertz message was embedded in the larger chaotic carrier and transmitted to a receiver system where the message was recovered from the chaos. Chaotic optical waveforms can thus be used to communicate masked information at high bandwidths.

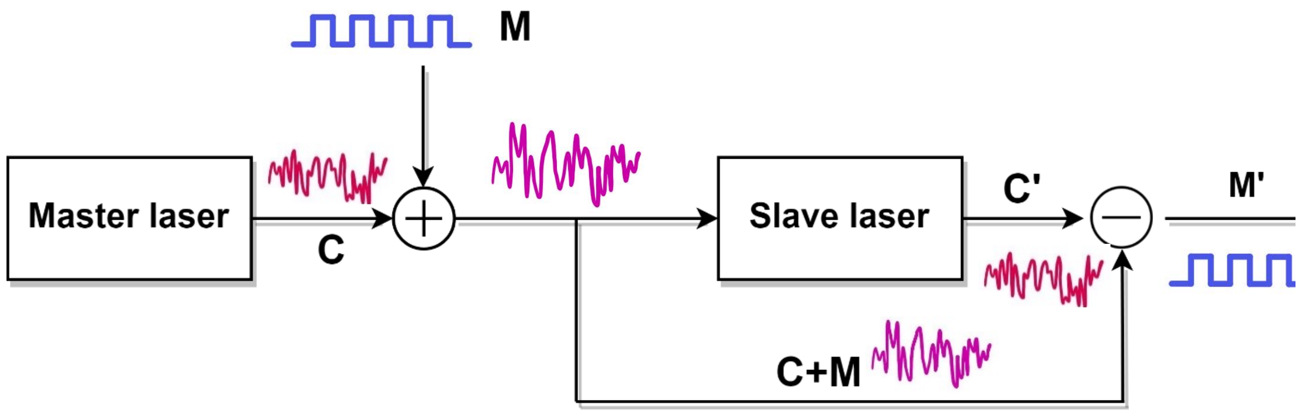
By spreading the narrow band signal (message) into a wideband signal (chaotic carrier), chaos-based communications can both create desired jamming and avoid malicious jamming

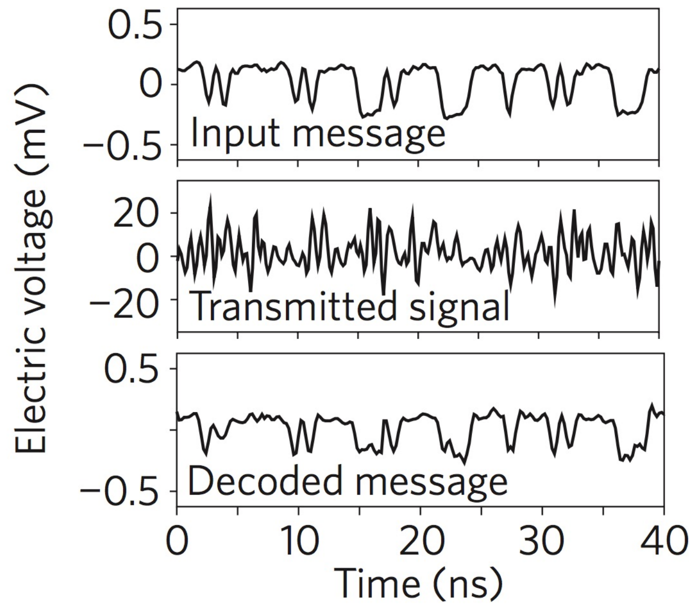
Note that here the secure system is totally different than a quantum communication channel whereby the key is generated and distributed between Alice and Bob

G. D. Van Wiggeren & R. Roy Science (1998)

Private communications

The generation of the chaos is based on the input signal, so only the receiver that has knowledge on how the chaos is generated can reproduce the chaos and cancel it to recover the signal





$$M + C - C' = M' \approx M \qquad C - C' \ll 1$$

Matched parameters (or with a very small mismatch) between the two physical systems is assumed for synchronized chaos

A. Uchida, Optical Communication with Chaotic Lasers, Wiley, (2012)

Private communications

Privacy in chaos communications results from the fact that an eavesdropper must have the proper hardware and parameter settings in order to decode and recover the message

In conventional encryption techniques, a key is used to alter the symbols used for conveying information. Tx/Rx share a key so that the information can be recovered

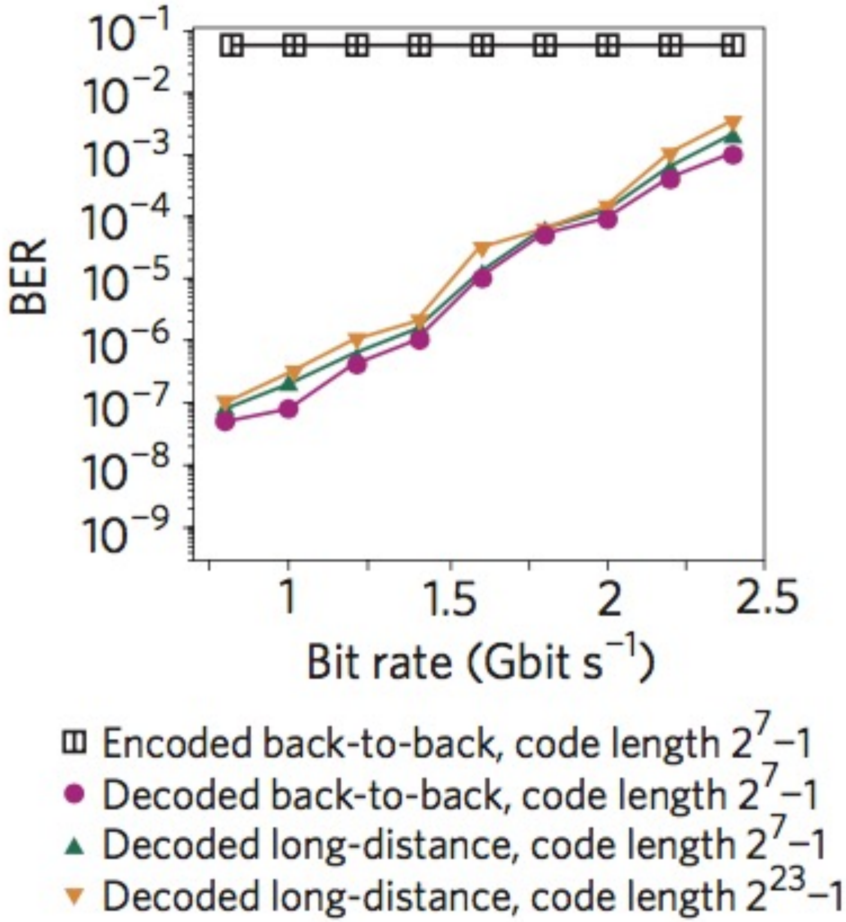
The key in chaos communications is a set of parameter values and a similar hardware structure required for chaos synchronization. The key is static even if a time-varying chaotic carrier is used

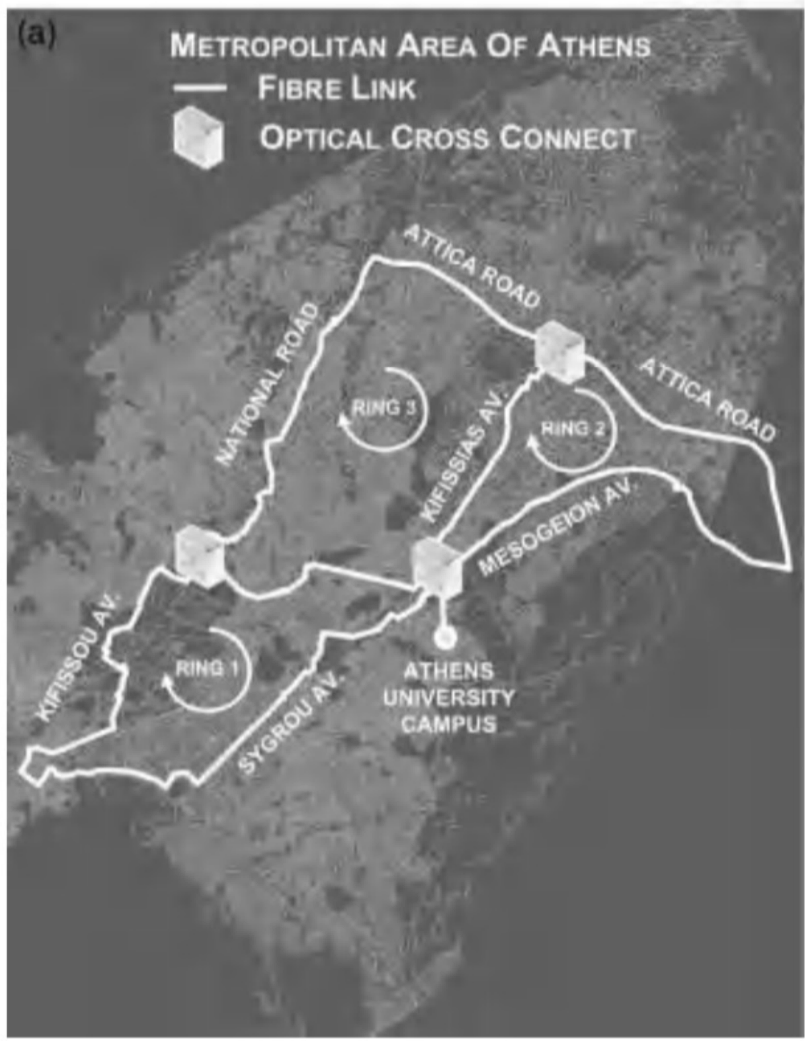
The two important factors for privacy considerations in chaos communication are the dimensionality of the chaos and the effort required to obtain the necessary parameters for a matched Rx

Y. Hong et al., Power loss resilience and eavesdropper detection in optical chaos communications systems, SPIE Photonics Europe (2010)

Fiber Optic-based private communications

The optical chaos-covered signal was transmitted over a 120 km distance in a commercial optical network over the metropolitan area of Athens



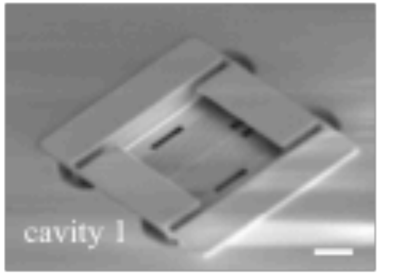


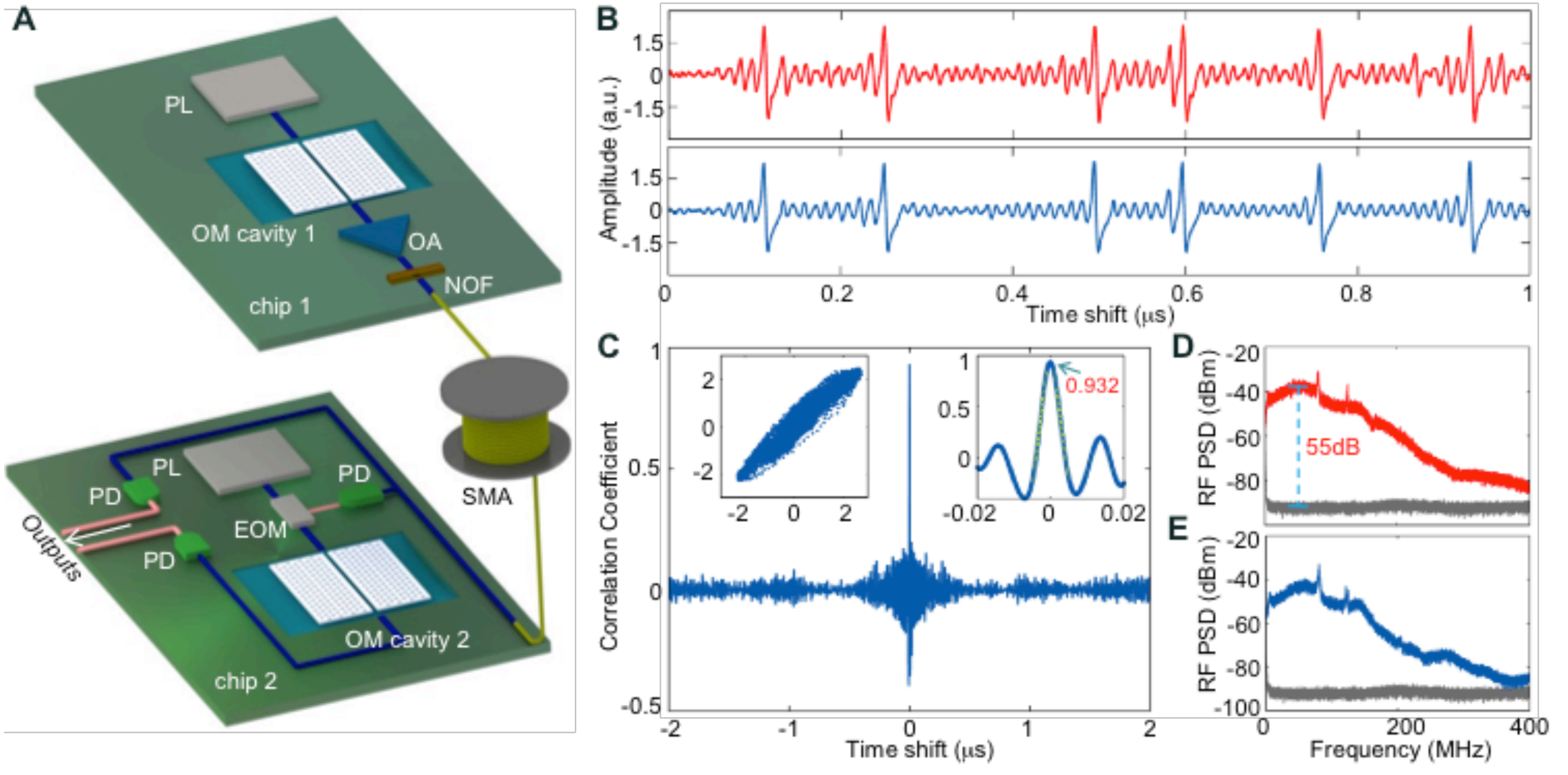
Argyris, A. et al., Nature Communications (2005)

On-chip private communications

Optomechanical chaos with silicon microcavities

- Made with photonics crystals
- Integrable on a photonics integrated circuit



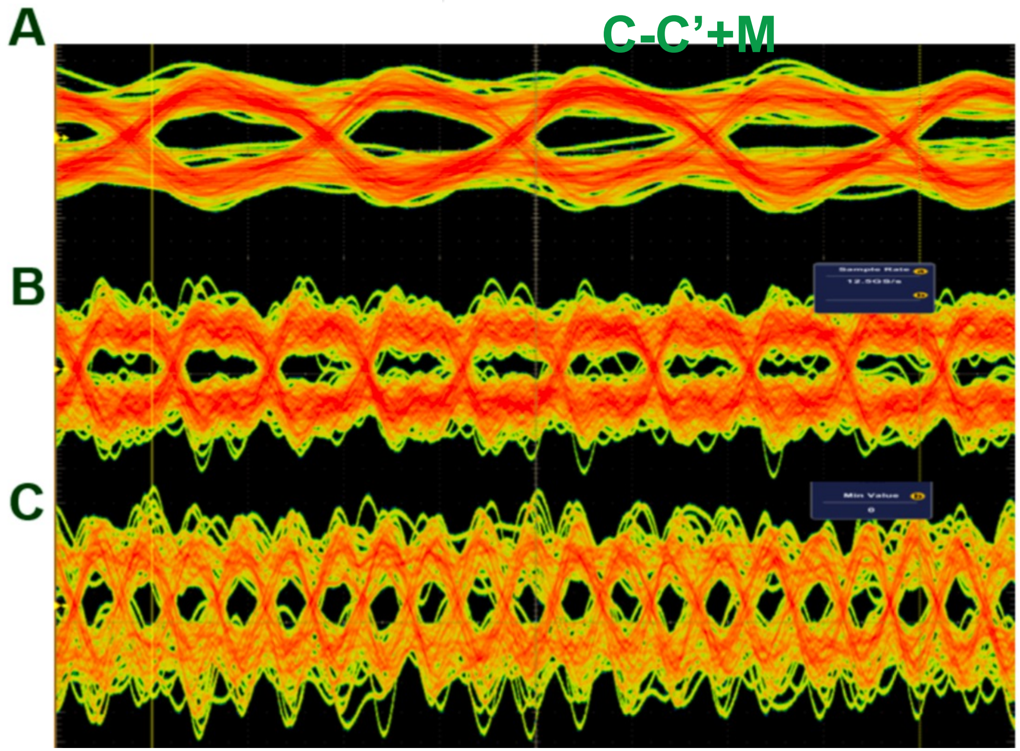


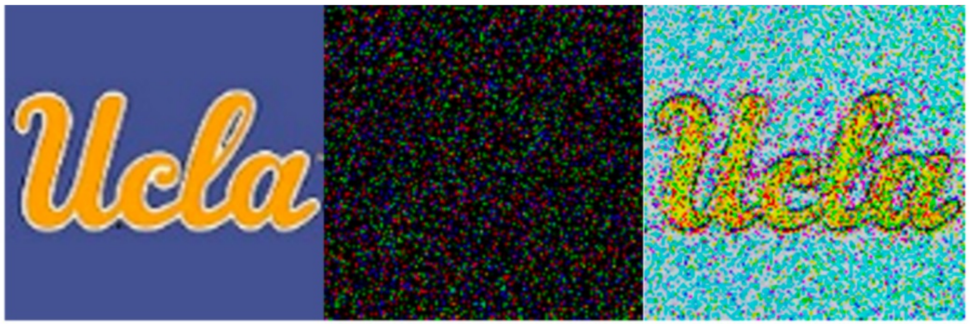
On-chip private communications

C-C'+M
0.5 Mbit/s digital message

C-C'+M
1 Mbit/s digital message

C-C'+M
2 Mbit/s digital message





Encrypted **Decrypted**

Test image with the logo of University of California Los Angeles

Free-space private communications

Private communications in the MIR

How?

Photonics chaos

Quantum cascade lasers

Thermal infrared window (9-11 microns)

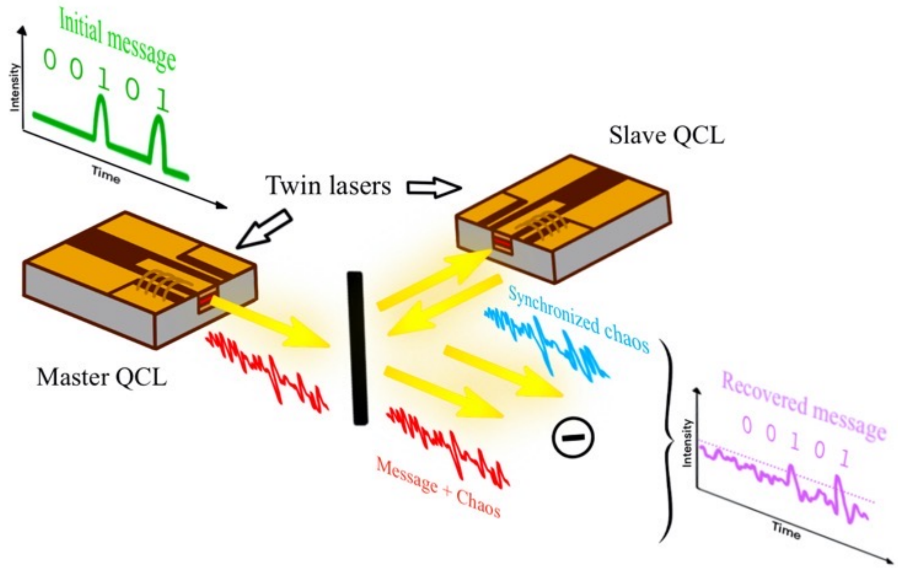
Why?

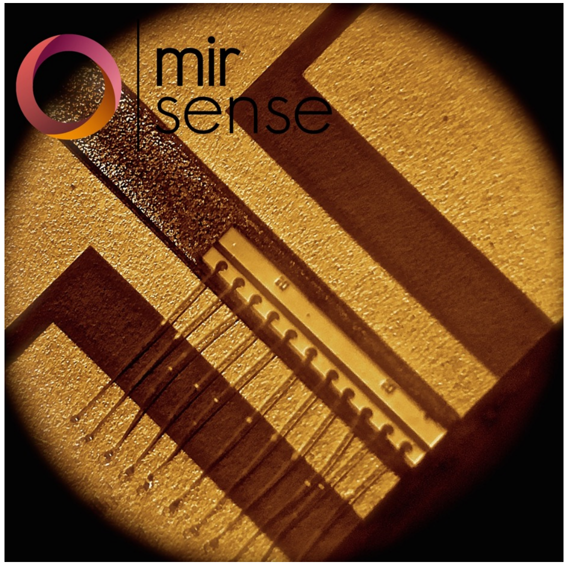
Stealthy mid-infrared radiation

Unpredictability of chaos

Simple implementation

No hardware quantum tech

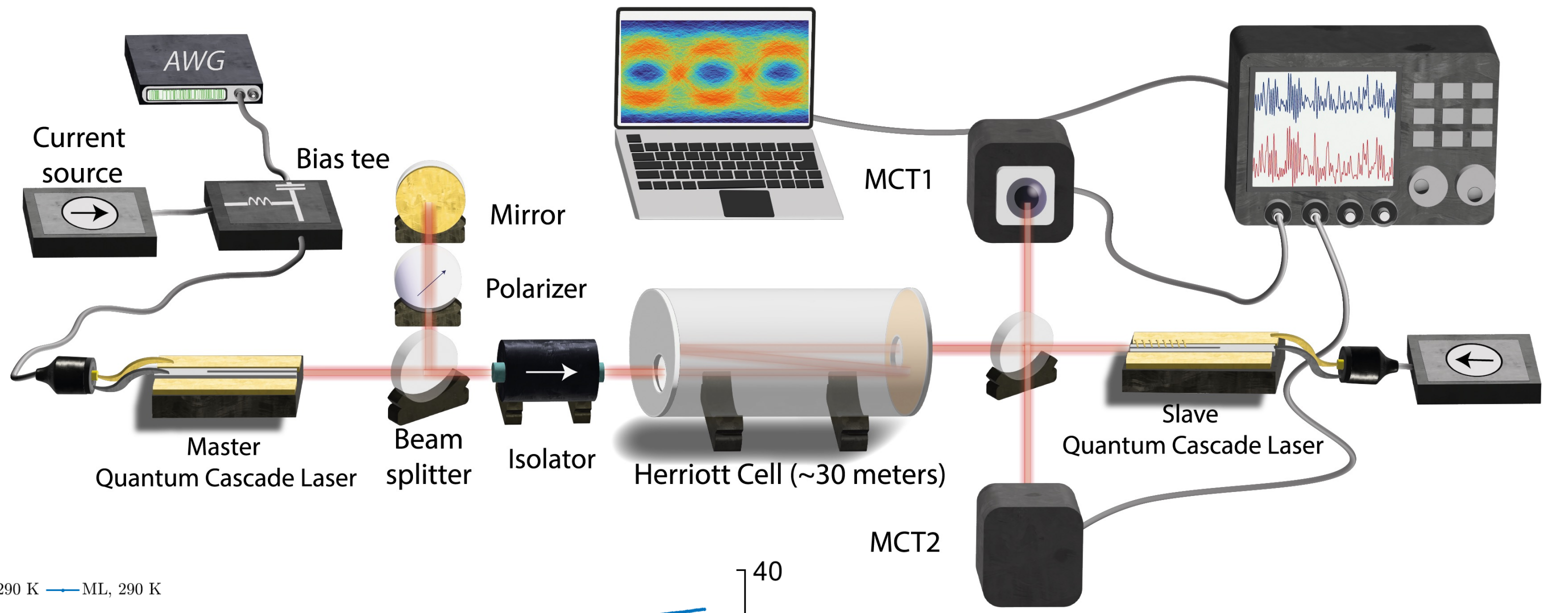


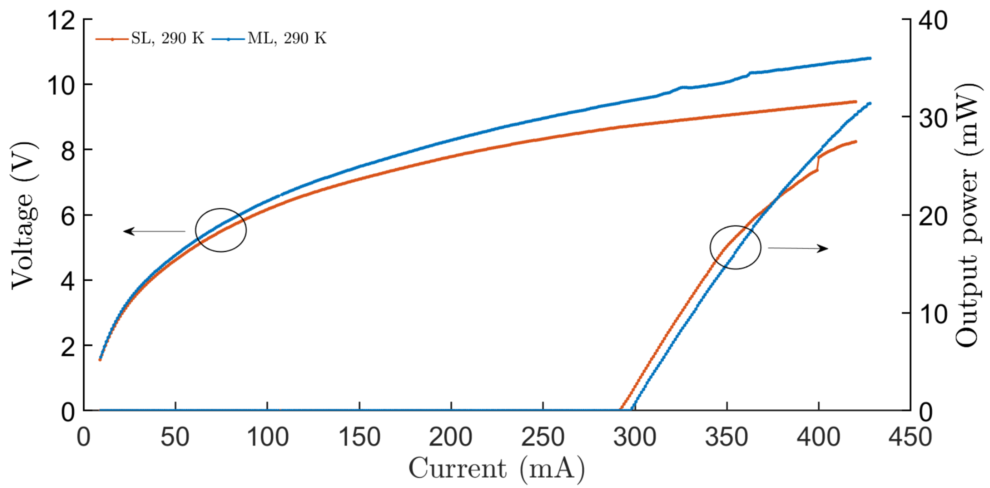


Grillot, F. et al., Optics and Photonics News (2021)

