Channel estimation and Superresolution in UWB system

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Outline

1. UWB system
   - Impulse Radio
   - Multi-band
   - Channel Model

2. Channel estimation
   - Cramer-Rao Bound
   - Existing estimates
   - Comparison

3. Superresolution
Digital communications system satisfies the following spectral mask:

![Graph showing spectral mask with frequencies and P IRE values for Indoor and Outdoor conditions.](image)

- **Interest**
  - Spread spectrum technique
  - Localization
Techniques

Approaches

- Impulse Radio (IR)
- Multi-band (MB)

We hereafter focus on Impulse-Radio technique

- Pierce and Hopper 1952
- Winthington and Fullerton 1992
- Win and Scholtz 1993
Time-Hopping (TH) IR-UWB signal associated with user $n$

- $N_f T_f$
- $d_n(i-1)$
- $d_n(i)$
- $d_n(i+1)$
- $N_f$ frames
- $T_f$
- Temps de garde
- $N_c$ chips
- $T_c$
- $d_n(i) = 0$
- $d_n(i) = 1$
- $d_n(i-1)$
- $d_n(i+1)$
- $d_n(i) = 1$ PAM
- $d_n(i) = 1$ PPM
Data stream

\[ s(t) = \sum_{i=0}^{M-1} d_i b(t - iN_f T_f) \]

where

- \( M \) is the number of transmit symbols
- \( d = [d_0, \cdots, d_{M-1}] \) belongs to PAM
- \( N_f \) is the number of frame per symbol
- \( T_f \) is the duration of each frame
The super frame composed by $N_f$ frames is structured as follows

$$b(t) = \sum_{j=0}^{N_f-1} g(t - jT_f - \tilde{c}_j T_c)$$

where

- $T_c$ is the chip duration
- $N_c$ is the number of chips in one frame
- Time-hopping code in the $j^{th}$ frame is given by $\tilde{c}_j \in \{0, \cdots, N_c - 1\}$
- $g(t)$ is the mono-cycle with the temporal support $[0, T_g)$
Developed code

For each frame $j$, let $c_j = [c_j(0), \cdots, c_j(N_c - 1)]$ defined as follows

$$c_j(i) = \begin{cases} 
1 & \text{if } i = \tilde{c}\_j \\
0 & \text{otherwise}
\end{cases}$$

Then $c = [c_0, \cdots, c_{N_f - 1}] = [c(0), \cdots, c(N_f N_c - 1)]$

$$s(t) = \sum_{i=0}^{M-1} d_i \sum_{j=0}^{N_f N_c - 1} c(j) g(t - j T_C - i N_f T_f)$$

- Status of the chip (occupied/free) outside $g(t)$
- Le Martret & Giannakis 2002
Channel model

- Multi-path random channel
- Molish 2003

Impulse response

\[ h(t) = \sum_{k=1}^{N_p} A_k \delta(t - \tau_k) \]

where

- \( A_k \) is the attenuation associated with the \( k^{th} \)-path
- \( \tau_k \) is the delay associated with the \( k^{th} \)-path
We focus on one cluster model

Statistical model

\[ p(\tau_k | \tau_{k-1}) = \lambda e^{-\lambda(\tau_k - \tau_{k-1})} \]

\[ A_k = (p_k \cdot b_k) e^{-\tau_k / \gamma} \]

where

- \( a_k \) independent of \( \tau_n \)
- \( p_k \) binary variable
- \( b_k \) log-normal variable

\( \lambda \) and \( \gamma \) are both deterministic parameters
Deterministic parameters

- $\lambda$ is the path density
- $\gamma$ is the RMS delay spread (i.e., length of impulse response)

$$\lambda = 0.1\text{ns}^{-1} \text{ and } \gamma = 20\text{ns}$$

$$\lambda = 1\text{ns}^{-1} \text{ and } \gamma = 200\text{ns}$$
Rake receiver (for sake of simplicity)

Correlation with the template $b(t) = \sum_{j=0}^{N_tN_c-1} c_j g(t - jT_c)$ synchronized at each path

