Imperial College
London

Timing Speculation in FPGAs: Probabilistic Inference of Data Dependent Failure Rates

Sumanta Chaudhuri, Justin S. J. Wong & Peter Y. K. Cheung

October 20, 2013

Outline

- Problem Formulation/Introduction
- Inference Method
- Experimental Setup & Results
- Conclusion

Problem Statement

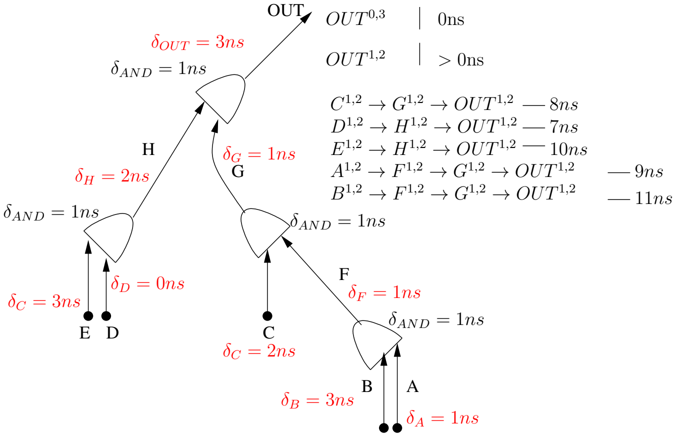
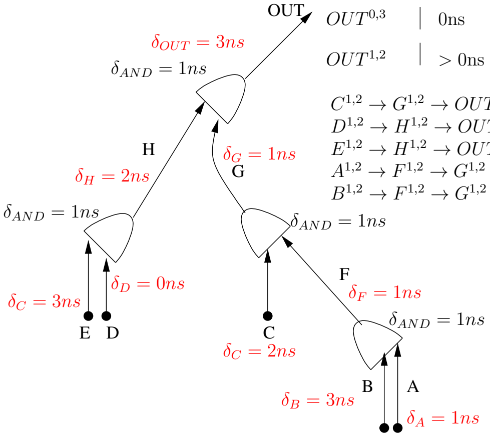


Figure 1: Data Dependent Path Excitation

Problem Statement



$OUT^{0,3}$	0ns
$OUT^{1,2}$	> 0ns

$C^{1,2} \rightarrow G^{1,2} \rightarrow OUT^{1,2}$ — $8ns$
$D^{1,2} \rightarrow H^{1,2} \rightarrow OUT^{1,2}$ — $7ns$
$E^{1,2} \rightarrow H^{1,2} \rightarrow OUT^{1,2}$ — $10ns$
$A^{1,2} \rightarrow F^{1,2} \rightarrow G^{1,2} \rightarrow OUT^{1,2}$ — $9ns$
$B^{1,2} \rightarrow F^{1,2} \rightarrow G^{1,2} \rightarrow OUT^{1,2}$ — $11ns$

Figure 2: Data Dependent Path Excitation

Worst Case:
$T_{clk} = 11ns$

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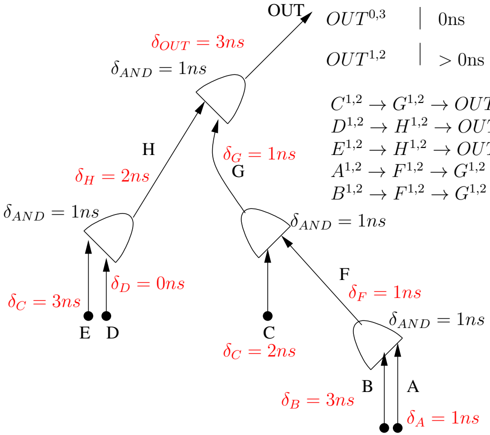

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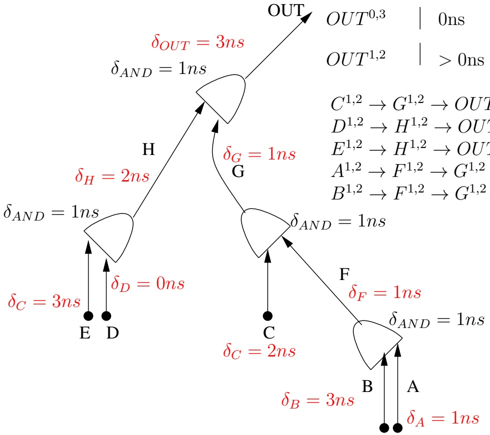
Figure 3: Data Dependent Path Excitation

We would like to find the probabilities of excitation of each path.



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$A^{1,2} \rightarrow F^{1,2} \rightarrow G^{1,2} \rightarrow OUT^{1,2}$ — $9ns$ $P(path4) = ??$
$B^{1,2} \rightarrow F^{1,2} \rightarrow G^{1,2} \rightarrow OUT^{1,2}$ — $11ns$ $P(path5) = ??$

Figure 4: Data Dependent Path Excitation

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Context: Timing Speculation

Taken from: Razor,Dan Ernst et. al. MICRO 36 2003

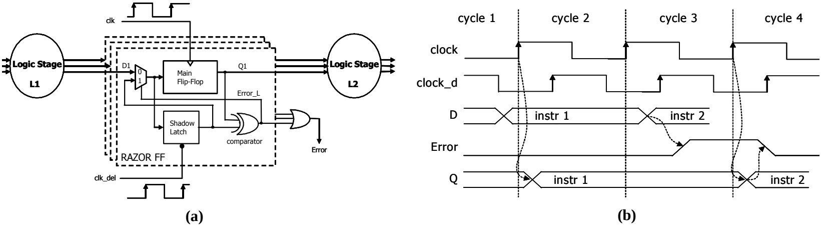

Figure 1. Pipeline augmented with Razor latches and control lines.

Throughput:

$$\frac{1}{(1 - P_{err}) \times T_{clk} + 2 \times P_{err} \times T_{clk}}$$

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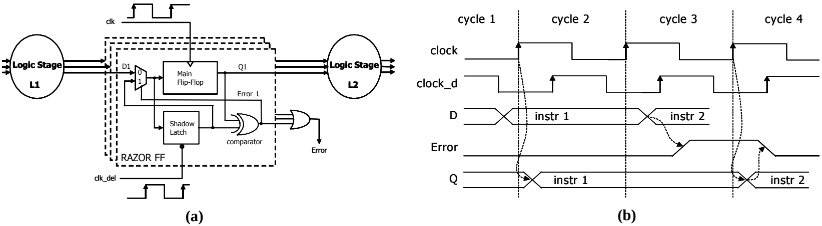

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We want to find P_{err} given a Logic Stage.