Free-space private communications

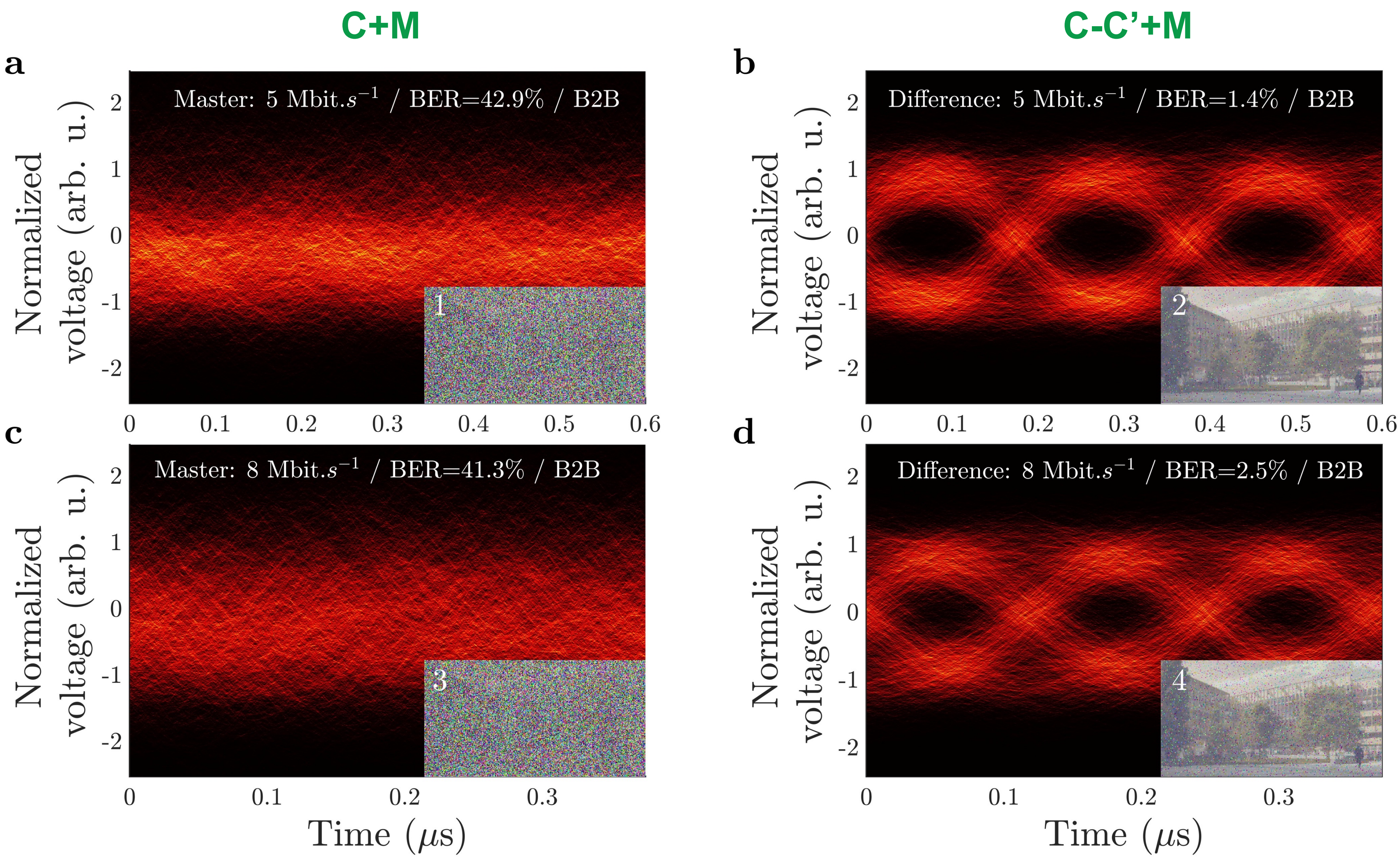
Unipolar quantum optelectronics@9 microns
Indoor transmission experiments over 30 meters





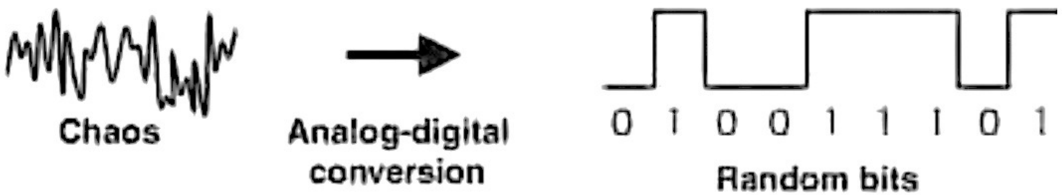
Both extensive and intensive parameters are matched to the best extent

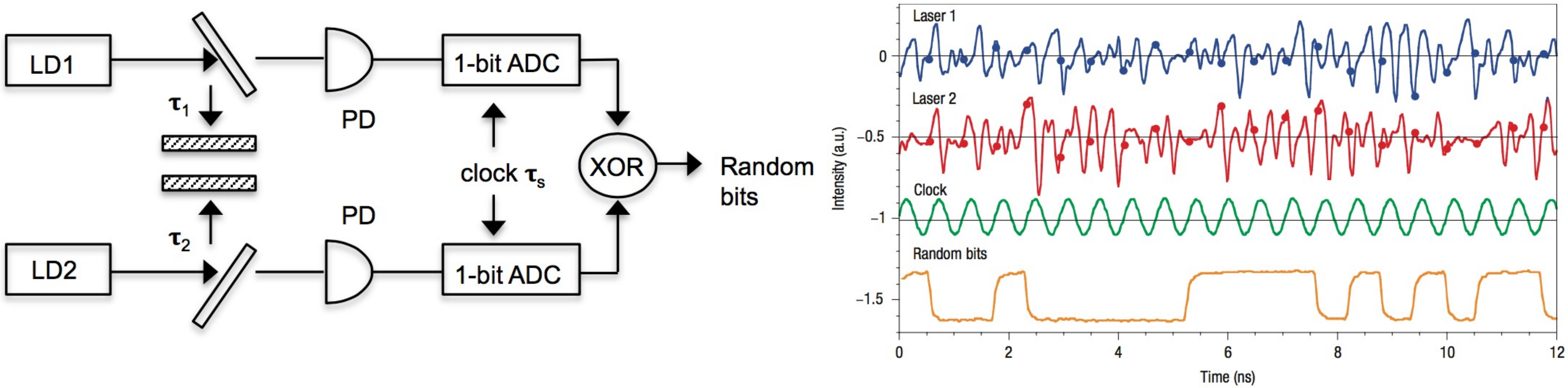
Free-space private communications



Chaos-based Random bit generation

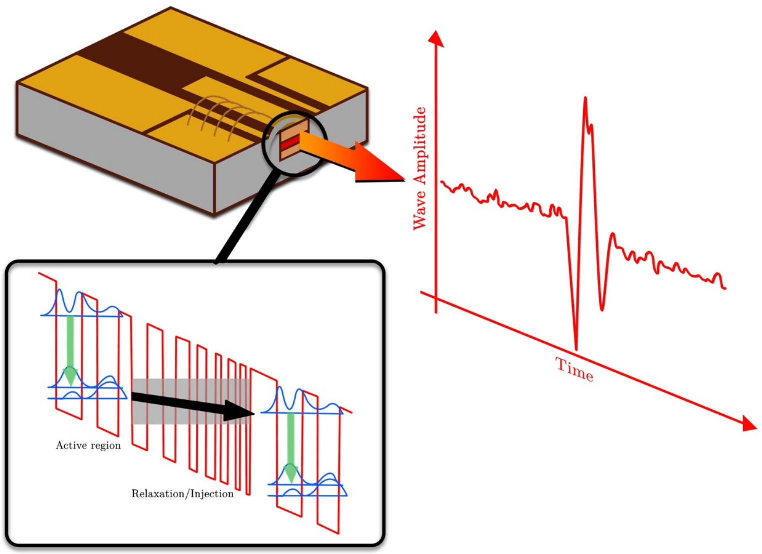
A chaotic signal of laser output is detected by a photo-detector and converted to a binary digital signal by an analog-to-digital converter (ADC)





M. Sciamanna and K. A. Shore, Nature Photonics (2015)

Ultrafast Optics of Semiconductor Lasers

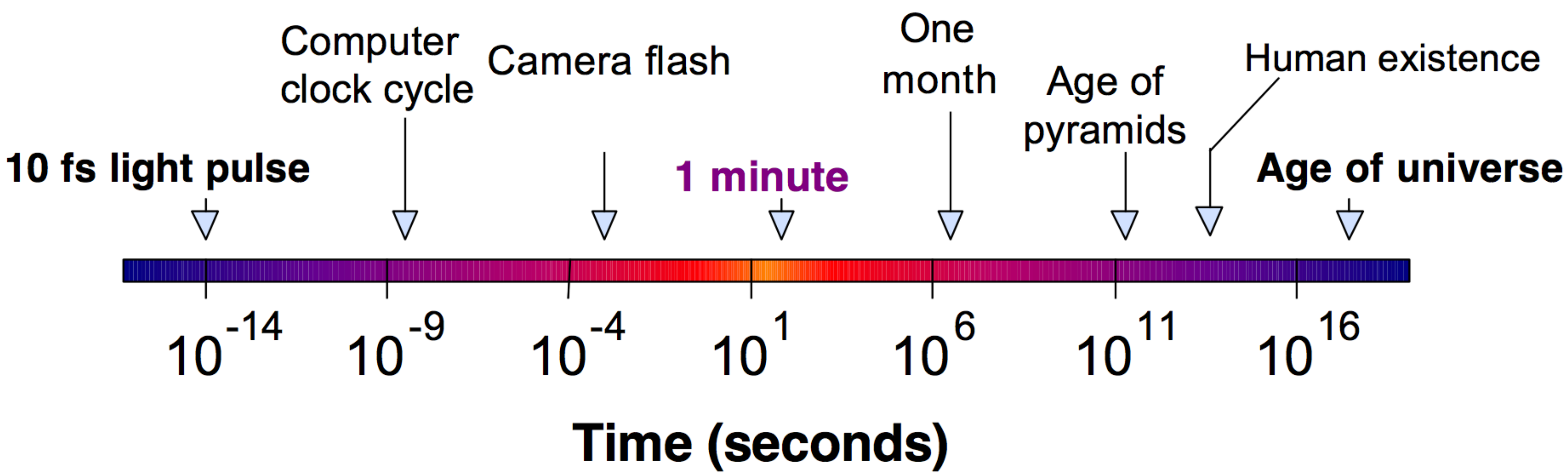


Ultrafast: optics vs electronics

FEW APHORISMS....

10 fs is to 1 minute as 1 minute is to the age of the universe

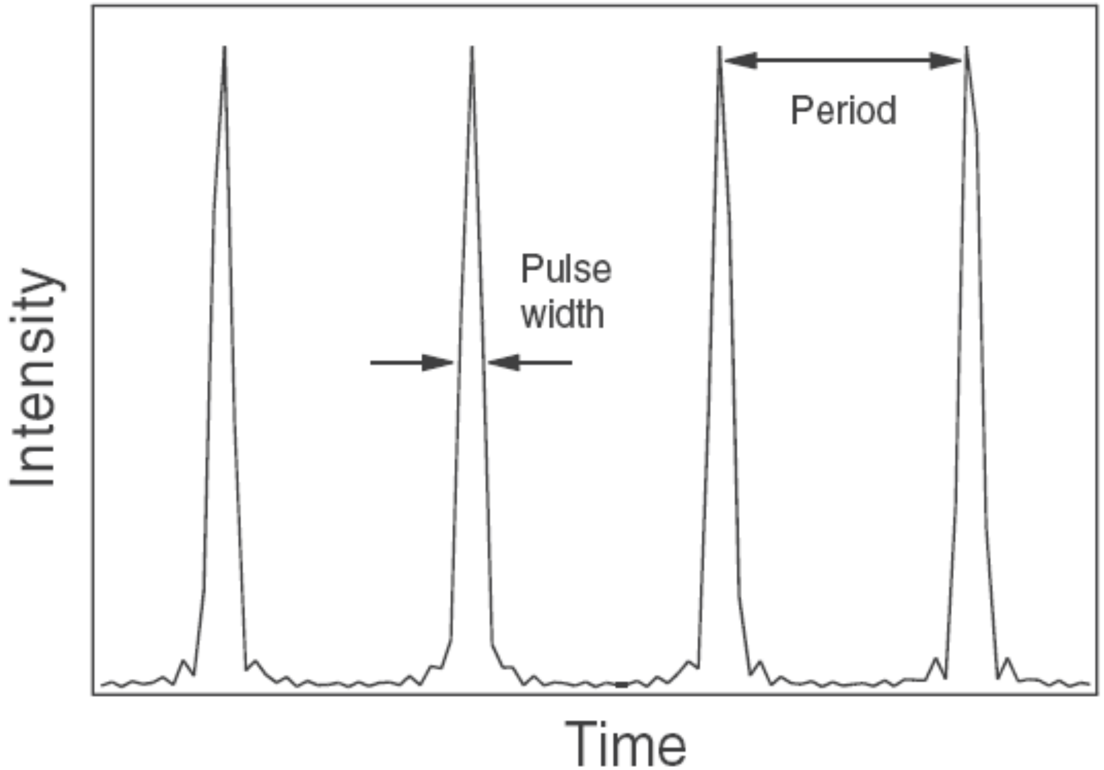
Alternatively, 10 fs is to 1 sec as 5 cents is to the US national debt



How fast is ultrafast?

Instead of emitting light in a continuous manner (like most lasers do), ultrafast lasers emit a regular train of ultra-short pulses, i.e., extremely short bursts of light with durations between…

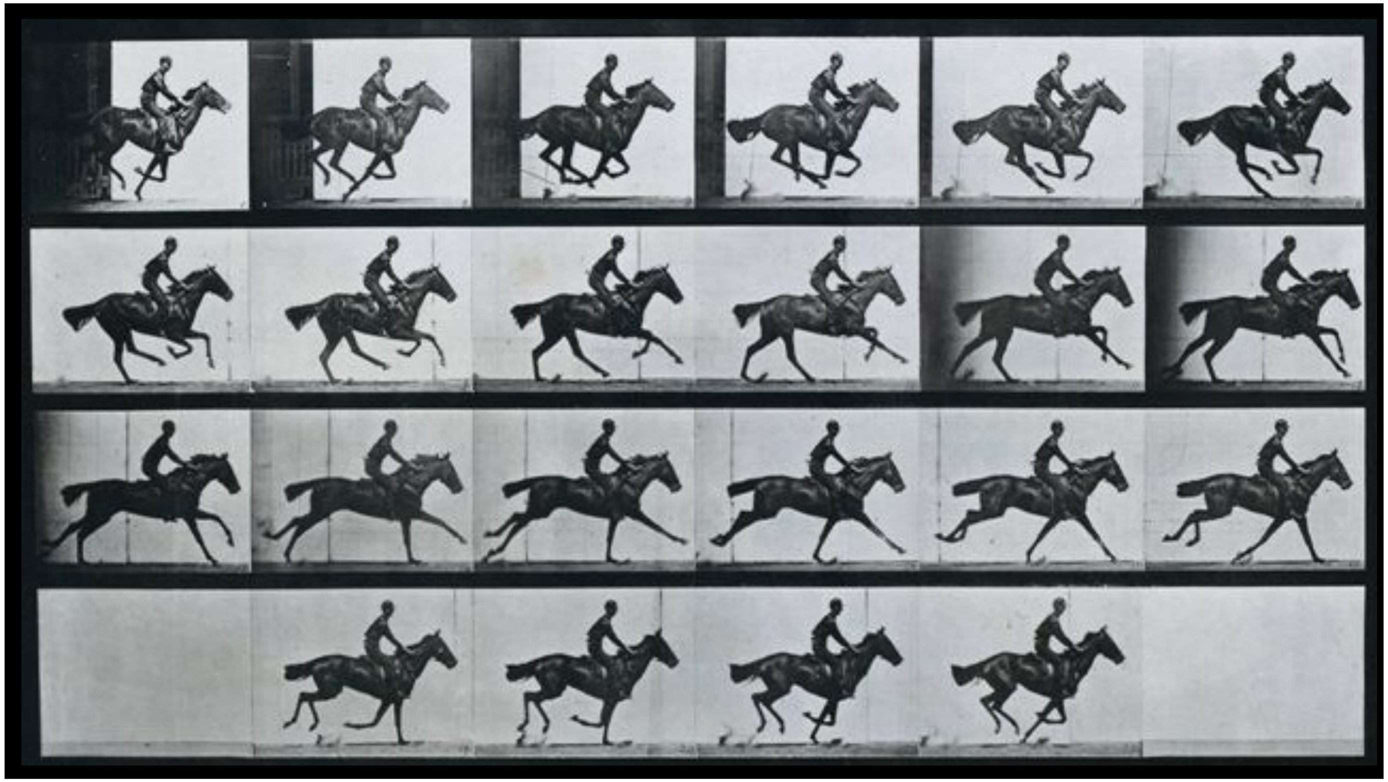
…a few femtoseconds (10^{-15} s) to a few picoseconds (10^{-12} s)



How fast is ultrafast?

Ultrafast lasers emit a regular train of ultra-short pulses, i.e., with durations between a few femtoseconds (10^{-15} s) to a few picoseconds (10^{-12} s)

Now compare this with snapshots that last a few milliseconds (10^{-3} s)…



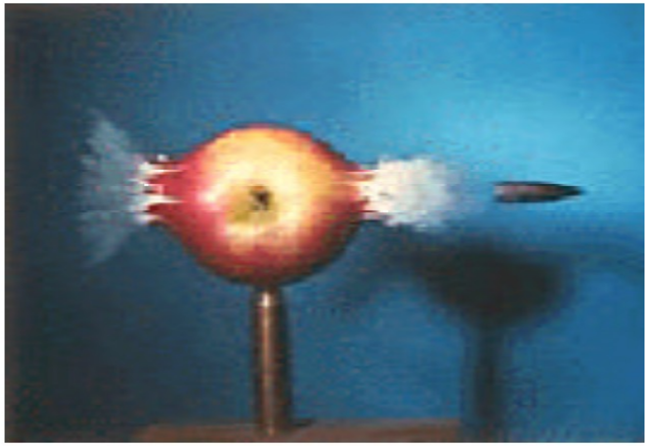
Ref: Eadweard Muybridge, Galloping Horse, (1887)

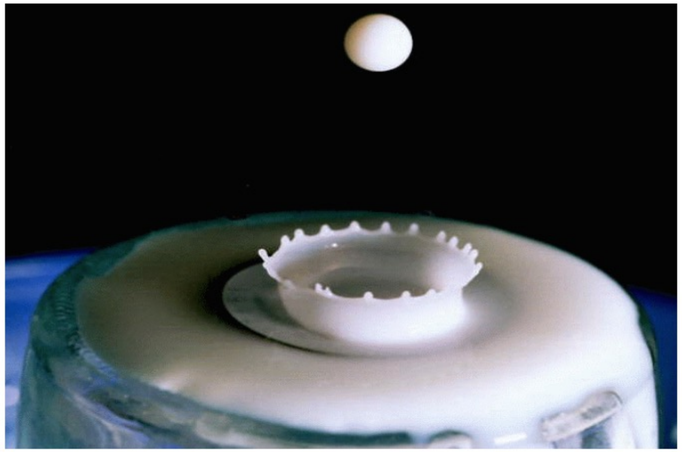
How fast is ultrafast?

Ultrafast lasers emit a regular train of ultra-short pulses, i.e., with durations between a few femtoseconds (10^{-15} s) to a few picoseconds (10^{-12} s)

Or compare with snapshots that last a few microseconds (10^{-6} s)…

"How to Make Apple sauce at MIT" 1964

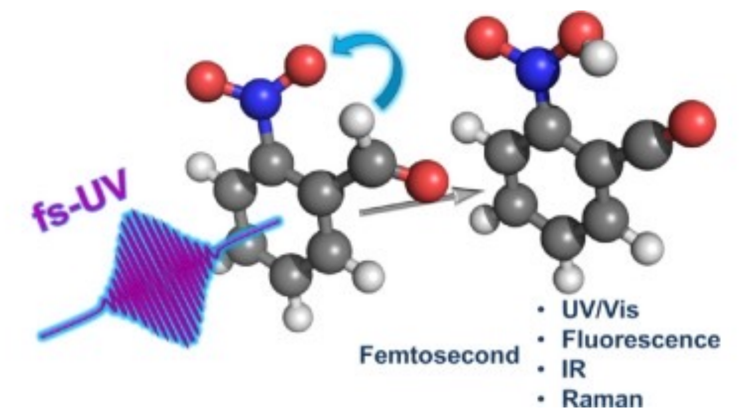




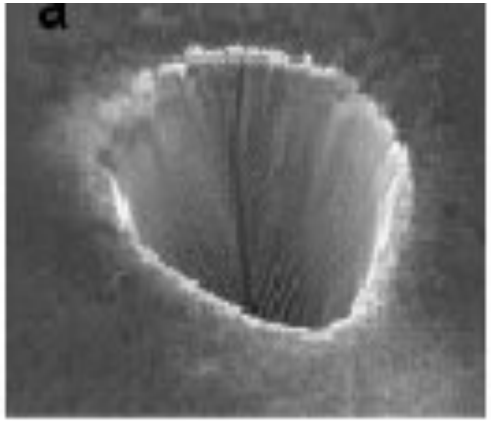
"Splash on a Glass" Curtis Hurley Junior High School student 1996

Why ultrashort pulses?

