Accurate Digital Frequency Offset Estimator for Coherent PolMux QAM Transmission Systems

Mehrez Selmi[(1)], Yves Jaouën[(1)], Philippe Ciblat[(1)]

[(1)] Institut Telecom/Telecom ParisTech, CNRS UMR LTCI, 46 rue Barrault 75634 Paris CEDEX 13 France, ✉ mehrez.selmi@telecom-paristech.fr

Abstract *An accurate blind frequency offset estimator adapted to QAM modulated signal is proposed. For coherent 100Gbit/s QAM PolMux transmission, frequency offset can be recovered with an accuracy of a few kHz.*

Introduction

M-ary quadrature amplitude modulation (M-QAM) formats combined with coherent detection and digital signal processing (DSP) are promising candidates for the implementation of next generation optical transmission systems. However, those modulation formats are more sensitive to signal distortions and phase errors than QPSK. These phase errors may correspond to constant phase offset, frequency offset (FO) and laser phase noise[1]. Several FO estimators have been already presented for QPSK based optical transmissions. These algorithms rely either on the phase difference between two adjacent receive samples[2,3] or the maximisation of the discrete-frequency spectrum of the fourth-power receive samples[4]. Let N be the number of available independent receive samples. The Mean Square Error (MSE) on the FO decreases as $1/N$ for the first kind of algorithms, and as $1/N^2$ for the second kind of algorithms. As M-QAM is more sensitive to FO, designing more accurate estimators is still required.

We here propose a new non-data-aided FO estimator for any QAM format in PolMux context. We especially show that its MSE decreases as $1/N^3$. Note that this algorithm can be adapted to PSK formats as well.

Frequency estimator description

The proposed estimator is carried out after the compensation of group velocity dispersion (GVD) and polarisation dispersion (PMD). As a consequence, assuming a perfect compensation, the receive signal on polarisation p (with $p \in \{X, Y\}$) takes the following form

$$y_p(k) = s_p(k)e^{2j\pi(\phi_{0,p}+k\phi_1)} + n_p(k) \qquad (1)$$

where $\{s_p(k)\}$ are independent sequences of QAM symbols, $n_p(k)$ is the additive channel noise. The term $\phi_{0,p}$ corresponds to the constant phase while $\phi_1 = \Delta f T_s$ is the discrete-time FO to be estimated where Δf is the continuous-time FO expressed in Hertz and T_s is the symbol period. Eq. (1) holds if $\Delta f \ll 1/T_s$.

If $s_p(k)$ is a QPSK modulated data stream, it has been remarked for a long time that $s_p^4(k)$ is constant and independent of the data stream. Consequently it is possible[5] to build FO estimate based on $y_p^4(k)$. When $s_p(k)$ is QAM modulated, $s_p^4(k)$ is not constant anymore. But a QAM modulated signal is fourth-order non-circular[6], *i.e.*, $\mathbf{E}[s_p^4(k)] \neq 0$. Based on this nice property, it is still possible to build FO estimate based on $y_p^4(k)$ in QAM context.

Indeed, by following the approach described in[7], one can remark that $y_p^4(k)$ can be decomposed as

$$y_p^4(k) = A_p e^{2j\pi 4(\phi_{0,p}+k\phi_1)} + e_p(k) \qquad (2)$$

where $A_p = \mathbf{E}[s_p^4(k)]$ is a constant amplitude and where $e_p(k)$ a zero mean process that can be viewed as a noise process. The most important thing is to remark that $y_p^4(k)$ is actually a constant-amplitude complex exponential with frequency $4\phi_1$ disturbed by a zero mean additive noise. Therefore one can deduce a FO estimate based on the maximization of the periodogram of $y_p^4(k)$ as proposed below

$$\hat{\phi}_{1,N} = \frac{1}{4} \arg \max_{\phi \in [-1/2, 1/2)} \sum_{p \in \{X,Y\}} f_p(\phi) \qquad (3)$$

with

$$f_p(\phi) = \left| \frac{1}{N} \sum_{k=0}^{N-1} y_p^4(k) e^{-2j\pi k\phi} \right|^2 \qquad (4)$$

and with N the number of available samples.

Unlike what is usually done in optical communications, we propose to compute the maximisation of periodogram into two steps as follows

1. a coarse step which detects the maximum magnitude peak which should be located at around the sought frequency . This is carried out by a N-Fast Fourier Transform (N-FFT). This step is been already implemented for QPSK format[4]. One can easily check that MSE associated with this step is of order of magnitude $1/N^2$.
2. a fine step which inspects the cost function around the peak detected by the coarse step. This step is implemented by a gradient-descent algorithm. MSE associated with this step[7] is of order of magnitude $1/N^3$.

Finally, to the best of our knowledge, our proposition of treating both polarisation ways jointly is new.

Numerical results

The performance of the algorithm is evaluated by using Monte-Carlo simulations. A 100Gbit/s transmission is achieved by multiplexing both polarisations with 16-QAM modulated signals which corresponds to 12.5Gbaud transmission per polarisation. The linewidths of lasers are set to zero. The polarisation dependent effects (PDE) are simulated using the concatenation of random birefringence matrices[8].

