Modulation Properties of Self-Injected Quantum-Dot Semiconductor Diode Lasers

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(Invited Paper)

***Abstract*—This paper investigates the modulation properties of self-injected quantum-dot semiconductor lasers. Using a semianalytical approach, the modulation characteristic of a quantum-dot nanostructure laser operating under the influence of optical feedback is successfully modeled. This novel approach derives a feedback-induced modulation response model based on the incorporation of the specific quantum nanostructure carrier dynamics as well as the effects of nonlinear gain. This study investigates the impacts of the carrier capture and relaxation time as well as other material parameters such as linewidth enhancement factor, differential gain, and gain compression factor for different feedback configurations. It is also shown that, under the short external cavity configuration, the dynamic properties such as the relaxation frequency as well as the laser's bandwidth can be improved through controlled optical feedback. On the other hand, numerical results show that under the long external cavity configuration, any small back-reflection from the laser's facets combined with the large variations of linewidth enhancement factor would significantly alter the laser's modulation response.**

***Index Terms*—Carrier dynamics, quantum dot (QD), semiconductor lasers, self-injection.**

I. Introduction

SEMICONDUCTOR lasers subjected to external optical feedback (i.e., self-injection) are known to exhibit very interesting nonlinear dynamics, which either lead to instabilities and chaotic behaviors of the laser output or result in practically useful impacts that can improve the device's intrinsic characteristics [1]. Commercial applications of the semiconductor laser in optical fiber links are the first practical motivation for studying the behavior of semiconductor lasers subject to optical feedback. Even a small back-reflection from the fiber pigtail tip or optical fiber connectors into the diode laser module was shown to degrade the modulation characteristics and increase the intensity noise [2], [3]. To prevent these undesired effects, the laser diode transmitter modules are usually accommodated with an optical isolator, which rejects any back-reflection but simultaneously increases the cost of using laser diodes in optical fiber links.

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In 1980, Lang and Kobayashi reported on some aspects of the statics and dynamics of the semiconductor laser exposed to the external optical feedback from a distant reflector [4]. In that study, intrinsic characteristics of the laser were investigated such as the gain media including the broad gain spectrum, the temperature dependence of the material refractive index, and the carrier density dependence of the refractive index, all of which exhibit complex behavior under external feedback conditions. It was also reported on the experimental observation of bistability and hysteresis characteristics in the laser output as a function of injected current. Since then, understanding the effects of the self-injected field on both dynamical and spectral features of semiconductor lasers has been pursued extensively [5]–[7]. Early studies on the spectral characteristics showed that under the influence of optical feedback, the laser linewidth could be either narrowed [6] or broadened [7], which were initially explained by the spectral sensitivity to the phase of the reflected light. In [8], it was shown that by changing the feedback parameters, multistability can be observed as the system performs *mode hops,* where a laser diode operates on one external cavity mode for some time, but then suddenly switches to another. Another form of bistability, referred to as *low-frequency fluctuations*,was studied in [9]. This form of instability can be observed when the laser is pumped close to the threshold and is subjected to a moderate feedback level. Under this condition, the laser output shows sudden drops followed by a gradual buildup. The noise properties of self-injected semiconductor lasers have also attracted considerable theoretical and practical interests [10], [11]. More generally, much effort has been devoted to modeling the impacts of optical feedback on the dynamical behavior of semiconductor lasers [12], [13].

A. Regimes of Optical Feedback

It has been experimentally and theoretically shown that the effects of external optical feedback on laser dynamics or spectral properties can be different depending on several factors such as the laser bias condition, strength and phase of optical feedback, as well as on the distance between the external reflector and the laser cavity [14]. Based on the observations in the experiments, five characteristic regimes of optical feedback in semiconductor lasers were introduced in [15]. In regime I, under weak feedback level, the laser linewidth can be either narrowed or broadened depending upon the phase of the optical feedback. Regime II is characterized by the appearance of longitudinal mode hopping. In regime III, the

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laser becomes stable and is locked to the mode with minimum linewidth. In regime IV, with increasing feedback level, the linewidth of the laser dramatically broadens, which is referred to as *coherence collapse*. Further increasing the feedback strength into regime V, the laser enters a stable external cavity mode operation. There have been extensive studies on the five feedback regimes of semiconductor lasers [10]. In particular, considerable effort has been given to determining the nature of the laser dynamics in the coherence collapse regime, or regime IV, as was first reported in [16].

B. Advantages and Applications of Controlled Optical Feedback

Although self-injection operation has been shown to produce deleterious effects on semiconductor lasers including significant linewidth broadening and mode hopping, it is demonstrated that controlled external feedback has much potential in stabilizing and improving laser performance according to the observed dynamical and spectral properties. Recent advancements in modeling the nonlinear dynamics of semiconductor lasers subject to optical feedback have provided a path to understand the associated instabilities and develop methods for controlling the useful underlying dynamics for practical applications [1], [12], [16]. Advantages of controlled feedback have been realized in earlier studies, where it was shown that adjusting the feedback level and phase matching can result in a stable operation with considerable spectral linewidth narrowing [17]. Coherent feedback control has also been found to be useful in enhancing the relaxation oscillations and reducing the signal distortion in the modulated laser output [18]. These effects are very important for the laser especially when it is implemented in coherent communication systems. In another control method of optical feedback, a frequency filter is typically used to access a desired dynamical behavior in a specific region by restricting the phase space [19]. Lately, the idea of controlling the nonlinear dynamical behavior in semiconductor lasers has been developed to utilize chaotic dynamics in applications such as chaos synchronization for secure communication systems [20]. The filtered optical feedback technique has become an interesting topic [21], since it can control the laser dynamics through two external parameters: the spectral width of the filter and frequency detuning of the free-running laser. The frequency filter method was shown to provide a mechanism for controlling the impact of relaxation oscillations on the dynamical response of the laser, as well as permitting an external control over the nonlinearities of the devices [19]. Using this approach, tunable and pure frequency oscillations in the solitary laser can be generated by detuning the frequency of the optical feedback through a Fabry–Perot resonator [22]. The external control through delayed optical feedback has recently found its way into the field of passively mode-locked semiconductor lasers. Recent studies have both experimentally and theoretically shown that controlled external optical feedback is a simple and efficient method to improve the radio-frequency (RF) linewidth and timing stability via reducing the RF phase noise in passively mode-locked lasers [23], [24].

Applications of semiconductor lasers with controlled external optical feedback are driving rapid developments in theoretical and experimental research. The very broad gain bandwidth of semiconductor lasers combined with frequency-filtered, strong optical feedback creates the tunable, single-frequency laser systems utilized in telecommunications, environmental sensing, measurement and control [25]. Those with weak to moderate optical feedback levels leading to the chaotic semiconductor lasers can be implemented in secure communication systems [26].

C. Objectives of this Paper

This paper aims to theoretically investigate the effects of the self-injection on the modulation properties of quantum-dot (QD) semiconductor lasers. Indeed, it is particularly known that QD semiconductor lasers have attracted a lot of interest in the last decade owing to their remarkable properties arising from charge carrier confinement in the three space dimensions [27]. Low threshold current density, high material gain, temperature insensitivity, and reduced linewidth enhancement factor (LEF or α_H-factor) at the lasing wavelength have been reported [28], [29]. The low LEF combined with a high damping factor was found to be of utmost importance because it should increase the tolerance to optical feedback in these devices and may also offer potential advantages for direct modulation without transmission dispersion penalty [29], [30]. Although the injection-locking technique has already shown superior improvements in the high-speed characteristics of directly modulated lasers [30], [31], the use of external optical feedback can also be powerful, since it relies on a simple, compact, and cheap solution which can be implemented in future integrated photonic circuits. This paper is organized as follows. In Section II, the theoretical model used to analyze the modulation properties of self-injected QD lasers is presented. Starting from the laser's rate equations, it is shown that the small-signal analysis allows us to extract the modulation response and to successfully predict the key features of the self-injected oscillator. The novelty presented in this paper relies on a semianalytical derivation, which directly incorporates the QD carrier dynamics, as well as on the nonlinear gain. Section III presents the numerical results as well as a discussion both for the short and long external cavity regimes. Although a pure numerical model including multipopulation rate equations and taking into account all the peculiar characteristics of self-injected QD lasers was previously reported in the literature [32], we believe that the semianalytical approach proposed in this paper would provide a new insight of the laser's dynamic characteristics. Numerical results depicted in this paper demonstrate that a proper combination of controlled optical feedback and optimization of key operating parameters can improve the current-modulation properties in QD laser diodes.