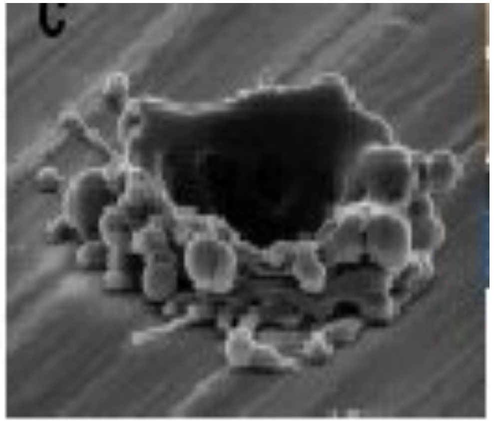
Femtochemistry



Micromachining

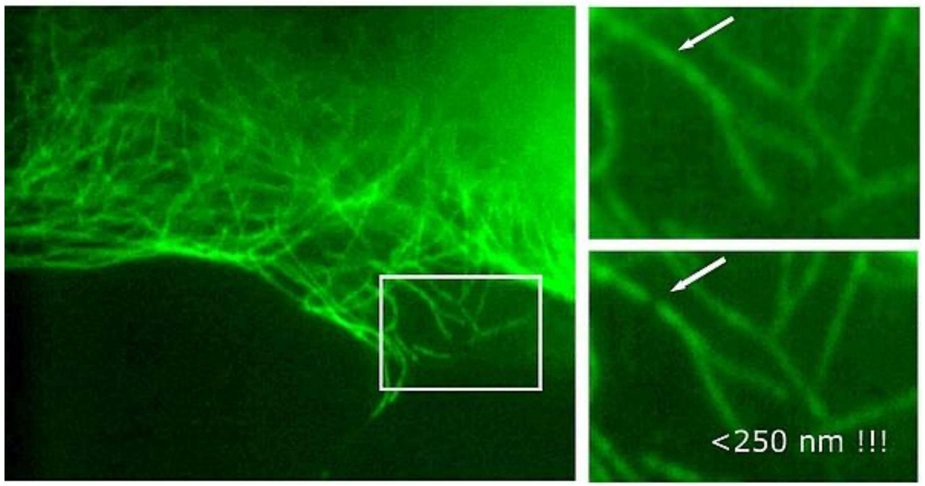


Ti: sapphire, **120 fs**

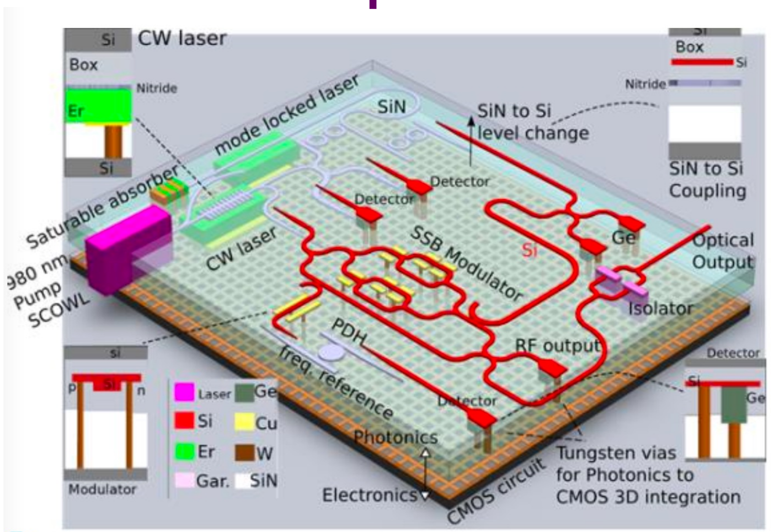


Nd:YAG, **100 ns**

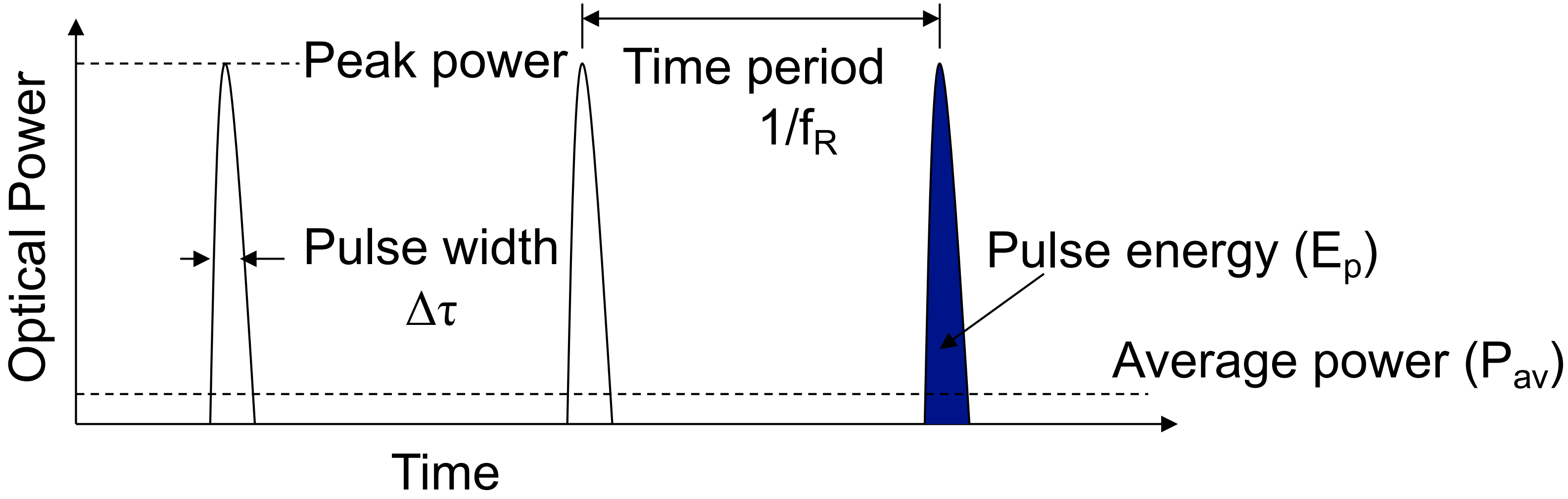
Nanosurgery



Silicon photonics



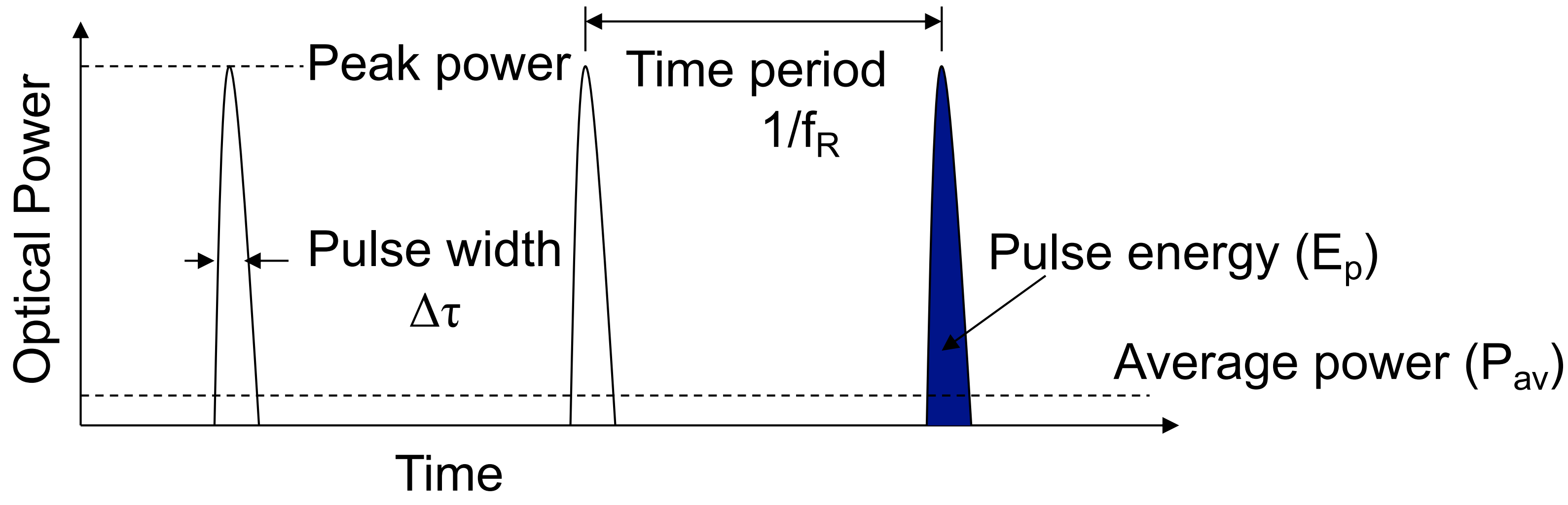
Anatomy of a laser pulse



Ultrashort pulses are generated with a certain periodicity. The interval of time between each pulse can be thousands of times longer than the duration of the pulse → the laser is switched off most of the time hence, in average, the optical power remains quite low

As all the optical energy of light is highly concentrated in time, the instantaneous or peak power can be incredibly high. And for a given energy, the shorter the pulse, the greater the peak power

Anatomy of a laser pulse



Example: passively mode-locked laser

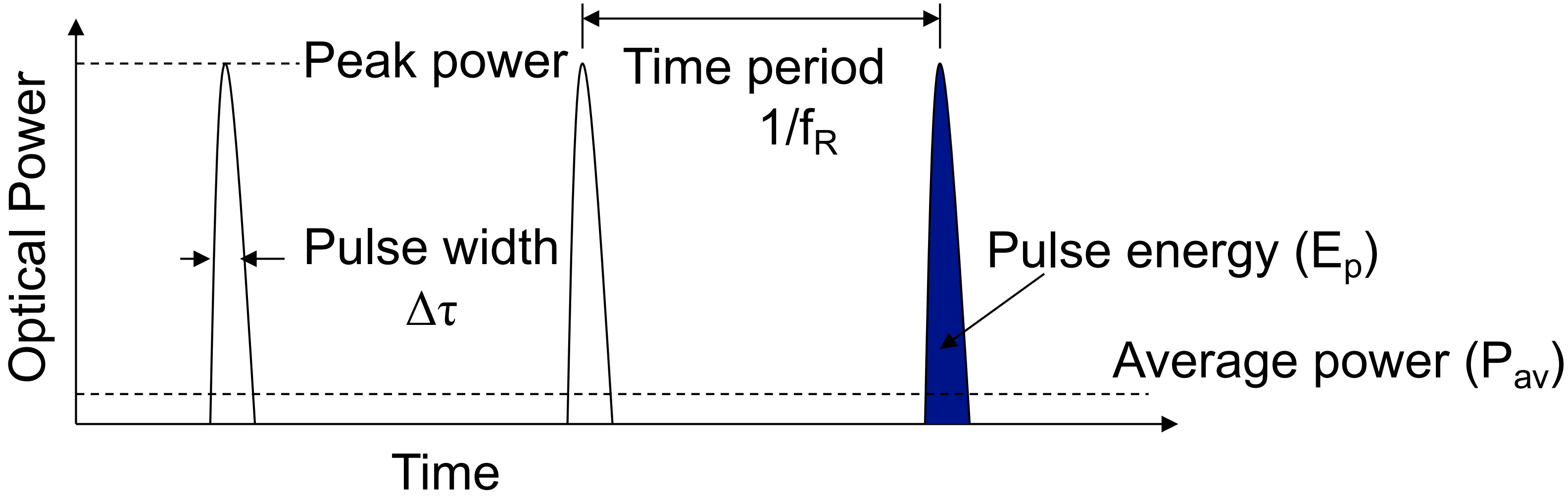
$$f_R = \frac{c}{2nL}$$

$$P_{peak} = \frac{E_p}{\Delta\tau}$$

$$P_{peak} = \frac{P_{av}}{f_R \Delta\tau}$$

The energy that is dispersed in several modes while in cw operation, is now concentrated in short pulses. Although the output average power P_{av} may be low, the peak power P_{peak} can be significantly higher

Anatomy of a laser pulse



The product of both the pulse duration $\Delta\tau$ and the optical frequency bandwidth Δv is called the time-bandwidth product (TBWP)

$$\boxed{\Delta\nu \times \Delta\tau = K}$$

For a given frequency bandwidth, there is a minimum pulse duration – if this is the case, then the pulse is said to be Fourier-transform limited, and the TBWP equals a constant K , whose value depends on the shape of the pulse (Gaussian, Lorentzian, etc)

Chirped pulses

In a semiconductor, when a pulse is amplified, the time bandwidth product deviates from the Fourier-transform limited conditions because of the linewidth broadening factor

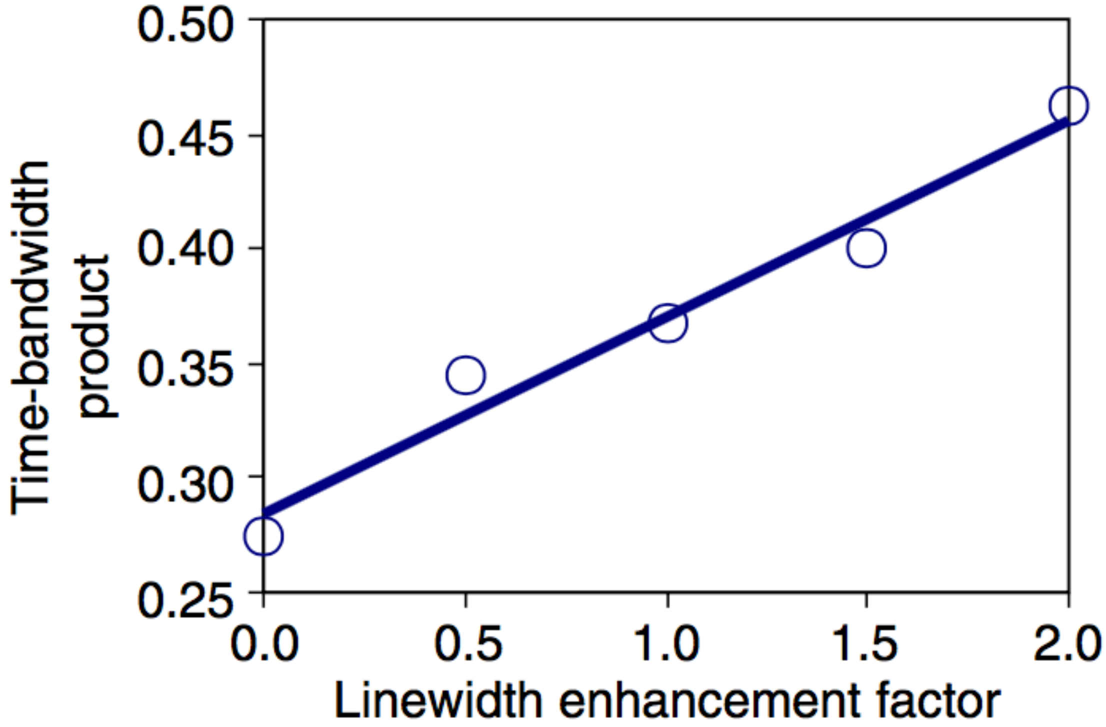
Gaussian pulse w/o chirp

$$\Delta\nu\Delta\tau = \frac{2\ln 2}{\pi} = 0.44$$

Gaussian pulse w/ chirp

$$\Delta\nu\Delta\tau = \frac{2\ln 2}{\pi}\sqrt{1+\alpha_H^2}$$

40 GHz passively mode-locked lasers



K. A. Williams et al., New Journal of Physics, vol. 6, pp. 170 (2004)

Current ultrafast lasers

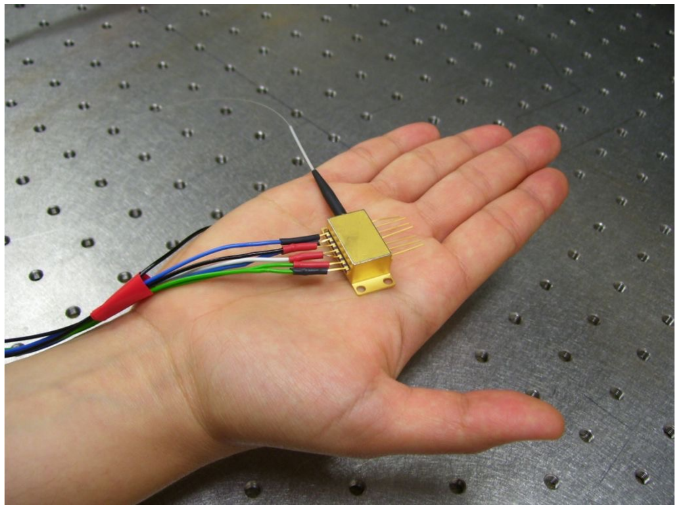
High complexity and footprint

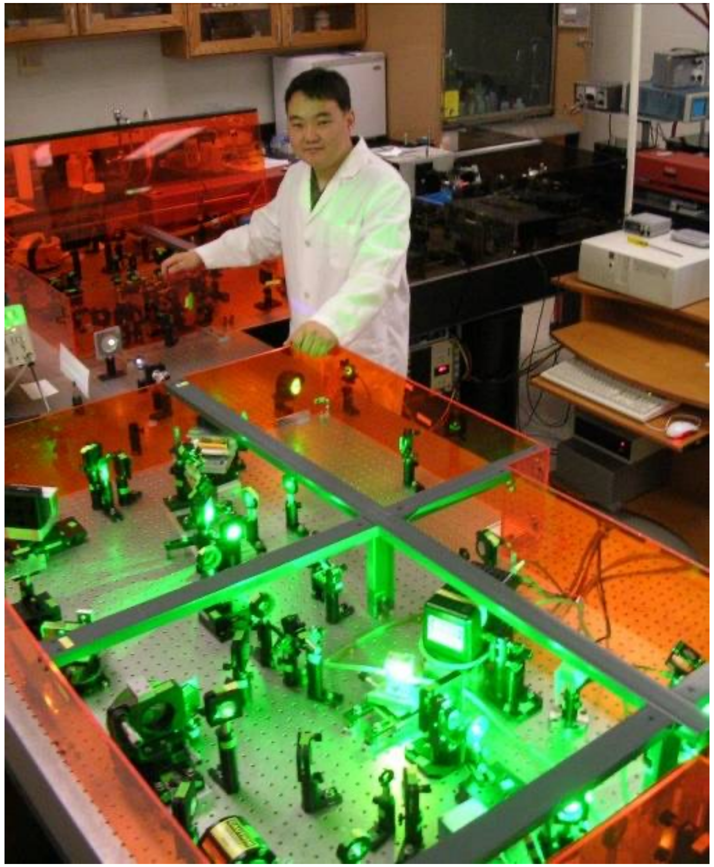
High cost of ownership (€100K - €500K)

High cost of maintenance

As a consequence, there is not a widespread use of ultrafast lasers outside research labs

How to develop efficient, compact and and low-cost ultrafast lasers?



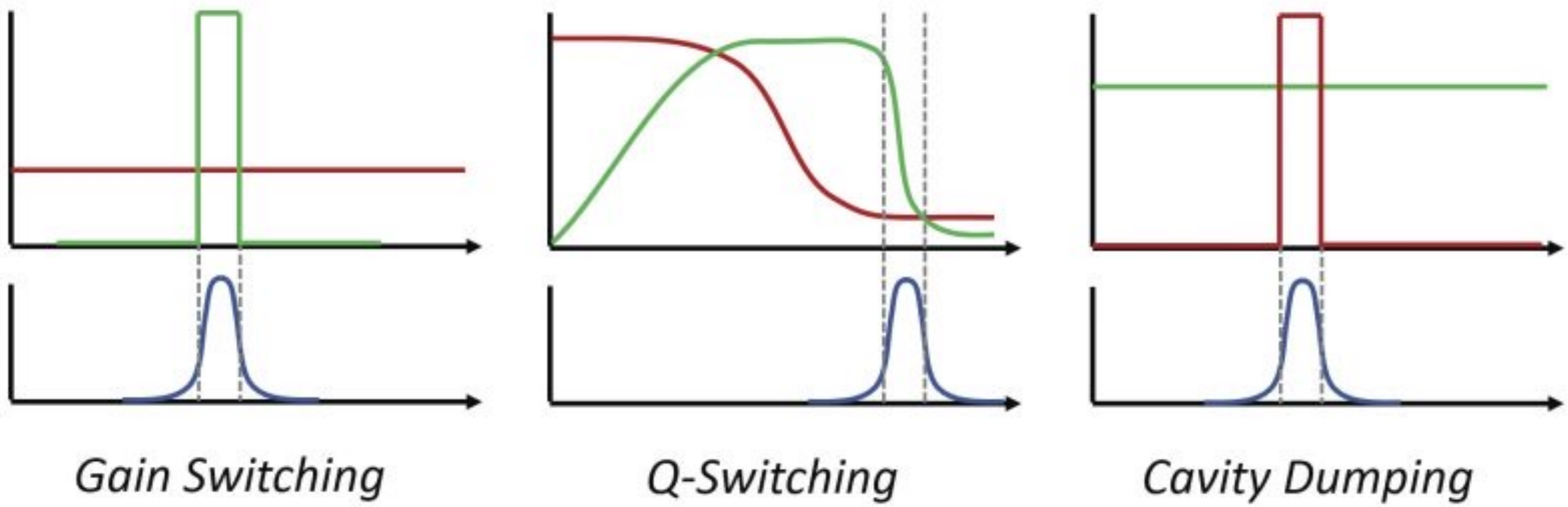


A typical bulky ultrafast laser!

How to produce pulses?

1. Gain switching
2. Q-switching
3. Cavity dumping
4. Mode-locking

Schematic illustrating some methods for producing pulsed lasers where loss (red), gain (green), and laser's output (blue) are shown as a function of time



Gain switching

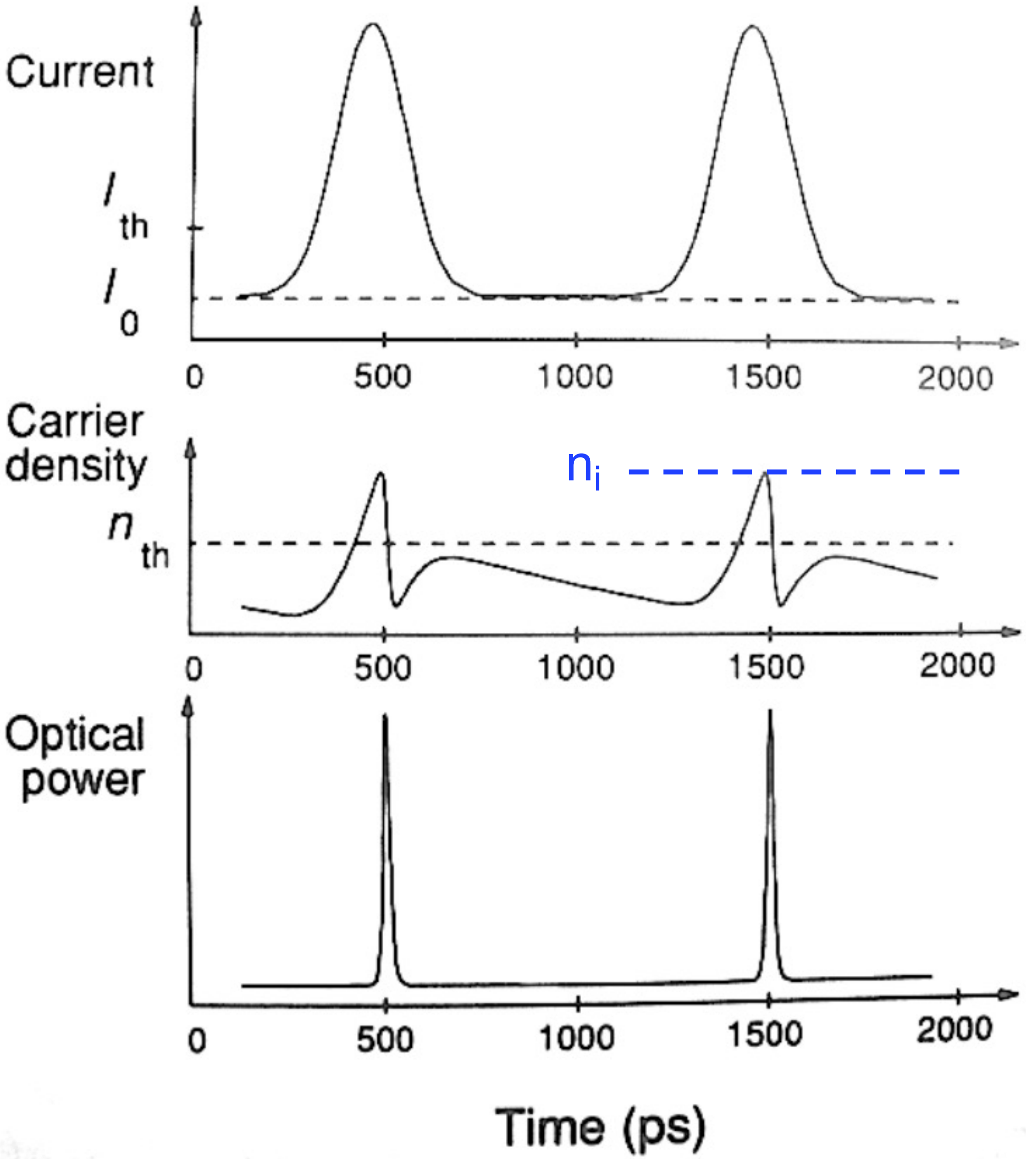
Gain switching originated from relaxations oscillations when turning on a laser diode from below threshold using electrical pulses with a fast leading edge

If the gain medium is pumped much faster than the steady-state value, a transient effect called gain switching occurs. In this case, the population inversion (and the gain coefficient) builds up much faster than the rate of stimulated emission within the laser cavity. The photons in the cavity experience an enormous amount of gain leading to a rapid increase in laser intensity

The electrical pulse width should be shorter and lie in the picosecond and nanosecond time range

P. Paulus et al., IEEE J. Quantum Electron. Vol. 24 (8), pp. 1519 (1988)
J. J. Zayhowski et al., Opt. Lett. Vol. 14 (23), pp. 1318 (1989)

Gain switching



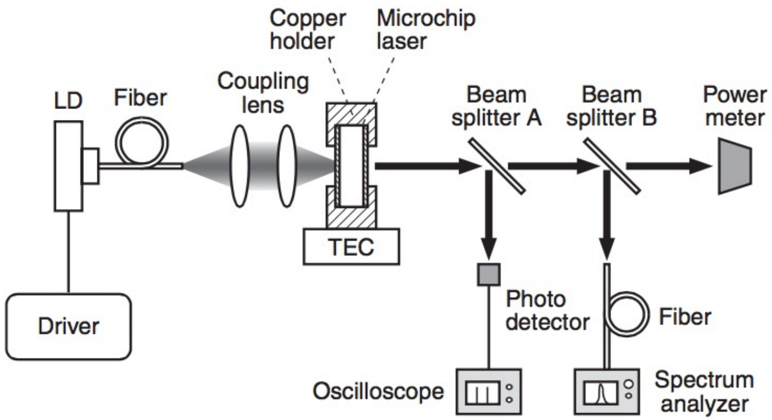
Pulse width

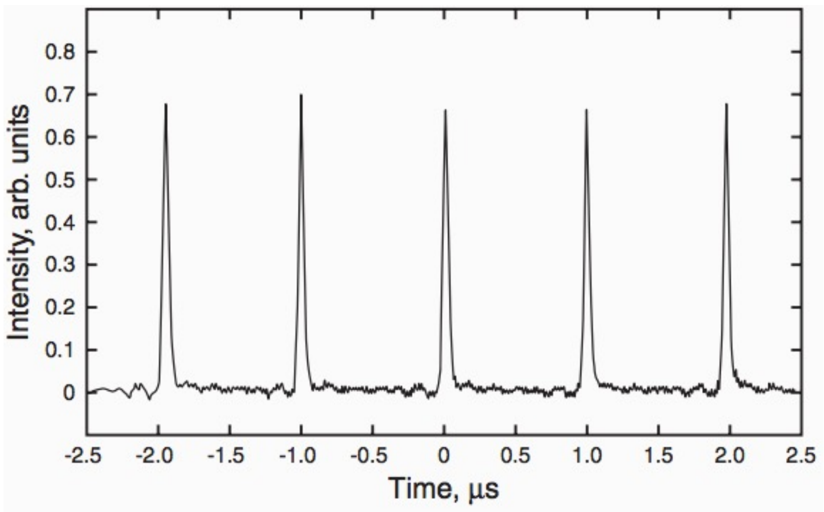
$$\Delta\tau = \tau_p + \frac{1}{\Gamma G_N n_i v_g}$$