Context: Probabilistic Computing

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No. of Errors/Cycle
given a module.

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- Our method is geared towards FPGA Implementations.

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- Can be used for symbolic calculation of probabilities instead of exhaustive Monte-Carlo simulation.

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    where $Pa^G_{X_i}$ denotes the parents of X_i in the BN.
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  - Collection of methods to infer the marginal(joint) probabilities of a set of event(s).

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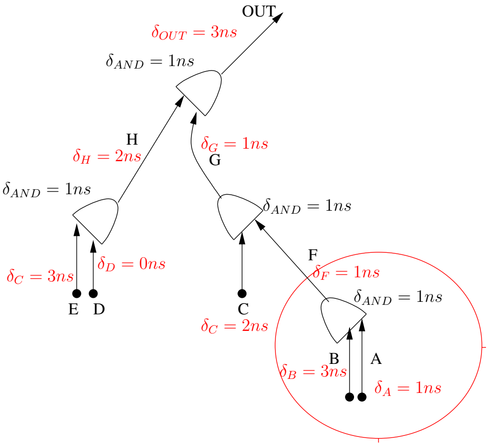
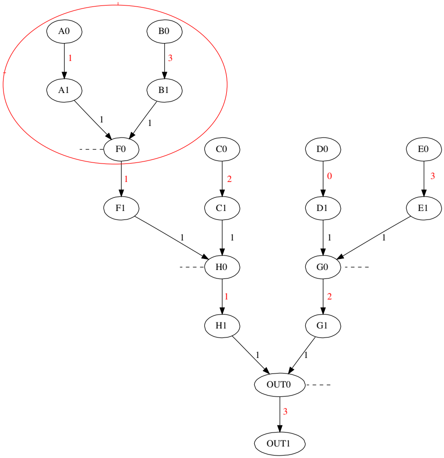
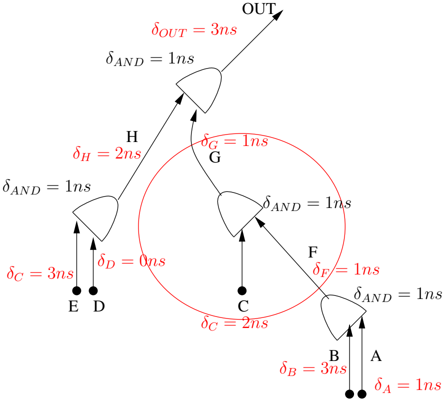
- some example events:
  $X^{1,2}$: either of transition 1 or transition 2 on X.
  (X^1, Y^1, Z^2): transition 1 on X, transition 1 on Y and transition 2 on Z.

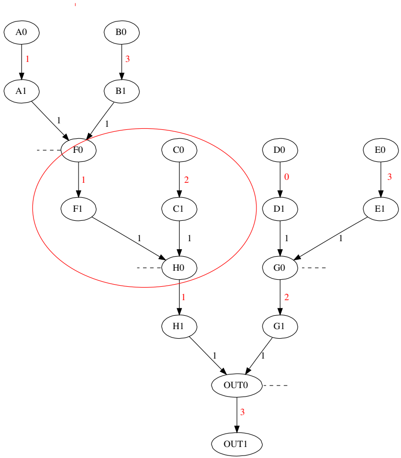




Nodes (RVs) corresp. to each terminal of nets in the circuit.
Edges for causal influence.

Bayesian Networks:Structure





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Edges for causal influence.

Table 1: Derivation of Transition Tables from Truth Tables.

Binary Format	Decimal Format

$a \backslash b$	0	1
0	0	0
1	0	1

$\bigotimes$

$a \backslash b$	0	1
0	0	0
1	0	1

$=$

$a \backslash b$	$0 \rightarrow 0$	$0 \rightarrow 1$	$1 \rightarrow 0$	$1 \rightarrow 1$
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$1 \rightarrow 0$	$0 \rightarrow 0$	$0 \rightarrow 0$	$1 \rightarrow 0$	$1 \rightarrow 0$
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CPD size $= 4 \times 4^{N_{inputs}}$

$$P(F^0|A,B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad P(F^1|A,B) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

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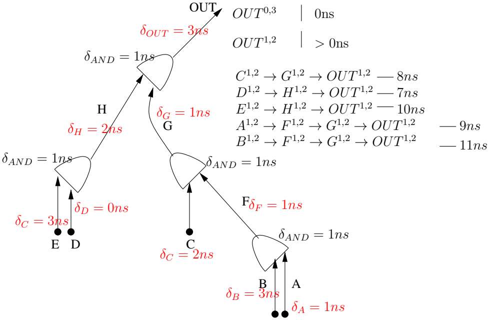
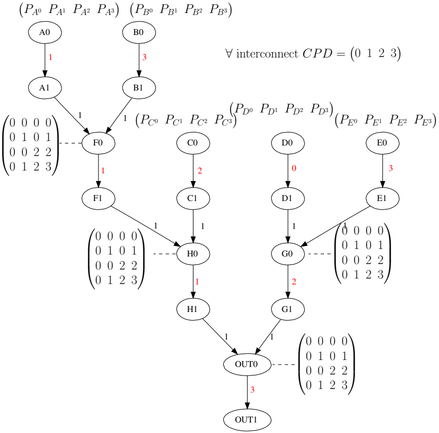
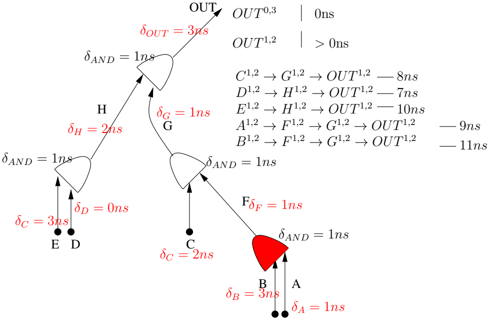
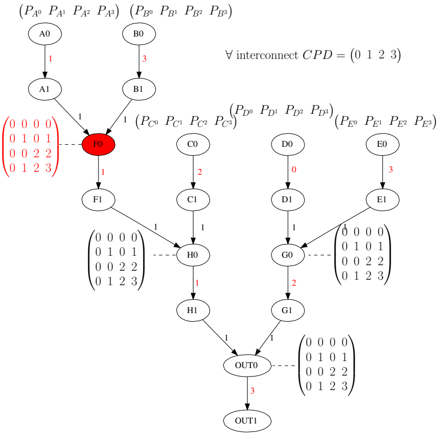




The Local Probability Models are attached to each gate output.