II. Theoretical Analysis

Fig. 1 shows a basic scheme of the self-injected QD laser with L_{in} the length of the laser cavity and L_{ex} the length of the external cavity. The model commonly used to describe the dynamics of semiconductor lasers with external optical feedback is the well-known LK model [4], in which one rate equation describes the complex electric field (amplitude and phase), while the other one accounts for the carrier density. In order to take into account the complex carrier dynamics occurring in QD lasers, Huyet *et al*. [33] coupled two additional carrier rate equations into the LK model, namely one

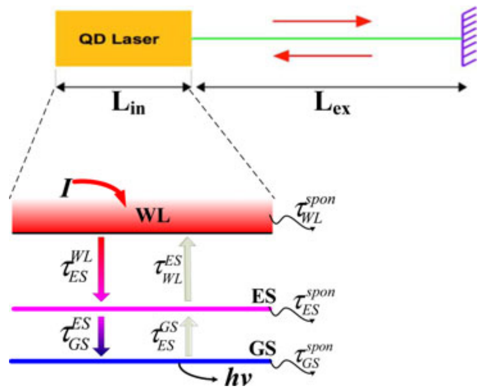

Fig. 1. Basic scheme of QD lasers operating under self-injection, and sketch of the corresponding carrier dynamics.

for the population in the wetting Layer (WL) and one for the population in the dots, in which the impacts of the Auger carrier capture rate and the Pauli blocking effect were also analyzed. To this end, it was shown that the insensitivity to optical feedback of QD lasers resulted from the low LEF and strongly damped relaxation oscillations. Employing a similar method, the bifurcation scenarios of QD lasers with optical feedback were also studied in [34].

In this paper, the numerical model of the QD laser holds under the assumption that the active region consists of only one QD ensemble, where nanostructures are interconnected by the WL. The QD ensemble includes two energy levels: a twofold degenerate ground state (GS) and a fourfold degenerate excited state (ES). The QDs are assumed to be always neutral, and electrons and holes are treated as electron–hole pairs, which mean that the system is in excitonic energy states. To this end, as shown in Fig. 1, it is assumed that the carriers are first injected into the WL before being captured into the ES within a capture time $\tau_{\mathrm{ES}}^{\mathrm{WL}}$, and then carriers will relax into the GS within a relaxation time $\tau_{\mathrm{GS}}^{\mathrm{ES}}$. On the other hand, carriers can also escape from the GS ($\tau_{\mathrm{ES}}^{\mathrm{GS}}$) and ES ($\tau_{\mathrm{WL}}^{\mathrm{ES}}$), which is governed by the Fermi distribution assumption.

With some approximations as those described in [35], the QD laser with optical feedback is described by the following set of differential rate equations:

$$\frac{dN_{\mathrm{WL}}}{dt} = \frac{I}{q} + \frac{N_{\mathrm{ES}}}{\tau_{\mathrm{WL}}^{\mathrm{ES}}} - \frac{N_{\mathrm{WL}}}{\tau_{\mathrm{ES}}^{\mathrm{WL}}} f_{\mathrm{ES}} - \frac{N_{\mathrm{WL}}}{\tau_{\mathrm{WL}}^{\mathrm{spon}}} \tag{1}$$

$$\frac{dN_{\mathrm{ES}}}{dt} = \frac{N_{\mathrm{WL}}}{\tau_{\mathrm{ES}}^{\mathrm{WL}}} f_{\mathrm{ES}} + \frac{N_{\mathrm{GS}}}{\tau_{\mathrm{ES}}^{\mathrm{GS}}} f_{\mathrm{ES}} - \frac{N_{\mathrm{ES}}}{\tau_{\mathrm{WL}}^{\mathrm{ES}}} - \frac{N_{\mathrm{ES}}}{\tau_{\mathrm{GS}}^{\mathrm{ES}}} f_{\mathrm{GS}} - \frac{N_{\mathrm{ES}}}{\tau_{\mathrm{ES}}^{\mathrm{spon}}} \tag{2}$$

$$\frac{dN_{\mathrm{GS}}}{dt} = \frac{N_{\mathrm{ES}}}{\tau_{\mathrm{GS}}^{\mathrm{ES}}} f_{\mathrm{GS}} - \frac{N_{\mathrm{GS}}}{\tau_{\mathrm{ES}}^{\mathrm{GS}}} f_{\mathrm{ES}} - \frac{N_{\mathrm{GS}}}{\tau_{\mathrm{GS}}^{\mathrm{spon}}} - \Gamma_p g v_g S \tag{3}$$

$$\frac{dS}{dt} = \left(\Gamma_p g v_g - \frac{1}{\tau_p}\right) S + \beta_{\mathrm{SP}} \frac{N_{\mathrm{GS}}}{\tau_{\mathrm{GS}}^{\mathrm{spon}}} + 2k_c \sqrt{S(t) S(t - t_{\mathrm{ex}})} \cos(\Delta\phi) \tag{4}$$

$$\frac{d\phi}{dt} = \frac{\alpha_H}{2} \left(\Gamma_p g v_g - \frac{1}{\tau_p}\right) - k_c \sqrt{\frac{S(t - \tau_{\mathrm{ex}})}{S(t)}} \sin(\Delta\phi) \tag{5}$$

where N_{WL}, N_{ES}, and N_{GS} are the carrier numbers in WL, ES, and GS, respectively. The symbol S represents the number of the photons emitted from the GS, and I the pump current. Stimulated emission from the ES is not taken into account in the model. In (3)–(5), ϕ denotes the phase, β_{sp} the spontaneous emission factor, Γ_p the confinement factor, τ_p the photon lifetime, v_g the group velocity, and α_H the LEF.

The GS gain is written as follows:

$$g = \frac{a_{\mathrm{GS}} \left(N_{\mathrm{GS}}/V_{\mathrm{QD}} - N_B/H\right)}{1 + \varepsilon S/V_p} \tag{6}$$

where a_{GS} is the differential gain, N_B is the QD surface density, H is the average height of the QD, V_p is the photon volume, V_{QD} is the total volume of the QDs, and ε accounts for the gain compression coefficient.

Besides, the Pauli blocking factors are given by

$$f_{\mathrm{GS}} = 1 - \frac{N_{\mathrm{GS}}}{2N_B}; f_{\mathrm{ES}} = 1 - \frac{N_{\mathrm{ES}}}{4N_B}. \tag{7}$$

The strength of the delayed field is defined as follows:

$$k_c = \frac{1}{\tau_{\mathrm{in}}} \frac{1 - R_1}{\sqrt{R_1}} \sqrt{f_{\mathrm{ext}}} \tag{8}$$

where τ_{in} is the round trip time in the laser cavity, R_1 is the laser facet reflectivity, and f_{ext} is the feedback ratio corresponding to the ratio of the returned power into the laser's facet to the emitted one.

The phase variation occurring in (4) and (5) is expressed as

$$\Delta\phi = \omega_0 \tau_{\mathrm{ex}} + \phi(t) - \phi(t - t_{\mathrm{ex}}) \tag{9}$$

with ω_0 being the solitary laser frequency, and τ_{ex} the round trip delay in the external cavity.

All the parameters above are listed in Table I, and the values used to seed the simulations are based on a 1.52-μm InAs/InP (311B) QD laser [36]. Let us remark that the capture time $\tau_{\mathrm{ES}}^{\mathrm{WL}}$ and the relaxation time $\tau_{\mathrm{GS}}^{\mathrm{ES}}$ are both set at half the measured values so as to clearly show the variations of the damping rate in the following sections.

