A Novel Mode Multiplexer/Demultiplexer for Multi-Core Fibers

Junhe Zhou and Philippe Gallion, *Senior Member, IEEE*

***Abstract*— In this letter, a mode multiplexer/demultiplexer (MUX/DEMUX) for multi-core fibers (MCFs) based on multimode interference (MMI) effect is proposed. Due to the coincidence of the rows of the transfer matrix of an MMI coupler and the modal profile of the MCFs with linearly aligned cores, an MMI coupler with the aid of a phase shifter array is able to distinguish different super-modes of the MCFs. The output port number indicates the number of the super-modes inside the MCFs. Detailed theoretical derivations and numerical simulations are provided to demonstrate the multiplexing/demultiplexing capability of the device. The proposed mode MUX/DEMUX can be fiber based or waveguide based and is easy to be integrated with the fiber system.**

***Index Terms*— Multimode waveguides, mode division multiplexing.**

I. INTRODUCTION

MODE division multiple access (MDMA) has recently attracted significant attention due to its higher transmission capacity [1]–[3]. Theoretically speaking, the capacity of the multimode transmission system is multiplied by the number of the transmission modes as compared to its single mode counterpart. One of the approaches to realize the multimode transmission system is to use the MCFs [3]–[6], which possess multiple cores for signal propagation.

Key devices to implement MCFs include the mode MUXs/DEMUXs for MCFs. Free space optics can be used to distinguish different modes [3]. However, it is of significant complexity and expense, especially while detecting the higher order modes. Another way to realize mode demultiplexing is to use the phase matching concept [5]. Although it is efficient and fiber based, the device is not straightforward to design for MCFs with a large number of cores. One more general method for MCF mode MUX/DEMUX design is to use arrayed waveguide gratings (AWGs) [6]. The order of the super-modes inside the MCF corresponds to the number of the output port of the AWG. The pioneering proposal is ground breaking, but it requires planar lightwave circuit (PLC) technology to fabricate the AWG, which might not be easy to be integrated with the fiber system [5]. Furthermore, the interference of the field inside the AWG requires the wave amplitudes at the input ports to be equal; however, the field distribution of the super-modes inside the MCF fulfills the sinusoidal function [7]. Therefore, the modal extinction ratio for the AWG based MUXs/DEMUXs will be degraded if the number of the cores is large.

An alternative way to achieve mode manipulation is to use the MMI effect [8]–[13], which has been proposed to generate [12] and convert [13] the modes in the multimode waveguides. In this letter, we propose to realize the mode MUXs/DEMUXs for MCFs using MMI couplers which are made of fibers with rectangle cores or multimode waveguides based on the PLC technology. It can be briefly described as follows: the super-modes of the MCF are passed on to a MMI coupler. The amplitudes of the waves in the cores of the MCF fulfill sinusoidal function and their phases are changed by a phase shifter array. The transfer matrix of the multimode waveguide [9] is modified by those phase shifters and its rows coincide with the amplitudes vectors of the super-modes of the MCF. When a certain super-mode arrives, it is phase shifted and passed on to the multimode waveguide, and the corresponding output port of the transformer will output the optical wave while other ports have no output at all. In this way, the MUX/DEMUX is realized.

It is worth noting that this method can multiplex/demultiplex modes with different amplitude/phase distribution, while polarization induced mode diversity is not considered here. However, TM and TE modes MUX/DEMUX are already discussed in the published literatures [14].

II. MATHEMATICAL DESCRIPTION

A. Super-Modes Inside the MCF

In this letter, it is assumed that the identical cores of the MCF are linearly aligned and equally spaced [5], [6]. If a single core is mono-mode, $N-1$ coupled cores will have $N-1$ super-modes, whose amplitudes on each core fulfill [7]

$$\sin\left(\frac{np\pi}{N}\right) \tag{1}$$

where n is the number of the super-mode, p the number of the cores, and they are integers between 1 and $N-1$. The core diameter is assumed to be a, and the distance between the core centers is assumed to be D.

B. MCF Mode MUX-DEMUX Based on MMI Couplers

Assuming that the cores of the MCF are connected with a fiber with a rectangle core or a rectangle multimode waveguide

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J. Zhou is with the Department of Electronics Science and Engineering, Tongji University, Shanghai 200092, China (e-mail: jhzhou@tongji.edu.cn).