Typical values for pulse widths are from 15 to 50 ps

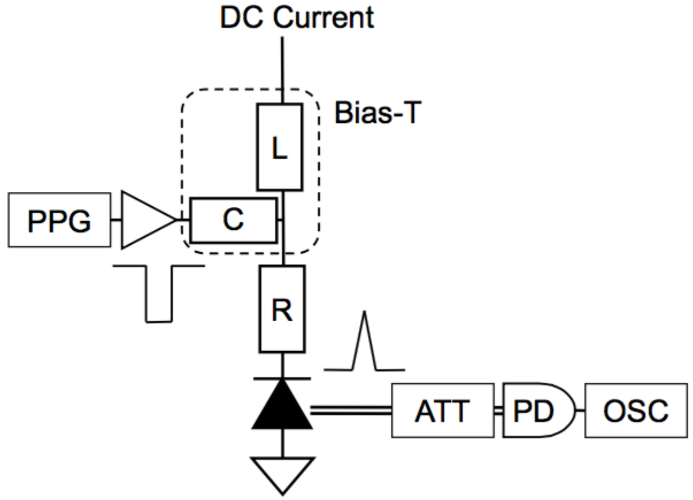
Gain switching

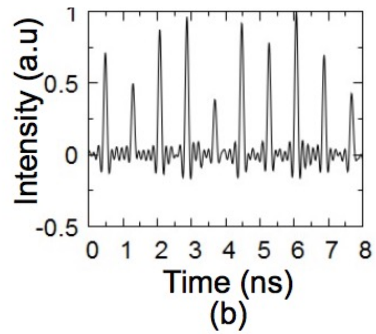
Nd:YAG microchip laser

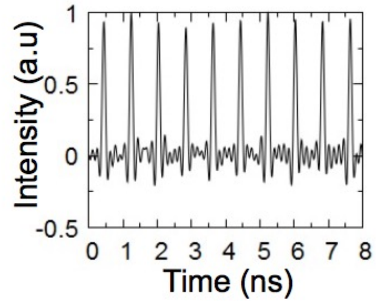




DFB laser







Y. Wang et al. Laser Phys. Lett. Vol. 4, pp. 580 (2007)
K. Nakata et al., Opt. Express. Vol. 25 (2), pp. 622 (2017)

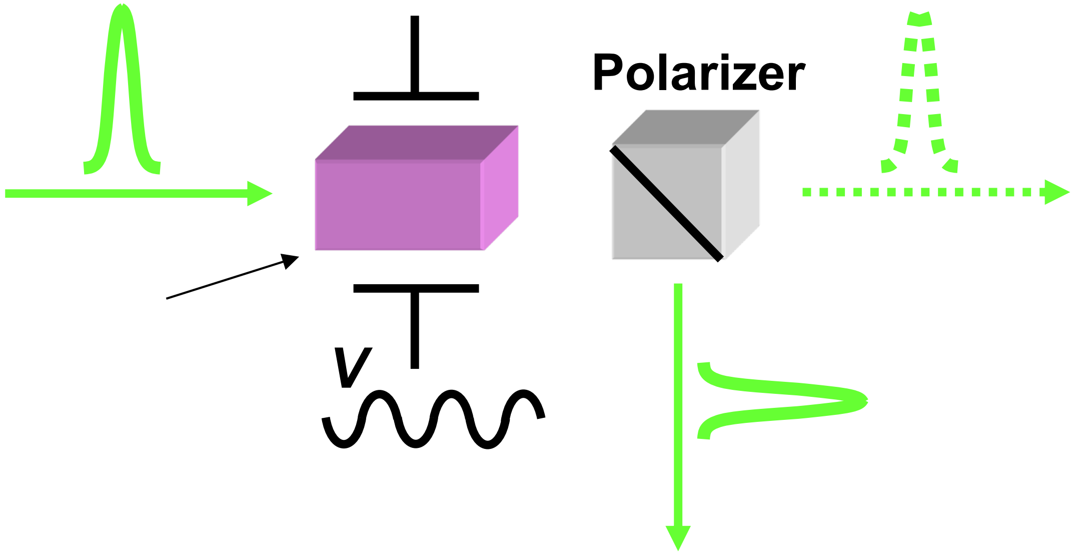
Q switching

Q-switching involves storing energy in the laser gain medium but not by modulating the pump source like in gain switching. Here, the laser pumping process is allowed to build up a population inversion far in excess of the typical threshold value by ensuring that the cavity losses are large, which prevents lasing. Inhibiting the optical feedback is accomplished by adding a loss in the laser cavity

After a large inversion is achieved, the cavity feedback is switched back on. The laser then experiences gain that greatly exceeds losses, and the stored energy is released as a short and intense light pulse

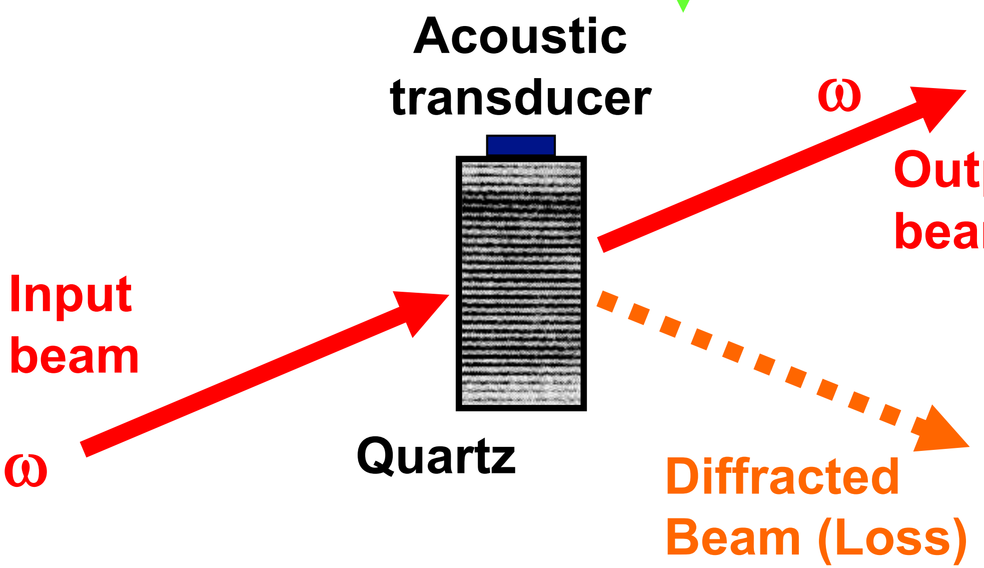
Q-switching methods include:
Active Q switching using electro-optic or acousto-optic switches
Passive Q switching using a saturable absorber (optical nonlinearity)

Electro-optic & acousto-optic switches



Electro-optic Q-switch
Birefringence effect from a Pockels cell (KDP, $LiNbO_3$)

If $V = V_p$, the pulse polarization switches to its orthogonal state (V=0, no change)

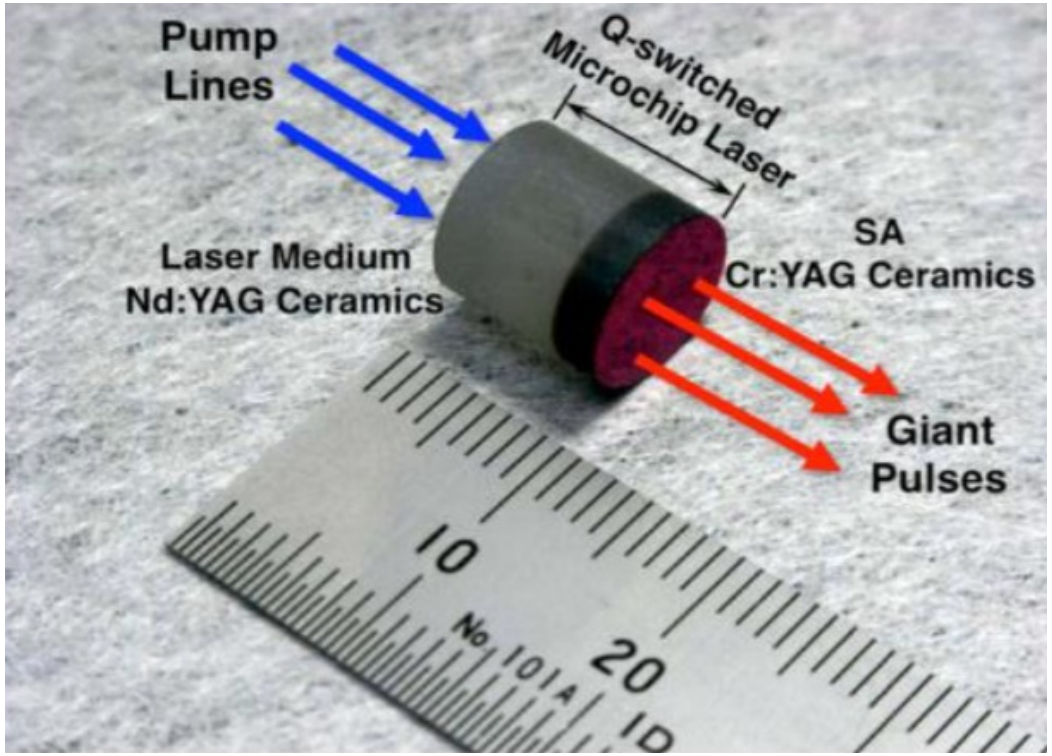


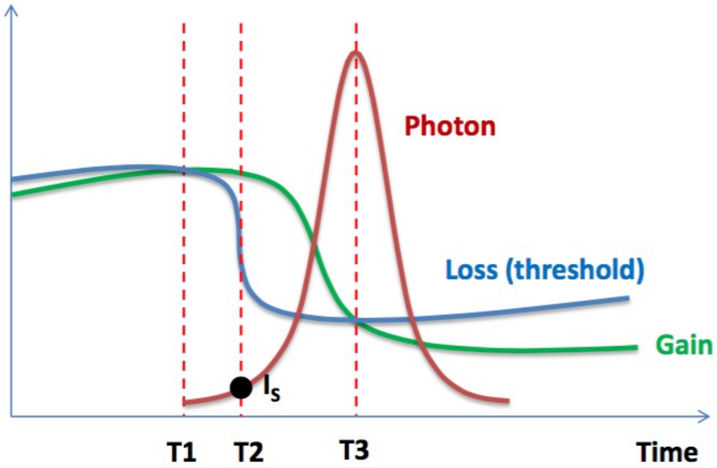
Acousto-optic Q-switch
Changes of the material refractive index through photo-elastic effect → diffracted beam

Ref: J. J. Degnan, IEEE J. Quantum Electron. Vol. 31, pp. 1890 (1995)
J. M. Liu, Photonic Devices, Cambridge University Press

Passive Q switching

The absorber exhibits a high loss (absorption) when the incident light is weak, while it becomes almost transparent (no absorption) when the light is strong (>I_s)





$$\alpha = \frac{\alpha_0}{1 + I/I_s}$$

$$I_s = \frac{h\nu}{2\sigma_a \tau_{sp}}$$

J. J. Degnan, IEEE J. Quantum Electron. Vol. 31, pp. 1890 (1995)
J. M. Liu, Photonic Devices, Cambridge University Press

Cavity dumping

Cavity dumping is a technique used is to keep the optical losses of the laser resonator as low as possible for some time, so that an intense pulse builds up in the resonator, and then to extract this pulse within about one cavity round-trip time using a cavity dumper

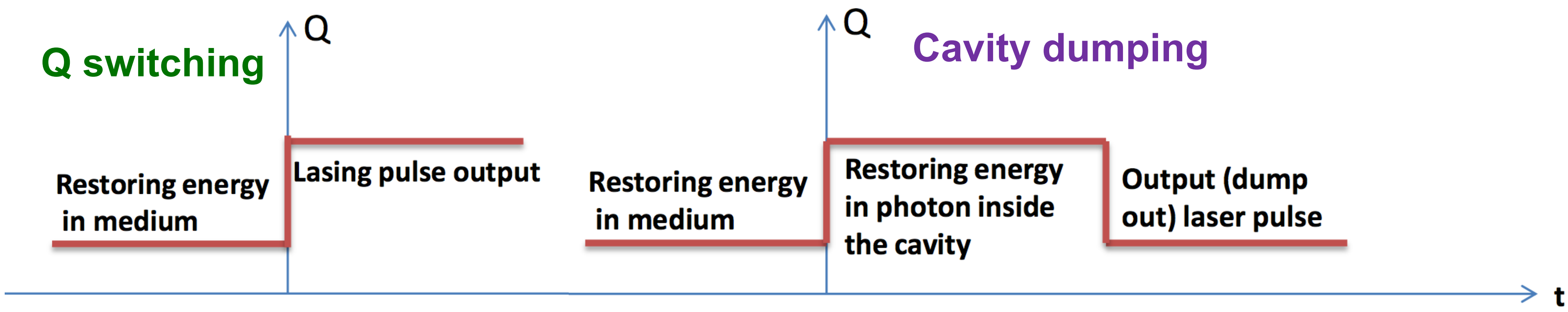
In contrast to prior techniques which store energy in the laser medium via a population inversion, cavity dumping stores energy in the photons within the resonator. The losses within the resonator are kept low for some time by keeping the cavity mirror transmittances negligible, effectively trapping the photons in the cavity and allowing an intense pulse to build up. This pulse is extracted by switching an intra-cavity element after one round trip and “dumping” the pulse out of the cavity

M. Grishin, J. Opt. Soc. Am. B vol. 28 (3), pp. 433 (2011)
X. Yu et al., Appl. Phys. B vol. 106 (2), pp. 309 (2012)
J. Myers et al., Opt. Lett. Vol. 42 (1), pp. 113 (2017)

Cavity dumping

Q switching: The medium restores energy when the cavity Q is low. Once the cavity Q is switched to a high level, the laser pulse will build up in the cavity, meanwhile the pulse is output from the cavity. The laser pulse will oscillates several round trips inside the cavity before the end

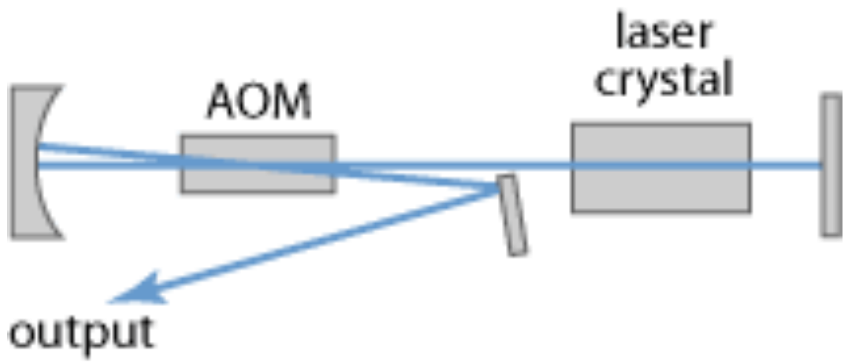
Cavity dumping: The medium restores energy when the cavity has a low Q, once the Q value is switched to a high level, the laser oscillates in the cavity but no output (T=0%). When the Q value is switched back to the low level, the photons are dumped out (T=100%) of the cavity within only one round trip

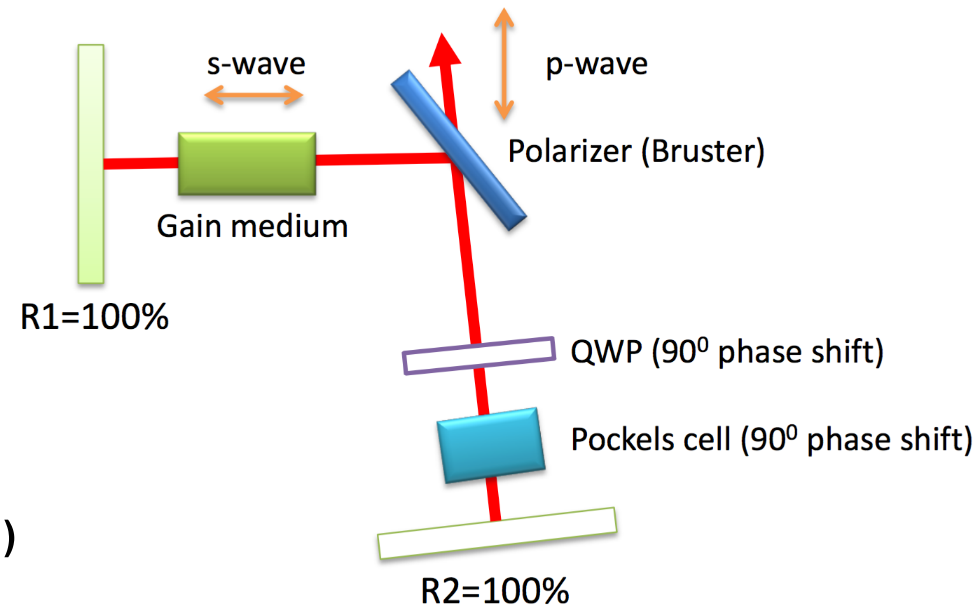


Cavity dumping

Cavity dumping eliminate some basic limitations of Q switching. In particular, it can be disturbing that the pulse duration achievable with a Q switched laser increases when the pulse repetition rate is increased; this is a consequence of the lower laser gain for a lower stored energy in the gain medium. Also, Q switching with high repetition rates may lead to pulse dropout

The optical switch typically is an acousto-optic modulator or electro-optic shutter like a Pockels cell

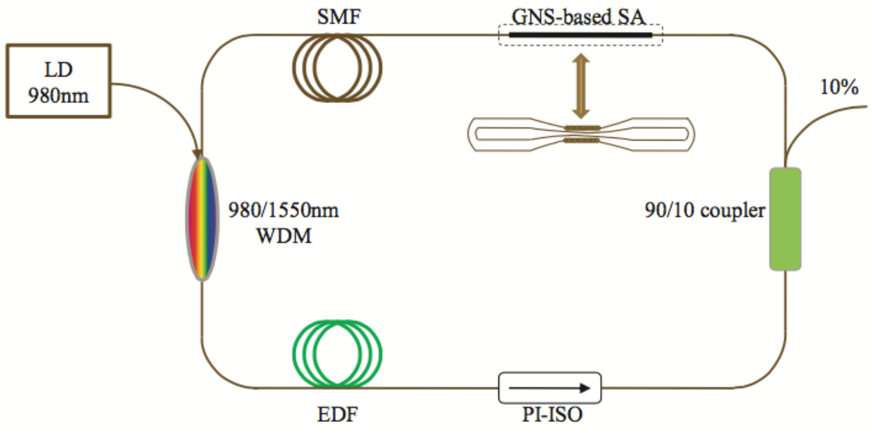


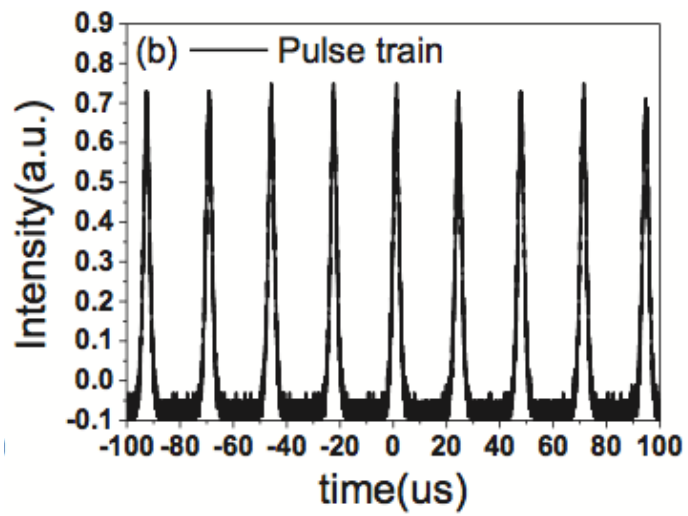


M. Grishin, J. Opt. Soc. Am. B vol. 28 (3), pp. 433 (2011)
X. Yu et al., Appl. Phys. B vol. 106 (2), pp. 309 (2012)
J. Myers et al., Opt. Lett. Vol. 42 (1), pp. 113 (2017)

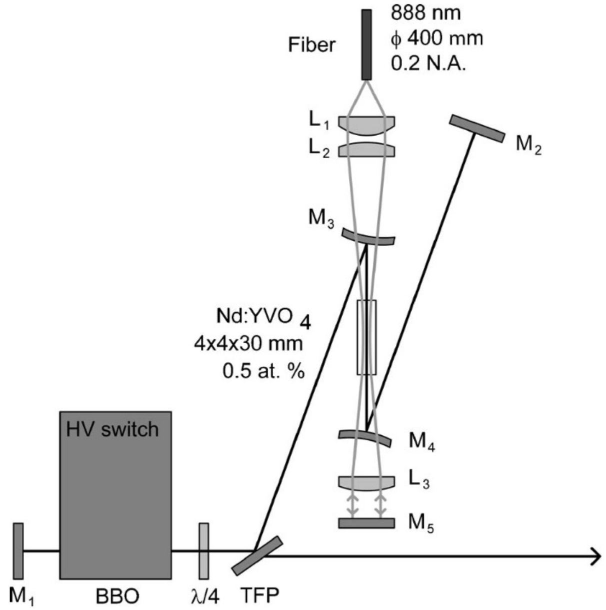
Q switching & Cavity dumping

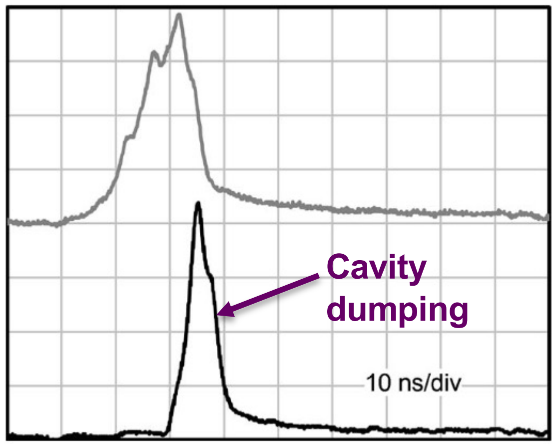
Passively Q-switched fiber laser





Actively Q-switched laser

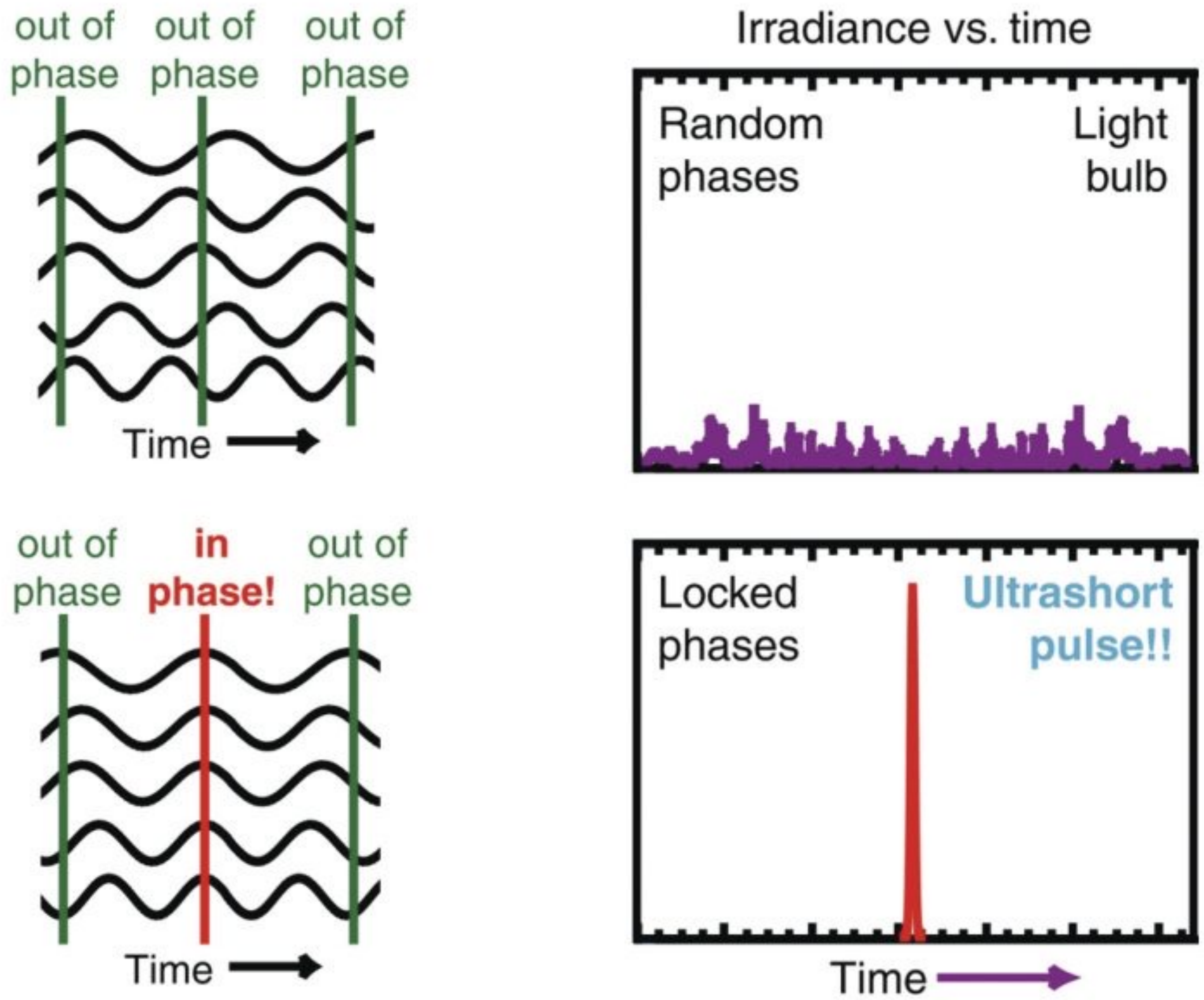




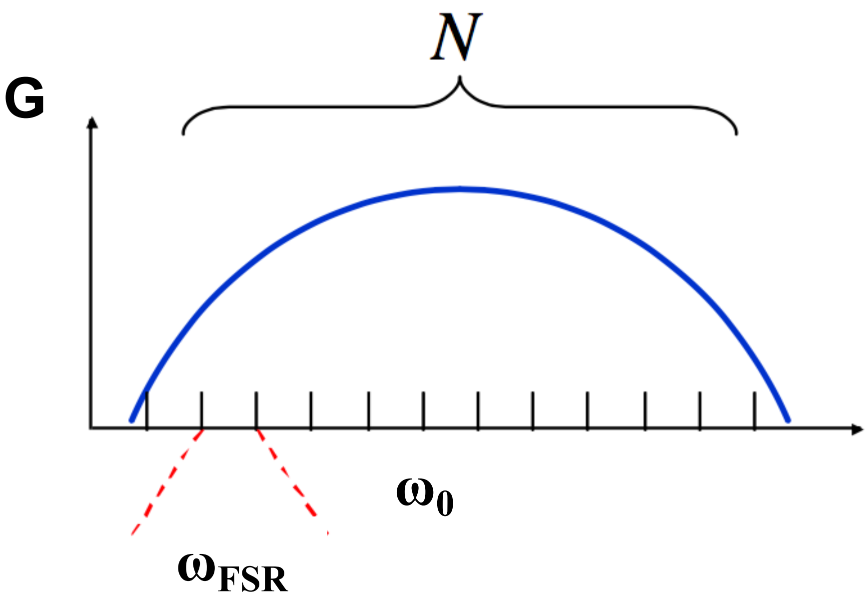
D. Fan et al. Opt. Express. Vol. 22 (15), pp. 18537 (2014)
L. McDonagh et al., Opt. Letts. Vol. 31 (22), pp. 3303 (2006)