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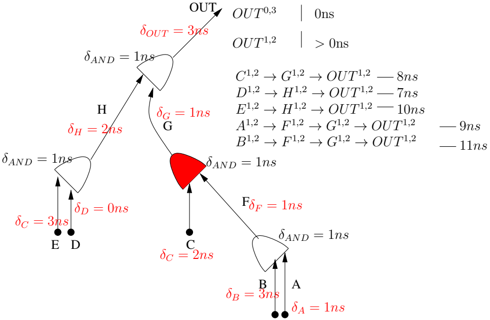

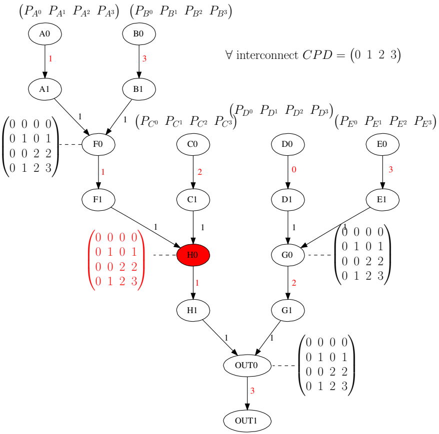

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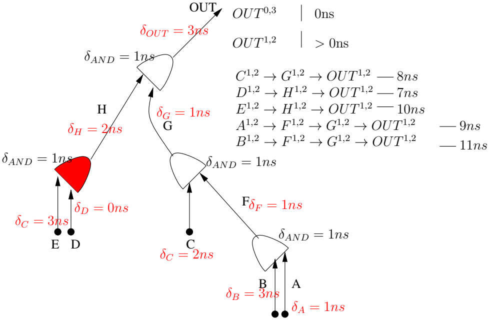

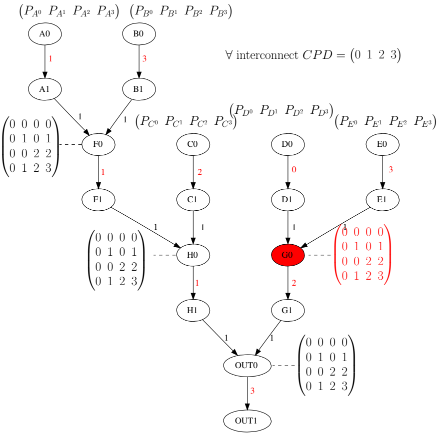

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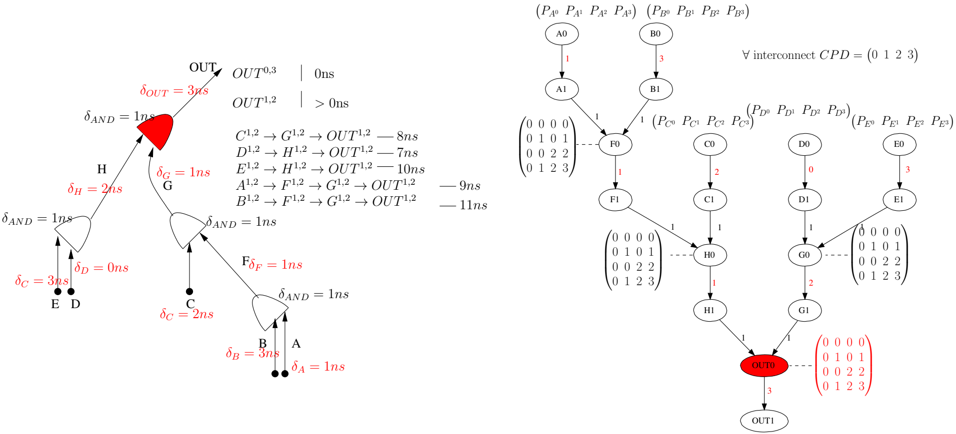

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Bayesian Networks:Probability Queries

Using MATLAB BNT Toolbox the BN can be queried for

- Joint Distribution:
  e.g $P(A0^1, A1^1, F0^1, F1^1, H0^1, H1^1, OUT0^1, OUT^1)$
- Marginal Distribution:
  e.g $P(OUT1^{0,3})$

- Exact Algorithms
  - Variable Elimination
  - Junction Tree
  - Quickscore
  - Pearl (for Polytree)
- Approximation Algorithms
  - Belief Propagation
  - Gibbs Sampling
  - Likelihood Weighting
  - Pearl (DAG)

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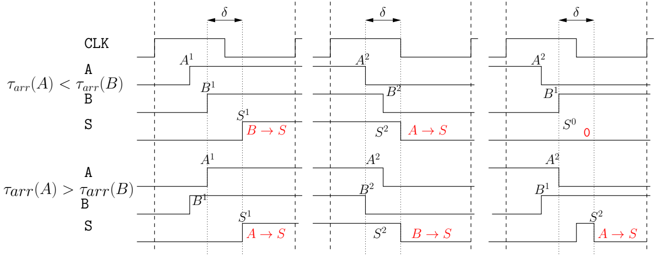

$A \to S$

$P(\tau_{arr}(S) = \tau_{arr}(A) + \delta_{AND} | A, B, \tau_{arr}(A), \tau_{arr}(B))$

$a \backslash b$	b^0	b^1	b^2	b^3
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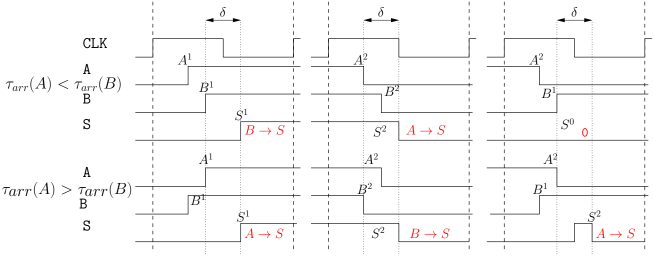