Path estimation is necessary
Fisher Information Matrix

\[
J_{A_l, A_k} = \frac{2}{N_0} f_1^{(k,l)}, \quad J_{A_l, \tau_k} = -\frac{2A_k}{N_0} f_2^{(l,k)}, \quad J_{\tau_l, \tau_k} = \frac{2A_k A_l}{N_0} f_3^{(k,l)}
\]

where

\[
f_1^{(k,l)} = \mathbb{E}_d \left[ \int s(t - \tau_k) s(t - \tau_l) dt \right]
\]

\[
f_2^{(k,l)} = \mathbb{E}_d \left[ \int s(t - \tau_k) s'(t - \tau_l) dt \right]
\]

\[
f_3^{(k,l)} = \mathbb{E}_d \left[ \int s'(t - \tau_k) s'(t - \tau_l) dt \right]
\]

with

- \( s'(t) = ds(t)/dt \) and \( \mathbb{E}_d[\phi(d)] = \phi(d) \) if \( d \) is a known sequence

\( \Rightarrow \) CRB for DA scheme and MCRB for NDA scheme
State-of-the-Art

1. Laurenti (September 2004) : one path
2. Huang (June 2004) : non-overlapping context (i.e., signal echoes are orthogonal)
   \[ f^{(k,l)}_m = 0 \quad \text{if} \quad k \neq l \]
3. Zhang (June 2004) : overlapping taken into account (but no closed-form expression for FIM)

Questions

- Non-overlapping assumption does not hold in realistic situation?
- Closed-form expressions for \( f^{(k,l)}_m \) even when \( k \neq l \)
Non-overlapping case

Straightforward derivations yield

\[
\text{CRB}_{\text{DA}}(A_l) = \text{MCRB}_{\text{NDA}}(A_l) = \frac{N_0}{MN_f} \frac{E_3}{2(E_1E_3 - E_2^2)}
\]

\[
\text{CRB}_{\text{DA}}(\tau_l) = \text{MCRB}_{\text{NDA}}(\tau_l) = \frac{N_0}{MN_f} \frac{E_1}{2A_l^2(E_1E_3 - E_2^2)}
\]

with \( E_1 = \int g(t)^2 dt \), \( E_2 = \int g(t)g'(t)dt \), and \( E_3 = \int g'(t)^2 dt \)

Remarks

\( \Rightarrow \) In DA scheme, performance does not depend on the training sequence

\( \Rightarrow \) Same expression in the context of single-path (when \( N_p = 1 \))
Overlapping case

Let
\[ \Delta \tau_{k,l} = \tau_k - \tau_l = Q_{k,l} N_f T_f + q_{k,l} T_c + \varepsilon_{k,l} \]
with the integer parts \( Q_{k,l} \) and \( q_{k,l} \), and the remainder \( \varepsilon_{k,l} \)

**Main result**

\[
\begin{align*}
    f_m^{(k,l)} &= M(C(q)A_m(\varepsilon) + C(q + 1)A_m(\varepsilon - T_c) \\
                 &+ D(q)B_m(\varepsilon) + D(q + 1)B_m(\varepsilon - T_c))
\end{align*}
\]

with
\[
C(q) = \sum_{j=0}^{N_f N_c - q - 1} c(j)c(j + q), \quad D(q) = \sum_{j=0}^{q - 1} c(j)c(j - q)
\]

\[
A_m(\varepsilon) = \frac{1}{M} \sum_{i=0}^{M-1} \mathbb{E}_d [d_{-Q-1+i} d_i] r_m(\varepsilon), \quad B_m(\varepsilon) = \frac{1}{M} \sum_{i=0}^{M-1} \mathbb{E}_d [d_{-Q+i} d_i] r_m(\varepsilon)
\]

\[
r_1(t) = g(t) \ast g(-t), \quad r_2(t) = g'(t) \ast g(-t), \quad r_3(t) = g'(t) \ast g'(-t)
\]
• Code collisions plays an important role.

• The more $f_m^{k,l}$ (for $k \neq l$) is high, the more the CRB is high.

• If $\varepsilon \in [T_g, T_c - T_g]$, there is no overlapping.

• The more the path is dense, the more the CRB taking into account the overlapping is larger than the (simplified) CRB.