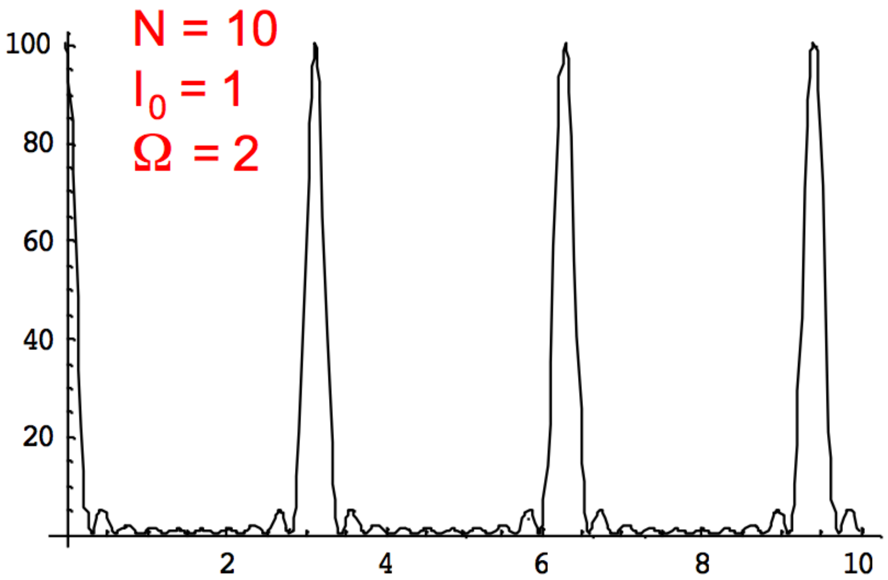
Mode-locking

Powerful technique to generate high repetition rate and short pulses
Semiconductor devices are meaningful owing to their small dimensions



Mode-locking





$$E(t) = \sum_{n=0}^{N-1} E_0 e^{i[\omega_0 + n\omega_{FSR}]t}$$

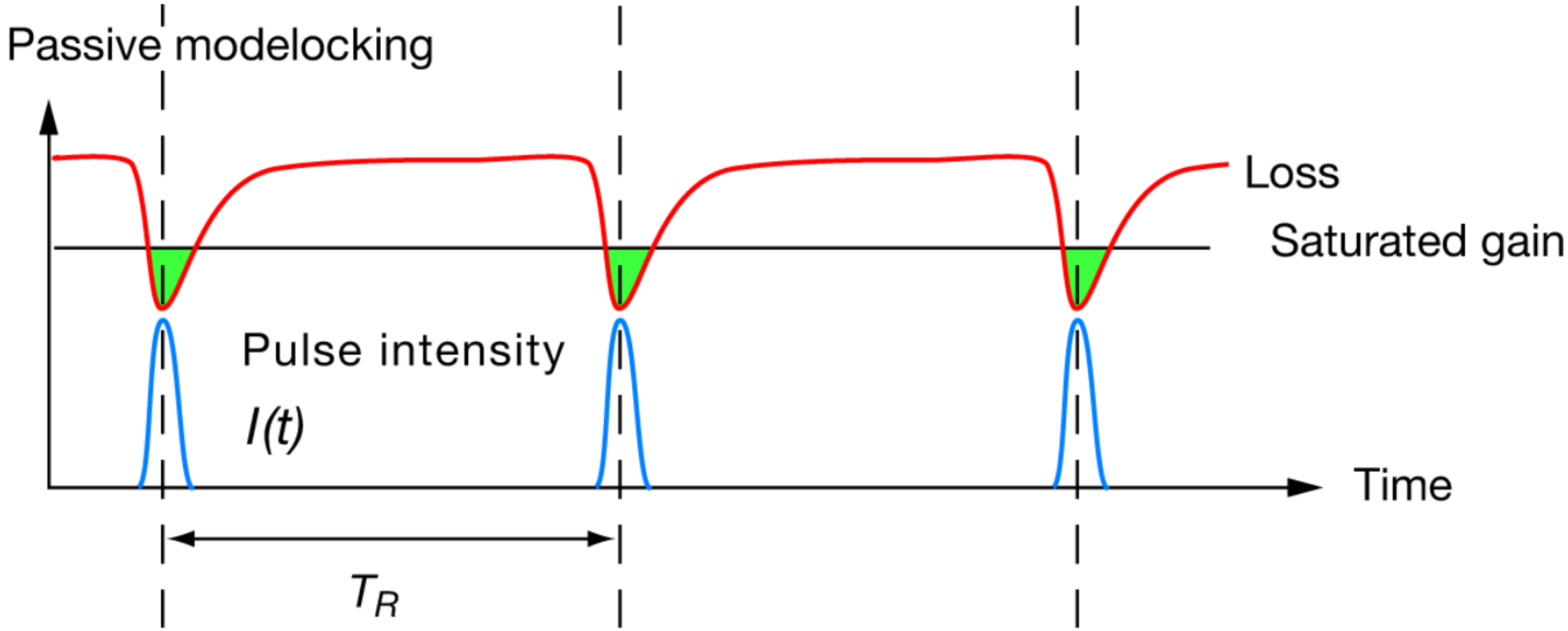
$$E(t) = E_0 \frac{e^{iN\omega_{FSR}t} - 1}{e^{i\omega_{FSR}t} - 1} e^{i\omega_o t}$$

$$\boxed{I(t) = I_0 \frac{\sin^2(N\omega_{FSR}t/2)}{\sin^2(\omega_{FSR}t/2)}}$$

Maximum	$I_{max} = N^2 I_0$
Period	$T = \frac{2\pi}{\omega_{FSR}} = \frac{2L}{c}$
Pulse width	$\Delta T = T/N$
Average intensity	$I_{max} \frac{\Delta T}{T} = N I_0$

Passively mode-locked lasers

Use a nonlinear passive element, such as a saturable absorber leads to the formation of an ultrashort pulse circulating in the laser cavity and causing self-modulation of light



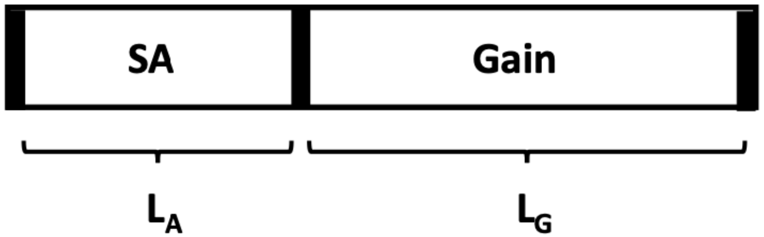
Mode locking needs saturated absorber with relaxation time less than round trip time of the cavity e.g. giant pulse generation develops better with slow relaxing absorber (totally different than Q switching!)

Passively mode-locked lasers

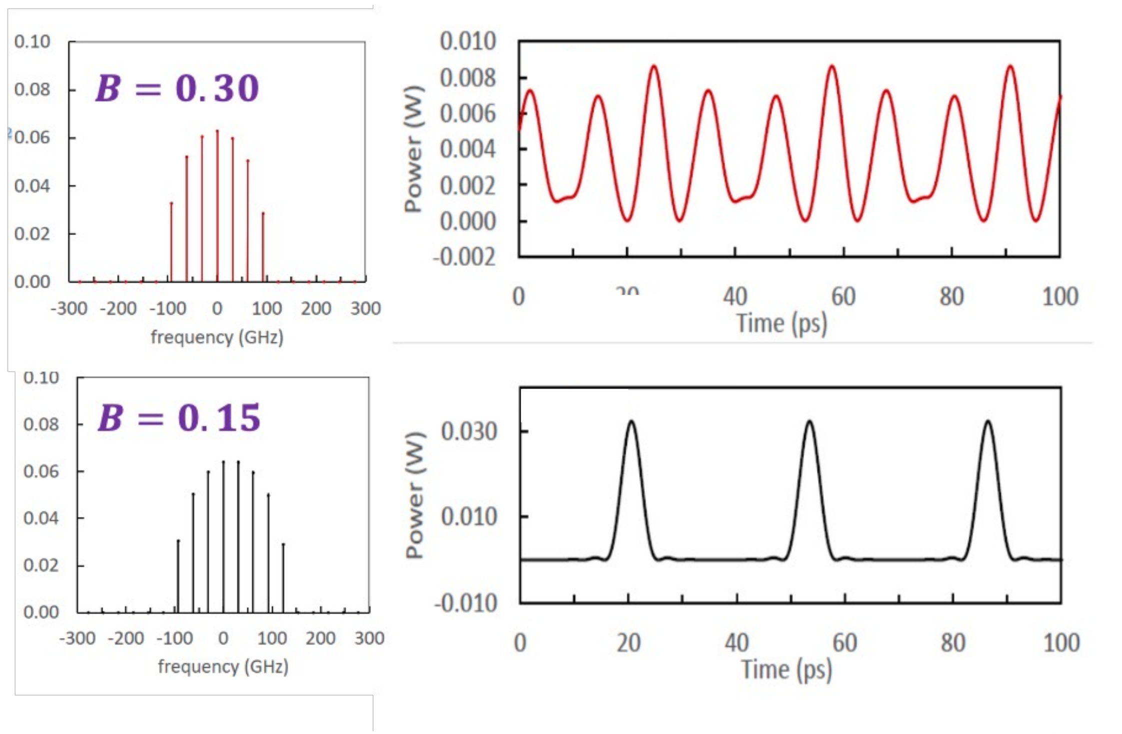
High performance mode - locked lasers

Importance of the design (quality of the pulses)

Many modes required (duration of the pulses)

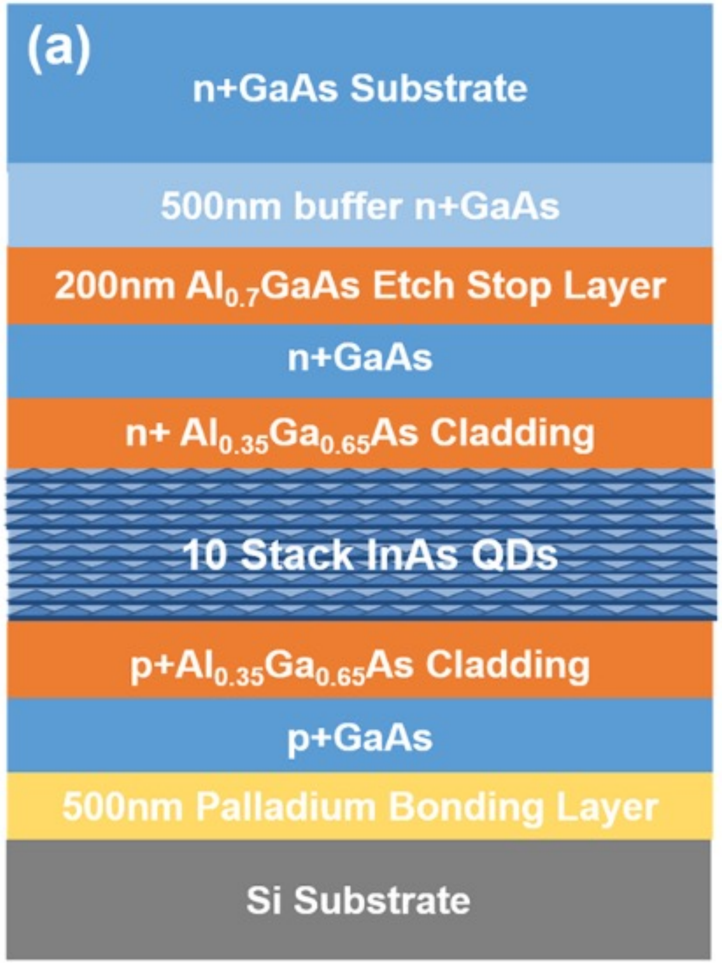


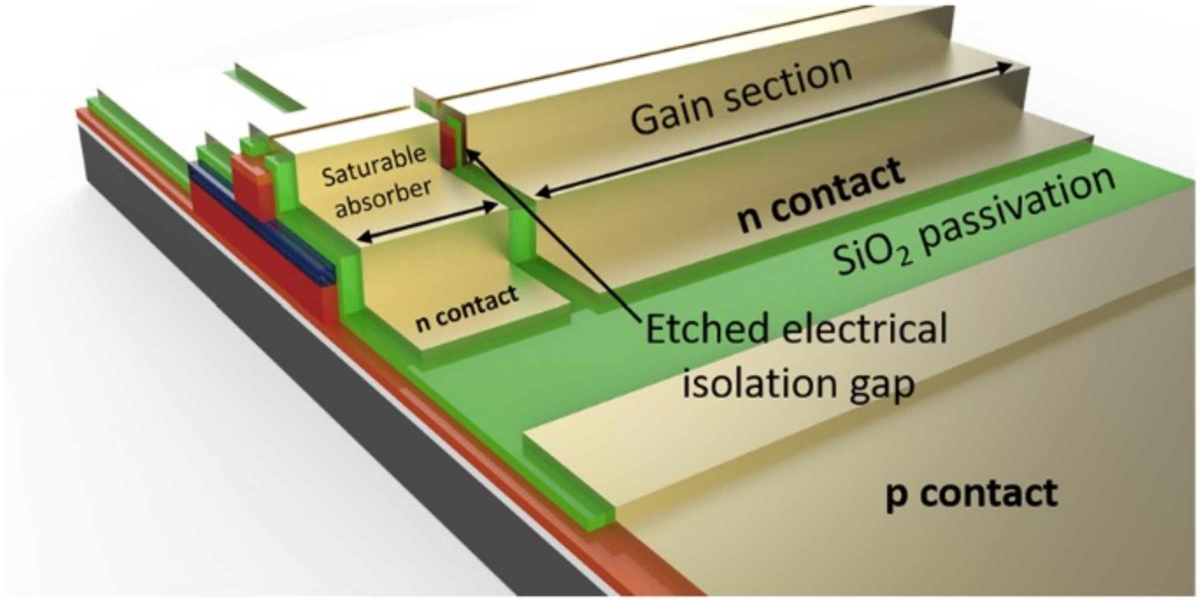
$$B = \frac{L_G}{L_G + L_A}$$



Passively mode-locked lasers

Quantum dot mode-locked lasers for silicon photonic integrated circuits

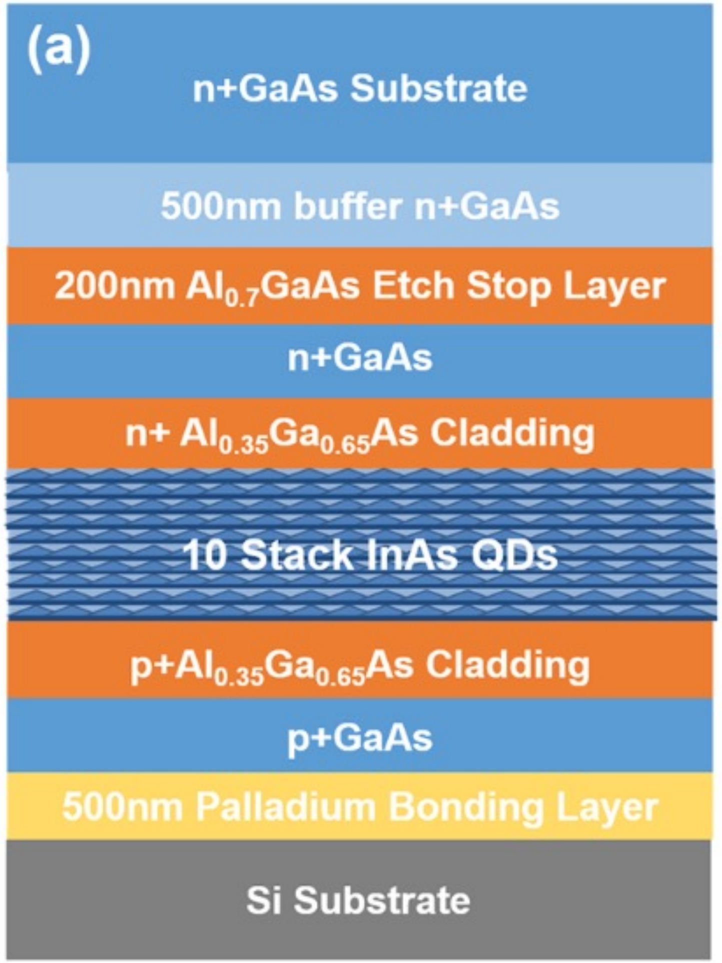


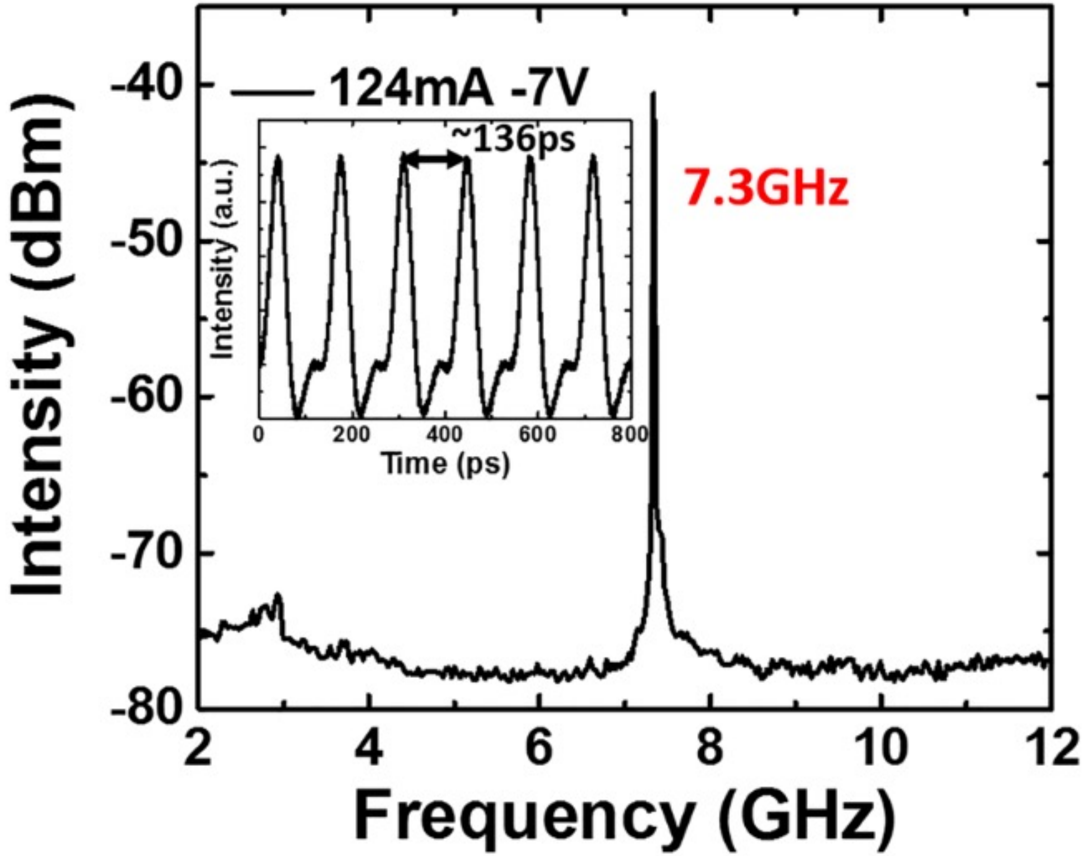


Z. Wang, Appl. Phys. Lett. vol. 110, pp. 141110 (2017)

Passively mode-locked lasers

Quantum dot mode-locked lasers for silicon photonic integrated circuits





Z. Wang, Appl. Phys. Lett. vol. 110, pp. 141110 (2017)

Passively mode-locked lasers

State-of-the-art of quantum dot mode-locked lasers

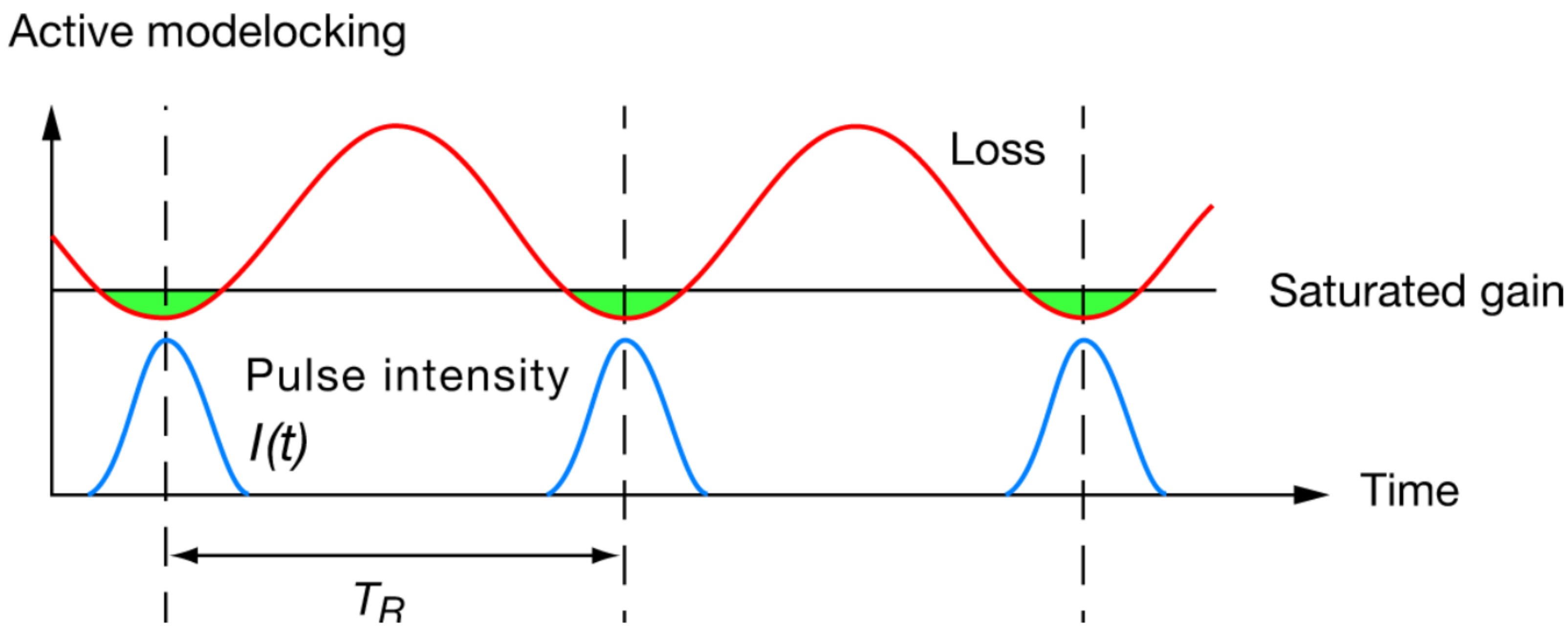
Substrate	Type	Repetition rate (GHz)	Pulse duration (ps)	TBP†	3-dB optical bandwidth (THz)	Year
InP	QDash	92	0.312	0.457	1.472	2008 [264]
InP	QDash	101	-	-	1.2	2009 [265]
InP	QDash	48-346	0.8	0.46	-	2011 [266]
InP	QD	50	0.43	-	1.65	2020 [267]
InP	QDash	34.2	-	-	1.6	2021 [268]
GaAs	QD	7.2-51	6.4	1.7	-	2007 [261]
GaAs	QD	39-237	0.36	0.5	-	2009 [262]
GaAs	QD	40	0.7	0.5	-	2010 [255]
GaAs	QD	60	-	-	2.46	2022 [269]
GaAs	QD	100	0.466	0.472	0.78	2022 [270]
GaAs	QD	100	0.81	0.45	1.9	2022 [189]
Si	QD	102	-	-	1.4	2018 [145]
Si	QD	20	5	-	1.14	2019 [257]
Si	QD	15.5	-	-	2.1	2022 [271]

† Time-bandwidth product

F. Grillot et al., Quantum Photonics (2023)

Actively mode-locked lasers

Modulation of the resonator loss via acousto-optic and electro-optic modulators or by external signal (diode laser)

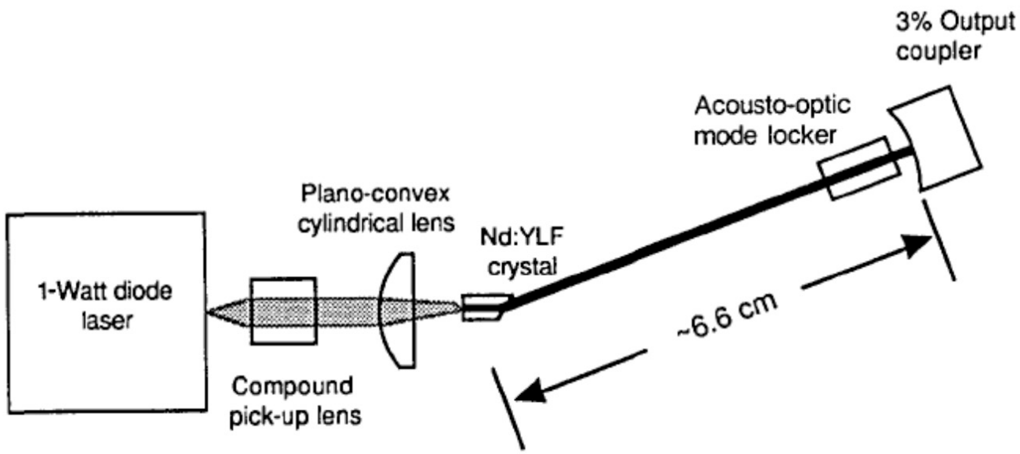


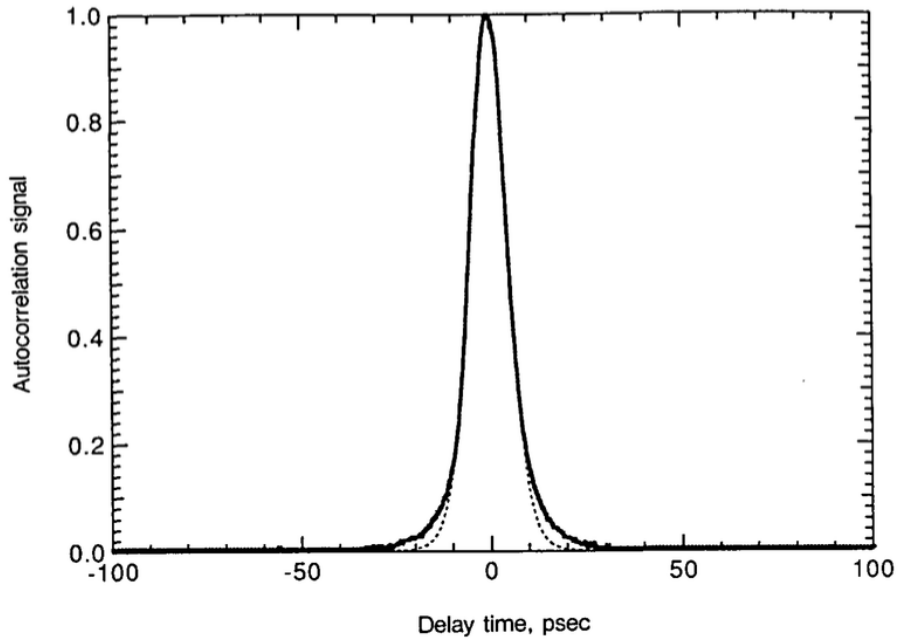
In diode lasers, low timing jitter is obtained, but high repetition frequencies difficult because fast modulation of the current is more difficult with increase in frequency