P. Gallion is with TELECOM ParisTech, Ecole Nationale Supérieure des Télécommunications, Paris 75013, France (e-mail: philippe.gallion@telecom-paristech.fr).

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fabricated by the PLC technology, multimode interference will take place. Without loss of generality, a rectangle core fiber is used as a MMI coupler and the following analysis will focus on the fiber technology based mode MUX-DEMUX. It has the height of a and an effective width of ND. It is single-mode in the y direction and multimode in the x direction. Therefore, during the analysis, the y direction modal contribution is represented by an effective index. The x dimensional modes inside a multimode waveguide can be represented by the sinusoidal function [8]–[12], which coincides with Eq. (1). We assume that the cores of the MCF, which are connected with the MMI coupler, are very thin, i. e. a approaches 0, and the modal distribution function of the cores is the ideal Dirac function (it is easy to extend to the thicker case and will be discussed later in this section). Each core is added with a phase shifter whose phase shift is $\frac{i^2\pi}{2N}$, where i is the number of the port. The center of the cores of the MCF are placed at [9]

$$x_i = \frac{iW_e}{N} = iD \quad (i = 1 \cdots N-1) \tag{2}$$

where W_e is the effective width of the MMI coupler. Considering the n^{th} super-mode of MCF is used as the input and phase shifted, the x dimensional modal field at the input of the MMI coupler becomes

$$E(0,x) = \sum_{i=1}^{N-1} \sin\left(\frac{ni\pi}{N}\right) \exp\left(j\frac{i^2\pi}{2N}\right) \delta(x - x_i) \tag{3}$$

The MMI coupler has the length of [11]

$$L_N = \frac{2n_{eff}W_e^2}{N\lambda_0} \tag{4}$$

where n_{eff} is the effective index of the fundamental mode of the MMI coupler and λ_0 the free space wavelength. The x dimensional mode propagation inside the multimode waveguide, can be described by the following equation [9]

$$E(z,x) = \exp(-jkz) \sum_{m=1}^{\infty} E_m \exp\left(j\frac{m^2\pi\lambda_0 z}{4n_{eff}W_e^2}\right) \sin\left(\frac{m\pi x}{W_e}\right) \tag{5}$$

where E_m is the amplitude of the m^{th} mode. With the input field as Eq. (3), E_m can be evaluated as

$$E_m = \frac{2}{W_e} \sum_{i=0}^{N-1} \sin\left(\frac{ni\pi}{N}\right) \exp\left(j\frac{i^2\pi}{2N}\right) \sin\left(\frac{mi\pi}{N}\right) \tag{6}$$

After the propagation of length L_N, one has

$$\begin{aligned} E(L_N,x) &= \tfrac{2}{W_e} \exp(-jkL_N) \\ &\times \sum_{m=1}^{\infty} \sin\left(\tfrac{m\pi x}{W_e}\right) \exp\left(j\tfrac{m^2\pi}{2N}\right) \sum_{i=1}^{N-1} \sin\left(\tfrac{ni\pi}{N}\right) \\ &\times \sin\left(\tfrac{mi\pi}{N}\right) \exp\left(j\tfrac{i^2\pi}{2N}\right) \end{aligned} \tag{7}$$

Let $m = lN + n_0$, where l is an integer and n_0 is an integer between 0 and $N-1$, and consider the last three terms in Eq. (7), one has

$$\begin{aligned} &\sum_{i=1}^{N-1} \sin\left(\tfrac{mi\pi}{N}\right) \exp\left(j\tfrac{i^2\pi}{2N}\right) \sin\left(\tfrac{ni\pi}{N}\right) \\ &= \sum_{i=1}^{N-1} \sin\left(\tfrac{n_0 i\pi}{N}\right) \exp\left(j\tfrac{i^2\pi}{2N}\right) \sin\left(\tfrac{ni\pi}{N}\right) \\ &= \sqrt{\tfrac{N}{2}}\, j \exp\left(j\tfrac{\pi}{4}\right) \sin\left(\tfrac{n_0 n\pi}{N}\right) \exp\left(-j\tfrac{(n_0^2+n^2)\pi}{2N}\right) \end{aligned} \tag{8}$$