In order to obtain the modulation response, the rate equations can be linearized by a modified small-signal analysis [37]. Considering a sinusoidal current modulation $I_1 e^{j\omega t}$ around the injection current I_0, the following laser values also vary around their steady-state solutions as

$$\begin{aligned} S(t) &= S_0 + S_1 e^{j\omega t} \\ \phi(t) &= \Delta\omega t + \phi_1 e^{j\omega t} \\ N_{\mathrm{WL,ES,GS}}(t) &= N_{\mathrm{WL0,ES0,GS0}} + N_{\mathrm{WL1,ES1,GS1}} e^{j\omega t}. \end{aligned} \tag{10}$$

Under small-signal modulation, the phase terms can be approximated given by [37]

$$\cos(\Delta\phi) \approx P[1 + \alpha_H \phi_1 (1 - e^{-jw\tau_{\mathrm{ex}}}) e^{jwt}] \tag{11}$$

$$\sin(\Delta\phi) \approx P[-\alpha_H + \phi_1 (1 - e^{-jw\tau_{\mathrm{ex}}}) e^{jwt}] \tag{12}$$

with $P = 1/\sqrt{1 + \alpha_H^2}$.

Inserting (10)–(12) into the rate equations (1)–(5) and neglecting higher order terms allows the derivation of the linearized rate equations with some Taylor polynomial

TABLE I
MATERIAL AND LASER PARAMETERS

Symbols	Definitions	Values
E_{WL}	WL energy	0.97 eV
E_{ES}	ES energy	0.87 eV
E_{GS}	GS energy	0.82 eV
τ_{ES}^{WL}	Capture time from WL to ES	12.6 ps
τ_{GS}^{ES}	Relaxation time from ES to GS	5.8 ps
τ_{WL}^{spon}	Spontaneous time of WL	500 ps
τ_{ES}^{spon}	Spontaneous time of ES	500 ps
τ_{GS}^{spon}	Spontaneous time of GS	1200 ps
a_{GS}	Differential gain	5×10^{-15} cm^2
ε	Gain compression coefficient	0 cm^3
n_r	Refractive index	3.5
L	Active region length	0.11 cm
W	Active region width	3×10^{-4} cm
N	Number of QD layers	5
N_B	QD density	10×10^{10} cm^{-2}
Γ_p	Optical confinement factor	0.06
β_{SP}	Spontaneous emission factor:	1×10^{-4}
α_i	Internal modal loss	6 cm^{-1}
R_1=R_2	Facet reflectivity	0.32
L_{ex}	External cavity length	0.35, 105 cm
n_{ex}	Refractive index in external cavity	1.5
α_H	Linewidth enhancement factor	1

approximations:

$$\begin{bmatrix} \gamma_{11}+jw & -\gamma_{12} & 0 & 0 & 0 \\ -\gamma_{21} & \gamma_{22}+jw & -\gamma_{23} & 0 & 0 \\ 0 & -\gamma_{32} & \gamma_{23}+jw & -\gamma_{34} & 0 \\ 0 & 0 & -\gamma_{43} & \gamma_{44}+jw & -\gamma_{45} \\ 0 & 0 & -\gamma_{53} & -\gamma_{54} & \gamma_{55}+jw \end{bmatrix} \times \begin{bmatrix} N_{\rm WL1} \\ N_{\rm ES1} \\ N_{\rm GS1} \\ S_1 \\ \phi_1 \end{bmatrix} = \frac{I_1}{q}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

with

$$\gamma_{11} = \frac{f_{\rm ES0}}{\tau_{\rm ES}^{\rm WL}} + \frac{1}{\tau_{\rm WL}^{\rm spon}}; \gamma_{12} = \frac{1}{\tau_{\rm WL}^{\rm ES}} + \frac{N_{\rm WL0}}{4N_B\tau_{\rm ES}^{\rm WL}}; \gamma_{21} = \frac{f_{\rm ES0}}{\tau_{\rm ES}^{\rm WL}}$$

$$\gamma_{22} = \frac{f_{\rm GS0}}{\tau_{\rm GS}^{\rm ES}} + \frac{1}{\tau_{\rm WL}^{\rm ES}} + \frac{N_{\rm WL0}}{4N_B\tau_{\rm ES}^{\rm WL}} + \frac{1}{\tau_{\rm ES}^{\rm spon}}; \gamma_{23} = \frac{f_{\rm ES0}}{\tau_{\rm ES}^{\rm GS}}$$

$$\gamma_{32} = \frac{f_{\rm GS0}}{\tau_{\rm GS}^{\rm ES}}; \gamma_{33} = \frac{f_{\rm ES0}}{\tau_{\rm ES}^{\rm GS}} + \frac{1}{\tau_{\rm GS}^{\rm spon}} + \frac{\Gamma_p v_g a_{\rm GS} S_0}{V_{\rm QD}}; \gamma_{34} = -\frac{1}{\tau_p}$$

$$\gamma_{43} = \frac{\beta_{\rm SP}}{\tau_{\rm GS}^{\rm spon}} + \Gamma_p v_g a_{\rm GS} S_0\left(1-\varepsilon S_0/V_p\right)/V_{\rm QD}$$

$$\gamma_{44} = -Pk_c\left(1+e^{-j\omega\tau_{\rm ex}}\right) + 2\frac{\varepsilon S_0}{\tau_p V_p}$$

$$\gamma_{45} = 2Pk_c\alpha_H S_0\left(1-e^{-j\omega\tau_{\rm ex}}\right)$$

$$\gamma_{53} = \frac{\alpha_H}{2}\Gamma_p v_g a_{\rm GS}\left(1-\varepsilon S_0/V_p\right)/V_{\rm QD}$$

$$\gamma_{54} = -\frac{\alpha_H}{2}\frac{Pk_c}{S_0}\left(1-e^{-j\omega\tau_{\rm ex}}\right) - \frac{\alpha_H}{2}\frac{\varepsilon}{\tau_p V_p}$$

$$\gamma_{55} = Pk_c\left(1-e^{-j\omega\tau_{\rm ex}}\right) \tag{14}$$

where the relationship $\Gamma_p v_g a_{\rm GS}(N_{\rm GS0} - N_B) \approx 1/\tau_p$ holds under the assumption of a weak optical feedback level. From (13), the modulation transfer function for QD lasers subjected to external optical feedback can be extracted as

$$H(\omega) = \frac{S(\omega)/J(\omega)}{S(0)/J(0)}. \tag{15}$$

In the free-running case, (15) can be approximately expressed as follows [35]:

$$H(\omega) \approx \left(\frac{\omega_R^2}{\omega_R^2 - \omega^2 + j\omega\Gamma}\right)\left(\frac{\omega_{R0}^2}{\omega_{R0}^2 - \omega^2 + j\omega\Gamma_0}\right) \tag{16}$$

where w_R is the resonance frequency and Γ is the damping factor, whose expressions are defined as follows:

$$w_R^2 = \frac{v_g a_{\rm GS} S_0}{\tau_p} + \left[\Gamma_p v_g a_{\rm GS}^p S_0 + \frac{\Gamma_p \beta_{\rm SP} N_{\rm GS0}}{\tau_{\rm GS}^{\rm spon} S_0}\right] \times \left(\frac{f_{\rm ES0}}{\tau_{\rm ES}^{\rm GS}} + \frac{1-\beta_{\rm SP}}{\tau_{\rm GS}^{\rm spon}}\right) + \frac{\beta_{\rm SP}}{\tau_{\rm GS}^{\rm spon}\tau_p} \tag{17}$$

$$\Gamma = v_g a_{\rm GS} S_0\left[1 + \frac{\Gamma_p a_{\rm GS}^p}{a_{\rm GS}}\right] + \frac{f_{\rm ES0}}{\tau_{\rm ES}^{\rm GS}} + \frac{1}{\tau_{\rm GS}^{\rm spon}} + \frac{\Gamma_p\beta_{\rm SP}N_{\rm GS0}}{\tau_{\rm GS}^{\rm spon}S_0} \tag{18}$$

with $a_{\rm GS}^p = -\partial g/\partial S$.