Actively mode-locked lasers

Diode-pumped Nd:YLF laser: actively mode-locked with an acousto-optic mode-locker (AOM)
AOM: Sapphire substrate, 0.5% loss modulation per watt

Average output power: 135 mW
Pulse duration: 7.1 ps
Pulse repetition rate: 2 GHz
Wavelength: 1.047 µm

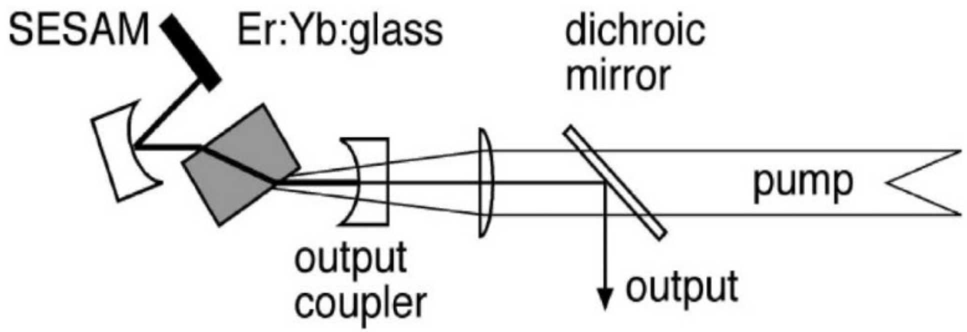


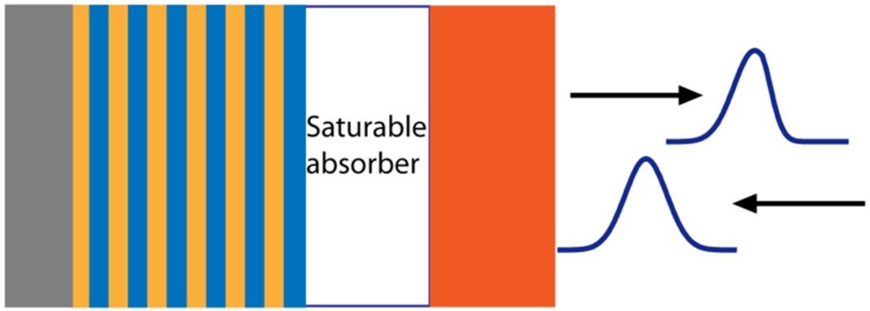


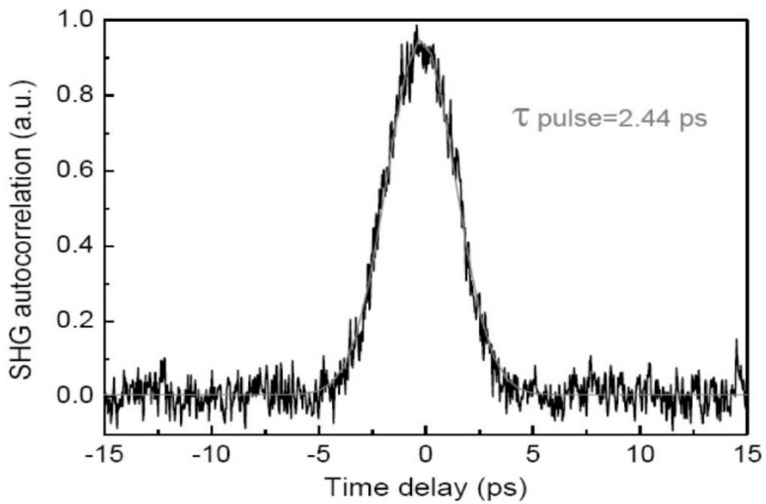
K. J. Weingarten et al, Opt. Lett., Vol. 15, pp. 962, (1990)
U. Keller et al, Opt. Lett., Vol. 15, pp. 45, (1990)

SESAM mode-locked lasers

Mode-locked lasers can use a SEmiconductor Saturable Absorber Mirror (SESAM) that is a saturable absorber operating in reflection i.e. the reflectivity increases with higher incoming pulse intensity







Mode-locked lasers using SESAMs usually exhibit low timing jitter, high pulse-to-pulse phase coherence, and high individual optical spectral mode signal to noise ratio →better to reach high repetition rate

Z.-Y. Zhang, Scientific Reports, vol 2, pp. 477 (2012)
U. Keller, Nature, vol. 424, pp. 831, (2003)

Appendices

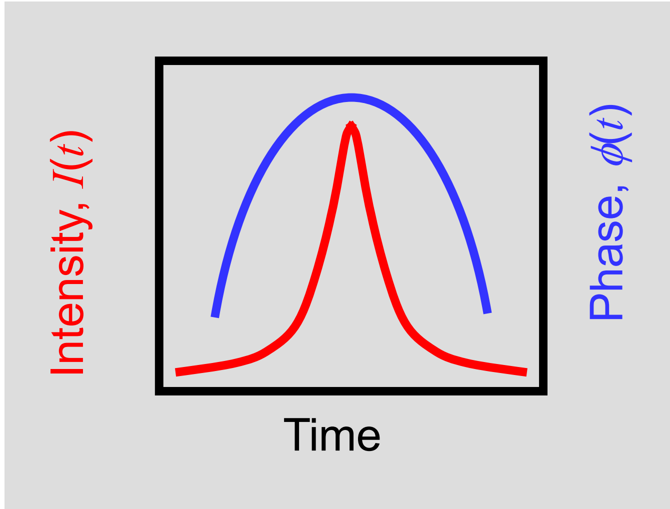
Pulse measurements

How to measure the intensity and phase vs. time or frequency?

The electric field can be written:

$$E(t) \propto \sqrt{I(t)}\exp\left[-i\phi(t)\right]$$

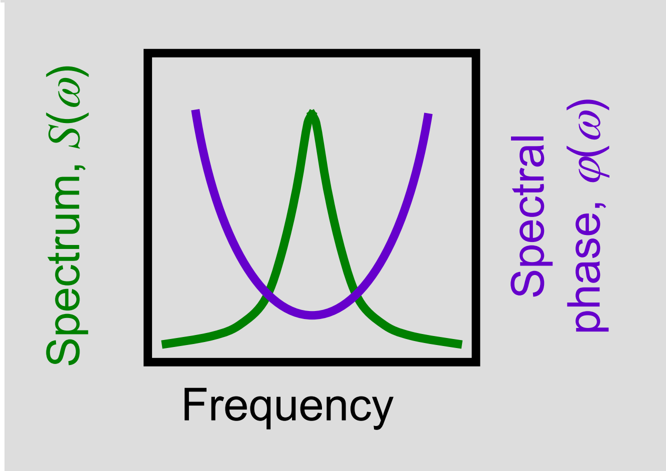
Intensity **Phase**



Alternatively, versus frequency:

$$\tilde{E}(\omega) \propto \sqrt{S(\omega)}\exp\left[-i\varphi(\omega)\right]$$

Spectrum **Spectral Phase**



We need to extract both the temporal (or spectral) intensity and phase

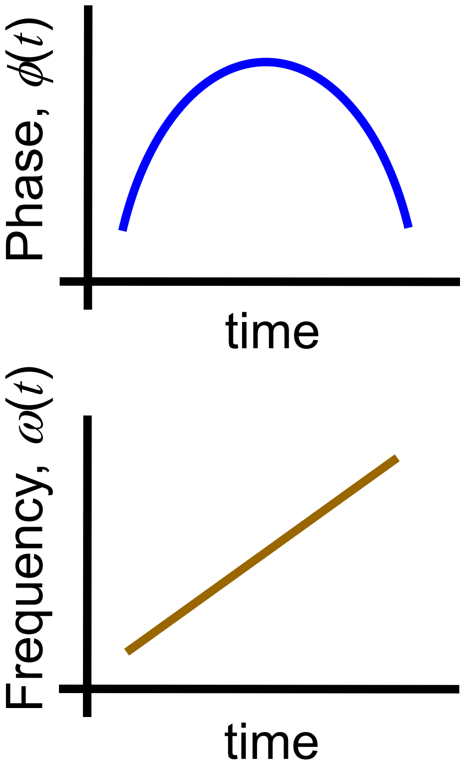
Adapted from Prof. Trebino, GeorgiaTech, USA

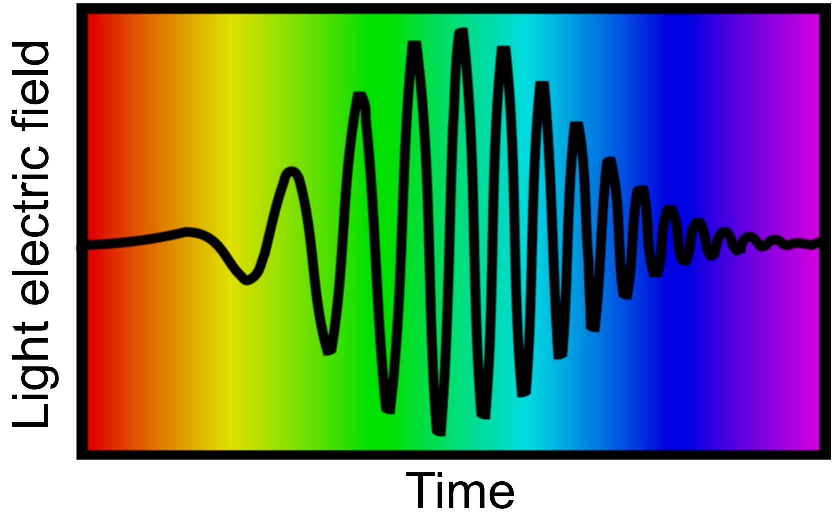
The phase determines the pulse frequency (color) vs. time

Instantaneous frequency

$$\omega(t) = \omega_0 - d\phi / dt$$

Linear chirp





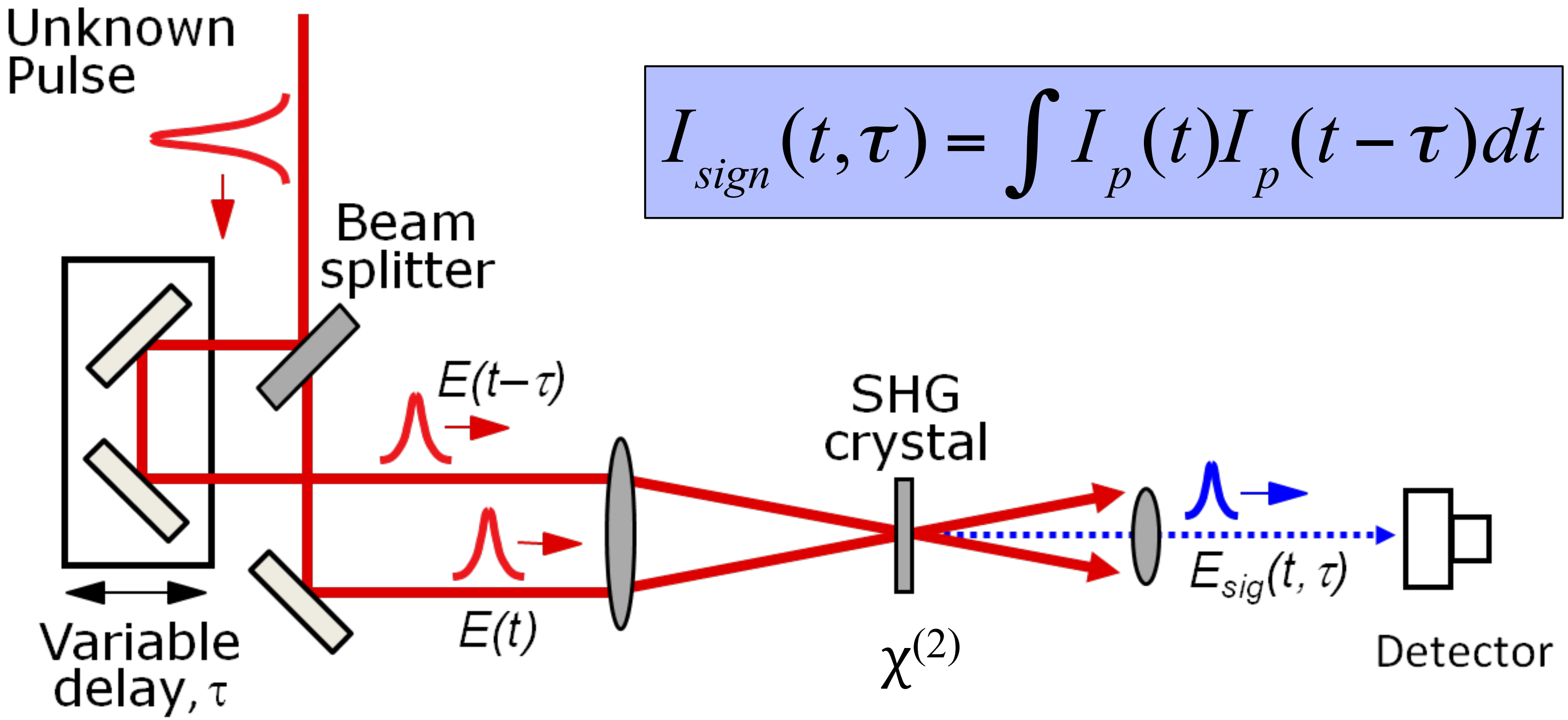
The objective is to measure not only linearly chirped pulses, but also pulses with arbitrarily complex phases and frequencies vs. time

Adapted from Prof. Trebino, GeorgiaTech, USA

SHG autocorrelation

Pulse interacts with a delayed replica of itself. The good temporal resolution and the low cost make this technique one of the most widely used methods for measuring ultrashort pulse durations

$$I_{sign}(t,\tau) = \int I_p(t) I_p(t-\tau) dt$$



K. W. DeLong et al. Journal of Optical Society of America B, vol. 11, pp. 2206, (1994)

SHG autocorrelation

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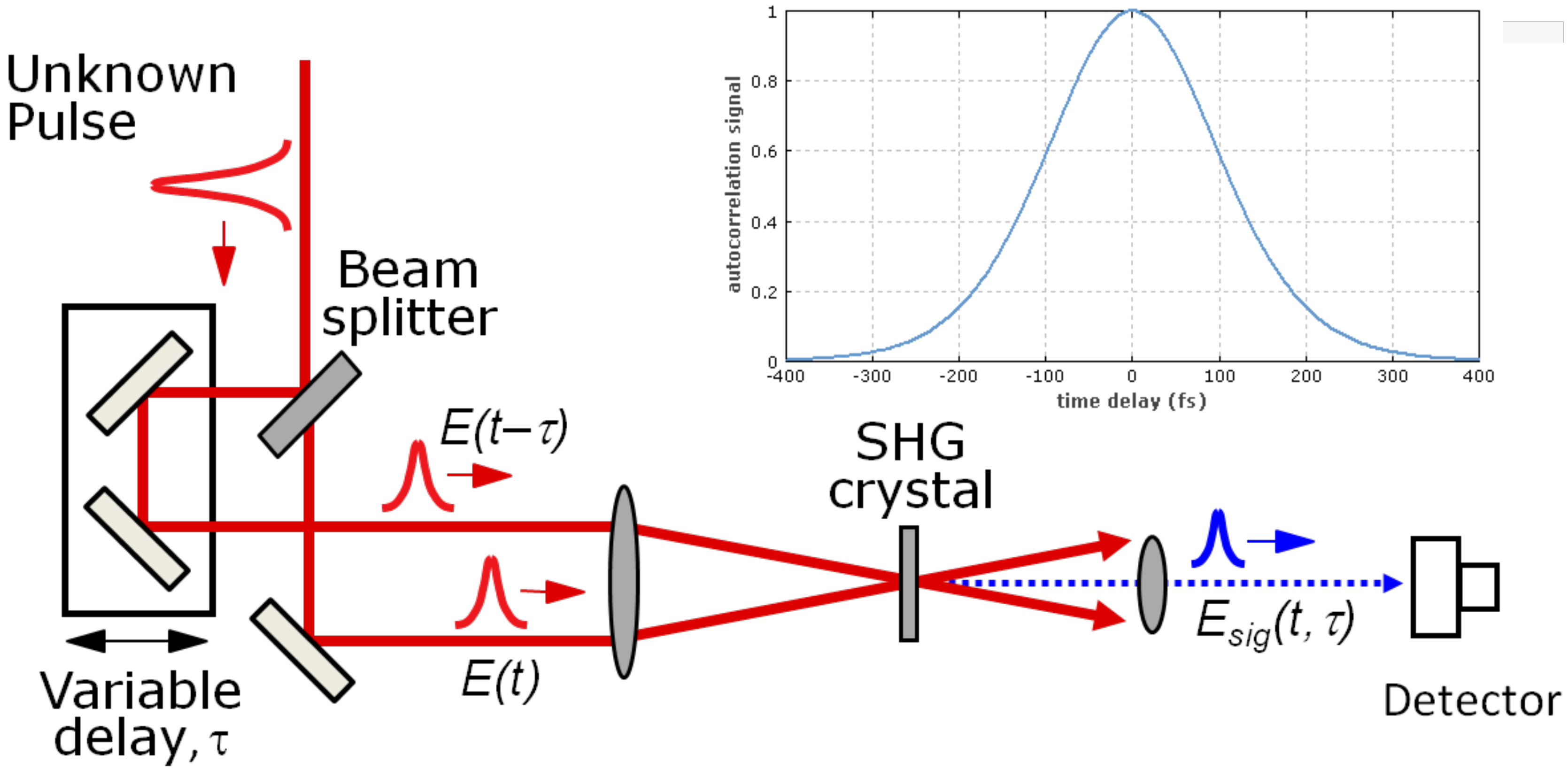
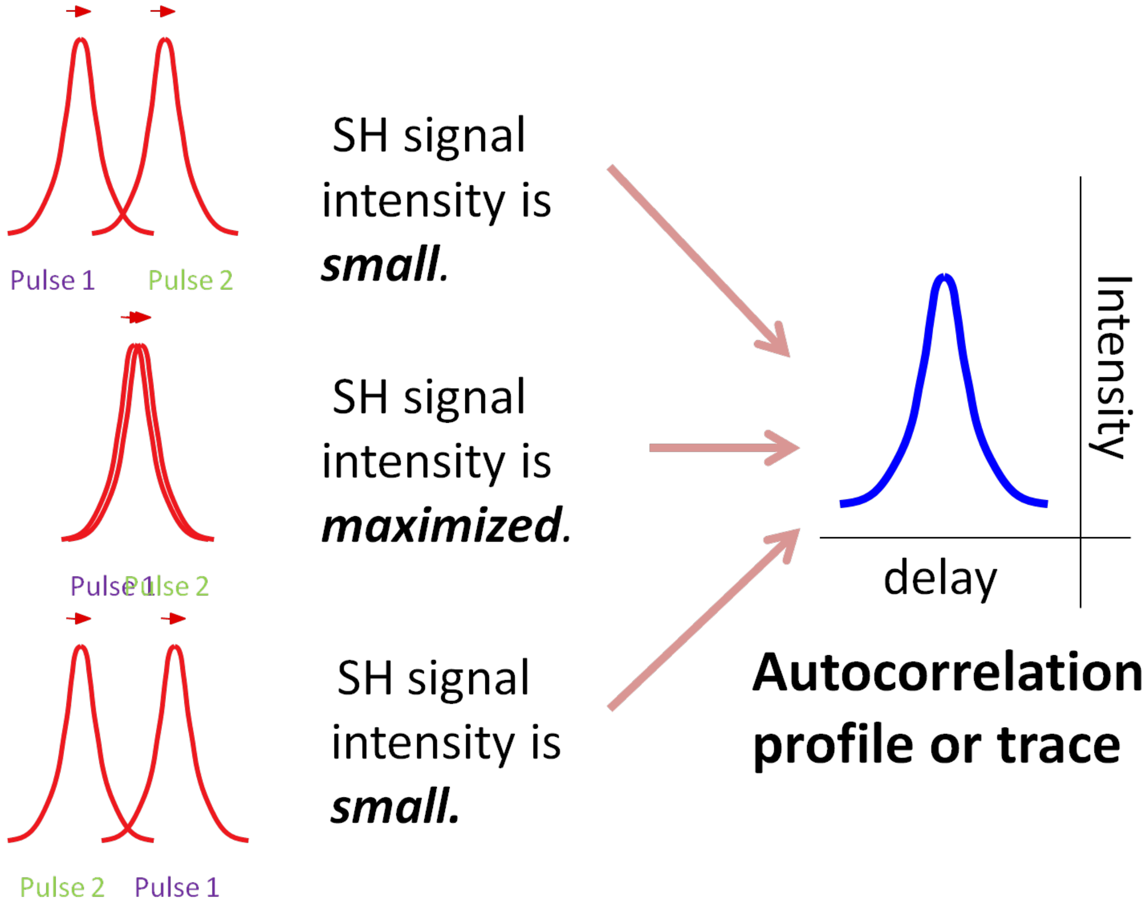


A light pulse takes 1ps to travel 300 µm in air – the temporal resolution is a few fs corresponding to a spatial accuracy of a few microns

SHG autocorrelation

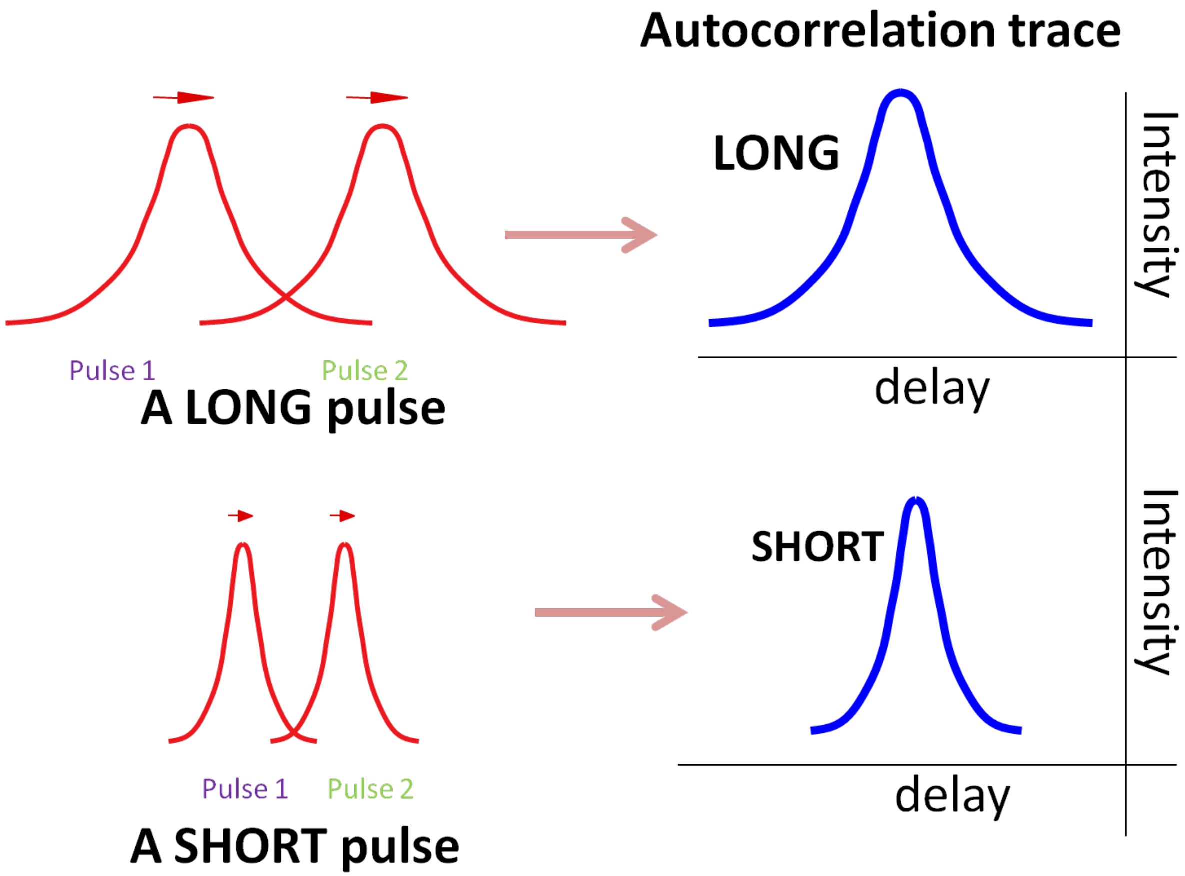
Varying the delay yields varying overlap between the two replicas of the pulse. The intensity autocorrelation is only nonzero when the pulses overlap