Combining Eq. (7), (8) and using $m = lN + n_0$, one has

$$\begin{aligned} E(L_N,x) &= \exp(-jkL_N) \sqrt{\tfrac{N}{2}}\, j \exp\left(j\tfrac{\pi}{4}\right) \exp\left(-j\tfrac{n^2\pi}{2N}\right) \\ &\times \tfrac{2}{W_e} \sum_{m=1}^{\infty} \sin\left(\tfrac{m\pi x}{W_e}\right) \sin\left(\tfrac{mn\pi}{N}\right) \end{aligned} \tag{9}$$

Considering the summation term inside Eq. (9), one has

$$\begin{gathered} \tfrac{2}{W_e} \sum_{m=1}^{\infty} \sin\left(\tfrac{mn\pi}{N}\right) \sin\left(\tfrac{m\pi x}{W_e}\right) = \delta(x - x_n) \\ x_n = n\tfrac{W_e}{N} \end{gathered} \tag{10}$$

Combining Eq. (9), (10) one has

$$\begin{aligned} E(L_N,x) &= \exp(-jkL_N)\, j \exp\left(j\tfrac{\pi}{4}\right) \\ &\times \sqrt{\tfrac{N}{2}} \exp\left(-j\tfrac{n^2\pi}{2N}\right) \delta(x - x_n) \end{aligned} \tag{11}$$

It can be seen from Eq. (11) that the output field concentrates on the n^{th} output port of the MMI coupler with the phase shift of $-j\frac{n^2\pi}{2N}$ if the constant phase term is dropped. Therefore, mode $n$generates the optical intensity at the n^{th} output port of the MMI coupler. It should be noted that since the mode profile expression is valid within the range of [0, W_e], $n$has to be an integer between 1 and $N-1$, which agrees with the modal number of the MCF.

Now, let's consider MCFs with thick cores. Assuming that the core is mono-mode and its field profile is $f(x)$ in the x dimension, we have the input as

$$E(0,x) = \left(\sum_{i=1}^{N-1} \sin\left(\frac{ni\pi}{N}\right) \exp\left(j\frac{i^2\pi}{2N}\right) \delta(x - x_i)\right) \otimes f(x). \tag{12}$$

It can be noted that the first three terms in Eq. (12) are exactly the same as Eq. (3). Since the MMI coupler is a linear system, the output should be

$$\begin{aligned} E(L_N,x) &= \exp(-jkL_N)\, j \exp\left(j\tfrac{\pi}{4}\right) \\ &\times \sqrt{\tfrac{N}{2}} \exp\left(-j\tfrac{n^2\pi}{2N}\right) f(x - x_n). \end{aligned} \tag{13}$$

Hence, it can be seen from Eq. (13) that the output field still concentrates on the n^{th} output port of the MMI coupler in the more general case.

III. NUMERICAL RESULTS AND DISCUSSION

Numerical simulations based on three dimensional beam propagation method (3DBPM) have been carried out for the proposed device. A 7-core MCF is used as an example in the simulation. The MCF has similar index distribution and geometrical parameters as Ref. [5], [6]. For readers' convenience, it is restated here. The MCF has the cladding index of 1.45 and the core index difference of 1.2%. The diameter of the

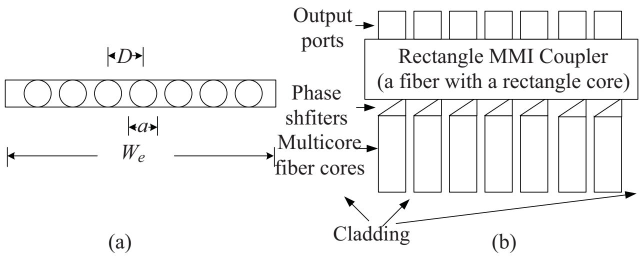

Fig. 1. Schematic of the device. (a) Cross section of the MCF and MMI coupler. (b) Top view of the device.

core is 5 μm. The 7 cores are equally spaced and linearly aligned. The distance between the adjacent core centers, i.e. D, is assumed to be 5 μm, 7 μm and 10 μm to test the robustness of the method.

As is indicated in the previous section, the rectangle multimode fiber, which is with the height of 5 μm and effective width of 8D, i.e, 40 μm, 56 μm and 80 μm, has the same core and cladding indexes as those of the MCF. The length of the MMI coupler is roughly estimated by Eq. (4) and adjusted during the calculation to get the best performance. The schematic of the device is illustrated in Fig. 1.