In (16), w_{R0} and Γ_0 correspond to new parameters, which are mostly characterized by the WL and the ES:

$$w_{R0}^2 = \left(\frac{f_{\rm ES0}}{\tau_{\rm ES}^{\rm WL}} + \frac{1}{\tau_{\rm WL}^{\rm spon}}\right)\left(\frac{f_{\rm GS0}}{\tau_{\rm GS}^{\rm ES}} + \frac{1}{\tau_{\rm ES}^{\rm spon}}\right) + \frac{1}{\tau_{\rm WL}^{\rm ES}}\frac{1}{\tau_{\rm WL}^{\rm spon}} \tag{19}$$

$$\Gamma_0 = \frac{f_{\rm ES0}}{\tau_{\rm ES}^{\rm WL}} + \frac{1}{\tau_{\rm WL}^{\rm spon}} + \frac{f_{\rm GS0}}{\tau_{\rm GS}^{\rm ES}} + \frac{1}{\tau_{\rm WL}^{\rm ES}} + \frac{1}{\tau_{\rm ES}^{\rm spon}}. \tag{20}$$

The free-running results reported in [35] point out that the finite carrier capture time $\tau_{\rm ES}^{\rm WL}$, finite carrier relaxation time $\tau_{\rm GS}^{\rm ES}$, as well as the Pauli blocking factor $f_{\rm ES}$ and $f_{\rm GS}$ are the underlying physical limitations of the laser's modulation bandwidth. Numerical results also show that carrier escape from the GS to the ES gives rise to a nonzero resonance frequency at low bias powers, as well as to a strong damping factor [35].

III. Numerical Results and Discussion

A. Critical Feedback Level

A significant part of the study dealing with optical feedback has been undertaken assuming long external cavities (i.e., $\omega_r\tau_{\rm ex} \gg 1$). The laser can become unstable above a certain critical feedback level $f_{{\rm ext},c}$ and then transit to the so-called coherence collapse regime [12]. As shown in [15] and [39], the onset of coherence collapse is typically determined at the point where the laser linewidth begins to

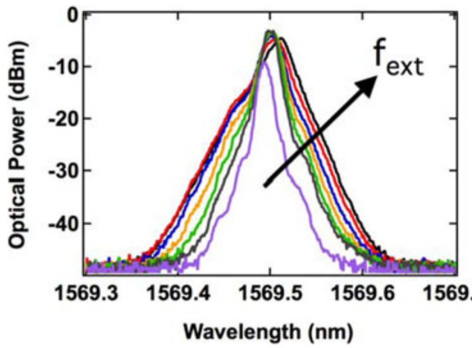

Fig. 2. Measured optical spectra of a 1.55-μm InAs/InP QD DFB laser for various feedback levels $f_{\rm ext}$. The last spectrum (black color) corresponds to the fully developed coherence collapse regime.

significantly broaden. As an example, Fig. 2 depicts the measured optical spectra of a 1.55-μm InAs/InP QD distributed feedback (DFB) laser recorded for various feedback levels $f_{\rm ext}$. The last spectrum (black color) corresponds to the fully developed coherence collapse regime. In this case, the spectral broadening can strongly alter the capacity for performing high-speed communications [39].

The critical feedback value corresponding to the onset of the coherence collapse can be estimated from the following relationship [38]:

$$f_{{\rm ext},c} = \frac{\Gamma^2}{(1+\alpha_H^2)} \frac{\tau_{\rm in}^2 R_1}{4\,(1-R_1)^2}. \tag{21}$$

The validity of (21) was evaluated from the microwave transfer function by numerically analyzing the stability of the external cavity mode with the minimum linewidth. Let us note that (21) which holds under the assumption of $\alpha_H > 1$ and $\omega_r \tau_{\rm ex} \gg 1$ depends on the solitary laser response and on the value of the LEF. In fact, it is important to stress that there is no explicit dependence on the external cavity length, since a long-cavity asymptotic assumption was used in the derivation. However, in the case of a short external cavity (i.e., $\omega_r \tau_{\rm ex} \ll 1$), the onset of the coherence collapse is found to be dependent on the external cavity length, as demonstrated in [40]. With sufficiently short external cavities, the laser can even remain stable for any feedback level without any coherence collapse regime. To this end, the short external cavity regime can even be used to enhance the laser modulation bandwidth, which is beneficial for high-speed communications [41], [42]. As mentioned previously, the strongly damped relaxation oscillation as well as the low LEF contributes to the reduced sensitivity of QD lasers to optical feedback [33], [38], [43]–[45]. It is also known that the differential gain plays a key role in the optical feedback tolerance [45] and the gain saturation contributes to the stability of self-injected lasers as well [46].

In validating (21) against numerical simulations of an external cavity laser diode, it was shown that the expression deviated from the numerical results for low LEF values. Consequently, the previous expression giving the critical feedback level was improved empirically as [40]

$$f_{{\rm ext},c} = \frac{\Gamma^2 (1+\alpha_H^2)}{\alpha_H^4} \frac{\tau_{\rm in}^2 R_1}{4\,(1-R_1)^2}. \tag{22}$$

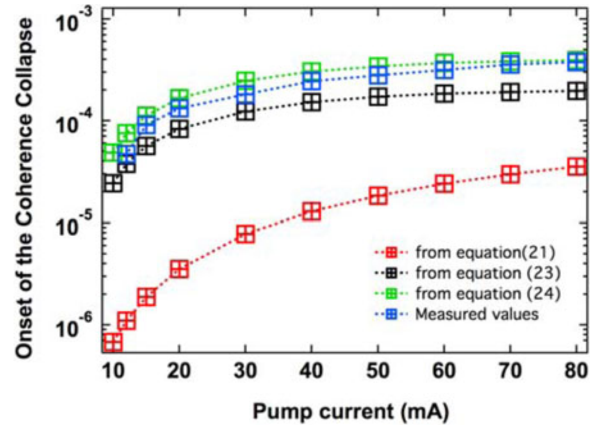

Fig. 3. Measured onset of the coherence collapse as a function of the pump current for a 1.55-μm InAs/InP QD DFB laser (blue squares). Red, black, and green squares correspond to the extracted onset of the coherence collapse from (21), (23), and (24). Dotted lines are added for visual help only.

In comparison with (21), this relation predicts that the coherence collapse does not occur if LEF$\rightarrow$ 0. Assuming a zero LEF, all the fixed points describing the stability of the system (*modes* and *antimodes*) are located on a circle around the solitary laser mode [47]. Under this condition, the mode with minimum gain and the mode with minimum linewidth do not compete with each other, so the coherence collapse cannot occur [47]. Despite a general agreement that the critical feedback level may be strongly up-shifted at low LEF values [48], the total suppression of the critical feedback regime has never been reported.

In comparison, Binder and Cormack proposed that the coherence collapse occurs when the maximum feedback-induced frequency shift exceeds the relaxation frequency. In their approach, the onset of the critical feedback level is written as [49]

$$f_{{\rm ext},c} = \frac{\omega_r^2}{1+\alpha_H^2} \frac{\tau_{\rm in}^2 R_1}{(1-R_1)^2}. \tag{23}$$

Another expression similar to (23) was also derived by analyzing the stability of the oscillation condition solutions for a laser under optical feedback [50]:

$$f_{{\rm ext},c} = \frac{\omega_r^2}{1+\alpha_H^2} \frac{2\tau_{\rm in}^2 R_1}{(1-R_1)^2}. \tag{24}$$

To this end, the coherence collapse is seen as a chaotic attractor and the chaos is reached when the feedback level increases through a quasi-periodic route, which is interrupted by frequency locking. In the case of long external cavity, expression (24) provides an approximation to the value of the feedback parameter k_c above which instability sets in. As (21), equations (23) and (24) predict that the coherence collapse can still occur even with a zero LEF. In spite of these differences, all theories predict that the feedback level corresponding to the onset of the coherence collapse increases for lasers with larger damping factor and smaller LEF.