K. W. DeLong et al. Journal of Optical Society of America B, vol. 11, pp. 2206, (1994)

SHG autocorrelation

A long pulse generates a wide trace and a short pulse generates a narrow trace. With this, it is possible to estimate the pulse duration



K. W. DeLong et al. Journal of Optical Society of America B, vol. 11, pp. 2206, (1994)

SHG autocorrelation

The SHG crystal produces a signal at twice the frequency of the input pulse and the detector measures an average power of the doubled frequency, and so it does not require a fast photo-detector to estimate durations of short pulses

The autocorrelation function is always symmetric and thus conceals any information about the symmetry properties of the original pulse (a deconvolution factor must be also considered)

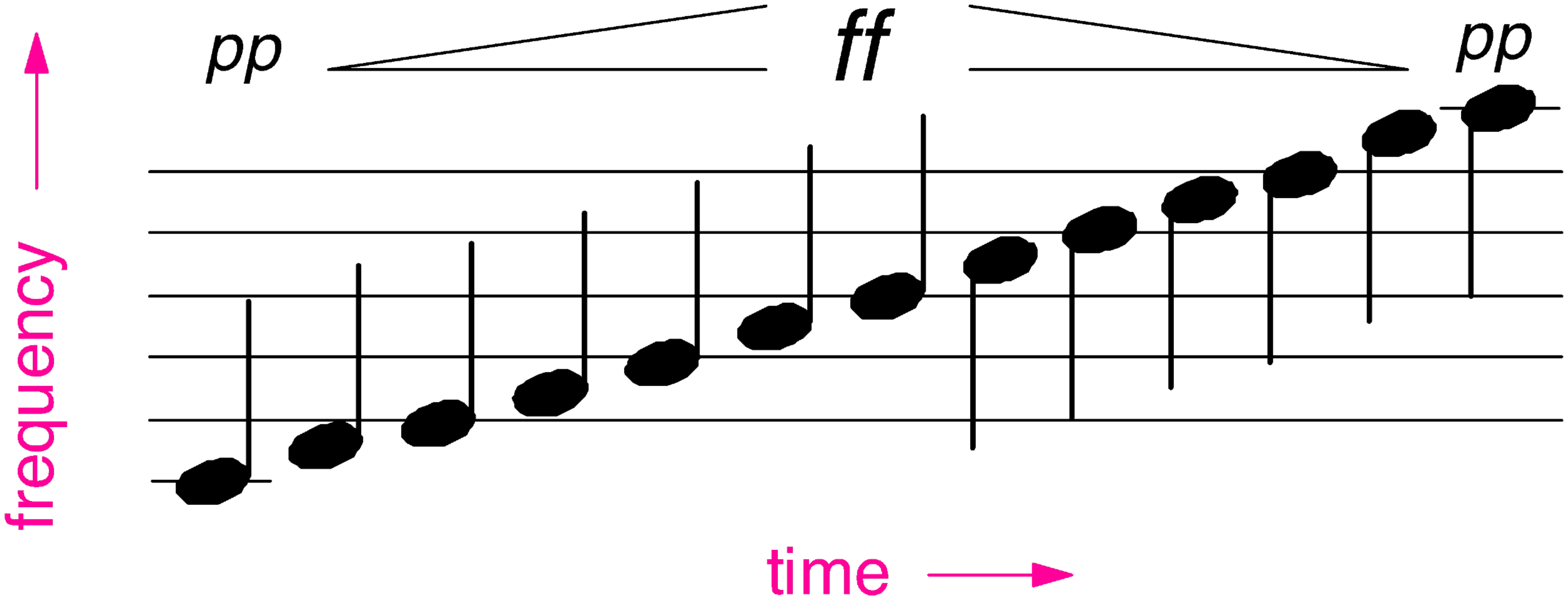
No information about the phase and the chirp of the pulse is available from intensity autocorrelation traces

Different pulses can result in similar autocorrelation functions, so there is a certain degree of ambiguity on matching a pulse shape to the autocorrelation trace

J. H. Chung and A. M. Weiner, IEEE J. Sel. Topics Quantum Electron., vol. 7, pp. 656, (2001)

SHG FROG

Most people think of acoustic waves in terms of a musical score



It's a plot of frequency vs. time, with information on top about the intensity. The musical score lives in the time-frequency domain

pp, standing for pianissimo and meaning "very quiet"

ff, standing for fortissimo and meaning "very loud"

Adapted from Prof. Trebino, GeorgiaTech, USA

SHG FROG

A mathematically rigorous form of a musical score is a spectrogram

If E(t) is the waveform of interest, its spectrogram is:

$$\Sigma_E(\omega,\tau) \equiv \left| \int_{-\infty}^{\infty} E(t)\, g(t-\tau)\, \exp(-i\omega t)\, dt \right|^2$$

where g(t-τ) is a variable-delay gate function and τ is the delay

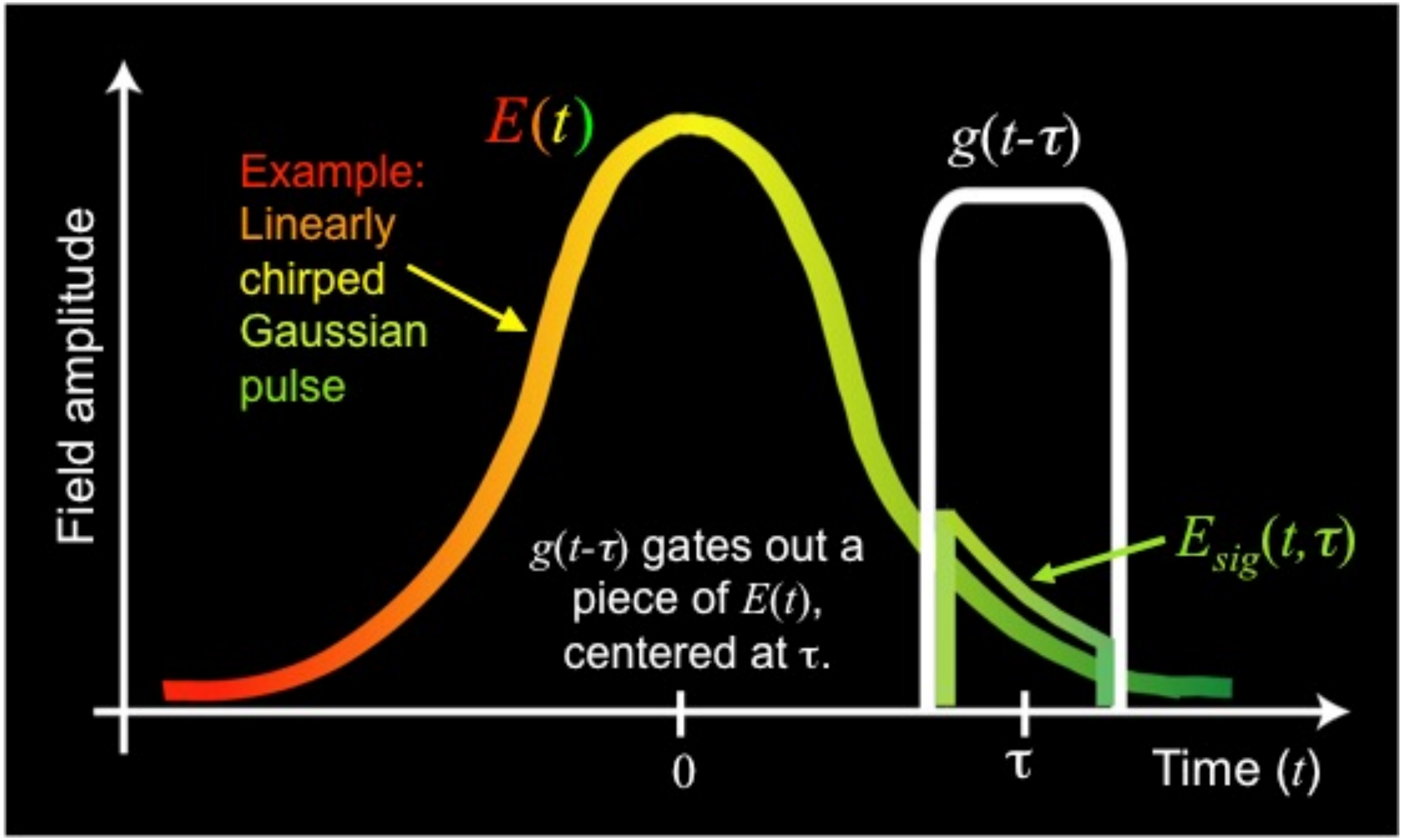
w/o g(t-τ), $\Sigma_E(\omega,\tau)$ would simply be the spectrum!!!

The spectrogram is a function of ω and τ and is a set of spectra of all temporal slices of E(t)

Adapted from Prof. Trebino, GeorgiaTech, USA

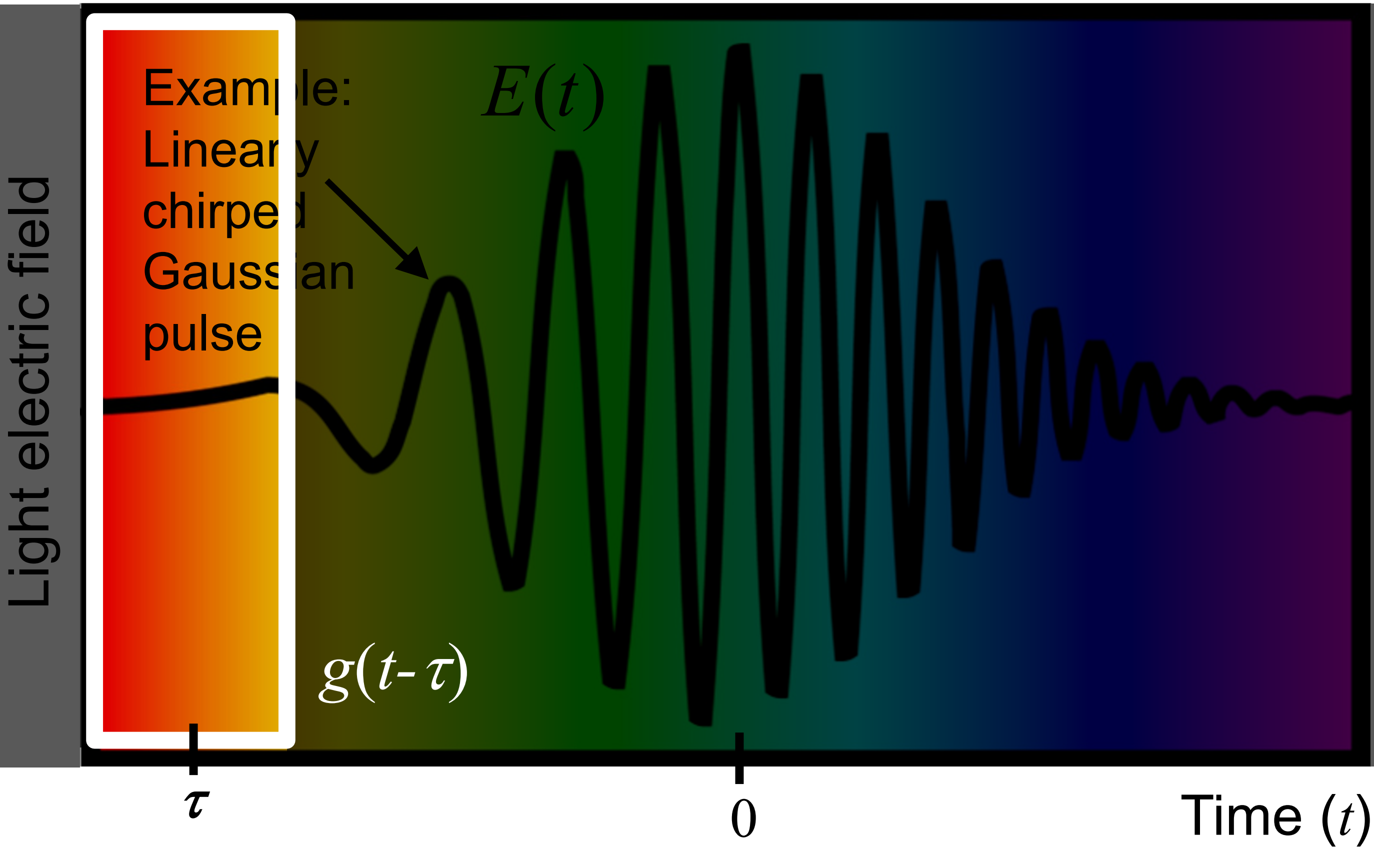
SHG FROG

The spectrogram tells the color and intensity of $E(t)$ at the time, τ



SHG FROG

The spectrogram tells the color and intensity of $E(t)$ at the time, τ



$g(t-\tau)$ gates out a piece of $E(t)$, centered at τ

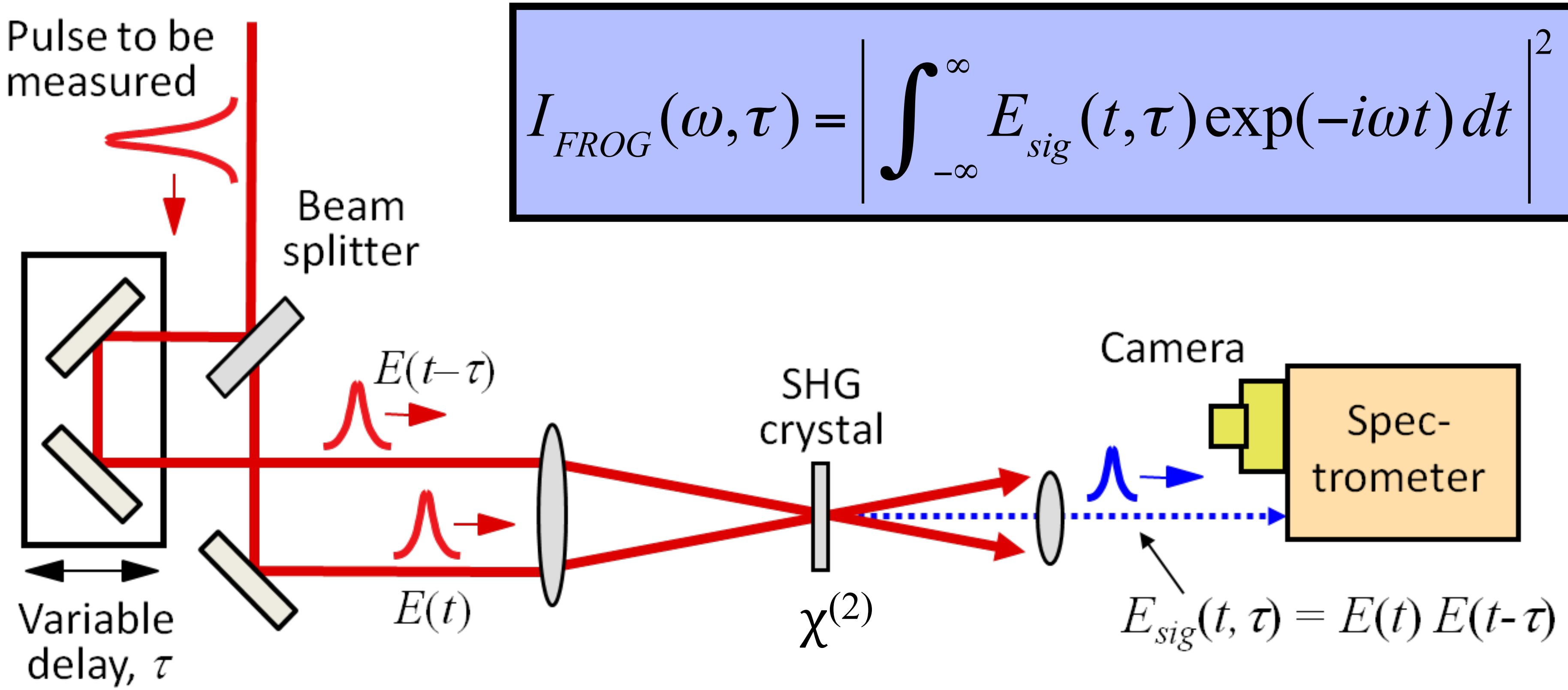
D. J. Kane, and R. Trebino, IEEE Journal of Quantum Electronics, vol. 29, pp. 571, (1993)
R. Trebino, "Frequency-resolved optical gating: the measurement of ultrashort laser pulses", Springer, (2000)

SHG FROG

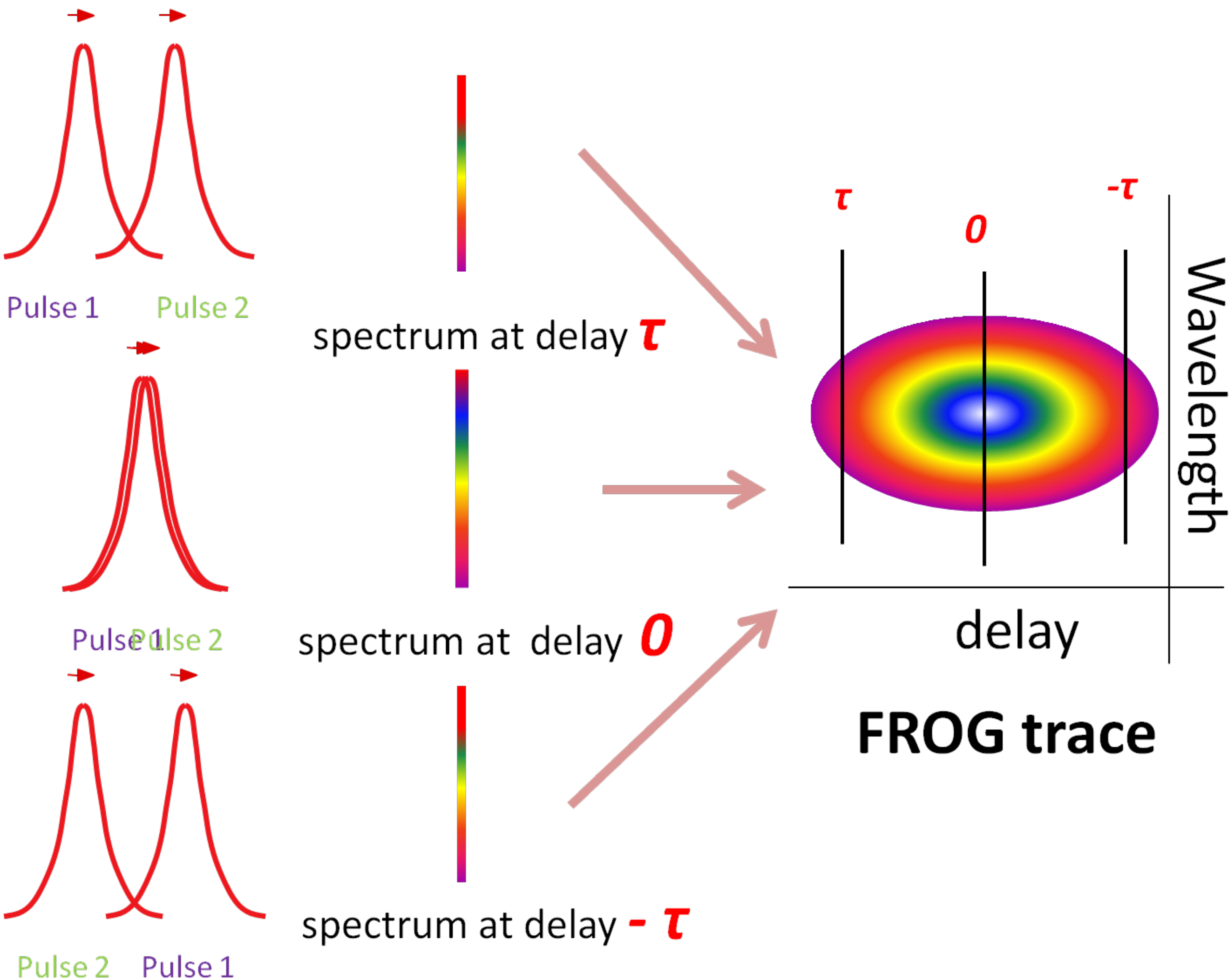
Frequency-Resolved Optical Gating (FROG) technique can measure the intensity and phase of the pulses in the time or/and the spectral domains

After getting a signal from the SHG crystal, the signal is sent to a spectrometer instead of a photo-detector

$$I_{FROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)\,dt \right|^2$$

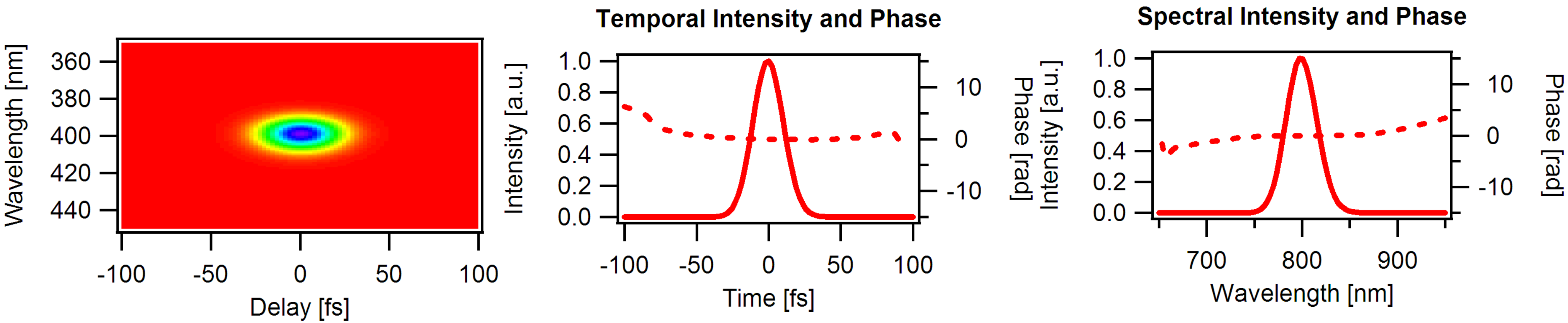


SHG FROG

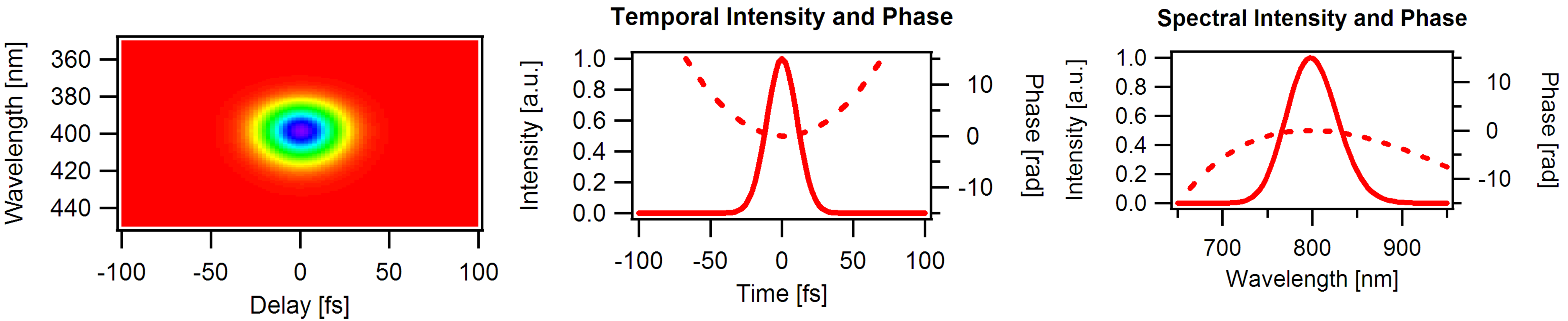

Pulse 1
Pulse 2
spectrum at delay τ
Pulse 1
Pulse 2
spectrum at delay 0
Pulse 2
Pulse 1
spectrum at delay - τ
τ
0
-τ
Wavelength
delay
FROG trace

SHG FROG

Transform limited Gaussian shape (25 fs)



Linearly chirped Gaussian pulse (25 fs)

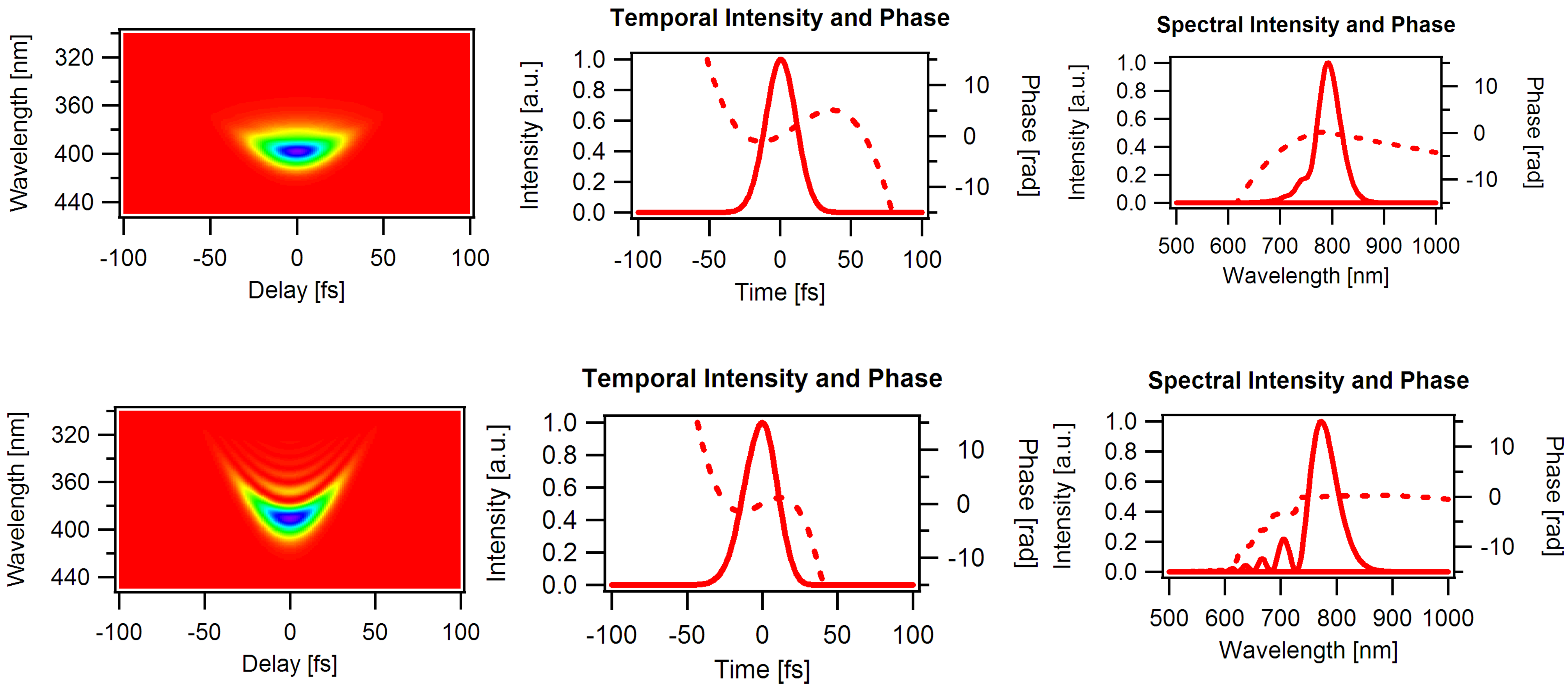


Like a musical score, the spectrogram visually displays the temporal/spectral intensity and phase

Femto Inc., http://advancedfemtotech.com

SHG FROG

Spectrograms for more complex pulses….