There are several ways to realize the phase shifters, either to introduce an external phase shifter array [15], to introduce bending array waveguides to achieve phase shift between the input and output planes [6], or to fabricate a MCF whose cores are with different refractive indexes [5]. There is no coupling between the fields inside different cores while using the first two methods. While using the last one, the coupling can also be neglected, because the waveguides can be spatially separated when introducing the phase shift. Without loss of generality, we assume that a perfect external phase shifter array is used to achieve the desired phase change on each of cores without affecting others. More specifically speaking, the fields within the cores are assumed to propagate with specific lengths without coupling from other cores in the simulation.

Firstly, we examine the validity and robustness of the proposed device. The core center distance D is varied and the fundamental modes of the different MCFs are assumed to be phase shifted and launched into the MMI coupler. The field profile of the fundamental super-mode and the output field of the MMI coupler are plotted in Fig. 2 with different Ds. It can be seen from the figure that the modal profiles are sinusoidal as indicated by [7] and therefore AWG might not have a good performance as a mode MUX/DEMUX. Also, it is illustrated that the output fields concentrate on the output port 1 of the MMI couplers, which demonstrates the capability of the device as a mode MUX/DEMUX. The modal extinction ratio is defined as the power at the 1st port over the power at other ports, and it is over 30dB. The comparison shows there is almost no significant performance difference among the three cases.

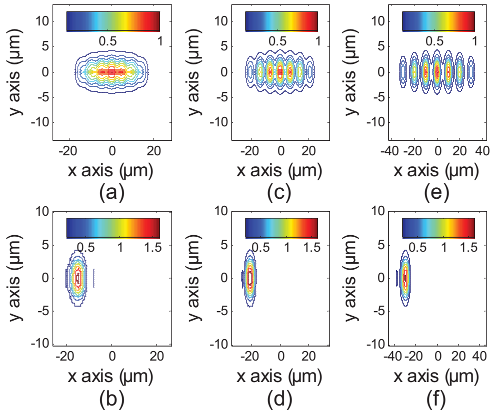

Fig. 2. The fundamental super-mode of the MCF and the field at the output of the DEMUX with different core distances. (a) The fundamental mode when $D = 5$ μm; (b) the field at the output of the DEMUX when $D = 5$ μm; (c) the fundamental mode when $D = 7$ μm; (d) the field at the output of the DEMUX when $D = 7\mu$m; (e) the fundamental mode when $D = 10\mu$m; (f) the field at the output of the DEMUX when $D = 10\mu$m.

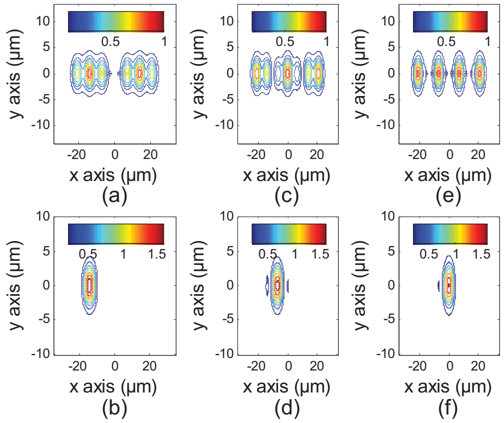

Fig. 3. The super-modes of the MCF and the fields at the output of the DEMUX with $D = 7$ μm. (a) The 2nd mode; (b) the field at the output of the DEMUX for the 2nd mode. (c) The 3rd mode; (d) the field at the output of the DEMUX for the 3rd mode. (e) The 4th mode; (f) the field at the output of the DEMUX for the 4th mode.

It is therefore reasonable to choose the case of $D = 7$ μm as an example to demonstrate the performance of the device for higher order super-modes. In Fig. 3 and Fig. 4., the 2nd to 7th super-modes of the MCF are plotted as well as the fields at the output of the MMI couplers. As the mode number varies, the output shift gradually from the left to the right just as predicted. The modal extinction ratio is also over 30 dB.