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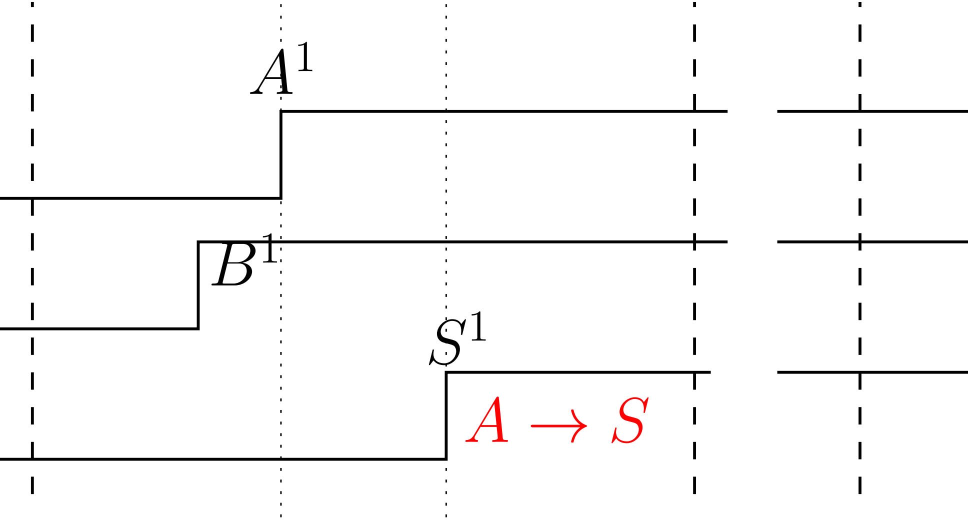
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S^1
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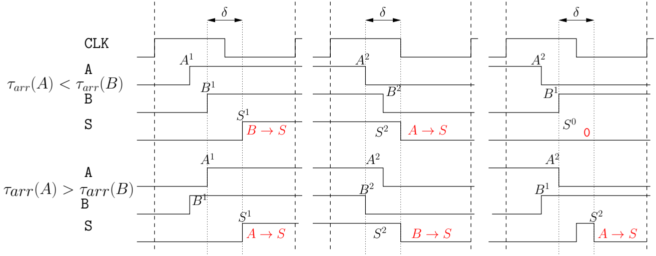

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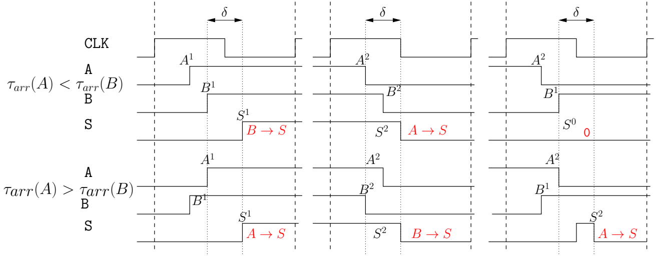
Bayesian Networks:Local Probability Model



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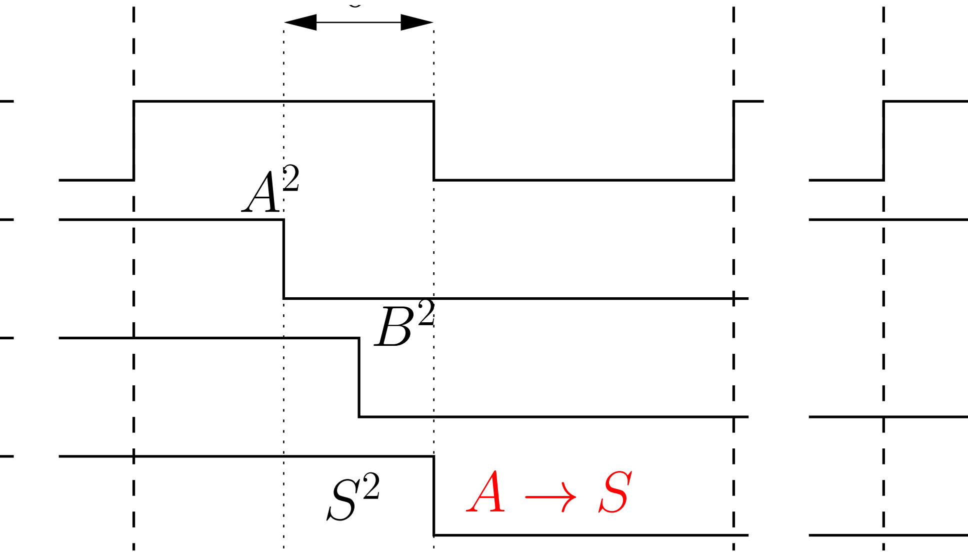
A^2
B^2
S^2
$A \to S$

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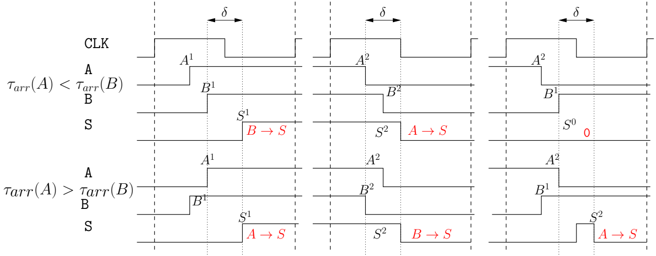

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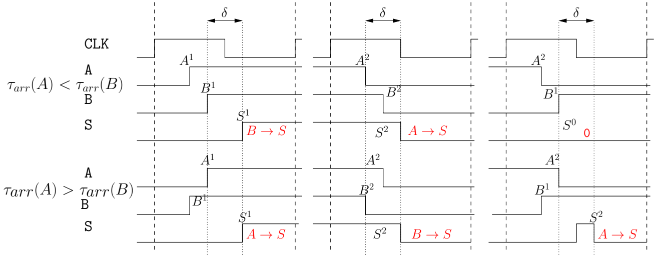
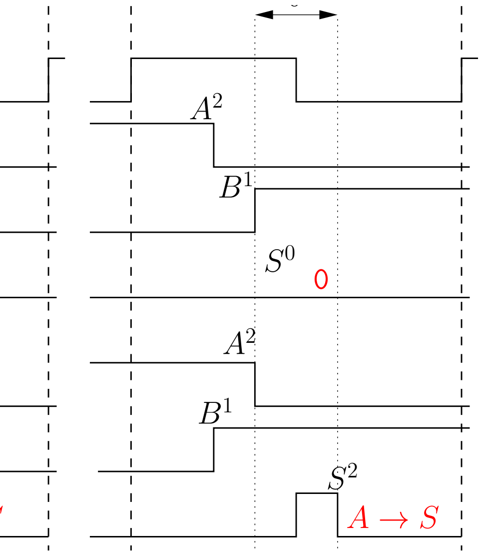