In Fig. 3, the measured onset of the coherence collapse for a 1.55-μm InAs/InP QD DFB laser is shown (blue squares) as a function of the pump current (long external cavity regime). Extracted values from (21), (23), and (24) are also depicted and represented by red, black, and green squares, respectively.

Dotted lines are added for visual help only. The critical feedback level is found to increase with the pump current [45]. Because (21) was derived after a cascade of approximations, the smallest discrepancy is obtained with relationships (23) and (24) [44]. Consequently, in order to avoid the occurrence of the coherence collapse regime, (23) and (24) will be used in the numerical simulations to maintain the feedback level below its critical value.

B. Modulation Properties of Self-Injected QD Lasers

Our previous work has demonstrated that the large damping factor of the free-running QD lasers is attributed to the carrier escape from the GS, as shown in (18) [35]. Let us also remark that in [51]–[53], the strong damping was attributed to the carrier scattering rate. These last conclusions are in agreement since the carrier escape time is proportional to the carrier lifetime. Besides, the finite carrier capture and relaxation times as well as the Pauli blocking are proved to act as limitations of the modulation bandwidth in QD lasers [35]. Finally, QD lasers suffer from larger gain compression effects (10^{-16}–10^{-15}cm^3) as compared to their QW counterparts (10^{-19}–10^{-17}cm^3) [36], [54]–[56], which can also alter the modulation dynamics [57]. In the following sections, we investigate the impacts of some crucial parameters such as carrier capture and relaxation times, LEF, differential gain, and gain compression on the intensity modulation (IM) properties of QD lasers, both for short and long external cavity configurations. In the simulations, the bias current is set at $I_{\text{bias}} = 1.1I\text{th}$ with $I\text{th}$ the threshold current of the free-running laser. The short external cavity length is fixed at 0.35 cm, while the long one is at 105 cm. Although these values correspond approximately to a phase condition of about 2π, let us note that practically it is hardly impossible to control the precise phase shift via the external cavity length. Assuming a resonance frequency of $f_r = 10$ GHz, the value $\omega_r\tau_{\text{ex}}$ is $\sim$2 for the short cavity configuration, and $\sim$600 for the long one. As previously mentioned, the feedback level associated with the LEF is controlled in the stable regime considering (23) and (24).

First, Fig. 4(a) depicts the calculated modulation response for various feedback levels with $f_{\text{ext}} = 10^{-4}$, 10^{-3}, 1×10^{-2}, 2×10^{-2}, and 5×10^{-2} in the case of the short external cavity regime. Horizontal dashed line corresponds to the 3-dB line. Thus, simulation shows that increasing the feedback level can indeed improve the modulation bandwidth. In the case under study, the modulation bandwidth is increased by a factor of 1.5 at $f_{\text{ext}} = 5 \times 10^{-2}$ as compared to the free-running case. Since the short external cavity is always stable to any large feedback level, the bandwidth can be further enhanced by increasing the feedback strength or by shortening the external cavity length [58]. As opposed to optical injection, calculations show that self-injection does not induce a preresonance frequency dip limiting the 3-dB bandwidth [31]. Indeed, interband lasers operating with optical injection traditionally exhibit a linear response at negative detuning without a resonance peak. At zero detuning, the laser is usually characterized by a broadband and flat response, while for positive detuning, the modulation response exhibits a higher resonance frequency associated with a sharp peak and a large frequency dip. To this end, the absence of frequency dip under optical feedback does correspond to a situation closer to the optical injection case at zero-detuning. Although the bandwidth enhancement remains less efficient as compared to what one can get under strong optical injection [31], we believe that the optical feedback technique can offer a low-cost solution and a simple implementation as long as the specifications do not require an ultra-large bandwidth.

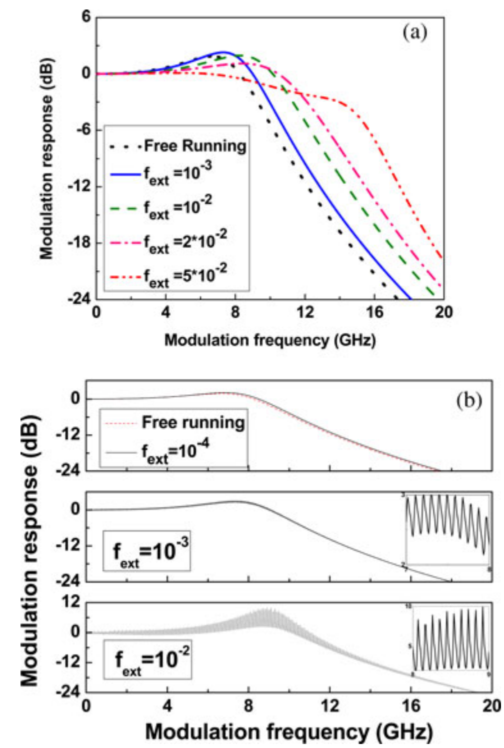

Fig. 4. Impacts of the feedback level on the IM response for (a) short external cavity and (b) long external cavity.

In comparison, no bandwidth improvement occurs for the long external cavity configuration, as shown in Fig. 4(b). On the contrary, assuming $f_{\text{ext}} = 10^{-4}$, 10^{-3}, and 1×10^{-2} oscillations progressively arise in the modulation response. This effect can be attributed to the large τ_{ex} value, which means that the variation of the term $1 \pm e^{-j\omega\tau_{\text{ex}}}$ in (14) remains large even if the modulation frequency is slightly changed. The periodicity of the oscillation is determined by the round trip time of the external cavity [59]. The ripple amplitude increases with the feedback level, especially near the resonance peak, as illustrated in the figures depicted in insets. For instance, this amplitude is as large as about 10 dB at $f_{\text{ext}} = 10^{-2}$. It is worth noting that this phenomenon also depends significantly on the laser's coherence and gets more significant for longer coherence length. To this end, these parasitic feedback effects can be reduced if the laser's spectrum is broadened. The spectral width of the laser can be enhanced by either increasing the intensity modulation index or by operating the laser at feedback levels higher than the critical external feedback level within the coherence-collapse regime. However, because of the large phase noise enhancement, the spectral broadening is usually detrimental for high-speed communications [39]. Besides, the numerical results presented in this paper hold for small-signal modulation with very small modulation indices. As previously pointed out for QW lasers, the sensitivity to optical feedback is more predominant at small-signal modulations because of the reduced

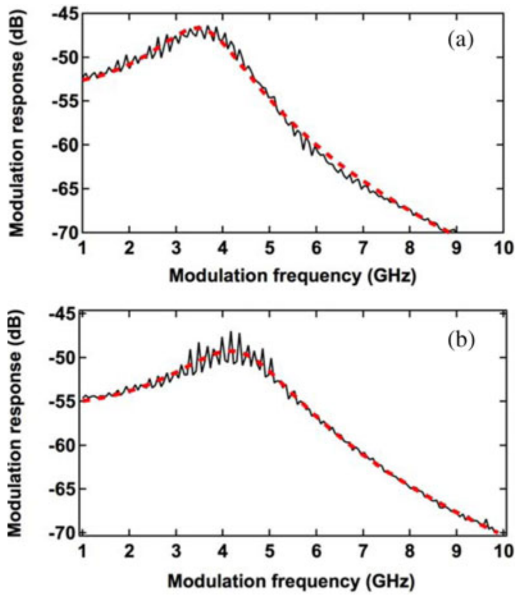

Fig. 5. Impacts of optical feedback ($f_{\mathrm{ext}} = 10^{-3}$) on the IM response of a 1.55-μm InAs/InP QD laser operating under a long external cavity at different pump current (a) 75 mA and (b) 115 mA. In both cases, red-dashed lines correspond to the free-running curve-fitting functions.