If different modes are mixed during the propagation inside the MCF, they will be demultiplexed at the output of the MMI coupler. To demonstrate this, a field consisting of the 1st mode and the 2nd mode is assumed to be propagating in the MCF. The 1st mode has the amplitude of 2 while the 2nd mode has the amplitude of 1, and they have the same phase. The combined field is depicted in Fig. 5(a). The field is phase shifted and passed on to the MMI coupler. As is shown in Fig. 5(b), it can be seen that mode 1 and mode 2 are separated with the right amplitudes at the right output ports. The example clearly demonstrates the mode demultiplexing capability of the device.

The discussion above illustrates the capability of the device to achieve mode demultiplexing. To achieve multiplexing, we can use the output ports as the input ports and the output fields as the input fields. It can be easily inferred that if these actions are taken, the super-modes of the MCF can be generated. Due to the page constraint, it is not illustrated here.

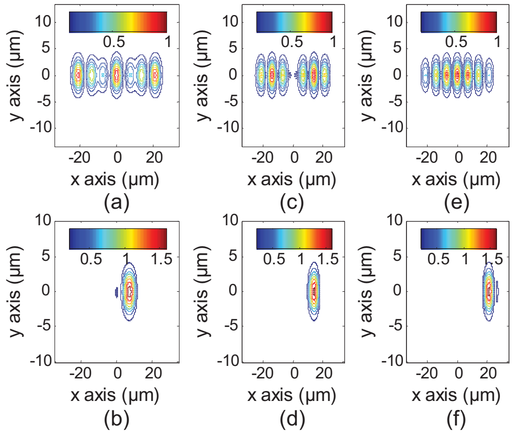

Fig. 4. The super-modes of the MCF and the fields at the output of the DEMUX with D = 7μm. (a) The 5th mode; (b) the field at the output of the DEMUX for the 5th mode. (c) The 6th mode; (d) the field at the output of the DEMUX for the 6th mode. (e) The 7th mode; (f) the field at the output of the DEMUX for the 7th mode.

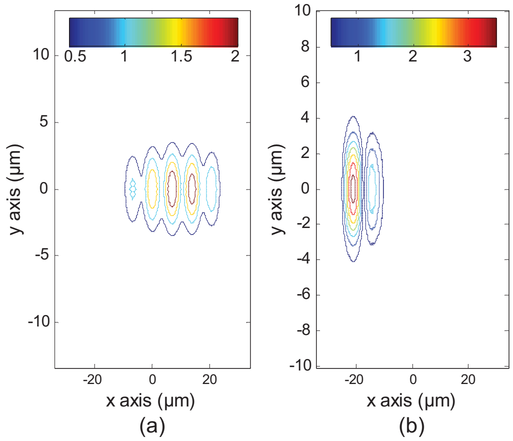

Fig. 5. The combination of super-modes of the MCF and the fields at the output of the DEMUX with D = 7 μm. (a) The combination of the 1st and the 2nd mode; (b) the field at the output of the DEMUX.

As an important factor, the fabrication error tolerance of the device is investigated. The parameters remain the same as those in the case of $D = 7$ μm. Two types of errors, i.e. the phase error of the phase shifter array and the fabrication error of the MMI coupler are studied. If the phase has 10% error (i.e. each of the phase term of the phase shifter array increases 10%), the modal extinction ratio will degrade to 12dB. When the MMI coupler has 40 μm length error (about 5% of the total length) or 1.4 μm width error (about 2.5% of the total width), the modal extinction ratio will degrade to 25dB and 26dB respectively. As indicated in [10], the MMI coupler fabrication errors, such as the errors in the width and the length of the MMI coupler can be proportionally related to each other [10]. The width error impact will be more significant as compared with the length error impact [10]. To mitigate those impacts, the phase shifter array should be fabricated precisely or designed to be adjustable via electro-optical tuning during the application. Also the length of the MMI coupler should be carefully adjusted like the case of the multimode fiber based band pass filter [16].

Finally, the temperature and stress will change the index of the device. If the device is fiber based and made of silica, those effects are negligible as suggested by the BPM simulations.

IV. CONCLUSION

In summary, a new method to realize the mode MUX/DEMUX for MCFs is proposed based on the multimode interference effect. The device can be either fiber based or PLC based, and is compact and feasible for the MCFs with arbitrary number of linearly aligned cores. Furthermore, the proposed device can be easily extended to the case of the MCFs with two dimensional cores. The implementation of the device will help to realize compact and low cost MDMA systems.

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