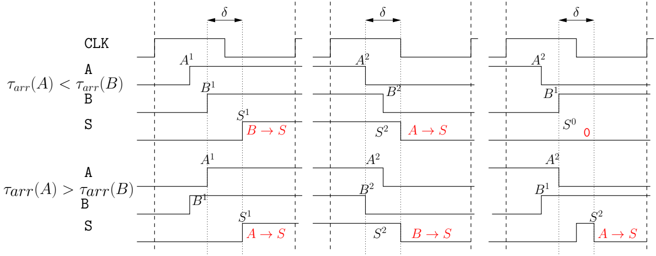
$A \to S$

$P(\tau_{arr}(S) = \tau_{arr}(A) + \delta_{AND} | A, B, \tau_{arr}(A), \tau_{arr}(B))$

$a \backslash b$	b^0	b^1	b^2	b^3
a^0	0	0	0	0
a^1	0	$P(\tau_{arr}(B) < \tau_{arr}(A))$	0	1
a^2	0	$P(\tau_{arr}(B) < \tau_{arr}(A))$	$P(\tau_{arr}(B) \geq \tau_{arr}(A))$	1
a^3	0	0	0	0



A^2
B^1
S^0
0
A^2
B^1
S^2
$A \to S$



$A \rightarrow S$

$$P(\tau_{arr}(S) = \tau_{arr}(A) + \delta_{AND} | A, B, \tau_{arr}(A), \tau_{arr}(B))$$

$a \backslash b$	b^0	b^1	b^2	b^3
a^0	0	0	0	0
a^1	0	$P(\tau_{arr}(B) < \tau_{arr}(A))$	0	1
a^2	0	$P(\tau_{arr}(B) < \tau_{arr}(A))$	$P(\tau_{arr}(B) \geq \tau_{arr}(A))$	1
a^3	0	0	0	0

Algorithms: BN Construction

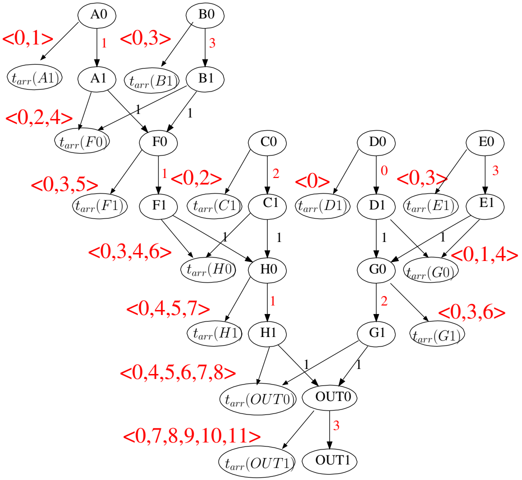
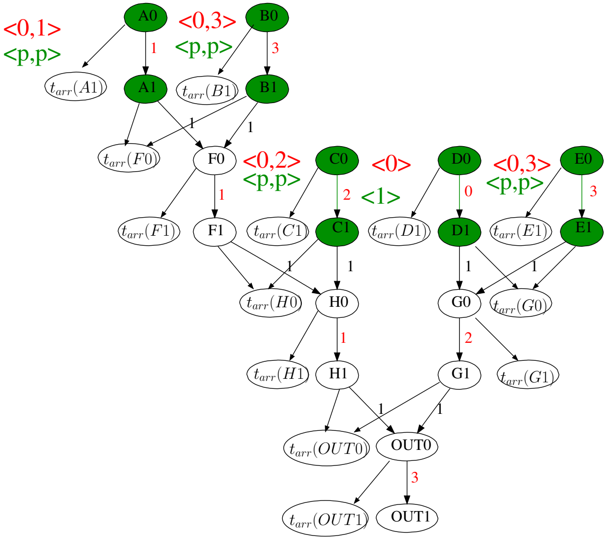
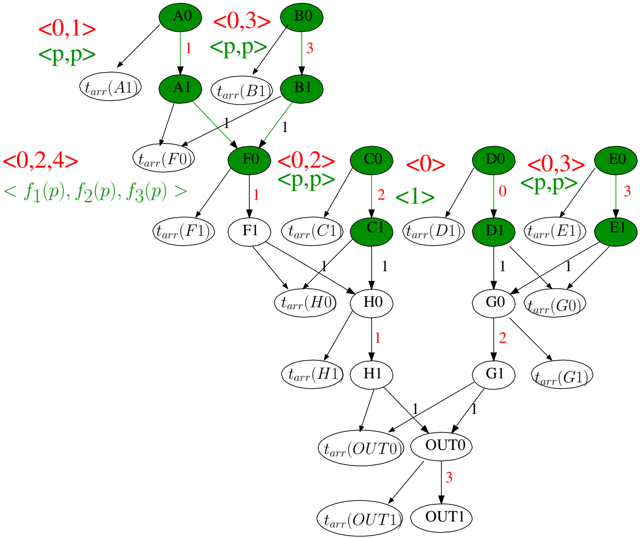


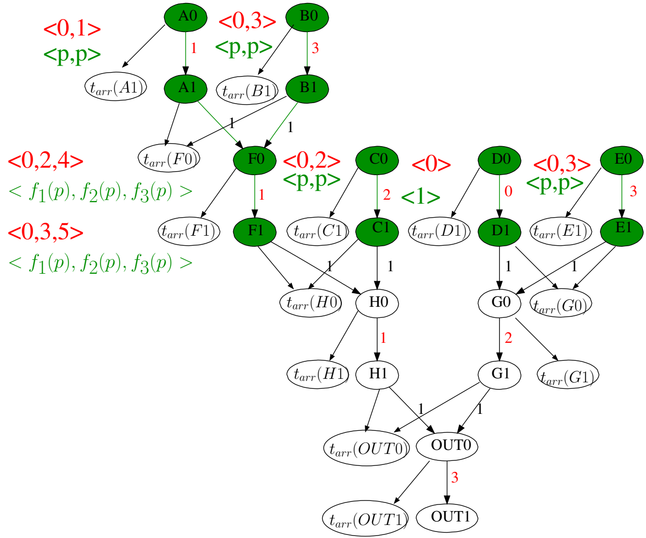
Figure 5: The same BN of figure 23 augmented with τ_{arr} nodes. The event space Ω for each RV τ_{arr} is shown in red.

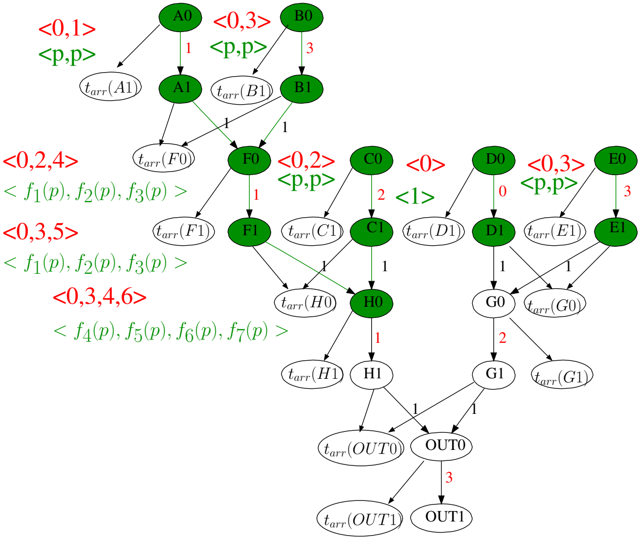

<0,1>
<p,p>
A0
1
A1
$t_{arr}(A1)$
<0,3>
<p,p>
B0
3
B1
$t_{arr}(B1)$
1
1
$t_{arr}(F0)$
F0
1
F1
$t_{arr}(F1)$
<0,2>
<p,p>
C0
2
C1
$t_{arr}(C1)$
<0>
<1>
D0
0
D1
$t_{arr}(D1)$
<0,3>
<p,p>
E0
3
E1
$t_{arr}(E1)$
1
1
$t_{arr}(H0)$
H0
1
H1
$t_{arr}(H1)$
1
1
G0
$t_{arr}(G0)$
2
G1
$t_{arr}(G1)$
1
1
$t_{arr}(OUT0)$
OUT0
3
OUT1
$t_{arr}(OUT1)$

Algorithms: BN Inference

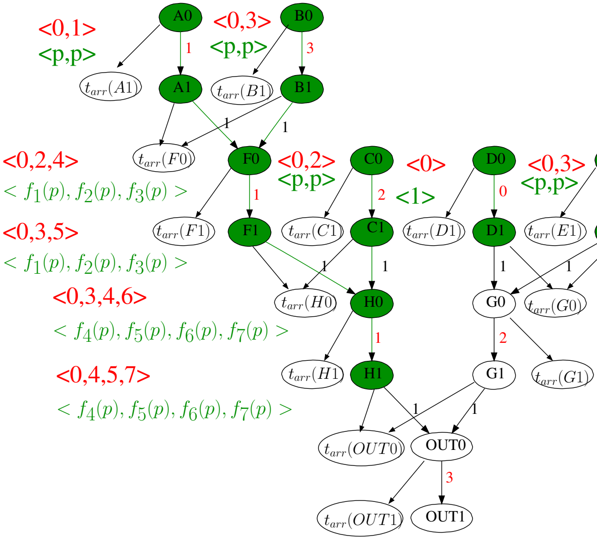


Algorithms: BN Inference



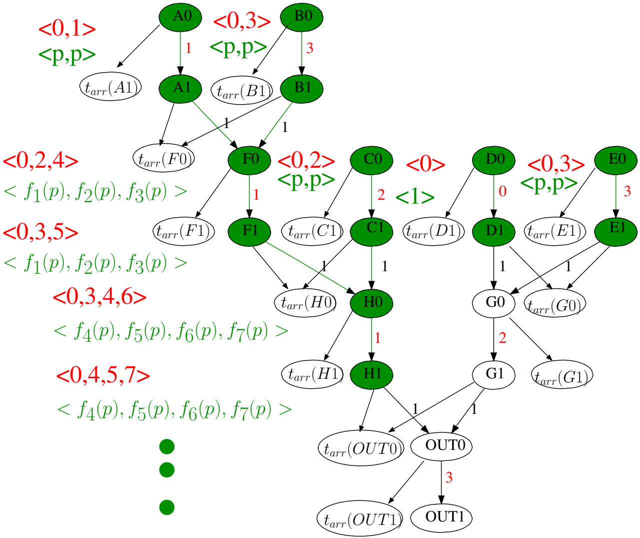


A0
B0
<0,1>
<p,p>
<0,3>
<p,p>
1
3
$t_{arr}(A1)$
A1
$t_{arr}(B1)$
B1
1
1
<0,2,4>
$< f_1(p), f_2(p), f_3(p) >$
$t_{arr}(F0)$
F0
<0,2>
<p,p>
C0
<0>
<1>
D0
<0,3>
<p,p>
E0
1
2
0
3
<0,3,5>
$< f_1(p), f_2(p), f_3(p) >$
$t_{arr}(F1)$
F1
$t_{arr}(C1)$
C1
$t_{arr}(D1)$
D1
$t_{arr}(E1)$
E1
1
1
1
1
<0,3,4,6>
$< f_4(p), f_5(p), f_6(p), f_7(p) >$
$t_{arr}(H0)$
H0
G0
$t_{arr}(G0)$
1
2
$t_{arr}(H1)$
H1
G1
$t_{arr}(G1)$
1
1
$t_{arr}(OUT0)$
OUT0
3
$t_{arr}(OUT1)$
OUT1



<0,1>
<p,p>
A0
<0,3>
B0
<p,p>
1
3
$t_{arr}(A1)$
A1
$t_{arr}(B1)$
B1
1
1
<0,2,4>
$t_{arr}(F0)$
F0
<0,2>
C0
<0>
D0
<0,3>
E0
$< f_1(p), f_2(p), f_3(p) >$
<p,p>
<p,p>
1
2
<1>
0
3
<0,3,5>
$t_{arr}(F1)$
F1
$t_{arr}(C1)$
C1
$t_{arr}(D1)$
D1
$t_{arr}(E1)$
E1
$< f_1(p), f_2(p), f_3(p) >$
1
1
1
1
<0,3,4,6>
$t_{arr}(H0)$
H0
G0
$t_{arr}(G0)$
$< f_4(p), f_5(p), f_6(p), f_7(p) >$
1
2
<0,4,5,7>
$t_{arr}(H1)$
H1
G1
$t_{arr}(G1)$
$< f_4(p), f_5(p), f_6(p), f_7(p) >$
1
1
$t_{arr}(OUT0)$
OUT0
3
$t_{arr}(OUT1)$
OUT1

Algorithms: BN Inference



Summary of Assumptions

- We assume that only one event occurs on any input to a gate in the netlist, within a single clock period. For outputs, if there are more than one events, we consider the latest event only.