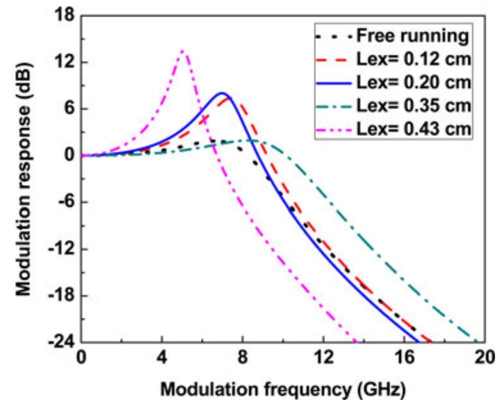

Fig. 6. Impacts of the feedback phase on the IM response for $\tau_{\mathrm{ES}}^{\mathrm{WL}} = 0.5$ ps, $\tau_{\mathrm{GS}}^{\mathrm{ES}} = 0.5$ ps, $\alpha_H = 1$; $f_{\mathrm{ext}} = 10^{-2}$; $L_{\mathrm{ex}} = 0.12$ cm ($\phi_0 = 2\pi \times 0.24$); $L_{\mathrm{ex}} = 0.20$ cm; ($\phi_0 = 2\pi \times 0.73$); $L_{\mathrm{ex}} = 0.35$ cm ($\phi_0 = 2\pi \times 0.02$); and $L_{\mathrm{ex}} = 0.43$ cm ($\phi_0 = 2\pi \times 0.51$).

frequency chirp [60]. In other words, increasing the modulation index can attenuate the parasitic effects occurring in the laser's frequency response. However, in QD lasers, we believe that these conclusions need further investigations because of a more complicated carrier dynamics associated with a larger discrepancy in the above-threshold LEF from chip to chip.

As the frequency of the external cavity is much smaller than the laser's relaxation frequency, the shape of the modulation response cannot be controlled and no improvements either in the relaxation peak or in the modulation bandwidth can be obtained under such a configuration as originally reported in quantum-well (QW) lasers [37].

As an example, Fig. 5 experimentally confirms the negative effects of optical feedback on the modulation response of a self-injected 1.55-μm InAs/InP QD laser (long external cavity regime). Measurements were done at a fixed feedback rate ($f_{\mathrm{ext}} = 10^{-3}$) and for two different pump currents [(a) 75 mA and (b) 115 mA]. In both cases, red-dashed lines correspond to the free-running curve-fitting functions. Thus, the results point out that a long external cavity leads to the occurrence of a periodic ripple mostly located close to the relaxation frequency peak and whose amplitude increases both with the feedback strength and with the value of the pump current so with the LEF.

Finally, it is also well known that the phase of the reflected wave wt_{ex} has a significant influence on the modulation response of self-injected semiconductor lasers [61]. As an example, Fig. 6 illustrates the impact of the phase on the modulation response under the short cavity regime for $\tau_{\mathrm{ES}}^{\mathrm{WL}} = 0.5$ ps, $\tau_{\mathrm{GS}}^{\mathrm{ES}} = 0.5$ ps, $\alpha_H = 1$, and $f_{\mathrm{ext}} = 10^{-2}$. The phase variation is obtained by varying the external cavity length such as $L_{\mathrm{ex}} = 0.12$ cm ($\phi_0 = 2\pi \times 0.24$), $L_{\mathrm{ex}} = 0.20$ cm ($\phi_0 = 2\pi \times 0.73$), $L_{\mathrm{ex}} = 0.35$ cm ($\phi_0 = 2\pi \times 0.02$), and $L_{\mathrm{ex}} = 0.43$ cm ($\phi_0 = 2\pi \times 0.51$). Simulations point out that the phase strongly modifies the position of the relaxation peak and so the value of the modulation bandwidth (by a factor of about 1.5 in the case under study). Let us stress that this effect is mostly predominant in the short cavity regime, while in the longer one, the coupled system remains phase independent.

Although QD lasers have been expected to exhibit near-zero LEFs because of their delta-like density of states [62], the measured above-threshold LEF can be extremely large, as already pointed out in [42], [56], and [63]. In QD lasers, the lasing wavelength can indeed switch from the GS to the ES as the injected current increases. The accumulation of carriers in the ES arises even though the GS lasing is still occurring. As a result, the filling of the ES inevitably enhances the effective LEF of the GS transition, which induces a nonlinear dependence with the injected current. It turns out that this interplay between the filling of lower energy transitions and higher ones is important to the above-threshold LEF. Thus, when the injection current increases, the LEF can balloon up to large values as the lower energy states of the QDs are saturated. In the calculations under study, the impact of the LEF on the modulation response is evaluated assuming values ranging from 1 to 7. Besides, let us note that for a given pump current, self-injection can also lead to slight modifications of the LEF as recently pointed out in [64]. To this end, it is important to stress that such optical feedback-induced variations are not taken into account in the calculations.

The feedback level is set to be $f_{\mathrm{ext}} = 10^{-3}$ for the short external cavity, and $f_{\mathrm{ext}} = 10^{-5}$ for the long cavity. To this end, Fig. 7 compares the different behaviors between the long and the short external cavities. Under the short cavity regime, simulations show that a larger LEF shifts the position of the resonance peak to a higher frequency range and enhances the peak amplitude. Besides, it is also pointed out that the modulation bandwidth can be increased, although the slope of the curve right above the peak frequency becomes larger. On the contrary, in the case of the long external cavity configuration, only the magnitude of the ripple is found to be increased which can be a detrimental effect for the signal detection.

Fig. 8 now illustrates the influences of the carrier capture time from the WL to the ES. In the simulations, values are set at $0.2\tau_{\mathrm{ES}}^{\mathrm{WL}}$, $1.0\tau_{\mathrm{ES}}^{\mathrm{WL}}$, and $2.0\tau_{\mathrm{ES}}^{\mathrm{WL}}$, where $\tau_{\mathrm{ES}}^{\mathrm{WL}} = 12.6$ ps. The feedback level is still $f_{\mathrm{ext}} = 10^{-3}$ for the short cavity, and $f_{\mathrm{ext}} = 10^{-5}$ for the long cavity. Simulations show that both under short and long cavities, the smaller the capture time, the

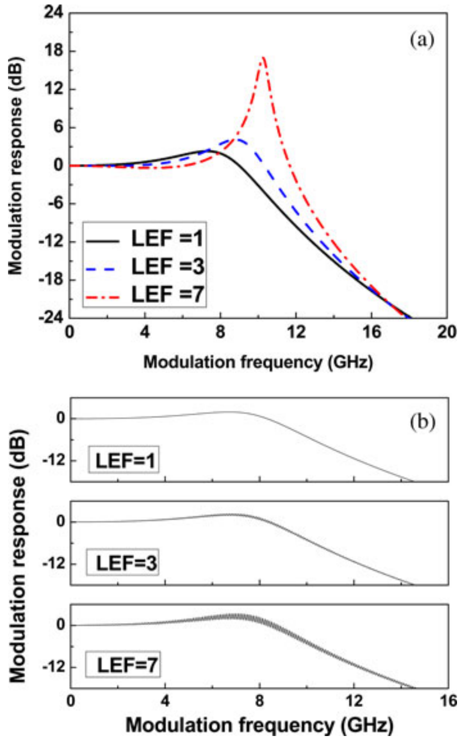

Fig. 7. Impacts of LEF on the IM response for (a) short external cavity ($f_{\mathrm{ext}} = 10^{-3}$) and (b) long external cavity ($f_{\mathrm{ext}} = 10^{-5}$).