• Deleuze & Ciblat & Le Martret (July 2004)
Average CRB (I)

\[ E_x[\text{CRB}(x)] = E_x[J(x)^{-1}] \geq (E_x[J(x)])^{-1} \]

Simplified expressions for \( A, B, C, D \) by averaging over
- symbol sequence
- time-hopping code

\[ \text{In DA scheme, average CRB over all possible training sequences} \]
\[ \text{In NDA scheme, MCRB is considered} \]
Average CRB (II)

- \( \{d(i)\}_i \) i.i.d. symbols belonging to 2-PAM

**Result**

\[
\mathbb{E}_d[A_m(\varepsilon)] = \delta_{Q,-1} r_m(\varepsilon), \quad \mathbb{E}_d[B_m(\varepsilon)] = \delta_{Q,0} r_m(\varepsilon)
\]

- \( c_j \) is the realization of i.i.d. random vector whose each component admits the following distribution

\[
p(c) = \left( (N_c - 1)\delta(c) + \delta(c - 1) \right) / N_c.
\]

**Result**

\[
\begin{align*}
\mathbb{E}_c[C(q)] &= \frac{N_f N_c - q}{N_c^2} & \text{if } q \neq 0 \\
\mathbb{E}_c[C(0)] &= N_f & \text{if } q = 0
\end{align*}
\]

\[
\begin{align*}
\mathbb{E}_c[D(q)] &= \frac{q}{N_c^2} & \text{if } q \neq N_f N_c \\
\mathbb{E}_c[D(N_f N_c)] &= N_f & \text{if } q = N_f N_c
\end{align*}
\]
Maximum Likelihood

- Lottici & Andrea & Mengali 2002
- No overlapping context
- Simulations done in a non-overlapping context
- ML carried out in DA and NDA schemes
  - DA scheme: derivations based on likelihood (for PAM or PPM)
  - NDA scheme: derivations based on true likelihood at low SNR (for PPM)

Algorithm

\[ J_{DA}(\tau) = \frac{1}{M E_b} \sum_{i=0}^{M-1} z_i(\tau) \]

with \[ z_i(\tau, d_i) = d_i(r(t) \ast b(-t) | t = i N_f T_f + \tau) \]

- Localizations of peaks provide \( \hat{\tau} \)
- Magnitudes of peaks provide \( \hat{A} \)
Undersampling based method (I)

- Maravic & Vetterli 2003
- DA scheme
- Undersampling at period $T_s \gg T_p$ preceded by Anti-Aliasing Filter

Let $\tilde{r}(t)$ the noiseless receiver signal at the output of AAF

$$\tilde{R}(m) = \text{F.T.}(t \mapsto \tilde{r}(t))|_{f=m_0} = \sum_{k=1}^{N_p} A_k \tilde{S}(m) e^{-2i\pi \tau_k m_0}$$

then

$$\tilde{R}_s(m) = \frac{\tilde{R}(m)}{\tilde{S}(m)} = \sum_{k=1}^{N_p} A_k z_k^m$$

with $z_k = e^{-2i\pi \tau_k f_0}$
Undersampling based method (II)

\[ R = \begin{bmatrix}
\tilde{R}_s(0) & \tilde{R}_s(1) & \cdots & \tilde{R}_s(N_p - 1) \\
\tilde{R}_s(1) & \tilde{R}_s(2) & \cdots & \tilde{R}_s(N_p) \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{R}_s(N_p - 1) & \tilde{R}_s(N_p) & \cdots & \tilde{R}_s(2N_p - 2)
\end{bmatrix} \]

\[ \Leftrightarrow [R]_{\ell,\ell'} = \sum_{k=1}^{N_p} A_k z_k^{\ell+\ell'} \]

Then

\[ R = V \Lambda V^H \quad \text{with} \quad V = \begin{bmatrix}
1 & \cdots & 1 \\
\vdots & \ddots & \vdots \\
1^{N_p - 1} & \cdots & 1^{N_p - 1}
\end{bmatrix} \]
Undersampling based method (III)

Shift invariance

\[ \overline{V} = V_{\text{diag}}([z_1, \ldots, z_{N_p}]) \]

where \( \overline{V} \) and \( V \) denote the omission of the first and last row of \( V \) respectively.