At the receiver, the continuous-time signal is sampled at symbol rate. The linear PDE is compensated using a 5 taps FIR MIMO filters calculated by means of CMA algorithm[9]. Afterwards, FO is estimated by using one of the four following methods: i) coarse step based on an unique periodogram associated with polarisation X (this algorithm is usually carried out in QPSK context), ii) coarse step based on the sum of both periodograms associated with polarisations X and Y, iii) coarse and fine step based on an unique periodogram associated with polarisation X, and iv) coarse and fine step based on the sum of both periodograms associated with polarisations X and Y. At each Monte-Carlo trial, FO is randomly located between 0 and 3.12GHz. The number of Monte-Carlo runs is fixed to 100.

In Fig. 1, MSE of FO (defined as $\mathbf{E}[|\phi_1 - \hat{\phi}_{1,N}|^2]$) is plotted versus OSNR for $N = 256$ and $N = 4096$. One observe that the outliers effect[10] at low SNR is stronger for the methods based on one polarisation than for those based on both polarisations. We also confirm that the convergence speed is faster with fine step than with coarse step. Notice that MSE for FO estimation by using Leven algorithm[2] is around 10^{-5} when $N = 4096$ whereas the proposed method achieves a MSE less than 10^{-12} for the same value of N.

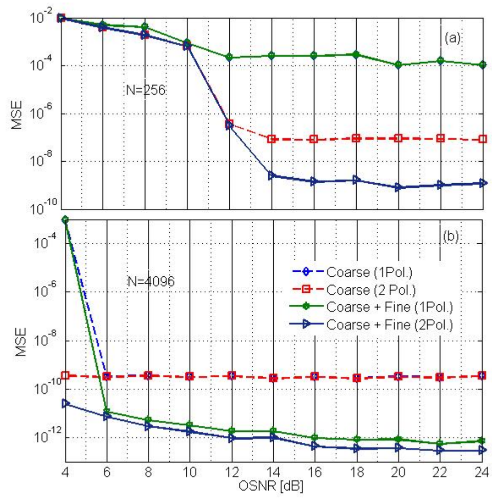

Fig. 1: MSE vs. OSNR for a) $N = 256$, b) $N = 4096$

In Fig. 2, Bit Error Rate (BER) is analysed with respect to OSNR when $N = 512$. BER of 10^{-4} is obtained at OSNR=20dB with the proposed method. Using the usual method leads to an error floor preventing to reach BER of 10^{-4}.

In Fig. 3, BER is plotted versus frequency offset Δf when OSNR=19dB and $N = 512$. The extrema of considered interval are chosen such that $4\Delta f T_s$ are two adjacent FFT points k_0/N and $(k_0 + 1)/N$ with $k_0 = 450$. Thanks to the fine step, our algorithm is insensitive to the location of the frequency offset.

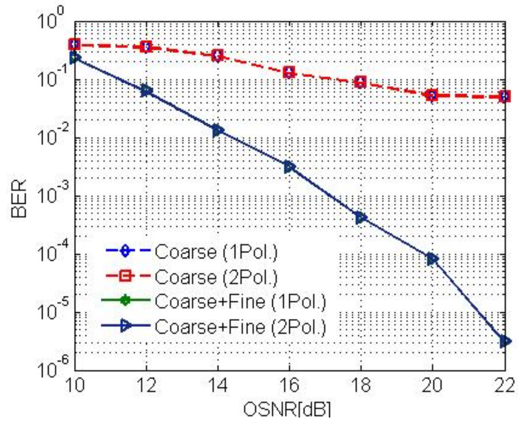

Fig. 2: BER vs. OSNR

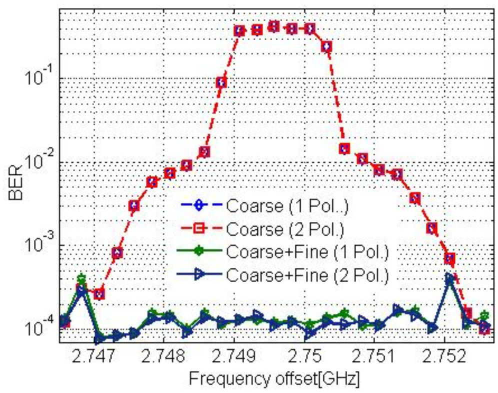

Fig. 3: BER vs. frequency offset Δf

Additional simulations (not plotted in the paper due to the space limitation) show that the proposed FO estimate tolerates more amounts of GVD and PMD. Nevertheless if the estimation algorithm is applied before CMA algorithm, we have observed a failure probability of about 5%.

Conclusion

We proposed an accurate non-data aided FO estimator when QAM modulated signals are considered. Consequently we showed that frequency offset can be properly mitigated which implies that coherent 16-QAM may remain an attractive candidate for 100Gbit/s transmission.

Acknowledgement

This work has been funded by Networks of Future Labs located at Institut TELECOM.

References

1 M. Seimetz, Proc. OSA/NFOEC, NWA4 (2006).
2 A. Leven et al., IEEE PTL., **6** (2007).
3 L. Li et al., Proc. OSA/NFOEC, OWT4 (2008).
4 S. Hoffmann et al., IEEE PTL., **18** (2008).
5 M. Morelli et al. , ETT., **2** (1998).
6 B. Picinbono, IEEE TSP., **12** (1994).
7 P. Ciblat et al., IEEE TCOM., **5** (2003).
8 A.O. Dal Forno et al., IEEE PTL., **3** (2000).
9 D.N. Godard, IEEE TCOM., **11** (1980).
10 P. Ciblat et al., IEEE TCOM., **10** (2006).