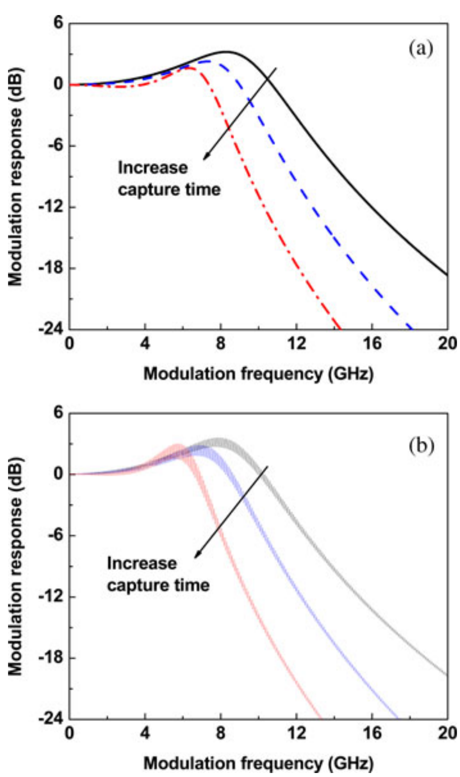

Fig. 8. Impacts of various carrier capture time 0.2 $\tau_{\mathrm{ES}}^{\mathrm{WL}}$, 1.0 $\tau_{\mathrm{ES}}^{\mathrm{WL}}$, and 2.0 $\tau_{\mathrm{ES}}^{\mathrm{WL}}$ on the IM response for (a) short external cavity ($f_{\mathrm{ext}} = 10^{-3}$, LEF $= 1$) and (b) long external cavity ($f_{\mathrm{ext}} = 10^{-5}$, LEF $= 5$).

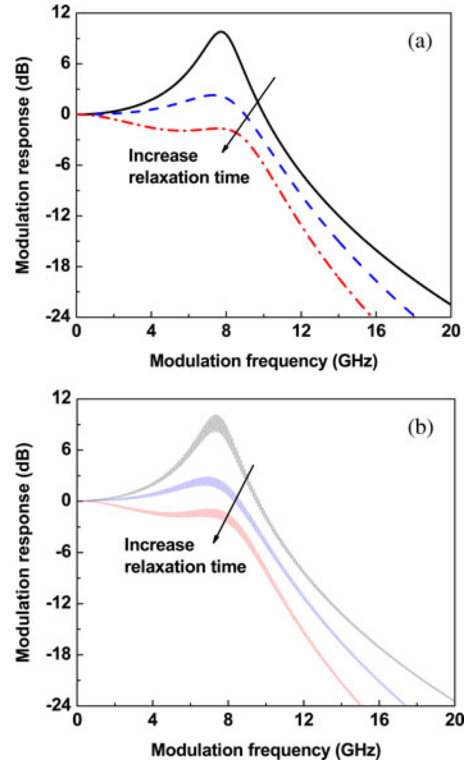

Fig. 9. Impacts of various carrier relaxation time 0.2 $\tau_{\mathrm{GS}}^{\mathrm{ES}}$, $1.0\tau_{\mathrm{GS}}^{\mathrm{ES}}$, and $2.0\tau_{\mathrm{GS}}^{\mathrm{ES}}$ on the IM response for (a) short external cavity ($f_{\mathrm{ext}} = 10^{-3}$, LEF $= 1$) and (b) long external cavity ($f_{\mathrm{ext}} = 10^{-5}$, LEF $= 5$).

higher the relaxation peak and the larger the modulation bandwidth enhancement. In this case, it is also important to stress that the peak amplitude is not sensitive to the carrier capture time although its magnitude remains slightly increased. In order to emphasize the effect of the capture time on the ripple occurring under the long external cavity regime, Fig. 8(b) is obtained assuming a larger LEF equal to 5 [instead of 1 in Fig. 8(a)]. Thus, the results demonstrate that the ripple amplitude is nearly not impacted by the capture time.

In comparison, the behaviors considering various carrier relaxation times from the ES to the GS ($0.2\tau_{\mathrm{GS}}^{\mathrm{ES}}$, $1.0\tau_{\mathrm{GS}}^{\mathrm{ES}}$, and $2.0\tau_{\mathrm{GS}}^{\mathrm{ES}}$) are presented in Fig. 9 assuming the same feedback conditions. Thus, a shorter relaxation time is clearly beneficial to enhance the modulation bandwidth and the peak amplitude both under the short and long external cavity configurations. However, attention has to be paid to the high peak amplitude, which can potentially contribute to increase the transient chirp of the QD laser operating with large-signal modulation format. Simulations also show that the position of the peak frequency becomes smaller with reduced relaxation time. The reason for this result, which is out of expectation, is still under investigation. As regards the long cavity configuration for which the LEF value is set at 5, simulations demonstrate that the parasitic ripple remains little impacted. From a general point of view, we found that the influence of the carrier capture and relaxation times on the modulation response of self-injected QD lasers is rather similar to those operating with optical injection but remains, however, clearly different from the free-running case. Such a comparison will be discussed in detail elsewhere.

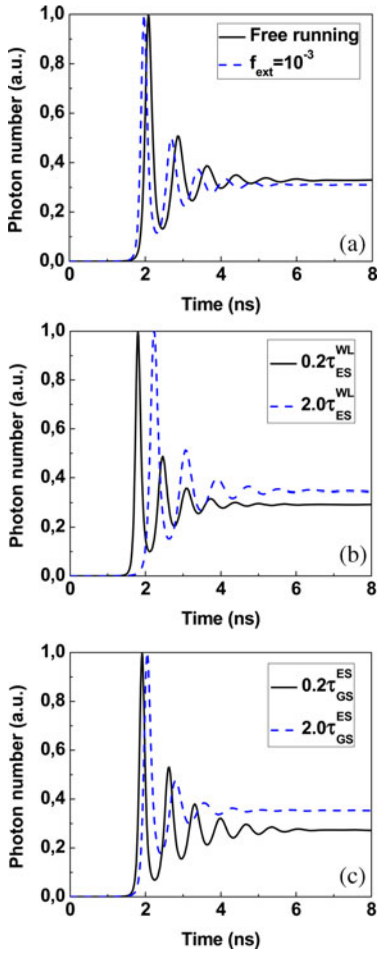

Fig. 10. TOD properties (a) without and with optical feedback ($f_{\rm ext} = 10^{-3}$) at $\tau_{\rm ES}^{\rm WL} = 12.6$ ps and $\tau_{\rm GS}^{\rm ES} = 5.8$ ps (b) for various capture times at $0.2\tau_{\rm ES}^{\rm WL}$ and $2\tau_{\rm ES}^{\rm WL}$ with $f_{\rm ext} = 10^{-3}$ and (c) for various relaxation times at $0.2\tau_{\rm GS}^{\rm ES}$ and $2\tau_{\rm GS}^{\rm ES}$ with $f_{\rm ext} = 10^{-3}$.

Fig. 10 now investigates the turn-on delay (TOD) dynamics under short external cavity configuration. Fig. 10(a) compares the TOD dynamics with and without external optical feedback ($f_{\rm ext} = 10^{-3}$). Although simulations demonstrate that the self-injection has little impact on the damping rate, it is found that this operation shortens the delay time required to reach the laser's threshold. This result agrees with previous works in which the threshold reduction was indeed pointed out in self-injected semiconductor lasers [1]. Fig. 10(b) and (c) also illustrates the influences on the TOD of the carrier capture and relaxation times for the laser operating with optical feedback ($f_{\rm ext} = 10^{-3}$). To this end, numerical results depicted in Fig. 10(b) show that the damping rate is nearly not affected when varying the carrier capture time namely the carrier capture rate does not contribute to the strong damping occurring in QD lasers. Fig. 10(c) demonstrates that a smaller carrier relaxation time can significantly decrease the damping rate. This result is in agreement with [65], where it is shown that the damping factor can be lowered with the increased electron scattering rate (in the range of $\sim 10^{12}$ s^{-1}). From a technological point of view, the TOD simulation means that the damping factor can be controlled by adjusting the confinement strength, which mostly affects the capture time and with a lesser extent the relaxation time.