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- We made an assumption that for any signal A, $P(\tau_{arr}(A^i)) = mean(\tau_{arr}(A))$ that is we ignore the individual arrival times of each event, replace it by the average.
- In actual operation there is an Aliasing Behaviour, that is the errors from one clock period are spilled into the next one. We don't take into account this effect.

Simulation Results

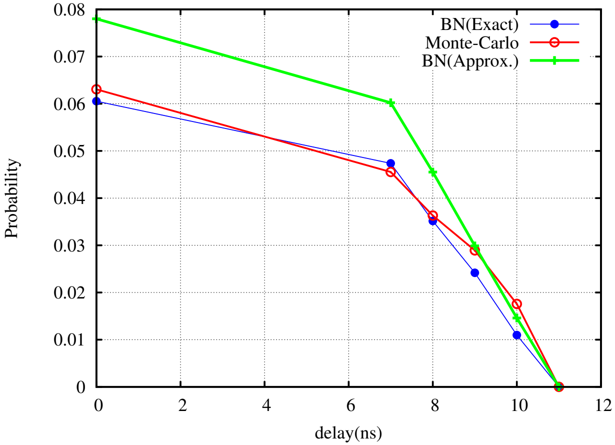
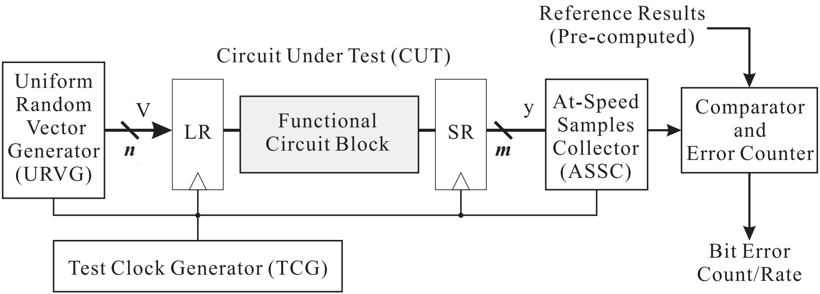


Figure 6: Comparison of MC simulations, and BN inference for example circuit

FPGA Implementation: Cyclone III



FPGA Implementation: Cyclone III

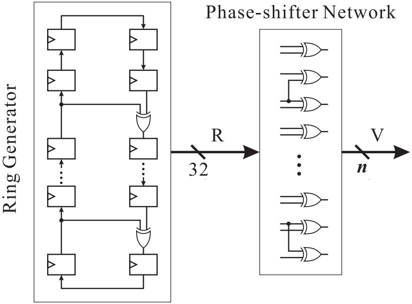


Figure 7: Circuit diagram of the Uniform Random Vector Generator (URVG).

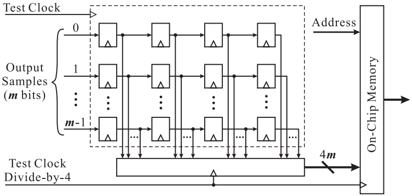


Figure 8: Circuit diagram of the At-Speed Samples Collector (ASSC).

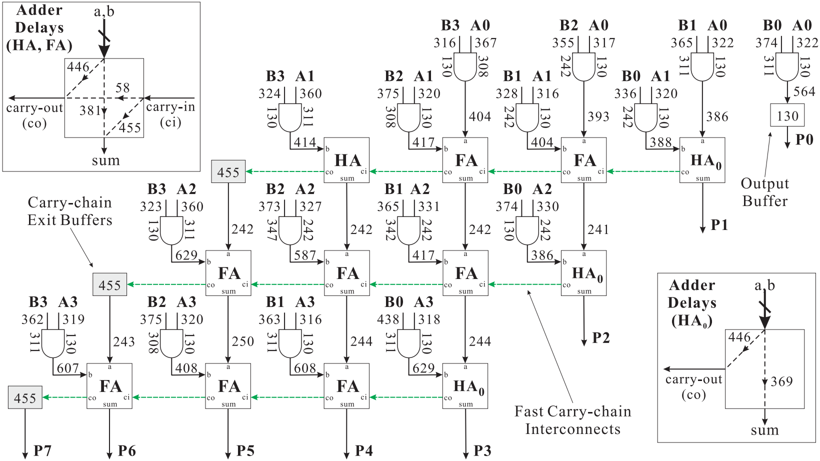
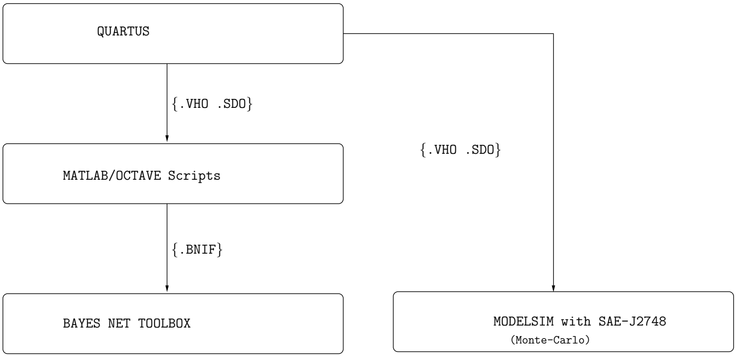
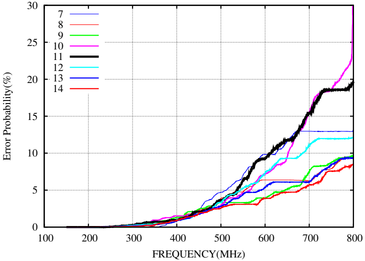


Figure 9: The adder based 4x4 multiplier circuit tested on the Cyclone III.

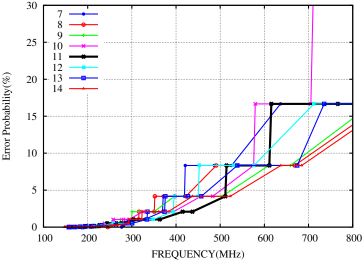
FPGA Implementation: Flow



Results and Comparison: 15 bit RCA 8 MSBs

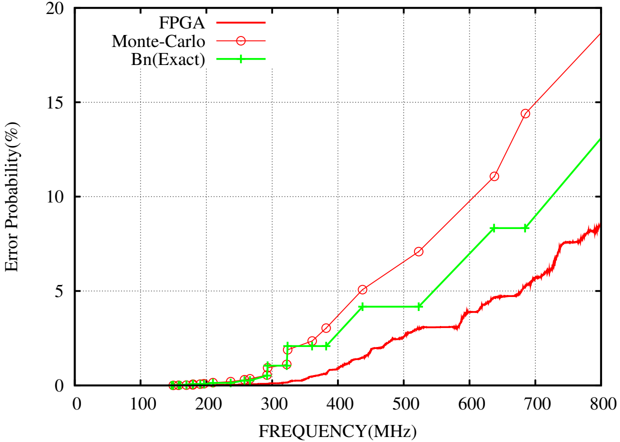


(a) 8 MSBs (CYCLONE III FPGA).



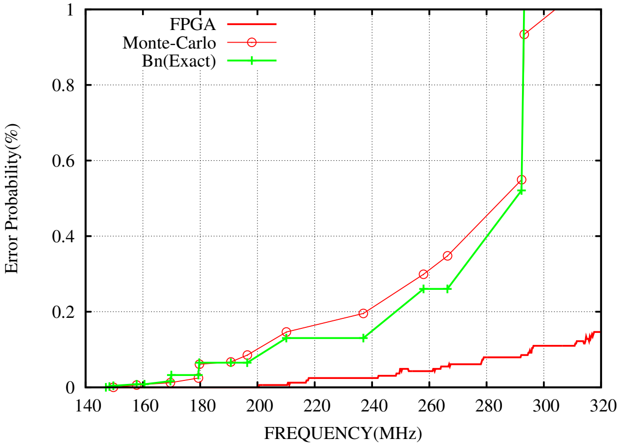
(b) 8 MSBs (BN inference).

Results and Comparison: 15 bit RCA MSB



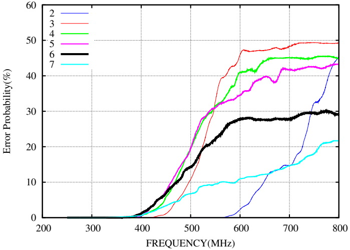
(c) MSB(14):comparison with BN and Monte-Carlo simulation.

Results and Comparison: 15 bit RCA MSB(Zoomed)

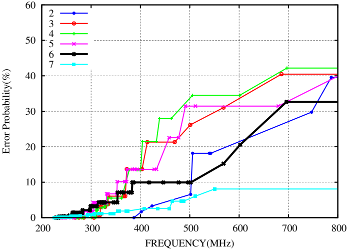


(d) MSB(14) Zoomed to (140-300) MHz Range, 140 MHz is the predicted f_{max} from STA

Delay Assumption 1: Carry always arrives late !



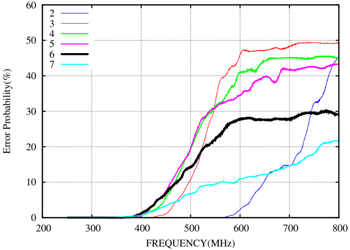
(e) 6 MSBs (CYCLONE III FPGA).



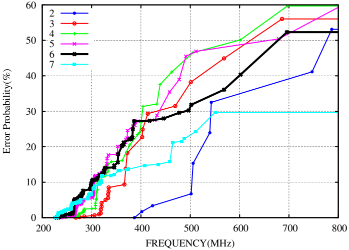
(f) 6 MSBs (Bn Inference).

Results and Comparison: 4x4 Array Multiplier 6 MSBs

Delay Assumption 2: $P(\tau_{arr}(C_{in}) \geq \tau_{arr}(A)) = P(\tau_{arr}(A) \geq \tau_{arr}(C_{in})) = 0.5.$

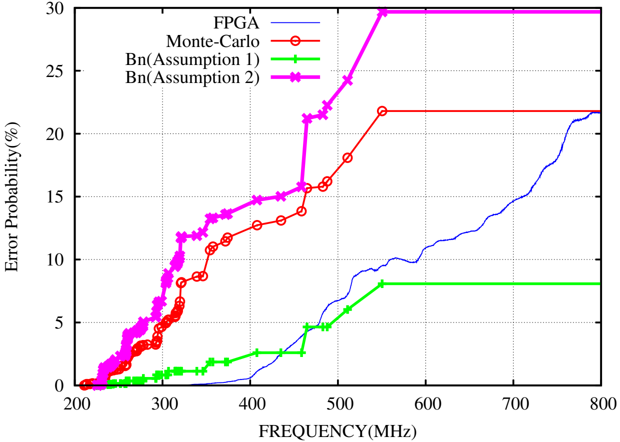


(g) 6 MSBs (CYCLONE III FPGA).



(h) 6 MSBs (Bn Inference).

Results and Comparison: 4x4 Array Multiplier MSB



(i) Failure rates for the MSB(7) and comparison with BN and Monte-Carlo simulation.210MHz is the predicted f_{max} from STA.

Conclusions & Future Work

- We modeled combinatorial datapaths with Bayesian Network, in order to infer failure probabilities given a certain clock frequency.
- Comparison of error profiles of different implementations is possible with this method.
- Useful for design/prediction of achievable throughput (timing speculation), and achievable accuracy (Probabilistic Computing).

Conclusions & Future Work

TODO (Chronological):

- Define a metric for goodness-of-fit.
- Analysis of Run-Time.
- Development of a generalised tool: BEST (BN based Error Speculation for Timing.)
- Including functional errors into the model.

BiblioGraphy & References

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Bon Apetit !!