IMAGE DENOISING BY ADAPTIVE LIFTING SCHEMES

M. Abid, M. Cagnazzo, and B. Pesquet-Popescu

Institut Télécom, Télécom ParisTech and CNRS LTCI
Signal and Image Processing Department,
46 rue Barrault, 75634 Paris Cedex 13, France

ABSTRACT

In this paper, we study the problem of image denoising by using an adaptive lifting scheme. Such a scheme can adapt itself well to the analyzed signal, which allows to keep important information for denoising applications. However, it results in a non-isometric transform which can be an important limitation as most of the denoising approaches rely on the estimation of the noise energy in the subbands. A previous study has been done to evaluate the subband energies of an uncorrelated signal, in the wavelet domain when using such an adaptive scheme. Based on this previous work, we propose in this paper an estimation of the noise energies in the subband and use it to perform image denoising. Experimental results illustrate that this approach is more effective, in image denoising, than the classical non adaptive lifting schemes both considering objective and subjective image quality measures.

I. INTRODUCTION

During its acquisition or transmission, an image is often corrupted by noise. The aim of denoising techniques is to remove this noise while keeping as much as possible the important features of the image. Recently, a particular interest has been dedicated to wavelet thresholding [1], [2], [3], [4]. The principal motivation is that the wavelet transform is appropriate in energy compactation : the small detail coefficients are more likely due to noise whereas the large ones are due to important signal features [5]. These small coefficients can be then thresholded without affecting the significant features of the image.

The lifting scheme (LS), introduced by Sweldenes [6] is a new wavelet constructing way, leading to the so-called *second generation wavelet*. It is popular because it has the capability of adjusting the wavelet transform to complex geometries and offers a simple yet efficient implementation of classical, first generation wavelet transform. However, an important limitation of this LS is that it cannot cope well with the sudden changes in the input signal, that hide important information in many applications, such as denoising. It becomes desirable to have a lifting scheme that is able to adapt itself to the data. The adaptive lifting schemes (ALS) have been designed for this particular purpose [7], [8], [9], [10], [11], [12]. The intuition behind using the ALS in the particular case of denoising via thresholding, is that these schemes allow to perfectly preserve the original characteristics of the input signal, offering thus a sparse representation, which makes the thresholding rules more effective than in the case of the traditional non adaptive LS. The most well-known thresholding methods include *VisuShrink* [1] and *SureShrink* [2]. In this paper, we focus in particular, on the soft thresholding method and use the universal threshold formula proposed by Donoho in [1] for *VisuShrink*, to derive a specific threshold for each subband when using an ALS. This approach relies on the estimation of the noise energy in each subband when an ALS is used, which is not a trivial issue. Usevitch [13] has shown, for generic linear wavelet filter banks, that for an uncorrelated signal, the energy in the spatial domain is the weighted sum of subband energies. This allows for example to estimate the distortion introduced by a quantization noise, but can be used to analyze other kinds of noise. This result has been generalized to the non linear ALS in [14], [15], [16] and the corresponding weights have been computed and used to perform optimal resources allocation. The contribution of this paper is to use these weights to derive estimates of the noise energy in each subband and then apply the soft thresholding procedure.

This paper is organized as follows: we first give in Section II a brief recall on the classical lifting scheme and its adaptive version. Then, we present the method for distortion estimation in the transform domain in Section III. In Section IV, we explain how to exploit this previous work in the context of image denoising. The experimental results are presented and discussed in Section V. Finally, Section VI concludes the paper and outlines future work.

II. ADAPTIVE LIFTING SCHEME

II-A. Classical lifting schemes

A typical lifting stage is composed of three steps : Split, Predict and Update as shown in Figure 1. The input signal x is first split into its even and odd polyphase components, respectively called the approximation signal x_a and the detail signal x_d. The odd samples of x are then predicted from the neighboring even ones. The predictor operator P is a linear combination of them and it is in general chosen such that it gives a good estimate of x_d. The new obtained

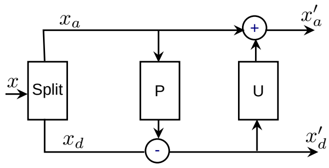

Fig. 1: Classical lifting scheme. Analysis.

signal $x'_d = x_d - P(x_a)$ is then smaller than x_d. Finally, the even samples of x are transformed into a low-pass filtered and subsampled version x'_a of the original signal x. This is performed by using an updating operator U which is a linear combination of the elements of x'_d. The approximation signal $x'_a = x_a + U(x'_d)$ is then obtained. The principal disadvantage of the LS described above, is that the linear filtering structure is fixed and thus, cannot match well the sharp transitions in the signal. The lifting schemes with adaptive prediction (APLS) [10], [11], [12] or adaptive update (AULS) [7], [8], [9] have been designed to overcome this limitation by the use of a filter that is able to adapt itself to the input signal it is analyzing. In the following, we will focus on the APLS and describe briefly its principle.