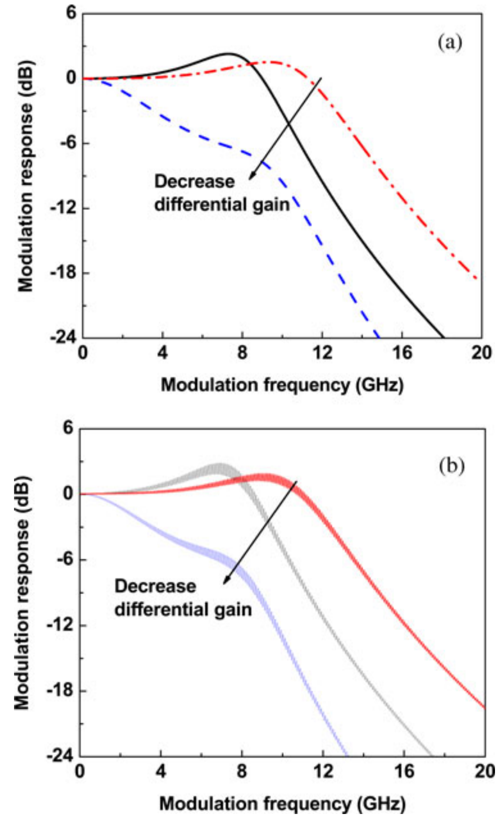

Fig. 11. Impacts of various differential gain $a_{\rm GS} = 3 \times 10^{-15}$, 5×10^{-15}, and $10 \times 10^{-15}\,{\rm cm}^2$ on the IM response for (a) short external cavity ($f_{\rm ext} = 10^{-3}$) and (b) long external cavity ($f_{\rm ext} = 10^{-5}$).

As a conclusion, it is clear that the strong damping usually observed in QD lasers is attributed to the large carrier relaxation time, as already demonstrated in [35], in which the analytical method showed that the carrier escape from the GS is responsible for the large damping rate. Finally, it is important to stress that QD lasers typically exhibit only one or two pronounced peaks in the TOD dynamics because of the strong damping. In the simulations under study, the exhibition of several peaks is due to the small carrier relaxation time as well as due to the low bias current ($1.1 \times I$th) [66].

From the illustrations of Fig. 11, we can see that the differential gain has a significant influence on the modulation response of the QD lasers operating with external optical feedback. In the calculations the differential gain values are 3×10^{-15}, 5×10^{-15}, and 10×10^{-15} cm^2, while the corresponding threshold currents are 93, 54, and 39 mA, respectively. Although the steady-state photon number gets smaller at larger differential gain, it is noted that the carrier populations in the GS, ES, and WL quantum levels are also reduced which results in a lower damping rate and a higher modulation bandwidth. The behavior for the long external cavity depicted in Fig. 11(b) is relatively similar to that of the short cavity except the occurrence of the unchanged ripple.

Regarding the impacts of the gain compression, we also verified the gain compression model (6) with the measured modulation response for the free-running laser [59]. As an example, Fig. 12(a) illustrates the calculated modulation response for various gain compression factors $\varepsilon_0 = 0$, $\varepsilon_1 = 3 \times 10^{-16}$ cm^3, and $\varepsilon_2 = 9 \times 10^{-16}$cm^3. Simulations obtained with a

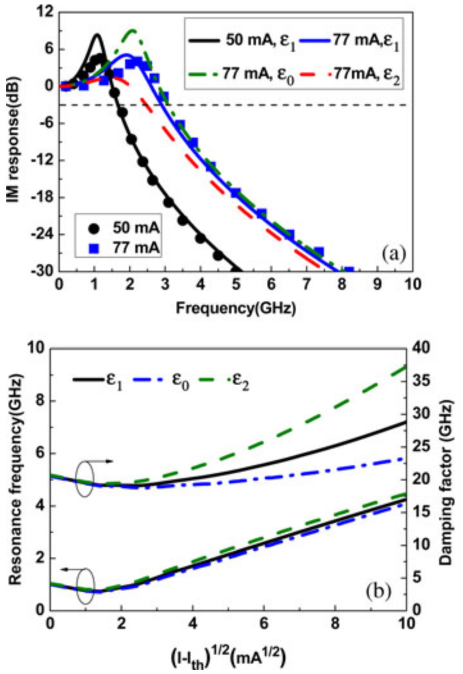

Fig. 12. (a) IM response for a free-running laser under various gain compression factor $\varepsilon_1 = 3 \times 10^{-16}\text{cm}^3$, $\varepsilon_0 = 0$, and $\varepsilon_2 = 9 \times 10^{-16}\text{cm}^3$. Dots represent experimental results and lines represent simulations. (b) Resonance frequency and damping factor as a function of the normalized current.

gain compression factor ε_1 (solid lines) are found in good agreement with the experimental results (at the bias currents 50 and 77 mA). Thus, as already pointed out in [67], increasing the gain compression factor induces a smaller modulation bandwidth. The resonance frequency and the damping factor are calculated via (17) and (18), and the results are depicted in Fig. 12(b) as a function of the normalized current [35]. Simulations show that both suffer from the nonlinear gain especially at high pump current. Although both the resonance frequency and the damping factor increase with gain compression, the latter one is much more impacted, which results in the reduction of the modulation bandwidth, as shown in Fig. 12(a). As for the self-injected QD laser, Fig. 13 shows that larger gain compression clearly suppresses the resonance peak and reduces the modulation bandwidth both for the short and the long external cavity regimes. In the latter configuration, let us stress that the amplitude of the ripple still occurring in the modulation response is little impacted as well.

IV. Conclusion

This paper has theoretically investigated the effects of the self-injection on the modulation properties of QD semiconductor lasers. Based on a semianalytical approach incorporating the QD carrier dynamics, as well as on the nonlinear gain, various numerical results taking into account the impacts of both the optical feedback and laser's intrinsic parameters have been presented. Using this approach, it is shown that a proper feedback level provided by a short external cavity can enhance both the relaxation frequency and the modulation bandwidth, which also benefits from fast carrier capture and relaxation times, a high differential gain, and a moderate LEF in QD active mediums. The QD laser relaxation time is found to be responsible for damping rate observed in IM response, which

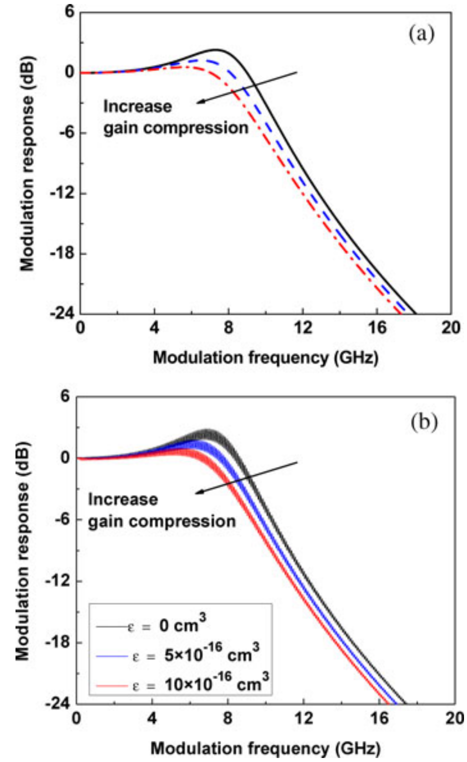

Fig. 13. Impacts of gain compression on the IM response with $\varepsilon = 0$, 5×10^{-16}, $10 \times 10^{-16}\text{cm}^3$ for (a) short external cavity ($f_{\text{ext}} = 10^{-3}$) and (b) long external cavity ($f_{\text{ext}} = 10^{-5}$).

in fact affects the sensitivity to external optical feedback. On one hand, the self-injection technique shown in this paper has a great advantage because it can be implemented in a simple and compact design, and moreover, it does not require an explicit control of the feedback level. On the other hand, the long external cavity regime shows the occurrence of ripples especially around the relaxation peak, which creates a noticeable overshoot in the modulation response. The observed ripple in the modulation response is shown to increase with both the feedback level and LEF. Finally, the presented model can be used to confidently extract microwave characteristics and operating parameters of the system in presence of external optical feedback. These preliminary results indicate the impact of QD lasers' key operating parameters on external feedback sensitivity and thereby are of first importance in diagnostics of optical telecommunication systems, as well as for future integrated photonic circuits.

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