Femto Inc., http://advancedfemtotech.com

SHG FROG

Iterative phase retrieval algorithm

Algorithms exist to retrieve $E(t)$ from its spectrogram. The spectrogram essentially determines the waveform intensity, $I(t)$, and phase, $\phi(t)$

The gate need not be and should not be much shorter than $E(t)$

Try by using a delta-function gate pulse!

$$\boxed{\left| \int_{-\infty}^{\infty} E(t)\, \delta(t-\tau)\, \exp(-i\omega t)\, dt \right|^2} = \left| E(\tau) \exp(-i\omega\tau) \right|^2$$

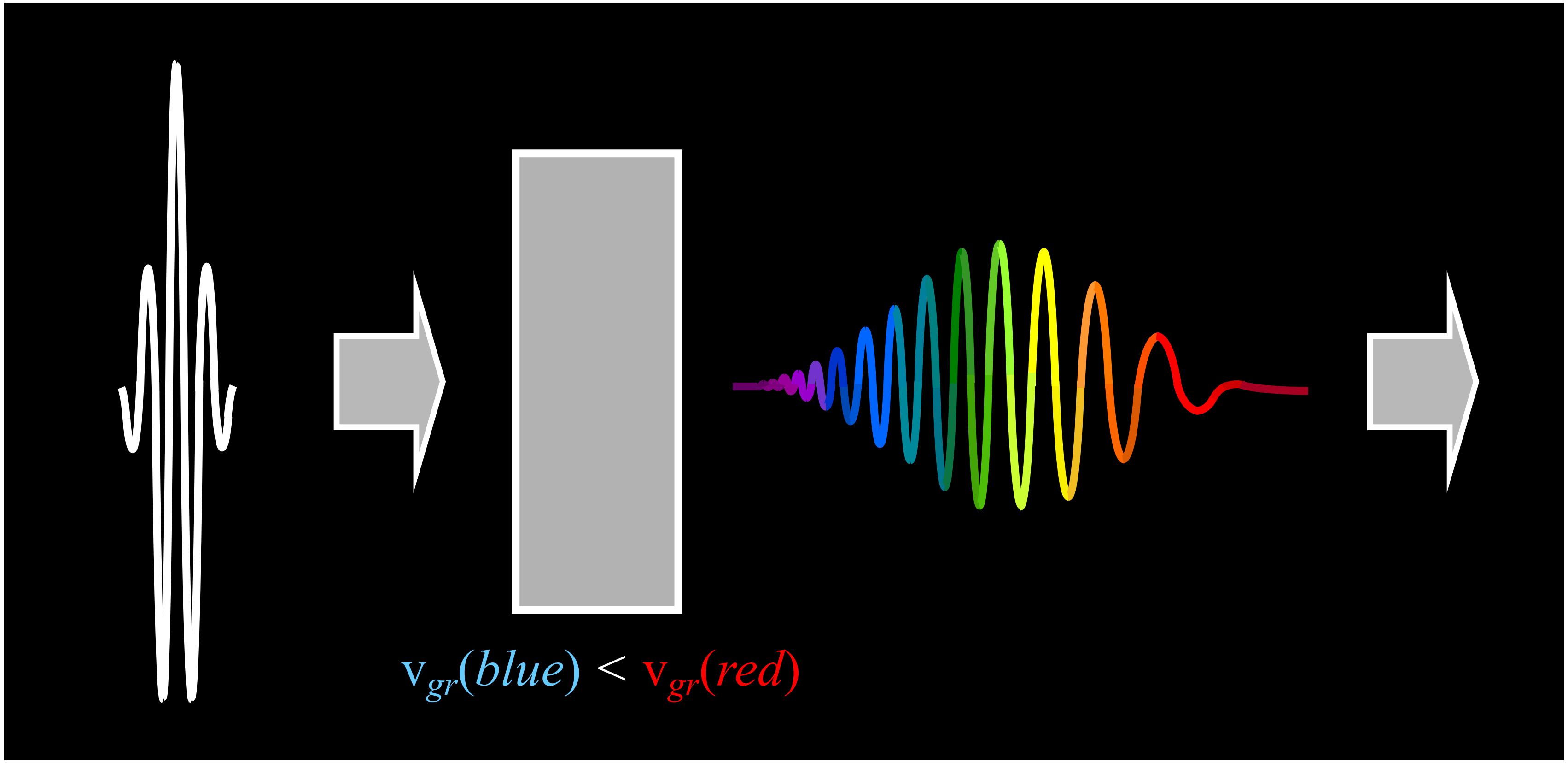
$$= \left| E(\tau) \right|^2$$ **= Intensity**

No phase information!

The algorithm projects it to a set of theoretical solution from the initial guess, a Gaussian intensity with random phase. The algorithm repeats the process to get the solution which satisfies both constrains which are from theoretical calculations

Group velocity dispersion

Group velocity dispersion means that the group velocity will be different for different wavelengths in the pulse



Because ultrashort pulses have such large bandwidths, GVD is a bigger issue than for cw light

Adapted from Prof. Trebino, GeorgiaTech, USA

Group velocity dispersion

Phase due to a medium is, $\varphi_H(\omega) = n(\omega)\, k_0\, L = k(\omega)\, L$

To account for dispersion, let us expand the phase (k-vector) in a Taylor series:

$$k(\omega)L = k(\omega_0)L + k'(\omega_0)[\omega-\omega_0]L + \tfrac{1}{2}k''(\omega_0)[\omega-\omega_0]^2 L + \ldots$$

$$k(\omega_0) = \frac{\omega_0}{\mathrm{v}_\phi(\omega_0)} \qquad k'(\omega_0) = \frac{1}{\mathrm{v}_g(\omega_0)} \qquad k''(\omega_0) = \frac{d}{d\omega}\left(\frac{1}{\mathrm{v}_g}\right)_{\omega=\omega_0}$$

The third term is the group velocity with frequency

$$\boxed{k''(\omega) = \frac{d}{d\omega}\left(\frac{1}{\mathrm{v}_g}\right)}$$

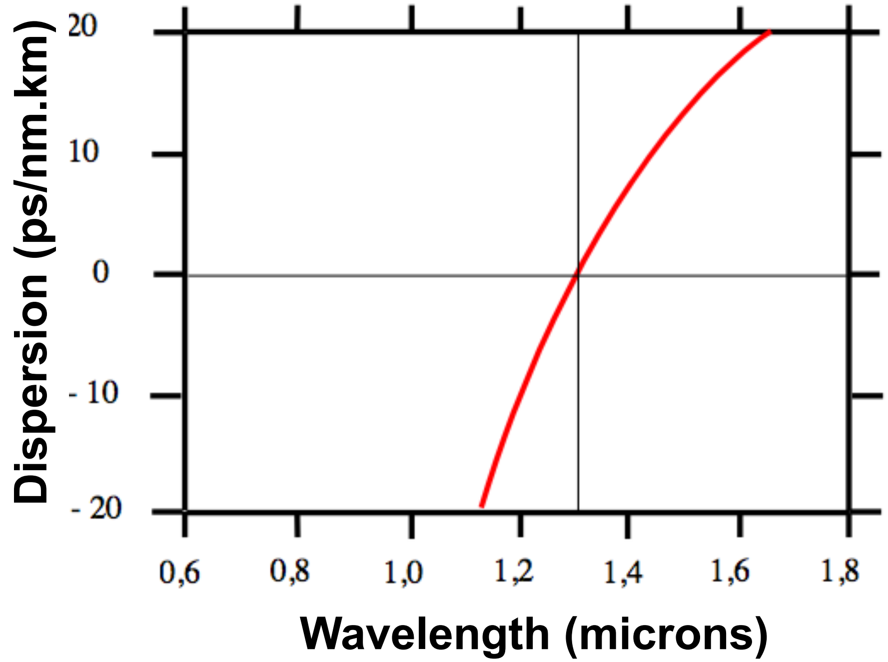
Group Velocity Dispersion (GVD)

Adapted from Prof. Trebino, GeorgiaTech, USA

Group velocity dispersion

$$\frac{d\lambda_0}{d\omega} = \frac{-\lambda_0^2}{2\pi c_0} \qquad \frac{d}{d\omega} = \frac{d\lambda_0}{d\omega}\frac{d}{d\lambda_0} = \frac{-\lambda_0^2}{2\pi c_0}\frac{d}{d\lambda_0}$$

$$\mathrm{v}_g = c_0 \Big/ \left(n - \lambda_0 \frac{dn}{d\lambda_0} \right)$$



The GVD express becomes

$$\frac{d}{d\omega}\left[\frac{1}{\mathrm{v}_g}\right] = \frac{-\lambda_0^2}{2\pi c_0}\frac{d}{d\lambda_0}\left[\frac{1}{c_0}\left(n - \lambda_0\frac{dn}{d\lambda_0}\right)\right]$$

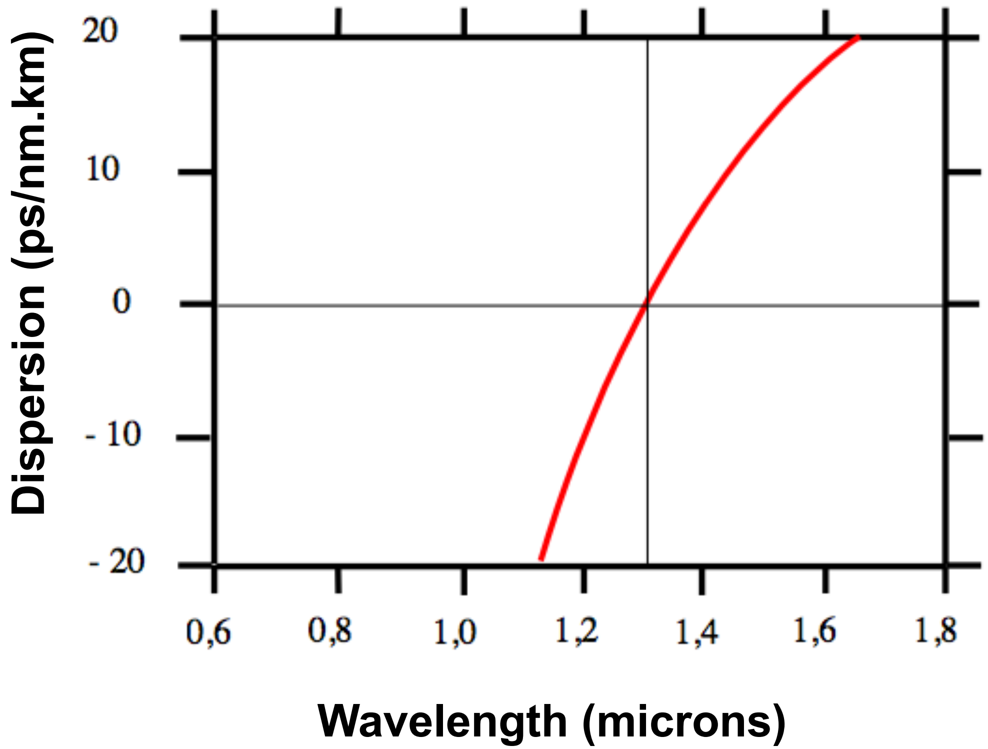
$$= \frac{-\lambda_0^2}{2\pi c_0^2}\frac{d}{d\lambda_0}\left(n - \lambda_0\frac{dn}{d\lambda_0}\right) = \frac{-\lambda_0^2}{2\pi c_0^2}\left(\cancel{\frac{dn}{d\lambda_0}} - \lambda_0\frac{d^2n}{d\lambda_0^2} - \cancel{\frac{dn}{d\lambda_0}}\right)$$

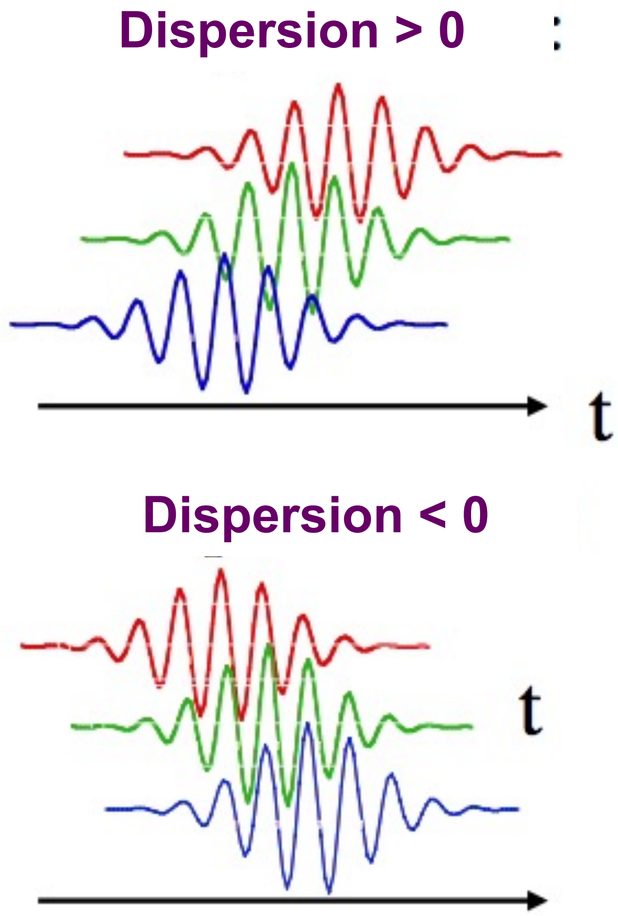
$$\boxed{GVD \equiv k''(\omega_0) = \frac{\lambda_0^3}{2\pi c_0^2}\frac{d^2n}{d\lambda_0^2}}$$

Adapted from Prof. Trebino, GeorgiaTech, USA

Group velocity dispersion

Positive dispersion broadens the pulse





Dispersion > 0 : blue faster than red

Dispersion < 0 : red faster than blue

Group delay dispersion

Phase delay

$$k(\omega_0) = \frac{\omega_0}{v_\phi(\omega_0)} \qquad t_\phi(\omega_0) = \frac{L}{v_\phi(\omega_0)} = \frac{k(\omega_0)L}{\omega_0}$$

Group delay

$$k'(\omega_0) = \frac{1}{v_g(\omega_0)} \qquad t_g(\omega_0) = \frac{L}{v_g(\omega_0)} = k'(\omega_0)L$$

Group delay dispersion (GDD)

GDD = GVD x L

$$k''(\omega) = \frac{d}{d\omega}\left(\frac{1}{v_g}\right) \qquad \boxed{GDD = \frac{d}{d\omega}\left(\frac{1}{v_g}\right)L = k''(\omega)L}$$

Adapted from Prof. Trebino, GeorgiaTech, USA

Phase manipulation

Spectral phase of the pulse expanded as a Taylor series

$$\varphi(\omega) \quad = \quad \varphi_0 + \varphi_1 \, [\omega - \omega_0] + \varphi_2 \, [\omega - \omega_0]^2 / 2! + \ldots$$

Same approach for the spectral phase of the optical medium,

$$\varphi_H(\omega) \quad = \quad \underbrace{\varphi_{H0}}_{\text{phase}} + \underbrace{\varphi_{H1}}_{\text{group delay}} \, [\omega - \omega_0] + \underbrace{\varphi_{H2}}_{\text{GDD}} \, [\omega - \omega_0]^2 / 2! + \ldots$$

To manipulate light, we must add or subtract spectral-phase terms

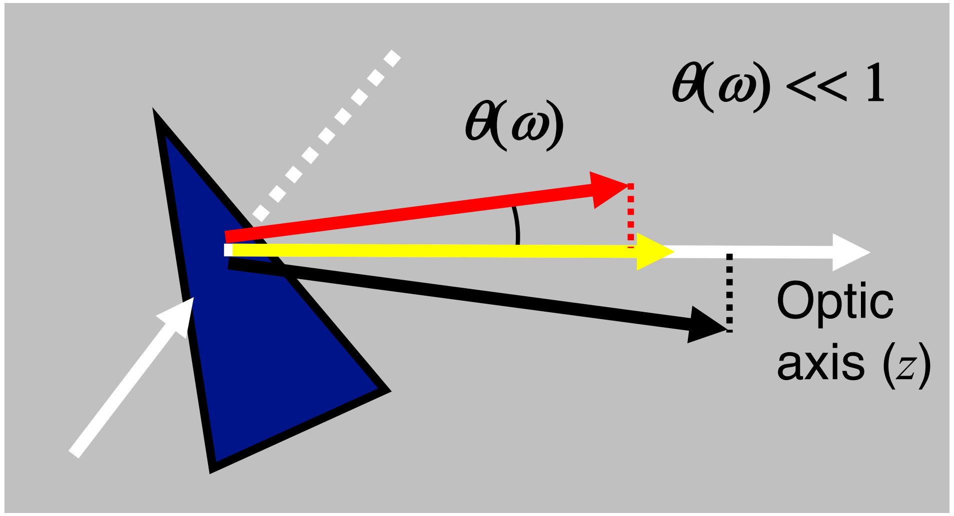
To eliminate a linear chirp (second-order spectral phase), one must design an optical device whose second-order spectral phase cancels that of the pulse

$$\boxed{\varphi_2 + \varphi_{H2} \; = \; 0} \qquad \text{i.e.,} \qquad \left(\frac{d^2\varphi_2}{d\omega^2}\right)_{\omega_0} + \left(\frac{d^2\varphi_{H2}}{d\omega^2}\right)_{\omega_0} = 0$$

Pulse compressor

The GDD due to angular dispersion is always negative!

$$\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -\frac{\omega_0 z}{c}\left(\left.\frac{d\theta}{d\omega}\right|_{\omega_0}\right)^2$$



Whatever the angular dispersion came from (prism or grating)

In most dielectric materials, k"L= φ_{H2} is positive

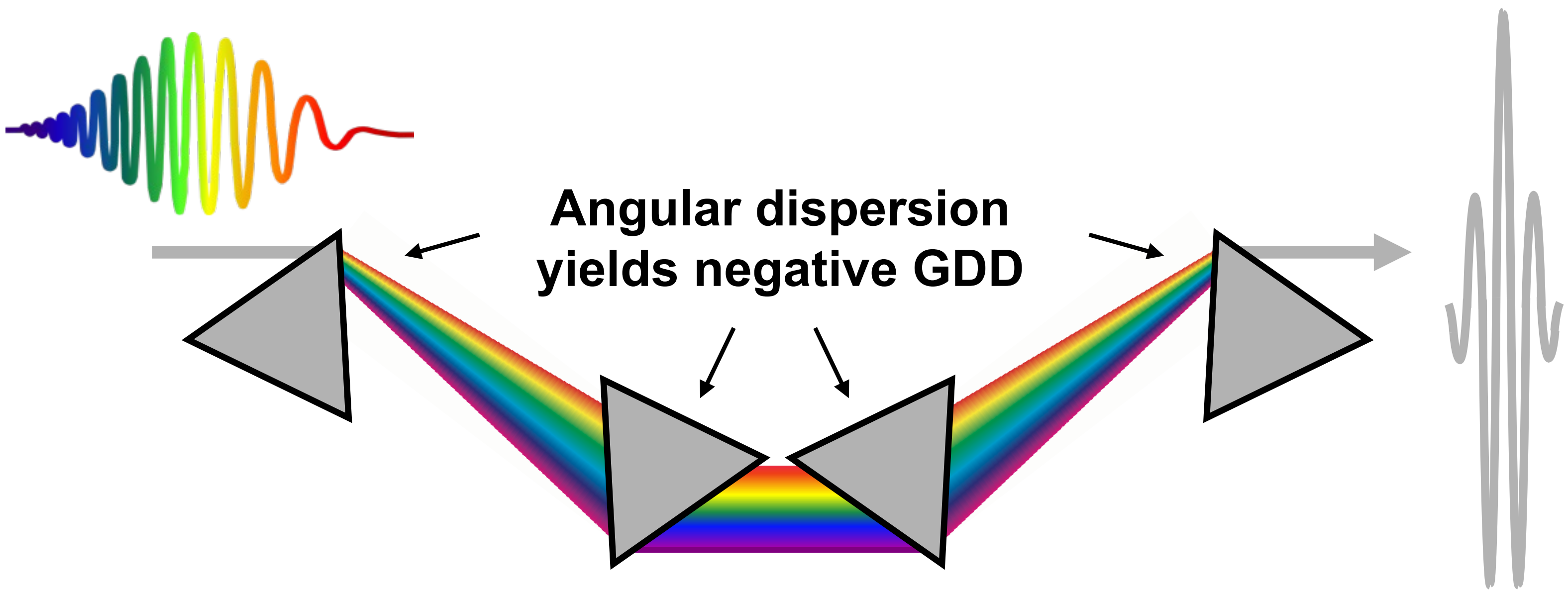
A device with a negative GDD can compensate for propagation through materials (i.e., for positive chirp)

$$\left(\frac{d^2\varphi_2}{d\omega^2}\right)_{\omega_0} + \left(\frac{d^2\varphi_{H2}}{d\omega^2}\right)_{\omega_0} = 0$$

Adapted from Prof. Trebino, GeorgiaTech, USA

Pulse compressor

This device, which also puts the pulse back together, has negative group-delay dispersion and hence can compensate for propagation through materials (i.e., for positive chirp)



Pulse compression can be up to a factor > 1000!!

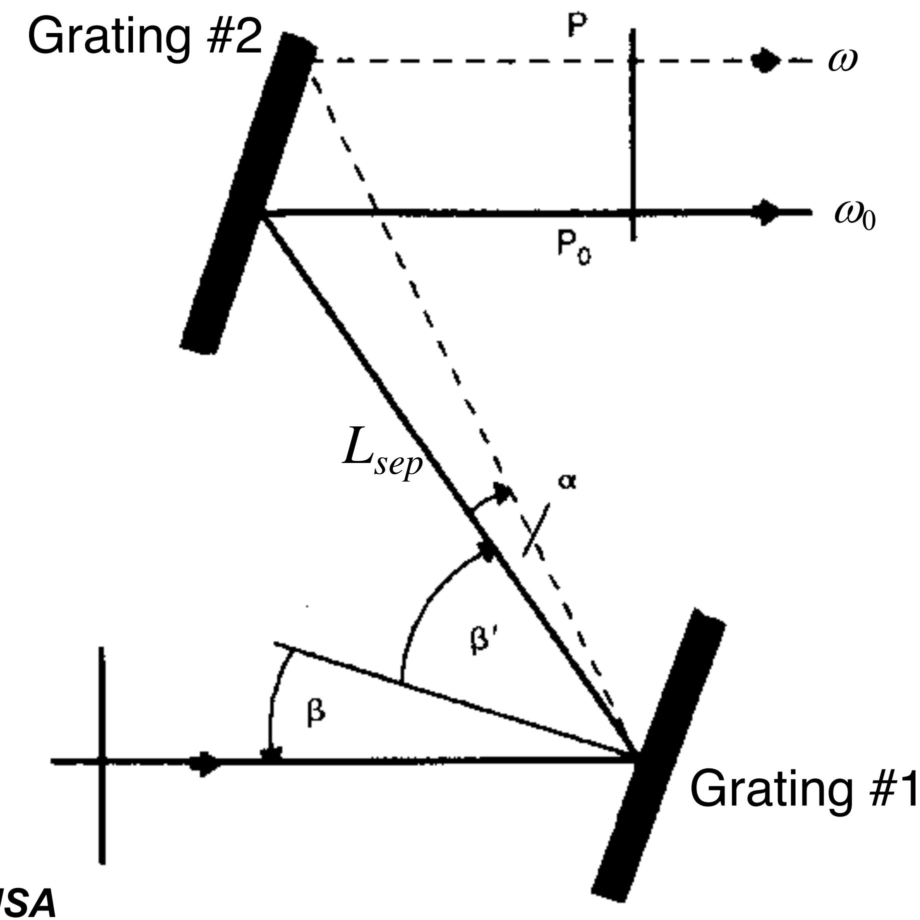
Adapted from Prof. Trebino, GeorgiaTech, USA

Pulse compressor

The grating pulse compressor also has negative GDD

$$\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -\frac{\lambda_0^3}{2\pi c^2 d^2}\frac{L_{sep}}{\cos^2(\beta')}$$

where d = grating spacing (same for both gratings)



As in the prism pulse compressor, the larger L_{sep}, the large the negative GDD

Adapted from Prof. Trebino, GeorgiaTech, USA