II-B. Adaptive prediction lifting scheme

Let x be the input signal and y_{ij} a wavelet subband, where $i \in \mathcal{I}$ identifies the decomposition level starting from 0, and $j \in \mathcal{J}$ identifies the channel. Usually $\mathcal{J} = \{0, 1\}$, with 0 used for the low-pass and 1 for the high-pass channel, but more channels can be used, for example in the case of multi-dimensional transforms. The subbands produced by one decomposition level are called y_{00} and y_{01}. In the

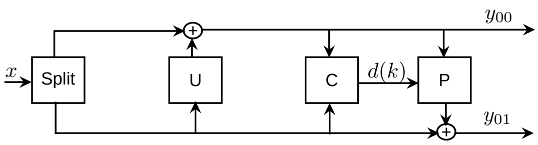

Fig. 2: Adaptive prediction lifting scheme

adapative prediction lifting schemes (APLS), the adaptivity is built into the prediction step of the lifting scheme as shown in Figure 2. They are based on the design of a data-dependent prediction filter in order to minimize the predicted detail signal. In this paper, we consider an adaptive prediction lifting scheme proposed by Claypoole *et al.*[11], which lowers the order of the prediction filter near jumps to avoid prediction across discontinuities, and uses higher order predictors where the signal is locally smooth. The choice of the prediction operator to be used at the position k, is made according to the decision map value at the position k, $d(k)$. The decision map $d(\cdot)$ allows to descriminate the smooth parts of the signal from its sharp parts. Once the decision map is calculated, the following equations are obtained for the analysis:

$$y_{00}(k) = x(2k) + \sum_{n\in\mathbb{Z}} \beta(n)x(2k+1-2n) \quad (1)$$

$$y_{01}(k) = x(2k+1) - \sum_{n\in\mathbb{Z}} \gamma_{d(k)}(n)y_{00}(k-n), \quad (2)$$

while the synthesis is described by:

$$x(2k+1) = y_{01}(k) + \sum_{n\in\mathbb{Z}} \gamma_{d(k)}(n)y_{00}(k-n) \quad (3)$$

$$x(2k) = y_{00}(k) - \sum_{n\in\mathbb{Z}} \beta(n)x(2k+1-2n). \quad (4)$$

As one can notice, this overall system is nonlinear since the prediction operator depends on the decision map which in its turn depends on the input signal x.

III. DISTORTION ESTIMATION IN THE TRANSFORM DOMAIN

For generic linear wavelet filter banks, Usevitch showed [13] that the energy σ^2 (in the spatial domain) of an uncorrelated one dimensional signal, is related to the energies σ^2_{ij} of the wavelet subbands y_{ij} by the linear relation:

$$\sigma^2 = \sum_{ij} \frac{1}{2^{i+1}} w_{ij}\sigma^2_{ij} \quad (5)$$

The weight w_{ij} is computed as norm of the reconstruction polyphase matrix columns for the subband y_{ij}.
This approach has been extended in [14], [15] to the case of the inherentely non linear ALS, for which no polyphase representation exists. The basic idea was to look at the overall ALS as a linear time-varying system, which is possible once the decision map $d(\cdot)$ is given. In facts, the authors have shown that the non linearity of the system depends only on the decision map and not on the whole input signal. Thus, the weights depend only on the values of $d(\cdot)$, and more precisely on the choices of the prediction filters. We give here directly the expression of the weights computed in the one dimensional case and for one decomposition level. For the details of the weights computation and its extension to the multi decomposition level and the multidimensional case, the reader is referred to [14], [15]. Let us first start by introducing the matrix $\mathbf{G}^{(h)}$. It is the polyphase synthesis matrix associated to the filter corresponding to the value h of the decision map $d(\cdot)$. It can be considered as the polyphase synthesis matrix used in the non adaptive scheme where the h-th filter is always used. As shown in [13], given the matrix $\mathbf{G}^{(h)}$ one can express the weight $w^{(h)}$ as:

$$w^{(h)} = \frac{2}{N} \sum_{n,m} \mathbf{G}^{(h)}(n,m)^2 . \quad (6)$$

When considering the adaptive case, where w_{ij} is the weight for the subband y_{ij} and N_h is the number of times the h-th filter is used in this subband, the authors have shown that:

$$w_{ij} = \sum_{h=0}^{\mathcal{D}-1} \frac{2N_h}{N} w_{ij}^{(h)} = \sum_{h=0}^{\mathcal{D}-1} \rho_h w_{ij}^{(h)}, \qquad (7)$$

where $\rho_h = \frac{2N_h}{N}$ is the relative frequency of filter h in the decision map for the current subband y_{ij}.

IV. APPLICATION TO IMAGE DENOISING

In the APLS approach, the prediction operator adapts itself to the input signal so that the characteristics of the original signal are very well preserved. This property has been successfully exploited to perform optimal resources allocation, by the mean of the estimation of the distortion, introduced by quantization, in the transform domain.
In this section, we propose to exploit the APLS properties as well as the weights for the purpose of image denoising.
Let the signal be $\{x(k,l), k,l = 1,\dots,N\}$ where N is an integer power of 2. It has been corrupted by an additive noise. The observed signal is then:

$$z(k,l) = x(k,l) + \varepsilon(k,l), \;\; k,l = 1,\dots,N \qquad (8)$$

where $\varepsilon(k,l)$ are independent and identically distributed (iid) as normal $N(0,\sigma^2)$ and independent of $x(k,l)$. The goal is to denoise $z(k,l)$ and to obtain an estimate $\hat{x}(k,l)$ of $x(k,l)$. Let us denote by I the coarsest scale in the decomposition. As in section II, we keep the same notation y_{ij} for the noisy subbands, where $i \in \mathcal{I}$, $j \in \mathcal{J}$, $\mathcal{I} = \{0,\dots,I-1\}$ and $\mathcal{J} = \{0,\dots,3\}$ since the two dimensional case is considered here. In the case of an orthogonal wavelet transform, the obtained noise wavelet coefficients in each subband y_{ij} are *iid* $N(0,\sigma^2)$. In our case, this result does not hold anymore since the considered APLS is neither isometric nor linear. The standard deviation σ_{ij} of the noise in the subband y_{ij} is not equal to the noise standard deviation σ in the spatial domain.

In what follows, we propose to use the wavelet thresholding procedure to remove the noise. It consists in thresholding only the wavelet coefficients of the details subbands while keeping the low resolution coefficients unchanged. We focus here on the soft thresholding method [4] because it gives the best performances when it is coupled with an undecimated discrete wavelet transform [17]. For a one-dimensional signal of length M, Donoho and Johnstone [1] proposed the universal threshold, $T_U = \sigma\sqrt{2\log M}$ which results in an optimal estimate in the minimax sense. In the case of our APLS, the soft thresholding is applied in each detail subband y_{ij}, with a specific threshold: $T_{ij} = \sigma_{ij}\sqrt{2\log(N_i^2)}$ where $N_i^2 = \frac{N^2}{4^{i+1}}$ is the number of coefficients in the subband y_{ij} and σ_{ij} is the noise standard deviation in the details subband y_{ij}. The problem is then to get a good estimation $\hat{\sigma}_{ij}$ of σ_{ij} in each subband. The equation (5) is only valid when σ_{ij} is the energy of an uncorrelated signal, which might not be the case when using the APLS on a noisy signal. Let us however consider this equation for the two-dimensional case, where we define the noise energy in the spatial domain by σ^2. One obtains: $\sigma^2 = \sum_{ij} \frac{1}{4^{i+1}} w_{ij} {\sigma_{ij}}^2$. As explained in [13], the use of a non orthogonal transform, results in a weighting of the energy in each subband. The weights can be seen as a measure of the closeness of the biorthogonal filters to the class of orthogonal filters. The introduction of these weights allows thus, to approach the behavior of the orthogonal transform in the sense that the equality between the energies in the subbands, which is verified by an orthogonal transform, is changed into an equality between the weighted energies when a non orthogonal transform is used. This can be expressed by:

$$w_{ij}\sigma_{ij}^2 \approx w_{i'j'}\sigma_{i'j'}^2, \;\text{ where } i,i' \in \mathcal{I} \text{ and } j,j' \in \mathcal{J} \qquad (9)$$

At the first resolution level, the noise energy in the subband of diagonal details y_{03} may be estimated by the formula [1], [3]: $\left(\frac{m}{0.6745}\right)^2$, where m is the median absolute deviation of the wavelet diagonal details at the finest decomposition level. From equation (9), an estimation of σ_{ij} can be:

$$\hat{\sigma}_{ij} = \sqrt{\left(\frac{w_{03}}{w_{ij}}\right)}\hat{\sigma}_{03}, \;\text{ where } \hat{\sigma}_{03} = \frac{m}{0.6745} \qquad (10)$$

One should point that equation (9) from which equation (10) is derived relies on the assumption of the equality between the weighted subband energies. In the following section, we will use the expressions obtained in (10) to achieve soft thresholding and thus evaluate the correctness of this assumption.

V. EXPERIMENTAL RESULTS

V-A. Noise standard deviation Estimation

In this subsection, our aim is to evaluate the correctness of our noise standard deviation estimation approach in each subband. We introduce a white gaussian noise with a standard deviation of σ in the original image that is further transformed using an APLS, as the one describe in the Subsecion II-B, with five decomposition levels. We use then the equation (10) to calculate the estimations $\hat{\sigma}_{ij}$ of the noise standard deviation in each subband y_{ij}. The per cent relative errors of this estimation for the first six subbands are reported in table I. For the first level of decomposition, we notice that the estimation is accurate, however the estimation errors become more important in the second level of decomposition. The reason is that the signal whose energy is estimated in each subband is not uncorrelated as it is supposed to be in [13] and as it is imposed in [15]. However, these estimations are good enough in the particular context of denoising by soft thresholding as will be shown in the next subsection.

V-B. Denoising by soft thresholding

In this subsection, we use the soft thresholding method on the noisy image, transformed with our considered APLS.

	Barbara			Lena		
σ	30	50	80	30	50	80
Subband y_{03}	0.86	0.51	0.12	0.51	0.20	0.03
Subband y_{02}	0.71	0.48	0.30	0.54	0.22	0.16
Subband y_{01}	0.98	0.55	0.32	0.58	0.32	0.17
Subband y_{13}	8.16	8.71	8.77	8.84	9.02	9.17
Subband y_{12}	7.25	7.57	7.52	7.61	7.82	7.77
Subband y_{11}	5.09	5.56	5.71	5.64	5.68	5.84

Table I: Relative error of the standard deviation estimation in some of the detail subbands for the images Barbara and Lena, corrupted with a spatial noise of standard deviation 30, 50 and 80.

A specific threshold is used for each detail subband as described in Section IV. We compare our denoising approach with the conventional soft thresholding approach realized on the same noisy image transformed with the classical non adaptive 9/7 wavelet transform. We considered four different images : Barbara, Lena, House and Peppers. The PSNR and SSIM curves as a function of the introduced noise, are reported in Figures 3 and 4. There is an improvement in PSNR and SSIM with our approach when compared to the 9/7 filter. When considering the image house, the obtained gain is up to 1 dB in PSNR and 0.03 in SSIM.

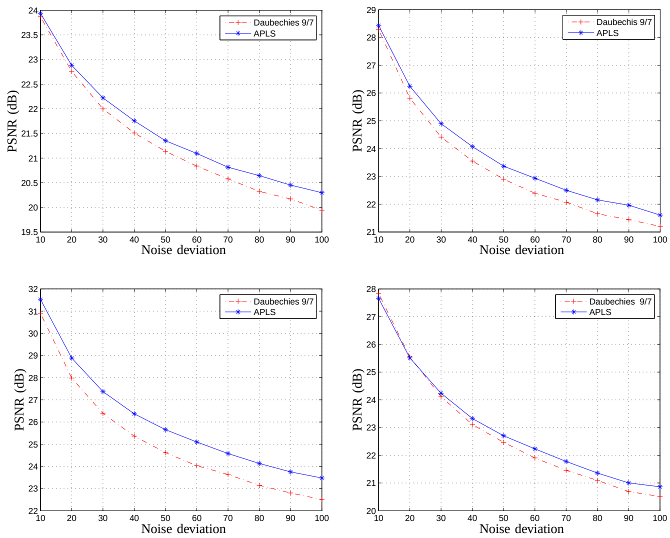

Fig. 3: PSNR as a function of the spatial noise introduced in the images: Barbara (top left), Lena (top right), House (bottom left) and Peppers (bottom right).

VI. CONCLUSION

In this paper, we have presented an approach for image denoising via soft thresholding, by using an adaptive lifting scheme. This approach, based on the estimation of the energy in the transform subbands, gives better performances than the classical non adaptive wavelet transform. Future work will be

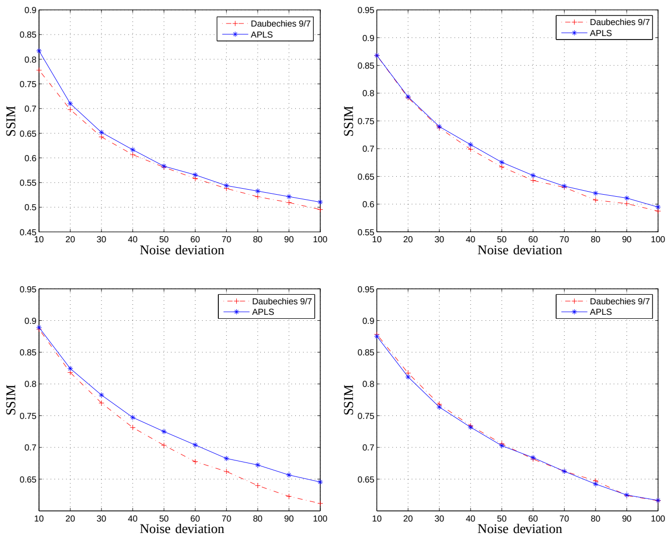

Fig. 4: SSIM as a function of the spatial noise introduced in the images: Barbara (top left), Lena (top right), House (bottom left) and Peppers (bottom right).

dedicated to other thresholding methods, such as *SureShrink*, which is a hybrid of the universal and the SURE threshold.

VII. REFERENCES

[1] D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation via wavelet shrinkage," in *Biometrika*, 1994, vol. 81, pp. 425455.

[2] ——, "Adapting to unknown smoothness via wavelet shrinkage," in *Journal of the American Statistical Assoc.*, 1995, vol. 90, no. 432, pp.12001224.

[3] D. Donoho and I. Johnstone, "Threshold selection for wavelet shrinkage of noisy data," in *Proc. 16th Annual Conf. of the IEEE Engineering in Medicine and biology society, Maryland, USA*, 1994, pp. 24a25a.

[4] D. Donoho, "De-noising by soft-thresholding," in *IEEE Trans. Inf. Theory*, 1995, 41, (3), pp. 613627.

[5] M. Jansen, "Noise reduction by wavelet thresholding," in *Springer Verlag New York Inc.-*, 2001.

[6] W. Sweldens, "The lifting scheme: A custom-design construction of biorthogonal wavelets," *Appl. Comput. Harmon. Anal.*, vol. 3, no. 2, pp. 186–200, 1996.

[7] H. J. A. M. Heijmans, B. Pesquet-Popescu, and G. Piella, "Building nonredundant adaptive wavelets by update lifting," *Applied Computational Harmonic Analysis*, no. 18, pp. 252–281, May 2005.

[8] G. Piella, B. Pesquet-Popescu, and H. J. A. M. Heijmans, "Adaptive update lifting with a decision rule based on derivative filters," vol. 9, pp. 329–332, Oct. 2002.

[9] ——, "Gradient-driven update lifting for adaptive wavelets," vol. 20, no. 9-10, pp. 813–831, Oct.-Nov. 2005.

[10] O. N. Gerek and A. E. Çetin, "Adaptive polyphase subband decomposition structures for image compression," vol. 9, no. 10, pp. 1649–1659, Oct. 2000.

[11] R. L. Claypoole, G. M. Davis, W. Sweldens, and R. G. Baraniuk, "Nonlinear wavelet transforms for image coding via lifting," vol. 12, no. 12, pp. 1449–1459, Dec. 2003.

[12] N. Mehrseresht and D. Taubman, "Spatially continuous orientation adaptive discrete packet wavelet decomposition for image compression," Atlanta, GA (USA), Oct. 2006, pp. 1593–1596.

[13] B. Usevitch, "Optimal bit allocation for biorthogonal wavelet coding," Snowbird, USA, Mar. 1996, pp. 387–395.

[14] S. Parrilli, M. Cagnazzo, and B. Pesquet-Popescu, "Distortion evaluation in transform domain for adaptive lifting schemes," in *IEEE Workshop on Multimedia Signal Processing*, Cairns, Australia, 2008.

[15] ——, "Estimation of quantization noise for adaptive-prediction lifting schemes," in *IEEE Workshop on Multimedia Signal Processing*, 2009.

[16] M. Cagnazzo and B. Pesquet-Popescu, "Perceptual impact of transform coefficients quantization for adaptive lifting schemes," in *International Workshop on Video Processing and Quality Metrics for Consumer Electronics*, 2010.

[17] R. R. Coifman and D. L. Donoho, "Translation invariant de-noising," in *Wavelets and Statistics, A. Antoniadis and G. Oppenheim, Eds. New York: Springer-Verlag,*, 1995, pp. 125150.