Then it exists a vector \( x_k \) such that

\[ \overline{V}x_k = z_k Vx_k \]

\( z_k \) is a generalized eigenvalue of \((\overline{V}, V)\)

Algorithm

For any \( k \), \( z_k \) is the root of the polynomial

\[ P(s) = \det(\overline{V} - sV) \]

This obviously provides \( \hat{\tau} \) and \( \hat{A} \)
First-order cyclostationarity based method (I)

- Luo & Giannakis 2004
- Asymmetric PAM \((d_i \in \{-1, \theta\})\)
- ISI-less context (delay spread < guard-time)

\[
 r(t) = \sum_{i=0}^{M-1} d_i b_r(t - \tau_1 - iN_f T_f) \quad \text{with} \quad b_r(t) = \sum_{k=1}^{N_p} A_k b(t - \Delta \tau_{k,1})
\]

If ISI-less, \(\{b_r(t - \tau_1 - iN_f T_f)\}_i\) is a orthogonal set and thus \(b_r(t)\) is a square-root Nyquist filter.

**Problem**

- **Optimal receiver is the matched filter** \(b_r(-t)\) **shifted by** \(\tau_1\)
- **Knowledge of** \(b_r(t)\) **and** \(\tau_1\) **is needed**
First-order cyclostationarity based method (II)

The cyclostationary mean contains information about \( b_r(t) \) and \( \tau_1 \)

Algorithm

If \( \tau_1 \) is associated with the strongest path, then

\[
\hat{\tau}_1 = \arg \max_{\tau \in [0, N_f T_f)} \left| \int_0^{2 N_f T_f} \E[r(t)] b(t - \tau) dt \right|
\]

and

\[
\hat{b}_r(t) = \frac{2}{\theta - 1} \E[r(t + \hat{\tau}_1)], \quad \text{for} \quad t \in [0, N_f T_f)
\]
Non-overlapping case

Set-up

- \( T_p = 1\text{ns}, \quad T_c = 2T_p, \quad N_c = 10, \) and \( N_f = 10, \quad T_s = 200\text{ns}, \quad M = 100 \)
- \( \tau = [5T_p, 10T_p, 15T_p] \) and \( \mathbf{A} = [0.73, 0.67, 0.35] \)

Such assumptions ensure the absence of overlapping

![Graph showing the relationship between \( \text{MSE/T}_p^2 \) and \( \text{Eb/N}_0 \)]

- Vetterli \( B = Bs/2 \)
- Vetterli \( B = Bs \)
- Giannakis
- ML NDA
- ML DA
- CRB
Overlapping case

Set-up

- $\tau = \{kT_p/2\}_{k=1, \ldots, 20}$
- $A$ obeys a normalized exponential decreasing profile

Such assumptions ensure the presence of overlapping

$\leadsto$ ML non optimal in overlapping case
Comparison

Question

Is there overlapping or not in realistic channel?

Two statistical models:
Molish ($\lambda = 0.2\text{ns}^{-1}, \gamma = 20\text{ns}$) and Lee ($\lambda = 2\text{ns}^{-1}, \gamma = 5\text{ns}$)

\[\text{Fig: MCRB : Delay vs SNR (M=100)}\]

\[\text{MSE vs SNR (M=100)}\]

$\rightarrow$ If path density is high, the non-overlapping model does not hold
Definition

- The superresolution is the smallest gap between two delays that we are able to distinguish from.
- The Cramer-Rao Bound $\text{CRB}(\tau)$ is the smallest mean square error that we may reach when the value of the sought delay is $\tau$.

Superresolution definition

The superresolution $\tau_{\text{res.}}$ satisfies the following equation:

$$\tau_{\text{res.}} = \sqrt{\text{CRB}(\tau_{\text{res.}})}$$

- When $\tau$ decreases, the overlapping increases.
- To evaluate accurately the superresolution, we need the closed-form expression of $\text{CRB}($ in overlapping case.
Superresolution versus SNR

Set-up

\( \tau = [0, \tau], \ A = [1, 0.5], \) and \( M = 100 \)

\( \Rightarrow \) Non-overlapping is too optimistic and does not make sense
Superresolution versus $T_p$

Set-up

$E_b/N_0 = 10\text{dB}$ and $M = 100$

$\rightarrow$ Resolution proportional to $T_p$
CRB derivations:


Estimator design:
